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**Basic Steel Design**
Session L4: Compression + Bending
March 18, 2021

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# AISC Live Webinars

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# AISC Live Webinars

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## AISC Live Webinars

**Course Description**

Compression + Bending

This lecture will discuss the behavior and design of beam-columns. The session will review elastic and plastic interaction principles, AISC interaction equations and design rules of thumb. The session will explore the design of members in single axis bending as well as the design of single angles for bending plus compression.

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## AISC Live Webinars

**Learning Objectives**

- Describe the behavior and design of steel beam-column members.
- Apply the AISC *Specification* interaction equations for the design of members with bending plus compression.
- List the design aids for beam-columns and demonstrate how to apply in design.
- Describe the design process for unsymmetric shapes with combined stress.

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Basic Steel Design: A review of the principles of steel design according to ANSI/AISC 360-16

Winter Webinar 2021

Lesson L4

Compression + Bending

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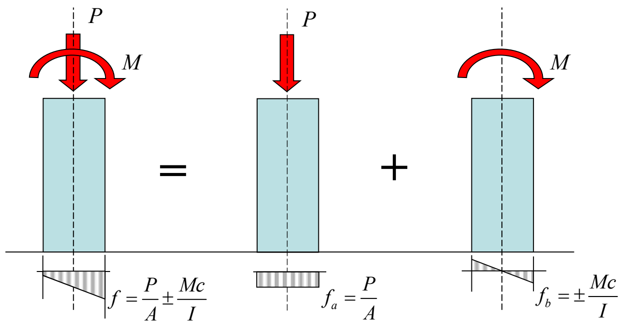
## Lesson L4 – Compression + Bending

- Combined force members
  - Interaction with elastic stress distribution
  - Interaction with plastic stress distribution
  - Specification interaction equations
  - Design aids for beam-columns
  - Initial beam-column selection
  - Single axis bending with axial load
  - Unsymmetric shapes with combined stress

L4.8

# Compression + Bending

- Elastic stress distribution




L4.9

# Compression + Bending

- Elastic stress distribution
  - Could limit bending stress to a specific value, $F_b$
  - Could limit axial stress to a specific value, $F_a$
  - But these limits are likely not the same value so what we really need is a way to limit the combination

$$f = \frac{P}{A} \pm \frac{Mc}{I} \le ?$$

L4.10

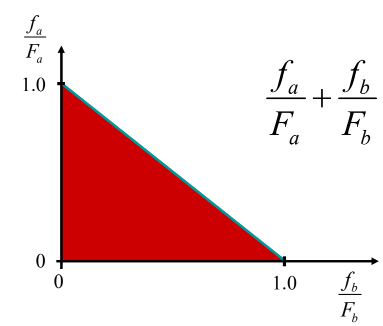
# Compression + Bending

- Elastic stress distribution
  - The usual way to apply these limits is through an interaction equation
  - The ratio of applied stress to the stress limit for axial, $f_a/F_a$, and bending, $f_b/F_b$, are added
  - The sum is limited to 1.0
  - Thus, you can never use more than 100% of the available stress

L4.11

# Compression + Bending

- Elastic stress interaction

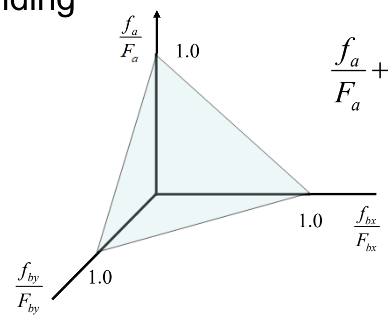



$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \le 1.0$$

L4.12

## Compression + Bending

- Elastic stress interaction for two axis bending




$$\frac{f_a}{F_a}+\frac{f_{bx}}{F_{bx}}+\frac{f_{by}}{F_{by}}\le 1.0$$

L4.13

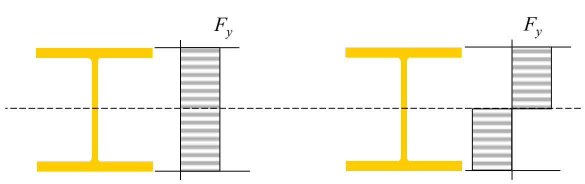
## Compression + Bending

- But there is a problem with all this.
  - We know that we are not looking at elastic behavior.
  - Columns may buckle elastically but they may also buckle inelastically. They also have yielding as their upper limit.
  - Beams may behave plastically, inelastically, or elastically.

L4.14

## Compression + Bending

- Consider a stub column with bending, a member in which length plays no part.



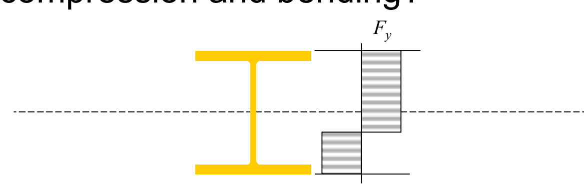

Pure Compression, $P_y$

Pure Bending, $M_{px}$

L4.15

## Compression + Bending

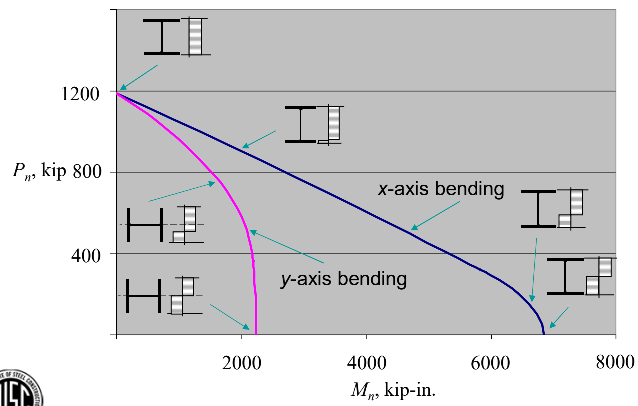
- What might the stress distribution look like if the column carried both axial compression and bending?




Combined Compression and Bending, $P_n$, $M_{nx}$

L4.16

## For a W14x82



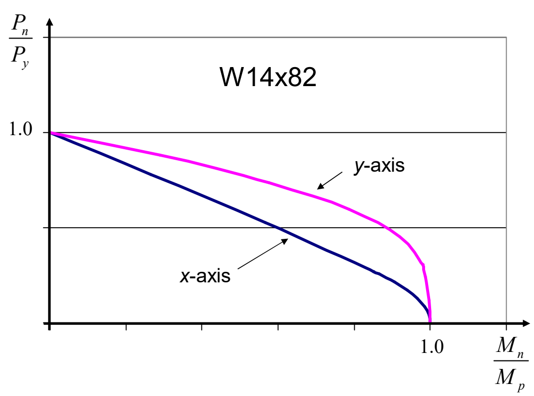

L4.17

## Compression + Bending

- To nondimensionalize
  - Divide the axial force by the pure axial strength, $P_y$
  - Divide the *x*-axis moment by the pure *x*-axis bending strength, $M_{px}$
  - Divide the *y*-axis moment by the pure *y*-axis bending strength, $M_{py}$

L4.18

## Compression + Bending




L4.19

## Compression + Bending

- To follow this approach for design
  - Each shape requires its own interaction diagrams for *x*- and *y*-axis bending.
  - Each material with different yield stress will require its own set of diagrams.
  - Shapes other than W-shapes are quite complex to deal with.
  - Thus, the Specification makes a simplification.

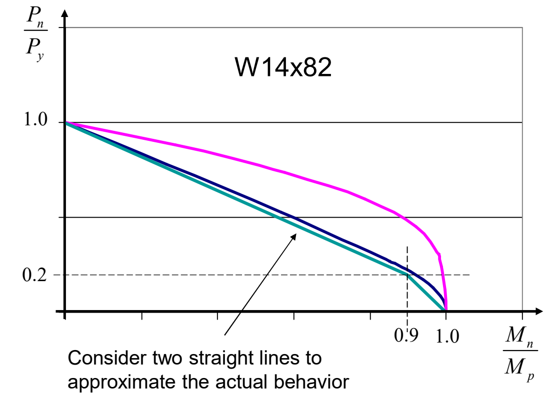
L4.20

## Compression + Bending

- After studying the full set of W-shapes, two straight line segments with a kink were selected to represent the interaction diagram.

L4.21

## Compression + Bending




L4.22

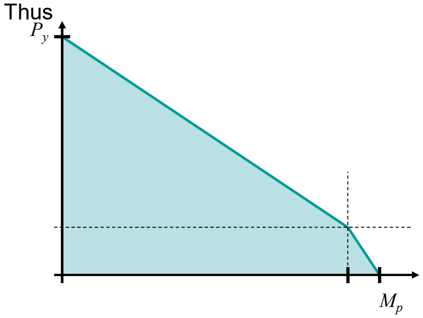
## Compression + Bending

- Notice that
  - The proposed straight lines are quite accurate, yet conservative, for *x*-axis bending of this W14x82
  - They are not very accurate for *y*-axis bending but are very conservative
    - Since the magnitude of moments for *y*-axis bending are relatively small, compared to *x*-axis bending, this error is not considered a critical shortcoming.

