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Design of Curved Members

Part 2: Design of Horizontally-Curved Members

December 12, 2019

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Course Description

Design of Horizontally-Curved Members
December 12, 2019

While curved members can be both structurally efficient and aesthetically interesting, their behavior can be much different than their straight counterparts. This two-part webinar series, presented by the author of AISC Design Guide 33 – Curved Member Design, provides guidance on how to use the AISC Specification to design curved members. This 'equivalent straight member' approach allows the use of existing commercial software for curved member design by modifying effective length factors and lateral-torsional buckling modification factors to account for the curvature. This course will provide a brief overview of the design guide and detailed design information on both vertically- and horizontally-curved members.

This session will examine the behavior and design of horizontally-curved members. The design of such members for flexural, torsional, and combined loading will be explained, as will the important issue of serviceability. The concepts will be illustrated with a design example.

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Learning Objectives

- Identify three methods for analyzing horizontally-curved members.
- Describe how to use the Isolated Flange Method for assessing the torsion on horizontally-curved members.
- Describe how to account for second-order effects when designing horizontally-curved members.
- List serviceability considerations for horizontally-curved members.

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Design of Curved Members

Part 2: Design of Horizontally-Curved Members
December 12, 2019

Bo Dowswell, PE, PhD
ARC International, LLC
Birmingham, AL

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Horizontally-Curved Members

Session Description

structural
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Session Description

- Introduction
- Structural analysis
- Flexural strength
- Torsional strength

(continued)

Photograph courtesy of the AISC Bender/Roller Committee

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Session Description

- Combined loads
- Serviceability
- Design example

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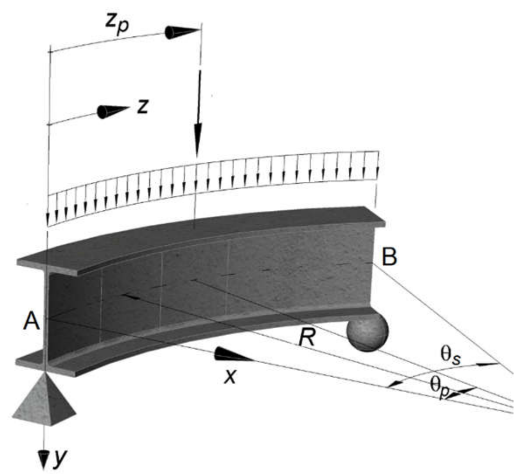
Horizontally-Curved Members

Introduction

structural STEEL

Introduction

- Curved Members ≠ Straight Members
- Curved beam → Flexure + Torsion



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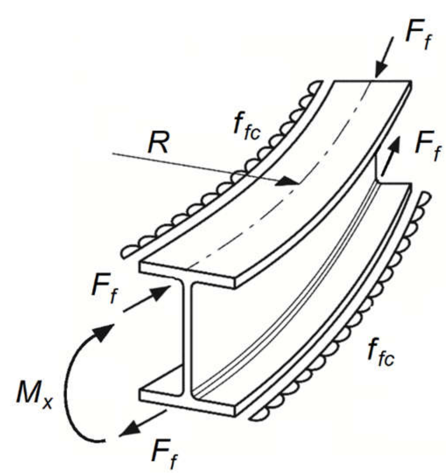
Introduction

Deflected Shape

- Vertical translation
- Torsional rotation



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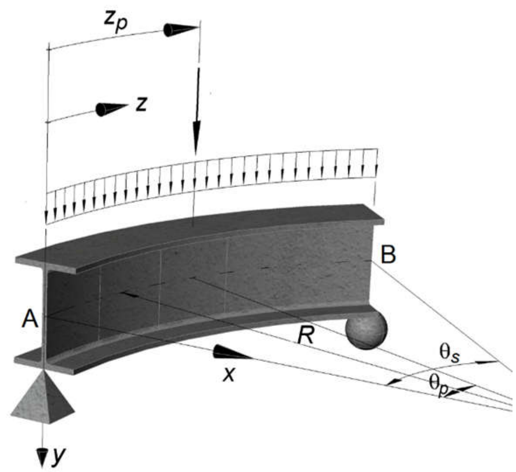
Introduction

Typical Behavior

- Dependent on θ_s
  - $\theta_s < 1°$: Flexure (F)
  - $1° \leq \theta_s \leq 20°$: F + T
  - $20° < \theta_s$: Torsion (T)



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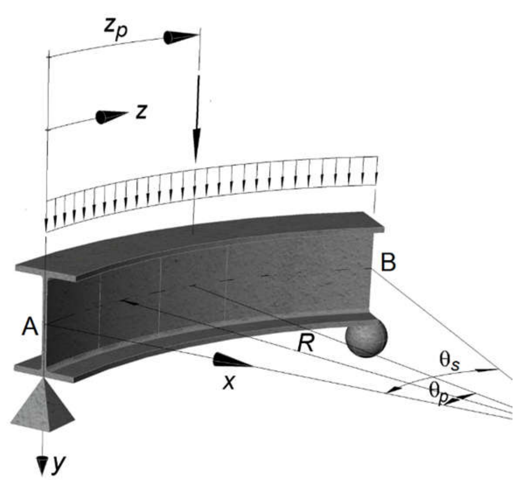
Introduction

Typical Limit States

- Excessive deformation
- Yielding



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Horizontally-Curved Members

Structural Analysis

Structural Analysis

Analysis Methods

- *M*/*R* method
- Finite element models
- Other methods (Section 7.3 of Design Guide 33)
  - – Theoretical Equations
  - – Eccentric load method

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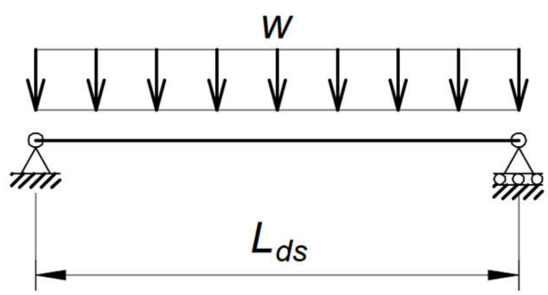
Design of Horizontally-Curved Members

Structural Analysis

M/*R* Method

M/*R* Method

- Curved beam is modeled as straight member



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M/*R* Method

- Developed lengths
  - Span length: L_{ds}
  - Length between torsional restraints: L_{db}