L4.23

## Compression + Bending

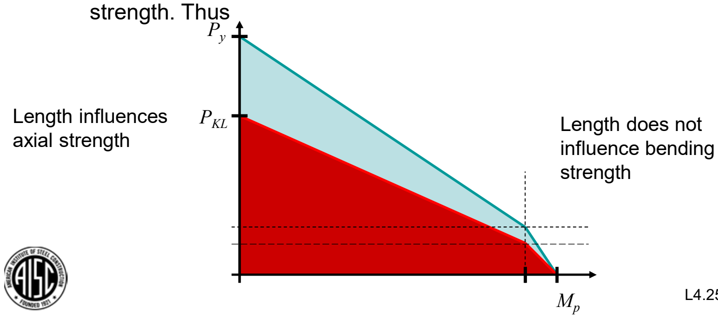
- How can we account for column length effects?
  - Look at plot for strength rather than nondimensionalized strength. Thus




L4.24

# Compression + Bending

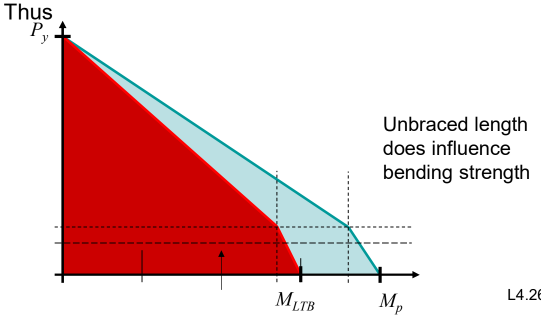
- How can we account for column length effects?
  - Look at plot for strength rather than nondimensionalized strength. Thus




L4.25

# Compression + Bending

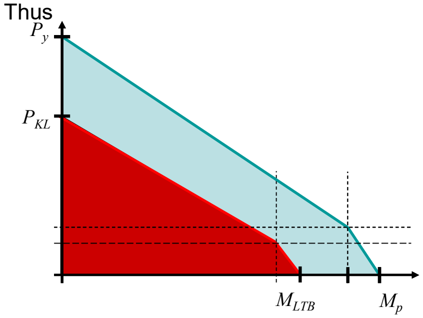
- How can we account for other bending limit states?
  - Look at plot for strength rather than nondimensionalized strength. Thus




L4.26

# Compression + Bending

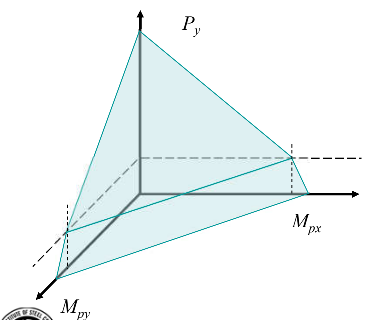
- Combine reductions in axial and flexural strengths.
  - Look at plot for strength rather than nondimensionalized strength. Thus



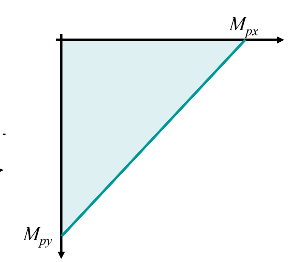

L4.27

# Compression + Bending

Now look at this same column with axial and bending about both axes.




And with bending only about both axes.




L4.28

## Compression + Bending

- Now we can look at the Specification equations and see that they are nondimensionalized with the available axial strength and the available bending strength.

$$\frac{P_r}{P_c} \text{ and } \frac{M_r}{M_c}$$

L4.29

## Design for Combined Forces

H1.1. Doubly and Singly Symmetric Members subject to Flexure and Axial Force

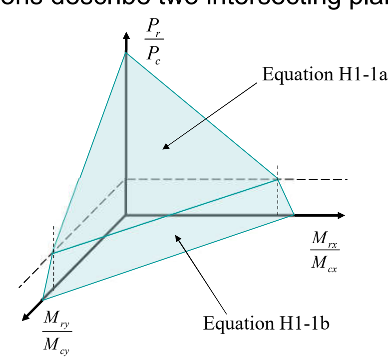
When $\frac{P_r}{P_c} \geq 0.2$ $\quad \frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (H1-1a)

When $\frac{P_r}{P_c} < 0.2$ $\quad \frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (H1-1b)

L4.30

## Compression + Bending

Thus, the Specification interaction equations describe two intersecting planes




L4.31

## Compression + Bending

- Definitions (ASD)

$P_r =$ required compressive strength (ASD)

$P_c = P_n / \Omega_c =$ allowable compressive strength

$M_r =$ required flexural strength (ASD)

$M_c = M_n / \Omega_b =$ allowable flexural strength

$\Omega_c = 1.67$

$\Omega_b = 1.67$

Determine required strength according to Chapter C

L4.32

# Compression + Bending

- Definitions (LRFD)

$P_r =$ required compressive strength (LRFD)

$P_c = \phi_c P_n =$ design compressive strength

$M_r =$ required flexural strength (LRFD)

$M_c = \phi_b M_n =$ design flexural strength

$\phi_c = 0.90$

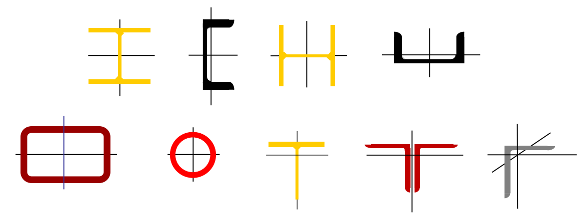
$\phi_b = 0.90$

Determine required strength according to Chapter C

L4.33

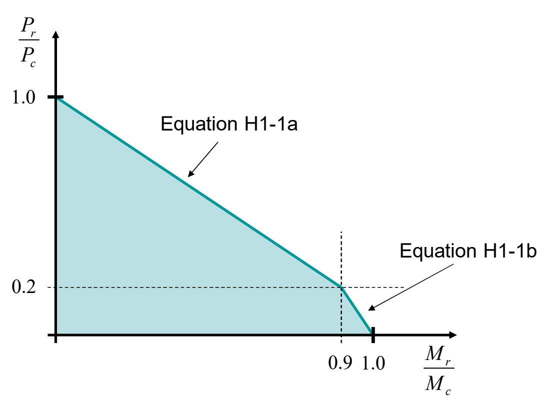
# Compression + Bending

- Equations H1-1a and H1-1b apply to all doubly and singly symmetric members



L4.34

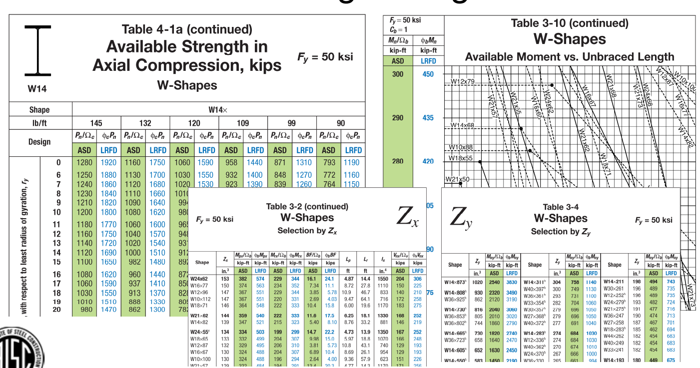
# Compression + Bending




L4.35

# Compression + Bending

- Beam-Column Design using Manual Tables




L4.36

# Compression + Bending

- Beam-Column Design using Manual Tables
  - Part 6 of the Manual contains a single table to assist in the design of members for combined forces.
  - Table entries included strength for all W-shapes.
  - Can be used to design for pure bending, pure compression, and pure tension.
  - Available for W-shapes only .

L4.37

# Compression + Bending

- These values can be found in Table 6-2 for all W-shapes, unlike the tables in Parts 3 and 4.
- Table 6-2 considers all appropriate limit states, including effective length for column buckling and unbraced length for beam lateral-torsional buckling.
- Table 6-2 also includes values for tension yield and tension rupture.
- All limit states considered in Lessons L1 through L3 are included.

L4.38

## Available Compressive Strength

The left half of Table 6-2 is the same as Table 4-1a.

However, it does include all W-shapes, thus more than Table 4-1a.

The length given in the middle column are the Effective Lengths with respect to the y-axis, just as for Table 4-1a

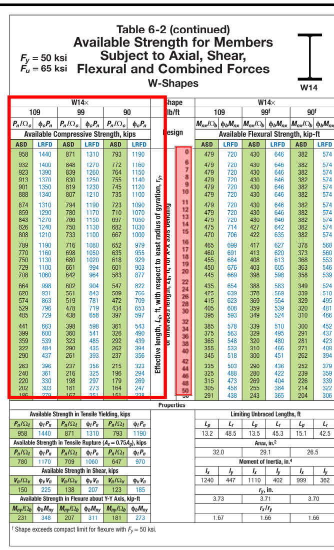


Table 6-2 (continued)
Available Strength for Members
Subject to Axial, Shear,
Flexural and Combined Forces
W-Shapes
$F_y$ = 50 ksi
$F_u$ = 65 ksi
W14

L4.39

## Available Flexural Strength

The right half of Table 6-2 does not have a comparable table in Part 3.

However, it provides the same information as given in Table 3-10. But again, for all W-shapes

The length given in the middle column are the Unbraced Lengths of the compression flange.