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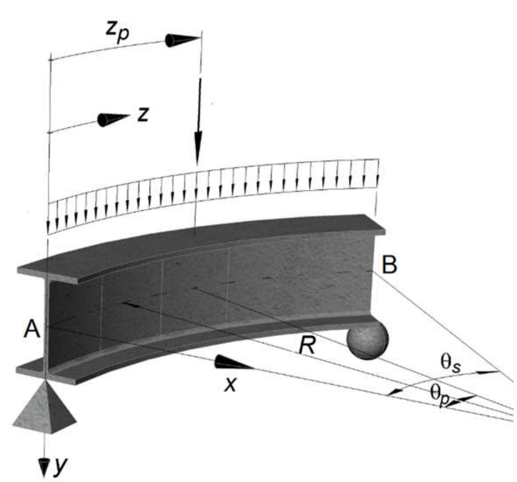
M/*R* Method

- Developed span length, $L_{ds} = R\theta_s$



θ_s = span angle, rad

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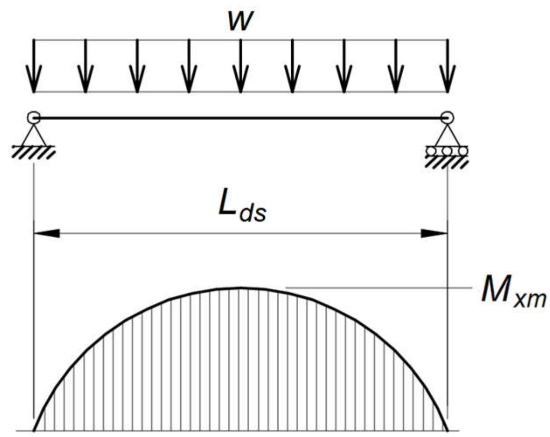
M/*R* Method

- Flexural (M_x) and shear (V) loads are calculated using L_{ds} with the flexural support conditions



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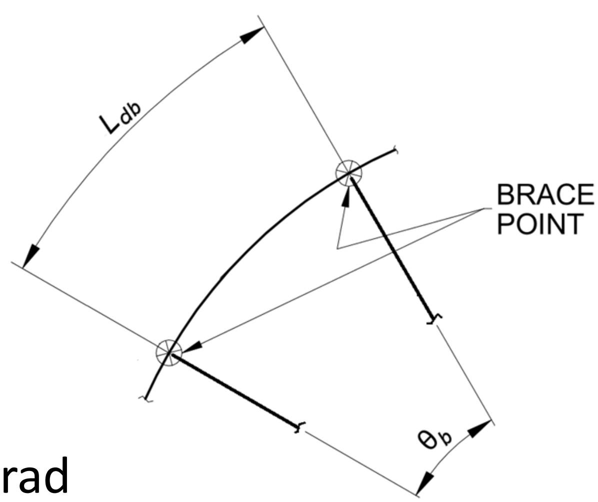
M/*R* Method

- Developed length between torsional restraints, $L_{db} = R\theta_b$



θ_b = angle between braces, rad

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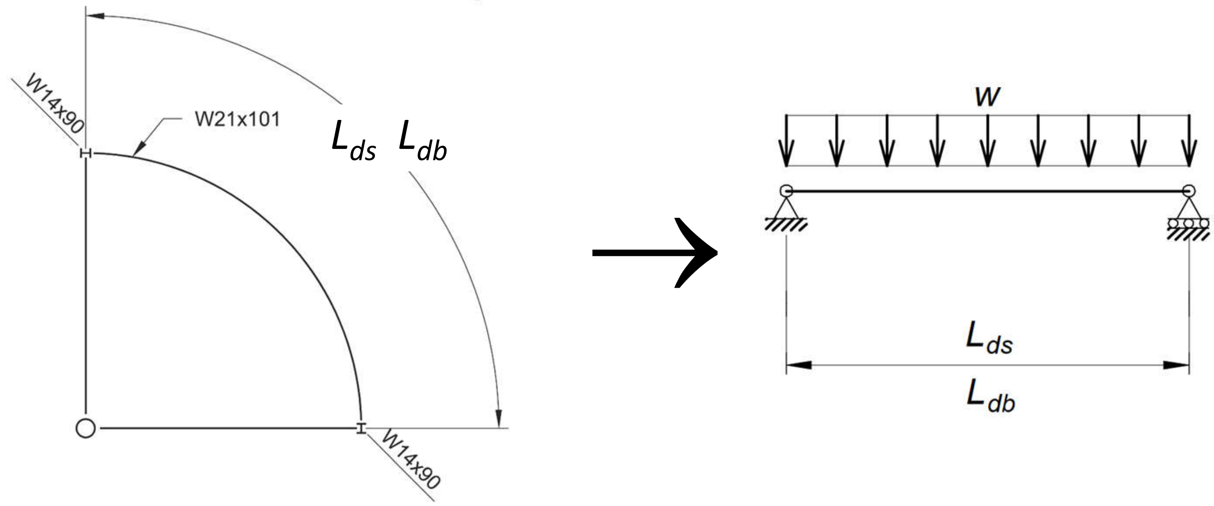
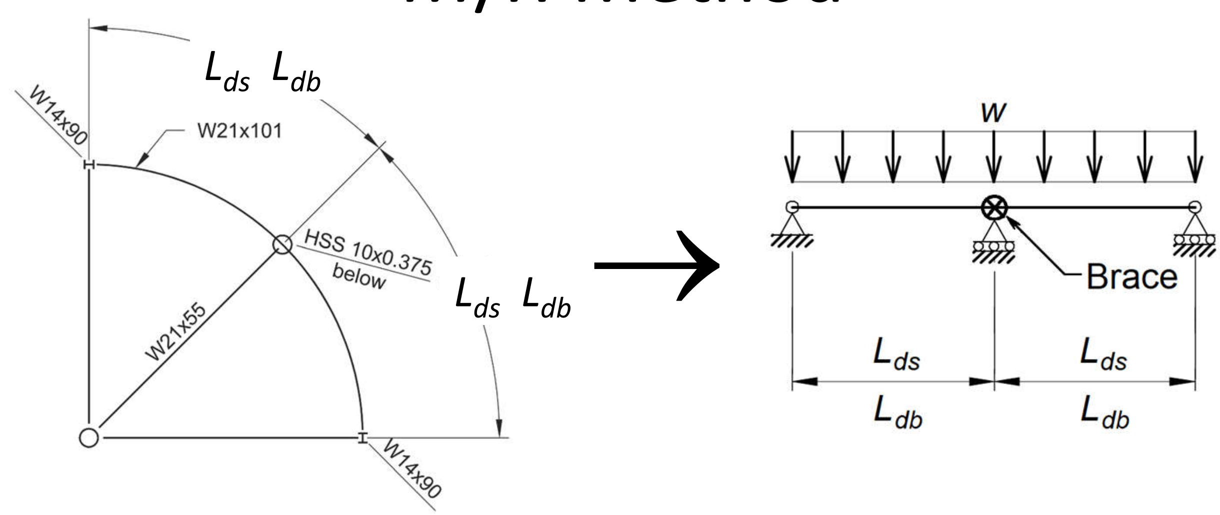
M/*R* Method