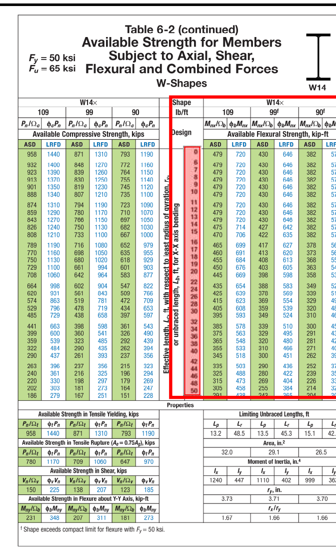


Table 6-2 (continued)
Available Strength for Members
Subject to Axial, Shear,
Flexural and Combined Forces
W-Shapes
$F_y$ = 50 ksi
$F_u$ = 65 ksi
W14

L4.40

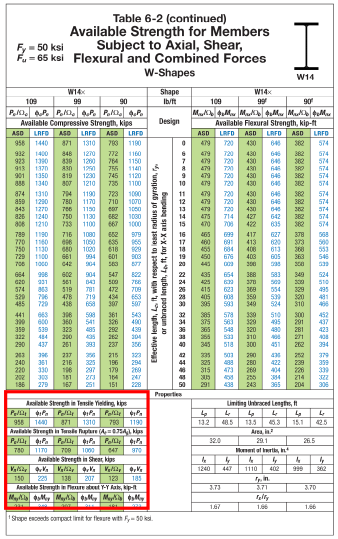
## Other Available Strengths

Strengths not a function of length:

Tensile Yield and Tensile Rupture, the same as Table 5-1.

Shear, the same as Tables 3-2 and 3-6.

Flexure about the y-axis, the same as Table 3-4.




L4.41

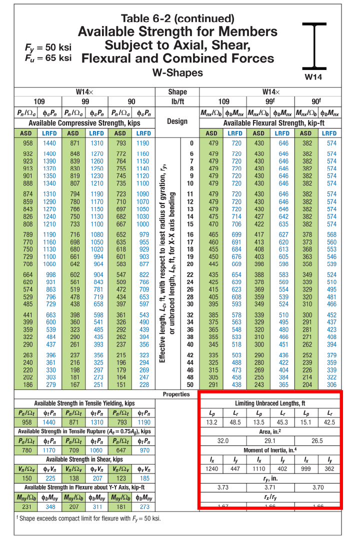
## Other Available Data

Unbraced lengths, $L_p$ and $L_r$

Area

Moment of Inertia, $I_x$ and $I_y$

Radius of gyration, $r_y$ and $r_x/r_y$

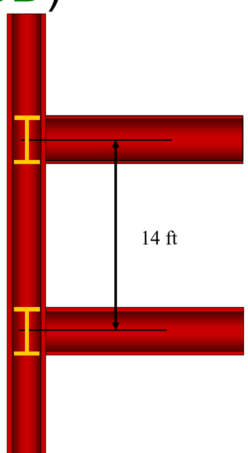



L4.42

# Example 1 (ASD)

- An ASTM A992 W14x90 column must carry an axial force of 333 kips, an *x*-axis bending moment of 169 ft-kips, and a *y*-axis bending moment of 20 ft-kips.
- These results are from a second-order Direct Analysis. (AISC 360-16 Chapter C)
- Will this column adequately support these loads?

The column is 14 ft long, is bending about both axes, has a length of 14 ft about the *x*- and *y*-axis and an unbraced length of the compression flange of 14 ft.



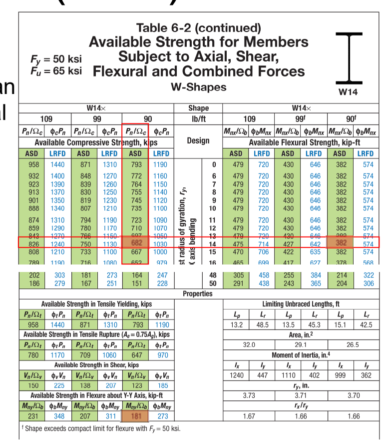

L4.43

# Example 1 (ASD)

The use of the direct analysis method means that we may use an effective length equal to the actual length, thus $K$ = 1.0

Also, since we used a second-order analysis there is no need to amplify forces or moments.

$$P_n/\Omega_c = 682 \text{ kips}$$
$$M_{nx}/\Omega_b = 382 \text{ ft-kips}$$
$$M_{ny}/\Omega_b = 181 \text{ ft-kips}$$




L4.44

# Example 1 (ASD)

Determine which equation to use:

$$P_r / P_c = 333/682 = 0.49 > 0.2$$

Therefore use Eq. H1-1a

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

$$\frac{333}{682} + \frac{8}{9}\left(\frac{169}{382} + \frac{20}{181}\right) = 0.98 \le 1.0$$

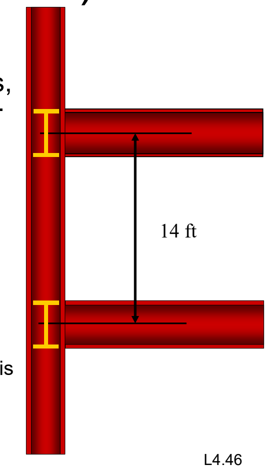
Therefore the W14x90 is adequate

L4.45

# Example 1 (LRFD)

- An ASTM A992 W14x90 column must carry an axial force of 500 kips, an *x*-axis bending moment of 253 ft-kips, and a *y*-axis bending moment of 30 ft-kips.
- These results are from a second-order Direct Analysis. (AISC 360-16 Chapter C)
- Will this column adequately support these loads?

The column is 14 ft long, is bending about both axes, has a length of 14 ft about the *x*- and *y*-axis and an unbraced length of the compression flange of 14 ft.




L4.46

# Example 1 (LRFD)

The use of the direct analysis method means that we may use an effective length equal to the actual length, thus $K = 1.0$

Also, since we used a second-order analysis there is no need to amplify forces or moments.

$$\phi_c P_n = 1030 \text{ kips}$$
$$\phi_b M_{nx} = 574 \text{ ft-kips}$$
$$\phi_b M_{ny} = 273 \text{ ft-kips}$$

**Table 6-2 (continued)**
**Available Strength for Members Subject to Axial, Shear, Flexural and Combined Forces**
**W-Shapes**

$F_y = 50$ ksi
$F_u = 65$ ksi

W14

| Shape | W14× 109 | | W14× 99 | | W14× 90 | | lb/ft | W14× 109 | | W14× 99f | | W14× 90f | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | Design | $M_{nx}/\Omega_b$ | $\phi_b M_{nx}$ | $M_{nx}/\Omega_b$ | $\phi_b M_{nx}$ | $M_{nx}/\Omega_b$ | $\phi_b M_{nx}$ |
| | Available Compressive Strength, kips | | | | | | | Available Flexural Strength, kip-ft | | | | | |
| | ASD | LRFD | ASD | LRFD | ASD | LRFD | | ASD | LRFD | ASD | LRFD | ASD | LRFD |
| st radius of gyration, $r_y$, / K axis bending | 958 | 1440 | 871 | 1310 | 793 | 1190 | 0 | 479 | 720 | 430 | 646 | 382 | 574 |
| | 932 | 1400 | 848 | 1270 | 772 | 1160 | 6 | 479 | 720 | 430 | 646 | 382 | 574 |
| | 923 | 1390 | 839 | 1260 | 764 | 1150 | 7 | 479 | 720 | 430 | 646 | 382 | 574 |
| | 913 | 1370 | 830 | 1250 | 755 | 1140 | 8 | 479 | 720 | 430 | 646 | 382 | 574 |
| | 901 | 1350 | 819 | 1230 | 745 | 1120 | 9 | 479 | 720 | 430 | 646 | 382 | 574 |
| | 888 | 1340 | 807 | 1210 | 735 | 1100 | 10 | 479 | 720 | 430 | 646 | 382 | 574 |
| | 874 | 1310 | 794 | 1190 | 723 | 1090 | 11 | 479 | 720 | 430 | 646 | 382 | 574 |
| | 859 | 1290 | 780 | 1170 | 710 | 1070 | 12 | 479 | 720 | 430 | 646 | 382 | 574 |
| | 843 | 1270 | 766 | 1150 | 697 | [illegible] | 13 | 479 | 720 | 430 | 646 | 382 | 574 |
| | 826 | 1240 | 750 | 1130 | 682 | 1030 | 14 | 475 | 714 | 427 | 642 | 382 | 574 |
| | 808 | 1210 | 733 | 1100 | 667 | 1000 | 15 | 470 | 706 | 422 | 635 | 382 | 574 |
| | 789 | 1190 | 716 | 1080 | 652 | 979 | 16 | 465 | 699 | 417 | 627 | 378 | 568 |
| | 202 | 303 | 181 | 273 | 164 | 247 | 48 | 305 | 458 | 255 | 384 | 214 | 322 |
| | 186 | 279 | 167 | 251 | 151 | 228 | 50 | 291 | 438 | 243 | 365 | 204 | 306 |

**Properties**

| Available Strength in Tensile Yielding, kips | | | | | |
|---|---|---|---|---|---|
| $P_n/\Omega_t$ | $\phi_t P_n$ | $P_n/\Omega_t$ | $\phi_t P_n$ | $P_n/\Omega_t$ | $\phi_t P_n$ |
| 958 | 1440 | 871 | 1310 | 793 | 1190 |
| **Available Strength in Tensile Rupture ($A_e = 0.75A_g$), kips** | | | | | |
| $P_n/\Omega_t$ | $\phi_t P_n$ | $P_n/\Omega_t$ | $\phi_t P_n$ | $P_n/\Omega_t$ | $\phi_t P_n$ |
| 780 | 1170 | 709 | 1060 | 647 | 970 |
| **Available Strength in Shear, kips** | | | | | |
| $V_n/\Omega_v$ | $\phi_v V_n$ | $V_n/\Omega_v$ | $\phi_v V_n$ | $V_n/\Omega_v$ | $\phi_v V_n$ |
| 150 | 225 | 138 | 207 | 123 | 185 |
| **Available Strength in Flexure about Y-Y Axis, kip-ft** | | | | | |
| $M_{ny}/\Omega_b$ | $\phi_b M_{ny}$ | $M_{ny}/\Omega_b$ | $\phi_b M_{ny}$ | $M_{ny}/\Omega_b$ | $\phi_b M_{ny}$ |
| 231 | 348 | 207 | 311 | 181 | 273 |

| Limiting Unbraced Lengths, ft | | | | | |
|---|---|---|---|---|---|
| $L_p$ | $L_r$ | $L_p$ | $L_r$ | $L_p$ | $L_r$ |
| 13.2 | 48.5 | 13.5 | 45.3 | 15.1 | 42.5 |
| **Area, in.$^2$** | | | | | |
| 32.0 | | 29.1 | | 26.5 | |
| **Moment of Inertia, in.$^4$** | | | | | |
| $I_x$ | $I_y$ | $I_x$ | $I_y$ | $I_x$ | $I_y$ |
| 1240 | 447 | 1110 | 402 | 999 | 362 |
| **$r_y$, in.** | | | | | |
| 3.73 | | 3.71 | | 3.70 | |
| **$r_x/r_y$** | | | | | |
| 1.67 | | 1.66 | | 1.66 | |

f Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

L4.47

# Example 1 (LRFD)

Determine which equation to use:

$$P_r / P_c = 500/1030 = 0.49 > 0.2$$

Therefore use Eq. H1-1a

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

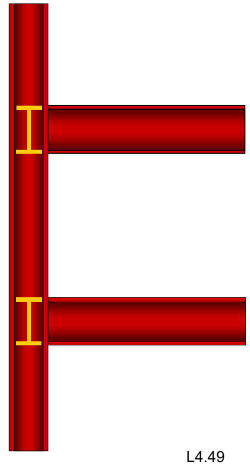
$$\frac{500}{1030} + \frac{8}{9}\left(\frac{253}{574} + \frac{30}{273}\right) = 0.97 \le 1.0$$

Therefore the W14x90 is adequate

L4.48

## Initial Beam-Column Selection

- How would we start a design if we had the force and moments from Example 1 but did not know what size column we were using?



L4.49

## Initial Beam-Column Selection

- Start with Equation H1-1a

$$\frac{P_r}{P_c}+\frac{8}{9}\frac{M_{rx}}{M_{cx}}+\frac{8}{9}\frac{M_{ry}}{M_{cy}}\le 1.0$$

- Multiply both sides by $P_c$

$$P_r+\frac{8}{9}\frac{M_{rx}P_c}{M_{cx}}+\frac{8}{9}\frac{M_{ry}P_c}{M_{cy}}\le P_c$$

L4.50

## Initial Beam-Column Selection

- Multiply last term by $M_{cx}/M_{cx}$ and reorganize

$$P_r+\frac{8P_c}{9M_{cx}}M_{rx}+\frac{8P_c}{9M_{cx}}\frac{M_{cx}}{M_{cy}}M_{ry}\le P_c$$

- Let

$$m=\frac{8P_c}{9M_{cx}}$$

$$U=\frac{M_{cx}}{M_{cy}}$$

L4.51

## Initial Beam-Column Selection

- Substitute these new terms back into the equation and

$$P_r+\frac{8P_c}{9M_{cx}}M_{rx}+\frac{8P_c}{9M_{cx}}\frac{M_{cx}}{M_{cy}}M_{ry}\le P_c$$

becomes

$$P_r+mM_{rx}+mUM_{ry}\le P_c$$

L4.52

## Initial Beam-Column Selection

- This can be thought of as

Effective required strength $\leq$ Available compressive strength

$$P_{eff} = P_r + mM_{rx} + mUM_{ry} \leq P_c$$

So what can we do about *m* and *U*?

L4.53

## Initial Beam-Column Selection

- Ignore the influence of length, thus no column buckling or lateral-torsional buckling
- Assume both axial and flexural stresses are $F_y$.

$$m = \frac{8P_c}{9M_{cx}} = \frac{8F_yA}{9F_yZ_x} = \frac{8A}{9Z_x}$$

$$U = \frac{M_{cx}}{M_{cy}} = \frac{F_yZ_x}{F_yZ_y} = \frac{Z_x}{Z_y}$$

L4.54

## Initial Beam-Column Selection

- Evaluate $m = \frac{8A}{9Z_x}$ and $U = \frac{Z_x}{Z_y}$ for all W-shapes

| Shape | $m_{avg}$ | $U_{avg}$ |
|---|---|---|
| W6 | 4.41 | 3.01 |
| W8 | 3.25 | 3.11 |
| W10 | 2.62 | 3.62 |
| W12 | 2.08 | 3.47 |
| W14 | 1.72 | 2.86 |

Calculated *m* was multiplied by 12 to permit working in kips and ft.

L4.55

## Initial Beam-Column Selection

- At this stage in design, the apparent level of accuracy of these numbers is unnecessary.
- Even these values are not all that accurate since they represent an average.
- These will be simplified in order to make them easy to remember and use.

L4.56

## Initial Beam-Column Selection

- Consider *U*
  - Only the smallest of the shapes in each group have *U* values appreciably greater than 3.
  - Thus, a reasonable value will be taken as *U* = 3.0 for all W-shapes up to W14.
- Consider *m*
  - Assume a moment arm of 0.89*d* for determination of the plastic section modulus.
  - Then *m* = 24/*d* (include the 12 for unit conversion)

L4.57

## Initial Beam-Column Selection

- The simplified multipliers become

| Shape | $m_{avg}$ | *m*=24/*d* | $U_{avg}$ | *U* |
|---|---|---|---|---|
| W6 | 4.41 | 4.0 | 3.01 | 3.0 |
| W8 | 3.25 | 3.0 | 3.11 | 3.0 |
| W10 | 2.62 | 2.4 | 3.62 | 3.0 |
| W12 | 2.08 | 2.0 | 3.47 | 3.0 |
| W14 | 1.72 | 1.71 | 2.86 | 3.0 |

Remember, the inaccuracy inherent here is not a concern since any final design must ultimately satisfy the interaction equations.

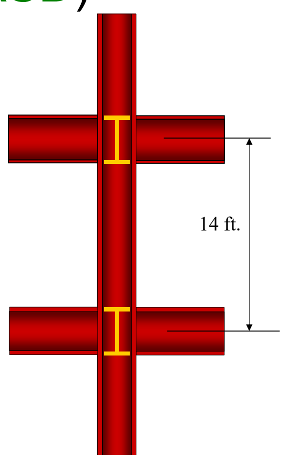
L4.58

## Example 2 (ASD)

An ASTM A992 column must carry an ASD axial force of 333 kips, an *x*-axis bending moment of 169 ft-kips, and a *y*-axis bending moment of 20 ft-kips. These results are from a second-order direct analysis.

The column is 14 ft long. Try a W14

$$m = 1.71$$
$$U = 3.0$$




L4.59

## Example 2 (ASD)

- Determine the effective axial force.

$$P_{eff} = 333 + 1.71(169) + 1.71(3.0)(20) = 725 \text{ kips}$$

- From Manual Table 4-1a, a W14x99 will support 750 kips and a W14x90 will support 682 kips.
  - We already know from Example 1 that the W14x90 works.

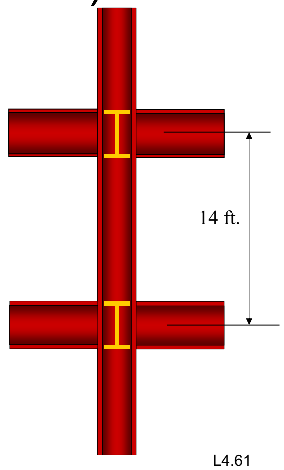
L4.60

# Example 2 (LRFD)

An ASTM A992 column must carry an LRFD axial force of 500 kips, an *x*-axis bending moment of 253 ft-kips, and a *y*-axis bending moment of 30 ft-kips. These results are from a second-order direct analysis.

The column is 14 ft long. Try a W14

$$m = 1.71$$
$$U = 3.0$$




L4.61

# Example 2 (LRFD)

- Determine the effective axial force.

$$P_{eff} = 500 + 1.71(253) + 1.71(3.0)(30) = 1090 \text{ kips}$$

- From Manual Table 4-1, a W14x99 will support 1130 kips and a W14x90 will support 1030 kips.
  - We already know from Example 1 that the W14x90 works.

L4.62

# Single Axis Bending

- Up to this point we have combined worst case column buckling with worst case flexure.
- However, it is possible to separate beam-column behavior into the in-plane effects and the out-of-plane effects.
- The Specification provides for the special case of doubly symmetric rolled compact members subject to single axis flexure and compression

L4.63

# Single Axis Bending

H1.3. Doubly Symmetric Rolled Compact Members Subject to Single-Axis Flexure and Compression

"For doubly symmetric rolled compact members with the effective length for torsional buckling less than or equal to the effective length for flexural-buckling, $L_{cz} \leq L_{cy}$, subjected to flexure and compression with moments primarily about their major axis, it is permissible to address the two independent limit states, in-plane instability and out-of-plane buckling or lateral-torsional buckling, separately in lieu of the combined approach provided in Section H1.1."