M/*R* Method



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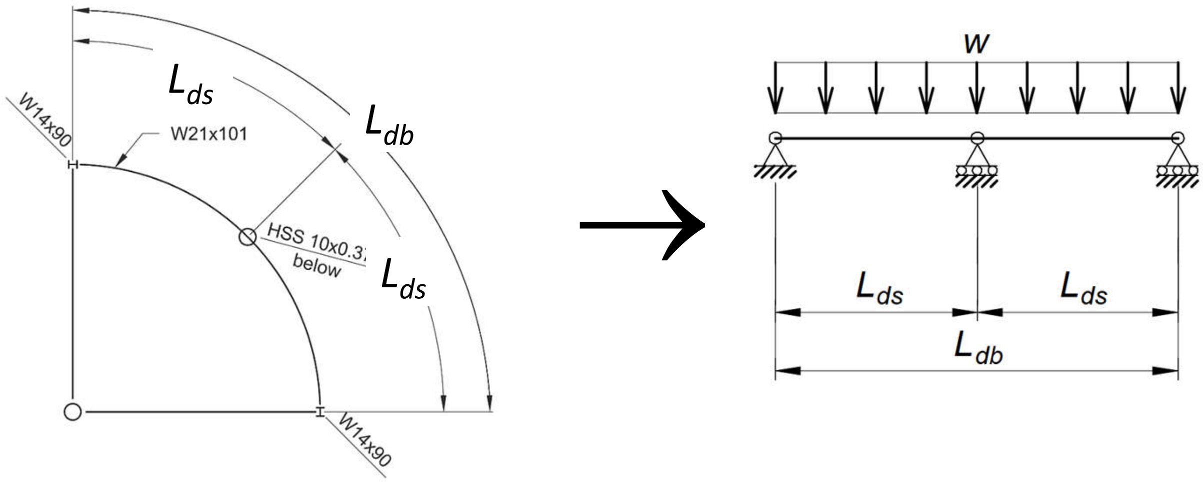

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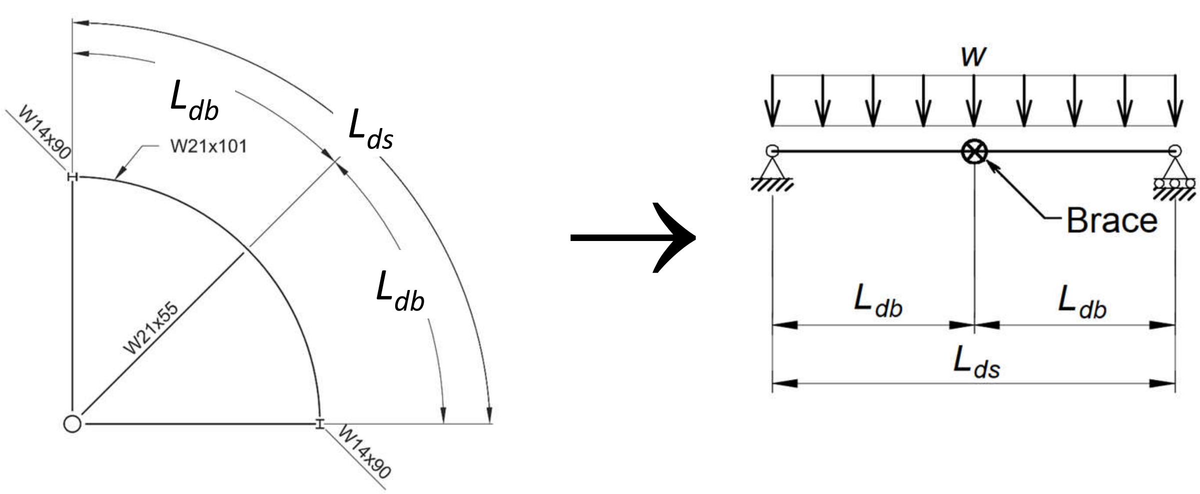
M/*R* Method



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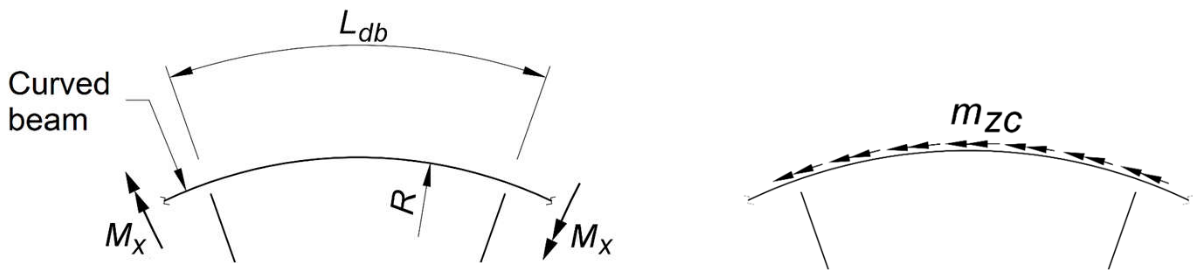
M/*R* Method

- Torsional moment per unit length: $m_{zc} = \dfrac{M_x}{R}$



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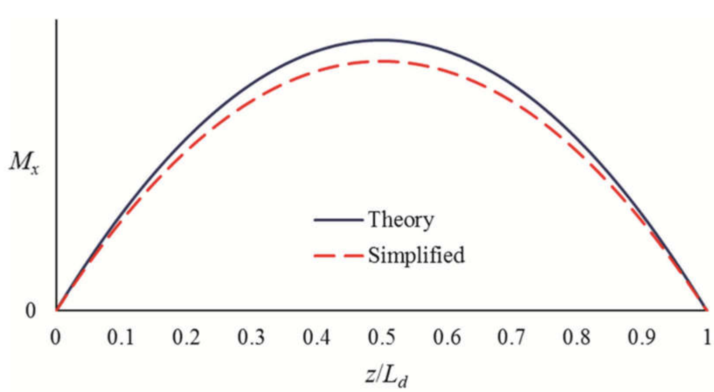
M/*R* Method