L4.64

# Single Axis Bending

- (a) For the limit-state of in-plane instability, Equations H1-1a and H1-1b are used with $P_c$ and $M_{cx}$ determined in the plane of bending.
  - This means the column strength is determined for *x*-axis buckling
  - The bending strength is $M_p$. (no consideration of lateral-torsional buckling)

L4.65

# Single Axis Bending

- (b) For the limit-state of out-of-plane plane buckling and lateral-torsional buckling:

$$\frac{P_r}{P_{cy}}\left(1.5-0.5\frac{P_r}{P_{cy}}\right)+\left(\frac{M_{rx}}{C_b M_{cx}}\right)^2 \le 1.0 \qquad \text{(H1-3)}$$

where

$P_{cy}$ = available compressive strength out of the plane of bending

$C_b$ = lateral-torsional buckling moment gradient factor

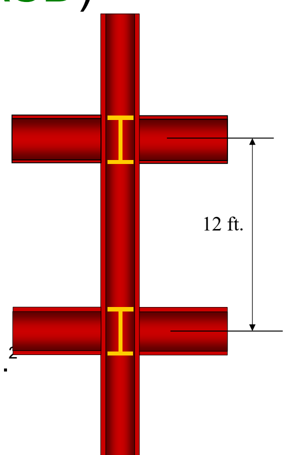
$M_{cx}$ = available lateral-torsional strength for strong axis flexure with $C_b = 1.0$

L4.66

# Example 3 (ASD)

- Check the adequacy of a W16x57 column in single axis bending using the alternate provisions of Section H1.3
- Compare the results to a solution if the alternate provisions are not used.

$r_x = 6.72$ in. $r_y = 1.60$ in. $A = 16.8$ in.$^2$




L4.67

# Example 3 (ASD)

- The column must carry an axial load, $P_a$ = 188 kips and moment about the strong axis, $M_a$ = 100 ft-kips, at each end bending the column in reverse curvature.
- The column has a length of 12 ft about the *x*- and *y*-axis and an unbraced length of the compression flange of 12 ft.
- Results are from a second-order Direct Analysis, thus use *K* = 1.0.

L4.68

## Example 3 (ASD)

- For the limit-state of in-plane instability, $P_c$ and $M_{cx}$ are determined in the plane of bending.

$$\frac{L_c}{r_x} = \frac{12(12)}{6.72} = 21.4 \le 113$$

$$F_{ex} = \frac{\pi^2(29{,}000)}{(21.4)^2} = 625 \text{ ksi}$$

Note that there are no column tables for the W16's and Table 6-2 does not include x-axis strength.

L4.69

## Example 3 (ASD)

Since $\frac{L_c}{r} \le 113$

Therefore

$$F_{cr} = (0.658)^{\left(\frac{50}{625}\right)}(50) = 48.4 \text{ ksi}$$

and

$$P_n = 48.4(16.8) = 813 \text{ kips}$$

$$\frac{P_n}{\Omega} = \frac{813}{1.67} = 487 \text{ kips}$$

L4.70

## Example 3 (ASD)

- In-plane bending strength is $M_p$. From Table 3-2

$$\frac{M_p}{\Omega} = 262 \text{ ft-kips}$$

- Thus, in the plane of bending

$$\frac{188}{487} + \frac{8}{9}\left(\frac{100}{262}\right) = 0.73 < 1.0 \qquad \text{(H1-1a)}$$

L4.71

## Example 3 (ASD)

For out-of-plane,

$$\frac{L_c}{r_y} = \frac{12(12)}{1.60} = 90.0 \le 113$$

$$F_{ey} = \frac{\pi^2(29{,}000)}{(90.0)^2} = 35.3 \text{ ksi}$$

L4.72

# Example 3 (ASD)

Therefore

$$F_{cr} = (0.658)^{\left(\frac{50}{35.3}\right)}(50) = 27.6 \text{ ksi}$$

and

$$P_n = 27.6(16.8) = 464 \text{ kips}$$

$$\frac{P_n}{\Omega} = \frac{464}{1.67} = 278 \text{ kips}$$

This could have been found in Table 6-2

L4.73

# Example 3 (ASD)

- From the beam curves (Table 3-10), with $C_b$ = 1.0 (or from Table 6-2)

$$M_{cx} = \frac{M_n}{\Omega} = 211 \text{ ft-kips}$$

- and, from Eq. F1-1, with equal and opposite end moments,

$$C_b = 2.27$$

L4.74

# Example 3 (ASD)

For out-of-plane,

$$\frac{P_r}{P_{cy}}\left(1.5 - 0.5\frac{P_r}{P_{cy}}\right) + \left(\frac{M_{rx}}{C_b M_{cx}}\right)^2 \leq 1.0 \qquad \text{(H1-2)}$$

$$\frac{188}{278}\left(1.5 - 0.5\left(\frac{188}{278}\right)\right) + \left(\frac{100}{2.27(211)}\right)^2 = 0.83 \leq 1.0$$

Eq. H1-1a = 0.73 < 1.0 and Eq. H1-2 = 0.83 < 1.0
Thus, the column is adequate

L4.75

# Example 3 (ASD)

- Without using the alternate provisions
- Eq H1-1a

$$\left(\frac{188}{278}\right) + \frac{8}{9}\left(\frac{100}{262}\right) = 1.02 > 1.0$$

Bending strength, *x*-axis, using $C_b$

$$M_{cx} = 2.27(211) = 479 > \frac{M_p}{\Omega} = 262$$

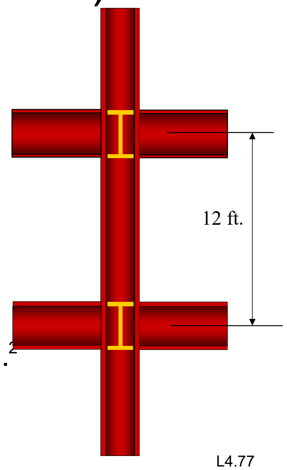
Compressive strength for buckling about *y*-axis

L4.76

# Example 3 (LRFD)

- Check the adequacy of a W16x57 column in single axis bending using the alternate provisions of Section H1.3
- Compare the results to a solution if the alternate provisions are not used.

$r_x = 6.72$ in. $r_y = 1.60$ in. $A = 16.8$ in.$^2$




L4.77

# Example 3 (LRFD)

- The column must carry an axial load, $P_u$ = 282 kips and moment about the strong axis, $M_u$ = 150 ft-kips, at each end bending the column in reverse curvature.
- The column has a length of 12 ft about the *x*- and *y*-axis and an unbraced length of the compression flange of 12 ft.
- Results are from a second-order Direct Analysis, thus use $K$ = 1.0.

L4.78

# Example 3 (LRFD)

- For the limit-state of in-plane instability, $P_c$ and $M_{cx}$ are determined in the plane of bending.

$$\frac{L_c}{r_x} = \frac{12(12)}{6.72} = 21.4 \leq 113$$

$$F_{ex} = \frac{\pi^2(29{,}000)}{(21.4)^2} = 625 \text{ ksi}$$

Note that there are no column tables for the W16's and Table 6-2 does not include x-axis strength.

L4.79

# Example 3 (LRFD)

Since $\frac{L_c}{r} \leq 113$

Therefore

$$F_{cr} = (0.658)^{\left(\frac{50}{625}\right)}(50) = 48.4 \text{ ksi}$$

and

$$P_n = 48.4(16.8) = 813 \text{ kips}$$

$$\phi P_n = 0.9(813) = 732 \text{ kips}$$

L4.80

## Example 3 (LRFD)

- In-plane bending strength is $M_p$. From Table 3-2

$$\phi M_p = 394 \text{ ft-kips}$$

- Therefore, in the plane of bending

$$\frac{282}{732} + \frac{8}{9}\left(\frac{150}{394}\right) = 0.72 < 1.0 \qquad \text{(H1-1a)}$$

L4.81

## Example 3 (LRFD)

For out-of-plane,

$$\frac{L_c}{r_y} = \frac{12(12)}{1.60} = 90.0 \le 113$$

$$F_{ey} = \frac{\pi^2(29{,}000)}{(90.0)^2} = 35.3 \text{ ksi}$$

L4.82

## Example 3 (LRFD)

Therefore

$$F_{cr} = (0.658)^{\left(\frac{50}{35.3}\right)}(50) = 27.6 \text{ ksi}$$

and

$$P_n = 27.6(16.8) = 464 \text{ kips}$$

$$\phi P_n = 0.9(464) = 418 \text{ kips}$$

This could have been found in Table 6-2

L4.83

## Example 3 (LRFD)

- From the beam curves (Table 3-10), with $C_b$ = 1.0 (or from Table 6-2)

$$M_{cx} = \phi M_n = 318 \text{ ft-kips}$$

- and, from Eq. F1-1, with equal and opposite end moments,

$$C_b = 2.27$$

L4.84

# Example 3 (LRFD)

For out-of-plane,

$$\frac{P_r}{P_{cy}}\left(1.5-0.5\frac{P_r}{P_{cy}}\right)+\left(\frac{M_{rx}}{C_b M_{cx}}\right)^2 \le 1.0 \qquad \text{(H1-2)}$$

$$\frac{282}{418}\left(1.5-0.5\left(\frac{282}{418}\right)\right)+\left(\frac{150}{2.27(318)}\right)^2 = 0.83 \le 1.0$$