Moment Diagram (Uniform Load)



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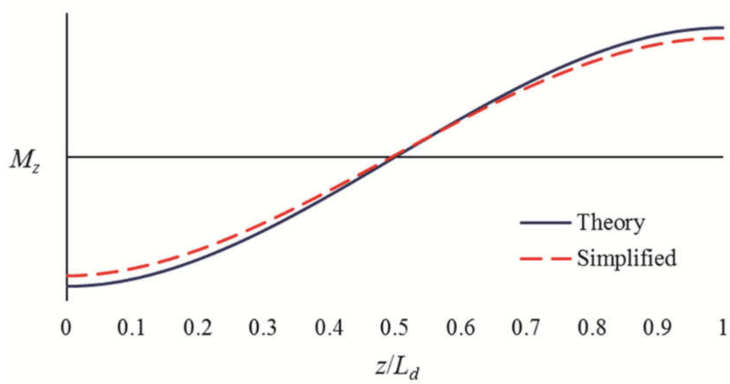
M/*R* Method

Torsion Diagram (Uniform Load)



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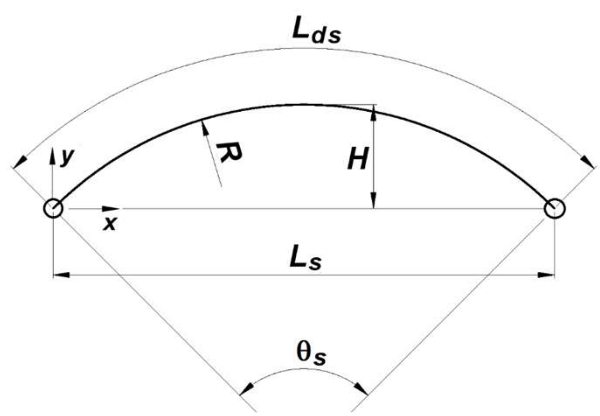
M/*R* Method

Corrected Moments

- Flexure: $M_{xc} = CM_x$
- Torsion: $M_{zc} = CM_z$

$$C = 1 - \frac{\theta_s}{30} + \frac{\theta_s^2}{6.2}$$



θ_s = span angle, rad

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Horizontally-Curved Members

Structural Analysis

Finite Element Models

Finite Element Models

- Curved members are usually modeled by segmenting a series of straight elements

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Finite Element Models

- 2-D Models: cross section is a single beam element
- 3-D Models: cross section is comprised of multiple elements (beam, plate, solid)

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Finite Element Models

2-D Models

- Basic beam finite element formulation
  - Used in most commercial finite element programs
  - Only St Venant stiffness
  - No warping stiffness
  - Over-estimates the rotation of open sections

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Finite Element Models

2-D Models

- The additional stiffness from warping can be approximated with an equivalent torsion constant, J_e

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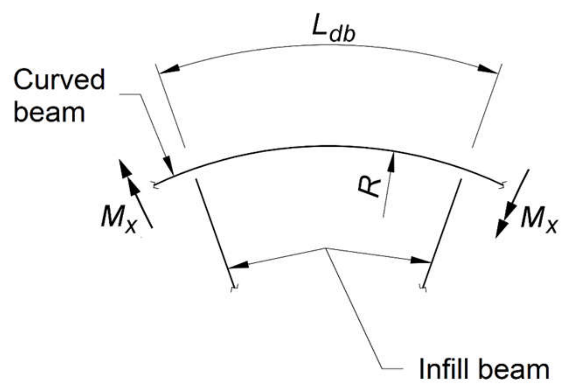
Finite Element Models

2-D Models

- Warping fixed at both ends of the span



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Finite Element Models

2-D Models

$$J_e = \frac{J}{1 - \frac{\sinh\gamma}{\gamma} + \frac{(\cosh\gamma - 1)^2}{\gamma\sinh\gamma}}$$

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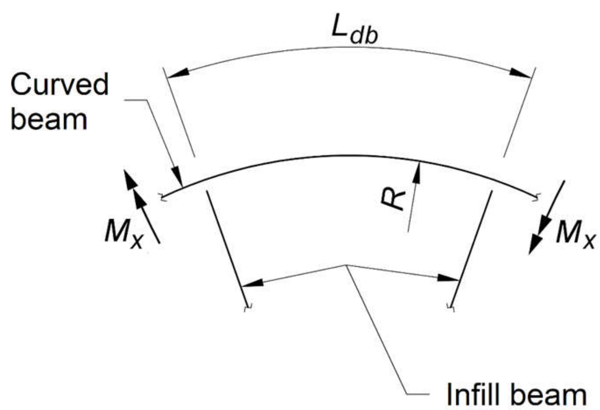
Finite Element Models

2-D Models

$$\gamma = L_{db}\sqrt{\frac{GJ}{EC_w}}$$



C_w = warping constant

J = torsional constant

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Finite Element Models

3-D Models

- Cross section is comprised of multiple elements

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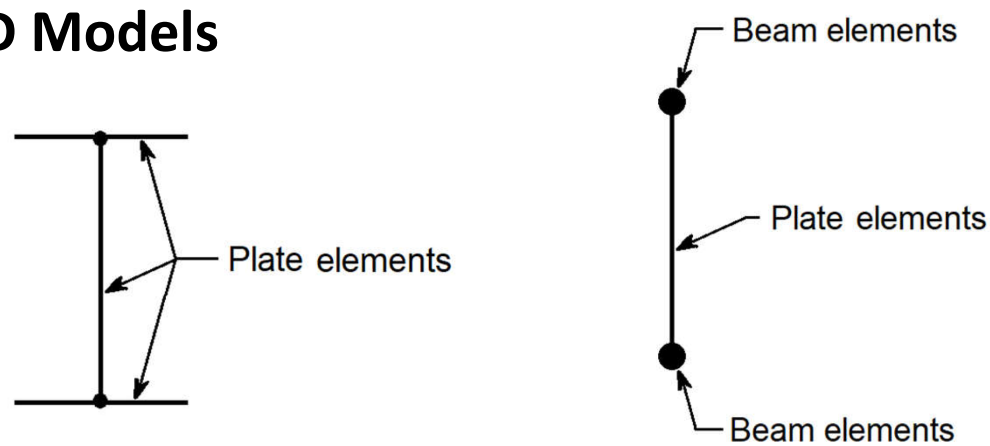
Finite Element Models

3-D Models



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Horizontally-Curved Members

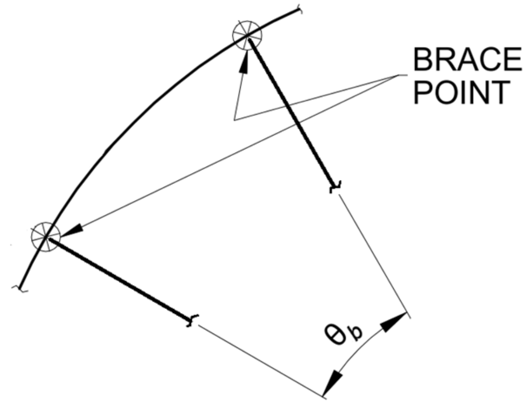
Flexural Strength

Flexural Strength

- Design as a straight beam
- AISC *Specification* Chapter F
- Unbraced length, $L_b \rightarrow L_{db}$
- Lateral-torsional buckling modification factor, $C_b \rightarrow C_{bo}$

Flexural Strength

$$C_{bo} = C_{bs}\left[1-\left(\frac{\theta_b}{\pi}\right)^2\right]^2$$



θ_b = angle between braces, rad

C_{bs} = C_b for an equivalent straight member

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Horizontally-Curved Members

Torsional Strength

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Torsional Strength

- Design as a straight beam
- Properly account for end conditions
  - Warping fixed
  - Warping free

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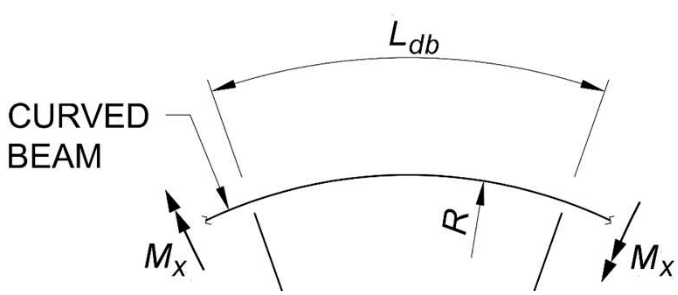
Torsional Strength

- Member length = developed length between torsional restraints, L_{db}



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Torsional Strength

- Analysis Methods
  - Isolated flange method
  - Elastic method (Section 7.5.1 of Design Guide 33)

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Horizontally-Curved Members

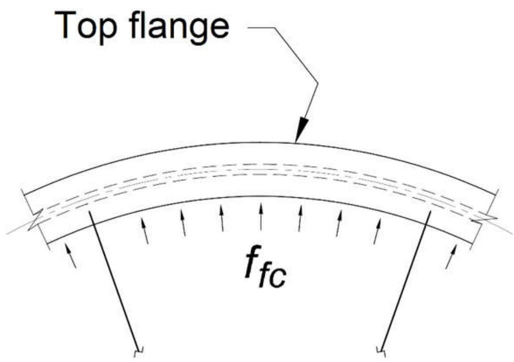
Torsional Strength

Isolated Flange Method

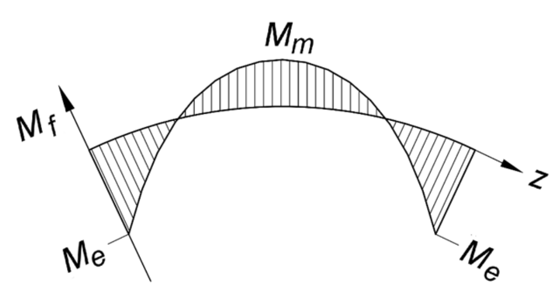
Isolated Flange Method

- I-shaped members
- Flanges are modeled as independent rectangular beams

Isolated Flange Method



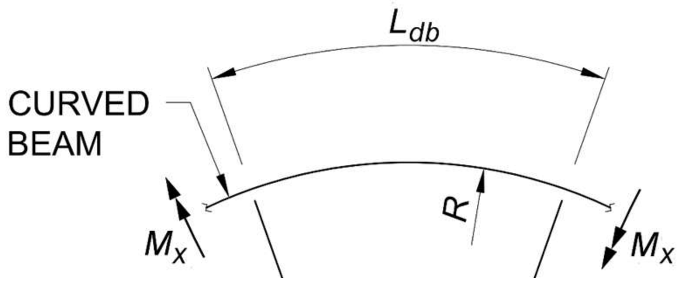
Radial Load



Moment Diagram

Isolated Flange Method

- Length of the isolated flange = developed (arc) length between torsional restraints, L_{db}



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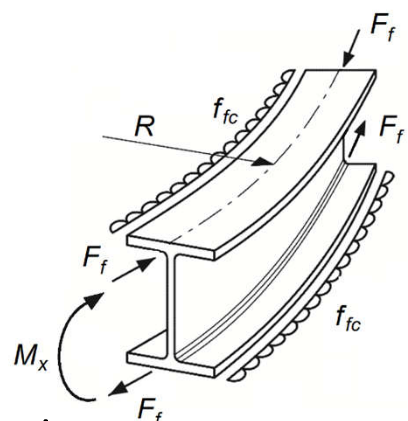
Isolated Flange Method

- Radial load, f_{fc}, is applied in the horizontal plane

$$f_{fc} = \frac{m_{zc}}{h_o} = \frac{M_x}{Rh_o}$$

h_o = distance between flange centroids



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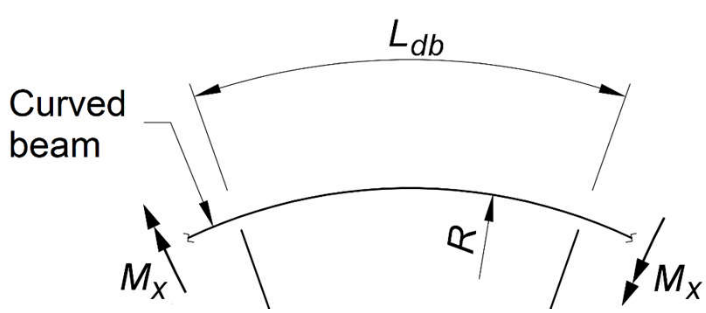
Isolated Flange Method