Eq. H1-1a = 0.72 < 1.0 and Eq. H1-2 = 0.83 < 1.0
Thus, the column is adequate



# Example 3 (LRFD)

- Without using the alternate provisions

$$\left(\frac{282}{418}\right)+\frac{8}{9}\left(\frac{150}{394}\right)=1.01>1.0$$

Bending strength, *x*-axis, using $C_b$

$$M_{cx}=2.27(318)=722>\phi M_p=394$$

Compressive strength for buckling about *y*-axis



# Compression + Bending

H2. Unsymmetric and Other Members Subject to Flexure and Axial Force

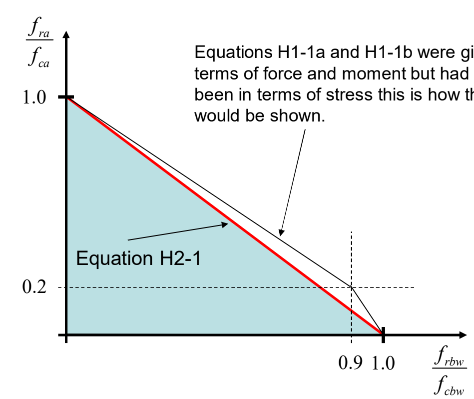
- For all unsymmetric members

$$\left|\frac{f_{ra}}{F_{ca}}+\frac{f_{rbw}}{F_{cbw}}+\frac{f_{rbz}}{F_{cbz}}\right| \le 1.0 \qquad \text{(H2-1)}$$

- This may be used for any member in place of the equations in Section H1.
- It requires the superposition of the stresses at critical points in the cross section

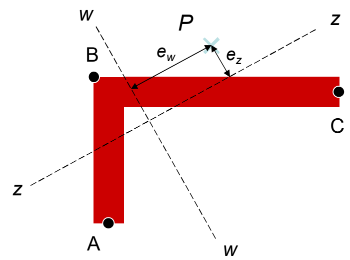


# Compression + Bending



# Single Angle

- Consider a single angle loaded as shown




*z* and *w* represent the major and minor principal axes.

$e_z$ and $e_w$ represent the eccentricity of load, *P*, from those axes.

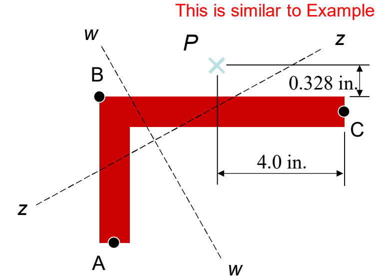
Points A, B, and C are the points on the section that must be checked through the interaction equation.



# Example 4 (ASD)

- Will a 5 ft long L8x4x7/16 A36 angle support a compressive load of $P_a$ = 40 kips at an eccentricity of 0.328 in. from the back face of the leg?

This is similar to Example E.14 in the Design Examples V15



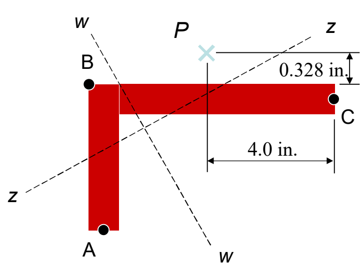

Determine sense of bending stress

| Point | $M_w$ | $M_z$ |
|---|---|---|
| A | T | T |
| B | T | C |
| C | C | T |



# Example 4 (ASD)

- Interaction equations at the 3 critical points




Compression is positive

$$\left| \frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}} \right|_A \le 1.0$$

$$\left| \frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}} \right|_B \le 1.0$$

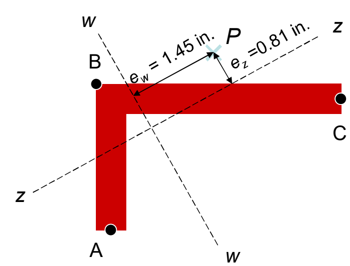
$$\left| \frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}} \right|_C \le 1.0$$

Eq. H2-1 applied at each point



# Example 4 (ASD)

- Required strength




$$P_a = 40.0 \text{ kips}$$

$$M_{aw} = B_{1w}(e_w)(P_a)$$

$$= 1.02(40)(1.45)$$

$$= 59.2 \text{ in.-kips}$$

$$M_{az} = B_{1z}(e_z)(P_a)$$

$$= 1.27(40)(0.810)$$

$$= 41.1 \text{ in.-kips}$$

$B_{1w}$ and $B_{1z}$ are second order amplifiers, AISC 360-16 Appendix 8.

# Example 4 (ASD)

- Determine the required stresses at points A, B, and C.
- These calculations will use the section modulus referred to each point about each axis.

$S_{wA} = 10.9 \text{ in.}^3$ $S_{zA} = 1.61 \text{ in.}^3$

$S_{wB} = 14.6 \text{ in.}^3$ $S_{zB} = 2.51 \text{ in.}^3$

$S_{wC} = 7.04 \text{ in.}^3$ $S_{zC} = 5.07 \text{ in.}^3$

These section properties are available in the AISC Shapes Database V15

and for axial stress, $A = 5.11 \text{ in.}^2$



# Example 4 (ASD)

- Determine the required stresses at points A, B, and C.

| | $P_a = 40.0$ kips | | $M_{aw} = 59.2$ in.-kips | | $M_{az} = 41.1$ in.-kips | |
|---|---|---|---|---|---|---|
| Point | $A$ | $f_a$ (ksi) | $S_w$ | $f_{bw}$(ksi) | $S_z$ | $f_{bz}$ (ksi) |
| A | 5.11 | **7.83** | 10.9 | **- 5.43** | 1.61 | **- 25.5** |
| B | 5.11 | **7.83** | 14.6 | **- 4.05** | 2.51 | **16.4** |
| C | 5.11 | **7.83** | 7.04 | **8.41** | 5.07 | **- 8.11** |



# Example 4 (ASD)

- Determine available strength
  - Flexural buckling about the z-axis

$$\frac{P_n}{\Omega} = 78.4 \text{ kips}$$

  - Lateral-torsional buckling about the *w*-axis

$$\frac{M_{nw}}{\Omega} = 166 \text{ in.-kips}$$

  - Yielding about the *z*-axis

$$\frac{M_{nz}}{\Omega} = 52.1 \text{ in.-kips}$$



# Example 4 (ASD)

- Determine the available stresses at points A, B, and C.

| | $\frac{P_n}{\Omega} = 78.4$ kips | | $\frac{M_{nw}}{\Omega} = 166$ in.-kips | | $\frac{M_{nz}}{\Omega} = 52.1$ in.-kips | |
|---|---|---|---|---|---|---|
| Point | $A$ | $F_a$ (ksi) | $S_w$ | $F_{bw}$(ksi) | $S_z$ | $F_{bz}$ (ksi) |
| A | 5.11 | **15.3** | 10.9 | **15.2** | 1.61 | **32.4** |
| B | 5.11 | **15.3** | 14.6 | **11.4** | 2.51 | **20.8** |
| C | 5.11 | **15.3** | 7.04 | **23.6** | 5.07 | **10.3** |

# Example 4 (ASD)

- Determine the results of Eq H2-1 at points A, B, and C.

$$\left|\frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}}\right|_A \leq 1.0$$

| Point | $f_a/F_a$ | $f_{bw}/F_{bw}$ | $f_{bz}/F_{bz}$ | | $\leq 1.0$ |
|---|---|---|---|---|---|
| A | **+ 0.512** | **- 0.357** | **- 0.778** | = | **- 0.623** |
| B | **+ 0.512** | **- 0.355** | **+ 0.788** | = | **0.945** |
| C | **+ 0.512** | **+ 0.356** | **- 0.787** | = | **0.081** |

Point B is the critical point on the angle and the column will support this load at this location.

L4.97

# Example 4 (ASD)

- Note that regardless of the point under consideration, the value of the ratio came out essentially the same, except for sign.
- This is because the same section modulus or area occurs in the numerator and denominator.
- Thus, all this could be simplified by taking just ratios of moment or force.

L4.98

# Example 4 (ASD)

- Look at point B with equation H2-1 in terms of force and moment.

$$\frac{P_a}{P_n/\Omega} - \frac{M_{aw}}{M_{nw}/\Omega} + \frac{M_{az}}{M_{nz}/\Omega} \leq 1.0$$

$$\frac{40.0}{78.4} - \frac{59.2}{166} + \frac{41.1}{52.1} =$$

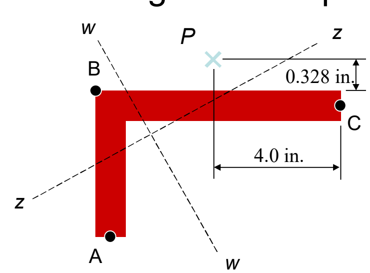
$$0.510 - 0.357 + 0.789 = 0.942 \leq 1.0$$

So why is equation H2-1 given in terms of stress?
To capture signs for tension and compression

L4.99

# Example 4 (ASD)

- Look again at the problem we solved




The location of the load was not selected by accident.

It is located at the midpoint of the 8 in. leg and at ¾ the thickness of the angle from the back of the angle.