- Flexural boundary conditions of the isolated flange are based on the warping boundary conditions of the curved member

Isolated Flange Method

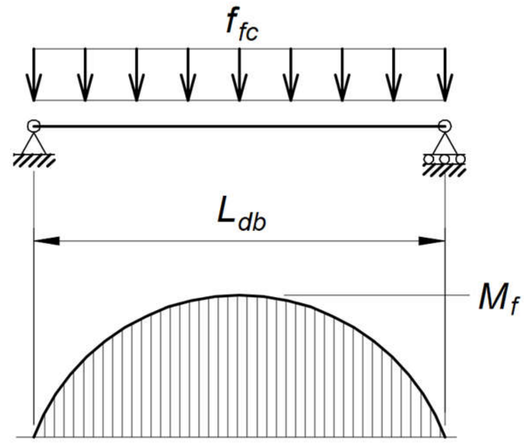
Equal End Moments



Isolated Flange Method

Equal End Moments

- Moment diagram for the isolated flange
- Warping free



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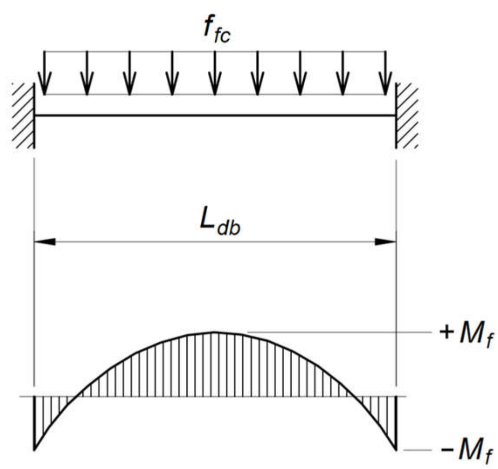
Isolated Flange Method

Equal End Moments

- Moment diagram for the isolated flange
- Warping fixed



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Isolated Flange Method

- Nominal flexural strength of the isolated flange: $M_{nw} = F_y Z_f$

$$Z_f = \frac{t_f b_f^2}{4}$$

b_f = flange width

t_f = flange thickness

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Horizontally-Curved Members

Combined Loads

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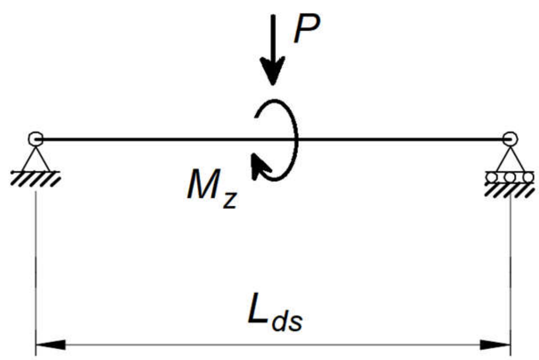
Combined Loads

Flexure-torsion Interaction



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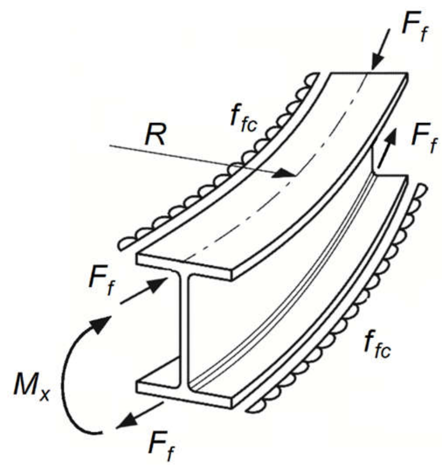
Horizontally-Curved Members

Combined Loads

Second-Order Effects

Second-Order Effects

- Increase torsional moments and torsional rotations
- Isolated compression flange is analogous to a beam-column



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Second-Order Effects

- Rigorous second-order analysis
- Amplified first-order analysis

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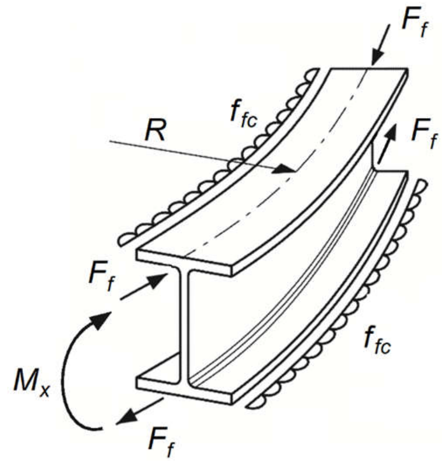
Second-Order Effects

Amplified First-Order Analysis

- Open sections subjected to torsion + strong-axis flexure



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Second-Order Effects

Amplified First-Order Analysis

- Second-order torsional rotation: $\theta_2 = B_o\theta_1$
- Second-order torsional moment: $M_{rz} = B_oM_z$

M_z = first-order torsional moment

θ_1 = first-order torsional rotation

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Second-Order Effects

Amplified First-Order Analysis

$$B_o = \frac{0.85}{1-\alpha\frac{M_{ro}}{M_{eo}}} \geq 1.0$$

M_{eo} = elastic lateral-torsional buckling moment

M_{ro} = required strong-axis flexural moment (M_{rx})

α = 1.00 (LRFD); 1.60 (ASD)

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Horizontally-Curved Members

Combined Loads

Member Strength

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Member Strength

I-Shaped Members

- Interaction method is based on the analysis method
  - Isolated flange method
  - Elastic method (Section 7.6.2 of Design Guide 33)
  - FE model (Section 7.6.2 of Design Guide 33)

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Member Strength

I-Shaped Members: Isolated Flange Method

$$\frac{M_{ro}}{M_{co}} + \frac{8}{9}\frac{M_{rw}}{M_{cw}} \leq 1.0$$

M_{co} M_{cw} = available flexural strengths: member, flange

M_{ro} M_{rw} = required flexural strength: member, flange

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Horizontally-Curved Members

Serviceability

Serviceability

- Large deformations at ultimate strength
- Member sizes are usually controlled by serviceability

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Serviceability

- Torsional rotation limits
  - Not in building codes
  - Some judgment may be required to define appropriate limits
  - Maintain geometry of the structure
  - Prevent damage to nonstructural elements