Manual Table 4-12 uses this location in tabulating the available strength of eccentrically loaded single angles

L4.100

# Example 4 (ASD)

**Table 4-12 (continued)**
**Available Strength in Axial Compression, kips**
**Eccentrically Loaded Single Angles**

$F_y$ = 36 ksi — L8–L7

| Shape | | L8×4× | | | | | | L7×4× | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $^9/_{16}$[c] | | $^1/_2$[c,f] | | $^7/_{16}$[c,f] | | $^3/_4$ | | $^5/_8$ | | $^1/_2$[c] | |
| lb/ft | | 21.9 | | 19.6 | | 17.2 | | 26.2 | | 22.1 | | 17.9 | |
| Design | | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ |
| | | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD |
| | 0 | 60.0 | 90.2 | 57.8 | 87.0 | 55.5 | 83.4 | 65.1 | 97.9 | 62.5 | 94.0 | 59.5 | 89.4 |
| | 1 | 59.4 | 89.3 | 57.2 | 86.0 | 54.8 | 82.5 | 64.4 | 96.8 | 61.8 | 93.0 | 58.8 | 88.4 |
| | 2 | 57.6 | 86.7 | 55.3 | 83.4 | 52.9 | 79.7 | 62.1 | 93.5 | 59.8 | 90.0 | 56.7 | 85.4 |
| | 3 | 54.7 | 82.5 | 52.4 | 79.2 | 50.0 | 75.5 | 58.7 | 88.6 | 56.5 | 85.3 | 53.4 | 80.6 |
| | 4 | 51.0 | 77.2 | 48.7 | 73.8 | 46.2 | 70.1 | 54.8 | 82.9 | 52.3 | 79.3 | 49.2 | 74.6 |
| | 5 | 46.7 | 70.9 | 44.6 | 67.7 | 42.1 | 64.0 | 50.3 | 76.4 | 47.7 | 72.4 | 44.4 | 67.5 |
| **Properties** | | | | | | | | | | | | | |
| $A_g$, in.$^2$ | | 6.49 | | 5.80 | | 5.11 | | 7.74 | | 6.50 | | 5.26 | |
| $r_z$, in. | | 0.859 | | 0.863 | | 0.867 | | 0.855 | | 0.860 | | 0.866 | |

| ASD | LRFD |
|---|---|
| $\Omega_c = 1.67$ | $\phi_c = 0.90$ |

[c] Shape is slender for compression with $F_y$ = 36 ksi; tabulated values have been adjusted accordingly.
[f] Shape exceeds compact limit for flexure with $F_y$ = 36 ksi.
Note: Heavy line indicates $L_c/r_z$ equal to or greater than 200.

For $L_c$ with respect to the *z*-axis equal to 5.0 ft

$$\frac{P_n}{\Omega} = 42.1 \text{ kips}$$

Since

$$P_a = 40.0 \text{ kips} < \frac{P_n}{\Omega}$$

the column will carry this load at the given eccentricities.

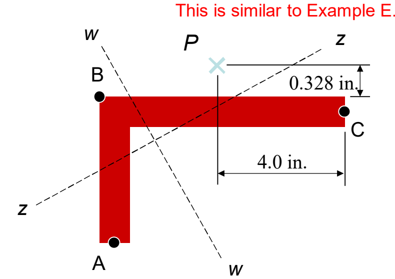
This was a lot less work!

L4.101

# Example 4 (LRFD)

- Will a 5 ft long L8x4x7/16 A36 angle support a compressive load of $P_u$ = 60 kips at an eccentricity of 0.328 in. from the back face of the leg?

This is similar to Example E.14 in the Design Examples V15



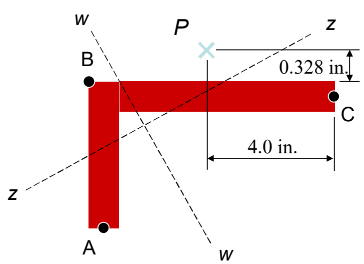

Determine sense of bending stress

| Point | $M_w$ | $M_z$ |
|---|---|---|
| A | T | T |
| B | T | C |
| C | C | T |

L4.102

# Example 4 (LRFD)

- Interaction equations at the 3 critical points




Compression is positive

$$\left|\frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}}\right|_A \le 1.0$$

$$\left|\frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}}\right|_B \le 1.0$$

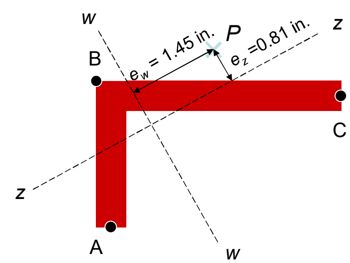
$$\left|\frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}}\right|_C \le 1.0$$

Eq. H2-1 applied at each point

L4.103

# Example 4 (LRFD)

- Required strength




$$P_u = 60.0 \text{ kips}$$

$$M_{uw} = B_{1w}(e_w)(P_u)$$
$$= 1.02(60)(1.45)$$
$$= 88.7 \text{ in.-kips}$$

$$M_{uz} = B_{1z}(e_z)(P_u)$$
$$= 1.24(60)(0.810)$$
$$= 60.3 \text{ in.-kips}$$

$B_{1w}$ and $B_{1z}$ are second order amplifiers, AISC 360-16 Appendix 8.

L4.104

# Example 4 (LRFD)

- Determine the required stresses at points A, B, and C.
- These calculations will use the section modulus referred to each point about each axis.

$S_{wA} = 10.9$ in.$^3$ $S_{zA} = 1.61$ in.$^3$
$S_{wB} = 14.6$ in.$^3$ $S_{zB} = 2.51$ in.$^3$
$S_{wC} = 7.04$ in.$^3$ $S_{zC} = 5.07$ in.$^3$

These section properties are available in the AISC Shapes Database V15

and for axial stress, $A = 5.11$ in.$^2$

L4.105

# Example 4 (LRFD)

- Determine the required stresses at points A, B, and C.

| | $P_u = 60.0$ kips | | $M_{uw} = 88.7$ in.-kips | | $M_{uz} = 60.3$ in.-kips | |
|---|---|---|---|---|---|---|
| Point | $A$ | $f_a$ (ksi) | $S_w$ | $f_{bw}$(ksi) | $S_z$ | $f_{bz}$ (ksi) |
| A | 5.11 | **11.7** | 10.9 | **- 8.14** | 1.61 | **- 37.5** |
| B | 5.11 | **11.7** | 14.6 | **- 6.08** | 2.51 | **24.0** |
| C | 5.11 | **11.7** | 7.04 | **12.6** | 5.07 | **- 11.9** |

L4.106

# Example 4 (LRFD)

- Determine available strength
  - Flexural buckling about the z-axis
    $$\phi P_n = 118 \text{ kips}$$
  - Lateral-torsional buckling about the *w*-axis
    $$\phi M_{nw} = 249 \text{ in.-kips}$$
  - Yielding about the *z*-axis
    $$\phi M_{nz} = 78.3 \text{ in.-kips}$$

L4.107

# Example 4 (LRFD)

- Determine the available stresses at points A, B, and C.

| | $\phi P_n = 118$ kips | | $\phi M_{nw} = 249$ in.-kips | | $\phi M_{nz} = 78.3$ in.-kips | |
|---|---|---|---|---|---|---|
| Point | $A$ | $F_a$ (ksi) | $S_w$ | $F_{bw}$(ksi) | $S_z$ | $F_{bz}$ (ksi) |
| A | 5.11 | **23.1** | 10.9 | **22.8** | 1.61 | **48.6** |
| B | 5.11 | **23.1** | 14.6 | **17.1** | 2.51 | **31.2** |
| C | 5.11 | **23.1** | 7.04 | **35.4** | 5.07 | **15.4** |

L4.108

# Example 4 (LRFD)

- Determine the results of Eq H2-1 at points A, B, and C.

$$\left|\frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}}\right|_A \le 1.0$$

| Point | $f_a/F_a$ | $f_{bw}/F_{bw}$ | $f_{bz}/F_{bz}$ | | $\le 1.0$ |
|---|---|---|---|---|---|
| A | **+ 0.506** | **- 0.357** | **- 0.772** | = | **- 0.623** |
| B | **+ 0.506** | **- 0.356** | **+ 0.769** | = | **0.919** |
| C | **+ 0.506** | **+ 0.356** | **- 0.773** | = | **0.089** |

Point B is the critical point on the angle and the column will support this load at this location.



# Example 4 (LRFD)

- Note that regardless of the point under consideration, the value of the ratio came out essentially the same, except for sign.
- This is because the same section modulus or area occurs in the numerator and denominator.
- Thus, all this could be simplified by taking just ratios of moment or force.



# Example 4 (LRFD)

- Look at point B with equation H2-1 in terms of force and moment.

$$\frac{P_u}{\phi_c P_n} - \frac{M_{uw}}{\phi_b M_{nw}} + \frac{M_{uz}}{\phi_b M_{nz}} \le 1.0$$

$$\frac{60.0}{118} - \frac{88.7}{249} + \frac{60.3}{78.3} =$$

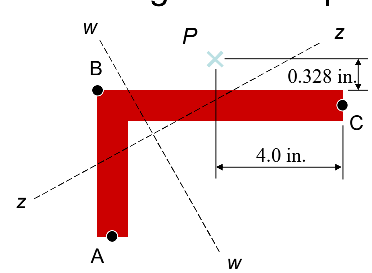
$$0.508 - 0.356 + 0.770 = 0.922 \le 1.0$$

So why is equation H2-1 given in terms of stress?
To capture signs for tension and compression



# Example 4 (LRFD)

- Look again at the problem we solved




The location of the load was not selected by accident.

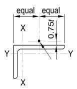
It is located at the midpoint of the 8 in. leg and at ¾ the thickness of the angle from the back of the angle.

Manual Table 4-12 uses these locations in tabulating the available strength of eccentrically loaded single angles

# Example 4 (LRFD)

Table 4-12 (continued)
**Available Strength in Axial Compression, kips**
**Eccentrically Loaded Single Angles**

$F_y$ = 36 ksi




L8–L7

| Shape | | L8×4× | | | | | | L7×4× | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | 9/16[c] | | 1/2[c,f] | | 7/16[c,f] | | 3/4 | | 5/8 | | 1/2[c] | |
| lb/ft | | 21.9 | | 19.6 | | 17.2 | | 26.2 | | 22.1 | | 17.9 | |
| Design | | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ | $P_n/\Omega_c$ | $\phi_c P_n$ |
| | | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD | ASD | LRFD |
| | 0 | 60.0 | 90.2 | 57.8 | 87.0 | 55.5 | 83.4 | 65.1 | 97.9 | 62.5 | 94.0 | 59.5 | 89.4 |
| | 1 | 59.4 | 89.3 | 57.2 | 86.0 | 54.8 | 82.5 | 64.4 | 96.8 | 61.8 | 93.0 | 58.8 | 88.4 |
| | 2 | 57.6 | 86.7 | 55.3 | 83.4 | 52.9 | 79.7 | 62.1 | 93.5 | 59.8 | 90.0 | 56.7 | 85.4 |
| | 3 | 54.7 | 82.5 | 52.4 | 79.2 | 50.0 | 75.5 | 58.7 | 88.6 | 56.5 | 85.3 | 53.4 | 80.6 |
| | 4 | 51.0 | 77.2 | 48.7 | 73.8 | 46.2 | 70.1 | 54.8 | 82.9 | 52.3 | 79.3 | 49.2 | 74.6 |
| | 5 | 46.7 | 70.9 | 44.6 | 67.7 | 42.1 | 64.0 | 50.3 | 76.4 | 47.7 | 72.4 | 44.4 | 67.5 |
| **Properties** | | | | | | | | | | | | | |
| $A_g$, in.$^2$ | | 6.49 | | 5.80 | | 5.11 | | 7.74 | | 6.50 | | 5.26 | |
| $r_z$, in. | | 0.859 | | 0.863 | | 0.867 | | 0.855 | | 0.860 | | 0.866 | |

| ASD | LRFD |
|---|---|
| $\Omega_c = 1.67$ | $\phi_c = 0.90$ |

[c] Shape is slender for compression with $F_y$ = 36 ksi; tabulated values have been adjusted accordingly.
[f] Shape exceeds compact limit for flexure with $F_y$ = 36 ksi.
Note: Heavy line indicates $L_c/r_z$ equal to or greater than 200.

For $L_c$ with respect to the $z$-axis equal to 5.0 ft

$$\phi P_n = 64.0 \text{ kips}$$

Since

$$P_u = 60.0 \text{ kips} < \phi P_n$$

the column will carry this load at the given eccentricities.

This was a lot less work!

L4.113

# Summary

- Looked at development of elastic and plastic approaches to the interaction.
- Used a single Manual table to determine all required strengths for combined forces.
- Derived a simple approach for initial selection of beam-column members.
- Investigated a special approach when bending is only about the $x$-axis.
- Addressed the approach for unsymmetric members.

L4.114

# Course Conclusion

- We have studied tension members, compression members, and flexural members.
- We have investigated the interaction of compression and bending.
- But, we have only touched on the basic principles of structural steel design.
  - It was our intent to make this a useful refresher for those who have not designed in structural steel for some time, we hope your capabilities have been improved because of your time in this course.

L4.115

# Thank You

American Institute of Steel Construction
130 East Randolph St., Suite 2000
Chicago, IL 60601

L4.116

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- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!

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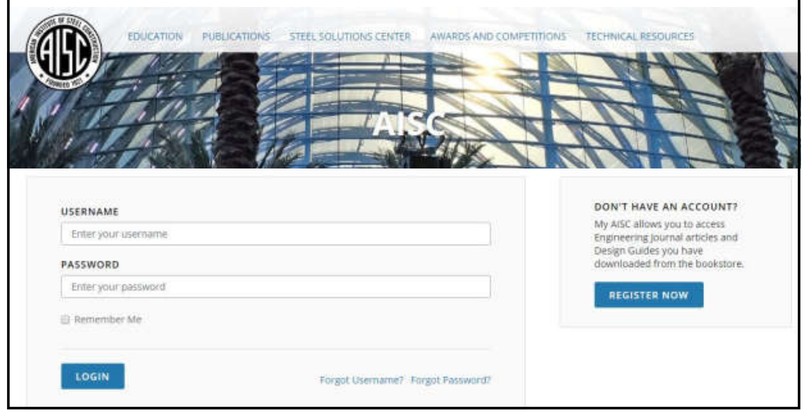


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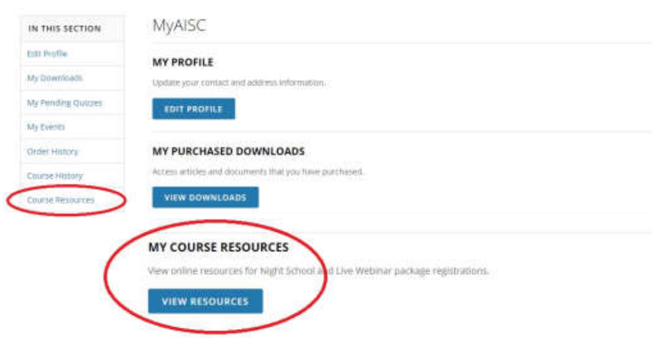


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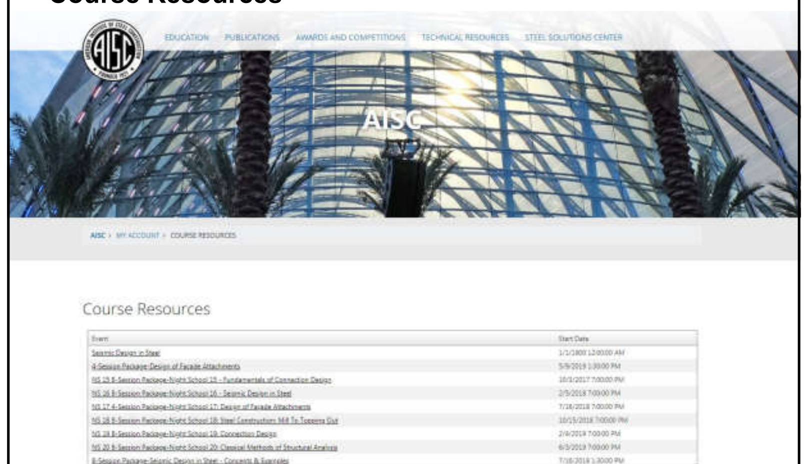

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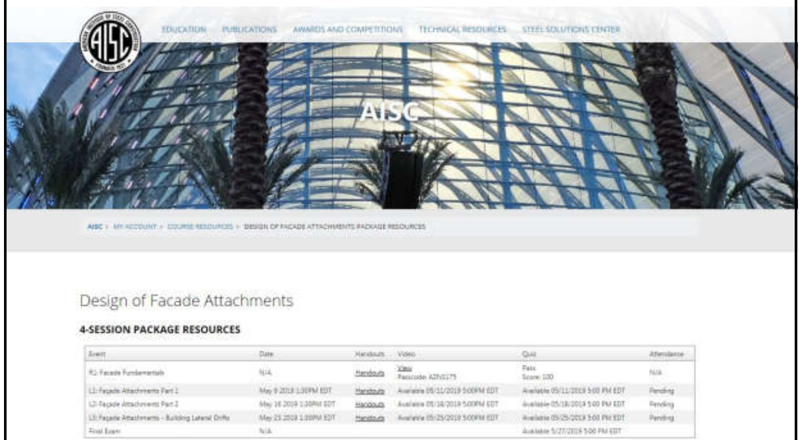

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Design of Facade Attachments

4-SESSION PACKAGE RESOURCES

| Event | Date | Handouts | Video | Quiz | Attendance |
|---|---|---|---|---|---|
| R1: Facade Fundamentals | N/A | Handouts | View<br>Passcode: AZN3175 | Pass<br>Score: 100 | N/A |
| L1: Facade Attachments Part 1 | May 9 2019 1:30PM EDT | Handouts | Available 05/11/2019 5:00PM EDT | Available 05/11/2019 5:00 PM EDT | Pending |
| L2: Facade Attachments Part 2 | May 16 2019 1:30PM EDT | Handouts | Available 05/18/2019 5:00PM EDT | Available 05/18/2019 5:00 PM EDT | Pending |
| L3: Facade Attachments - Building Lateral Drifts | May 23 2019 1:30PM EDT | Handouts | Available 05/25/2019 5:00PM EDT | Available 05/25/2019 5:00 PM EDT | Pending |
| Final Exam | N/A | | | Available 5/27/2019 5:00 PM EDT | |

**AISC** | Thank you.

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