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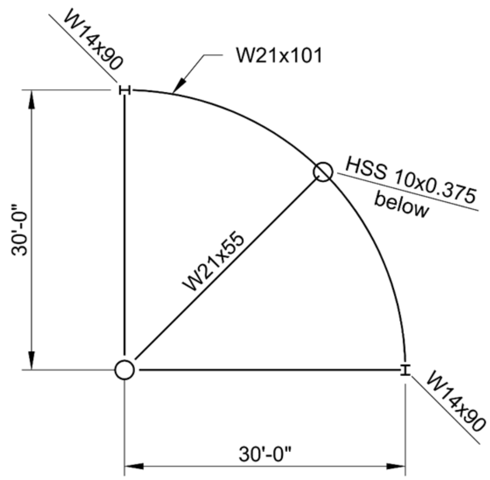
Horizontally-Curved Members

Design Example

Design Example



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Horizontally-Curved Members

Design Example

Problem Statement

Problem Statement

- Verify that the horizontally-curved beam is adequate for the imposed loading
- Use LRFD

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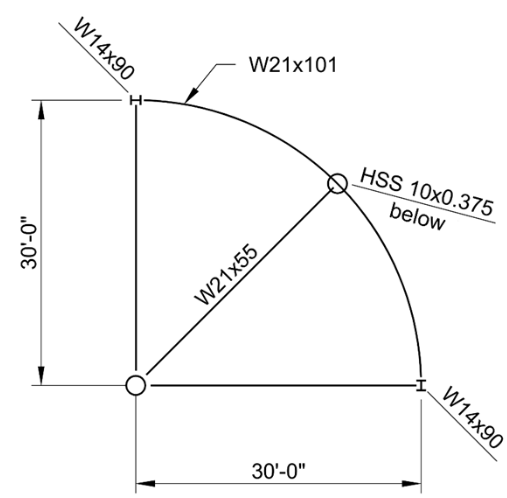
Problem Statement

- Curved member
  - W21×101
  - ASTM A992
  - Bent the easy way
  - Circular curve



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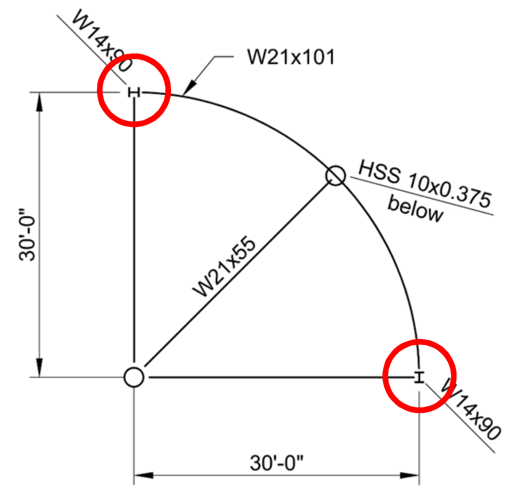
Problem Statement

- End connections
  - Torsional rotation is restrained
  - No warping restraint
  - No flexural restraint



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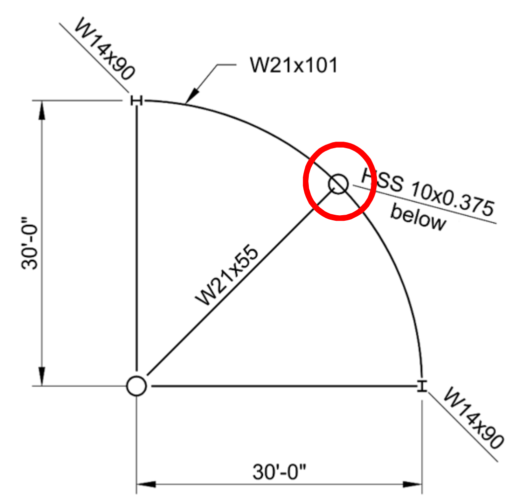
Problem Statement

- Connections @ the HSS10
  - Continuous for flexure
  - Torsion is restrained by the W21 beam
  - Continuous for warping



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Problem Statement

- The factored uniformly distributed load along the member circumference including the beam self weight is w_u = 0.750 kip/ft

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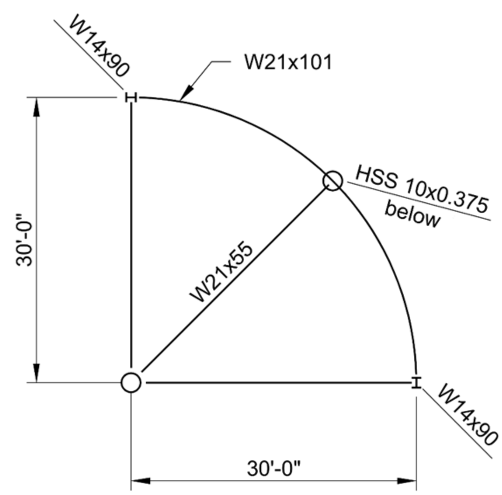
Problem Statement

- Assume the critical condition is for patch loading (one span loaded)



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Horizontally-Curved Members

Design Example

Properties

Properties

- Material properties of ASTM A992 (AISC *Manual* Table 2-4)

  F_y = 50 ksi

  F_u = 65 ksi

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Properties

- Dimensions of W21×101 (AISC *Manual* Table 1-1)

d = 21.4 in. t_w = 0.500 in.
b_f = 12.3 in. t_f = 0.800 in.
h_o = 20.6 in.

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Properties

- Section properties of W21×101 (AISC *Manual* Table 1-1)

I_x = 2,420 in.^{4} S_x = 227 in.^{3}
Z_x = 253 in.^{3}

J = 5.21 in.^{4} C_w= 26,200 in.^{6}
r_{ts} = 3.35 in.

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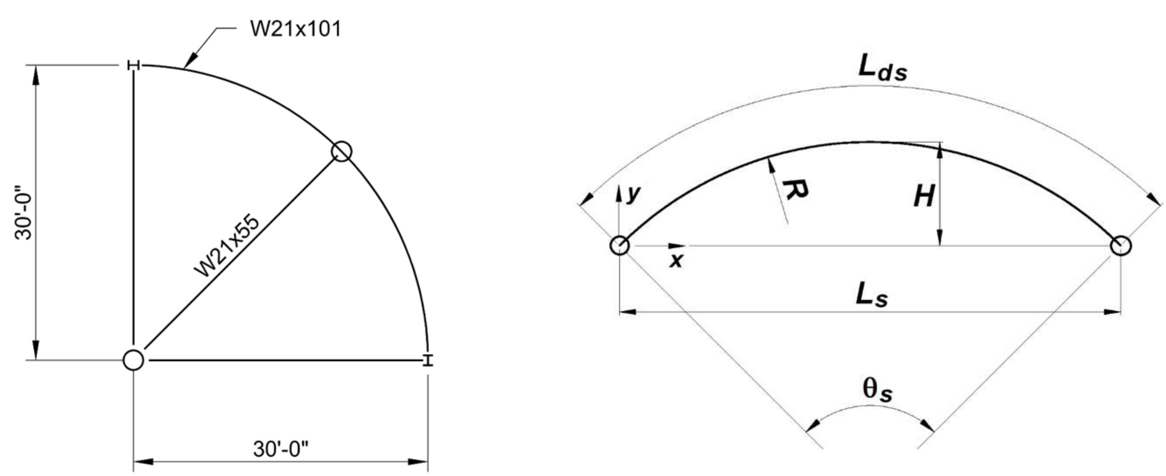
Horizontally-Curved Members

Design Example

Beam Geometry

Beam Geometry



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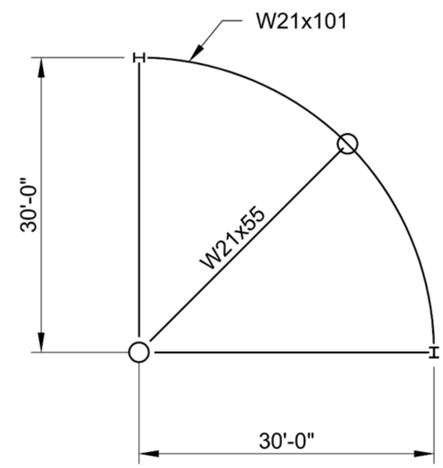
Beam Geometry

- Centroidal radius

$$R = (30\text{ ft})(12\text{ in./ft})$$
$$= 360\text{ in.}$$



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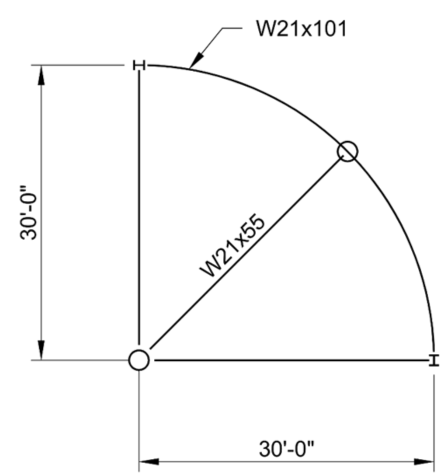
Beam Geometry

- Span angle

$$\theta_s = (45^\circ)\left(\frac{\pi\text{ rad}}{180^\circ}\right)$$
$$= \pi / 4\text{ rad}$$



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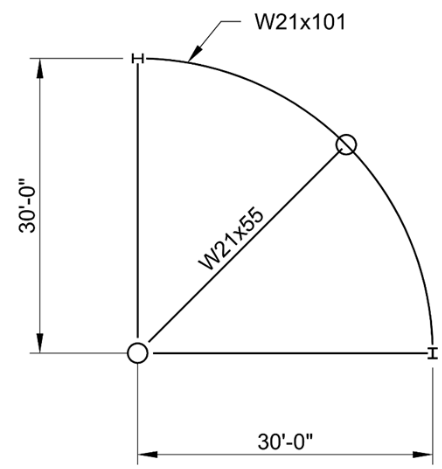
Beam Geometry

- Developed span length

$$L_{ds} = (30\text{ ft})(\pi/4\text{ rad})(12\text{ in./ft})$$
$$= 283\text{ in.}$$



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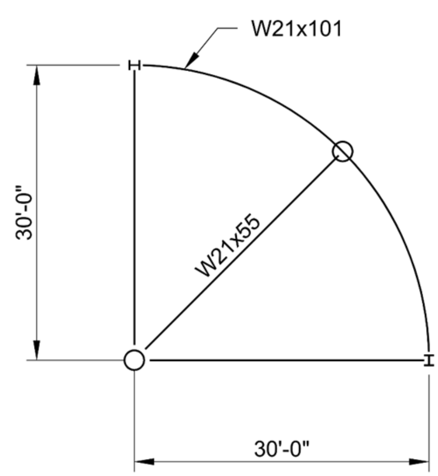
Beam Geometry

- Angle between torsional restraints

$$\theta_b = (45^\circ)\left(\frac{\pi\text{ rad}}{180^\circ}\right)$$
$$= \pi/4\text{ rad}$$



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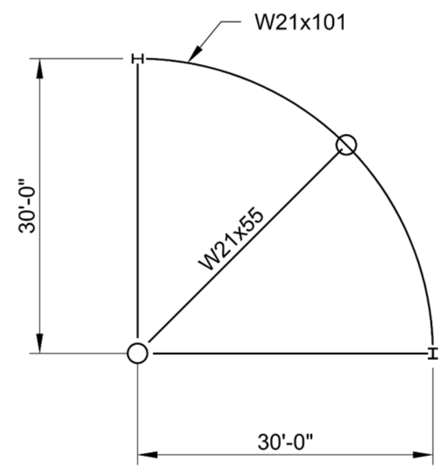
Beam Geometry

- Developed length between braces

$$L_{db} = (30 \text{ ft})(\pi / 4 \text{ rad})$$
$$= 23.6 \text{ ft}$$
$$= 283 \text{ in.}$$



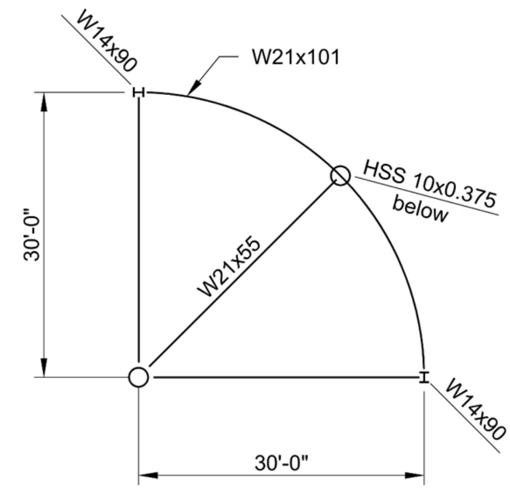
Horizontally-Curved Members

Design Example

Structural Analysis

Structural Analysis

- At the ends
  - No warping restraint
  - No flexural restraint
- At the HSS10
  - Continuous for flexure
  - Continuous for warping



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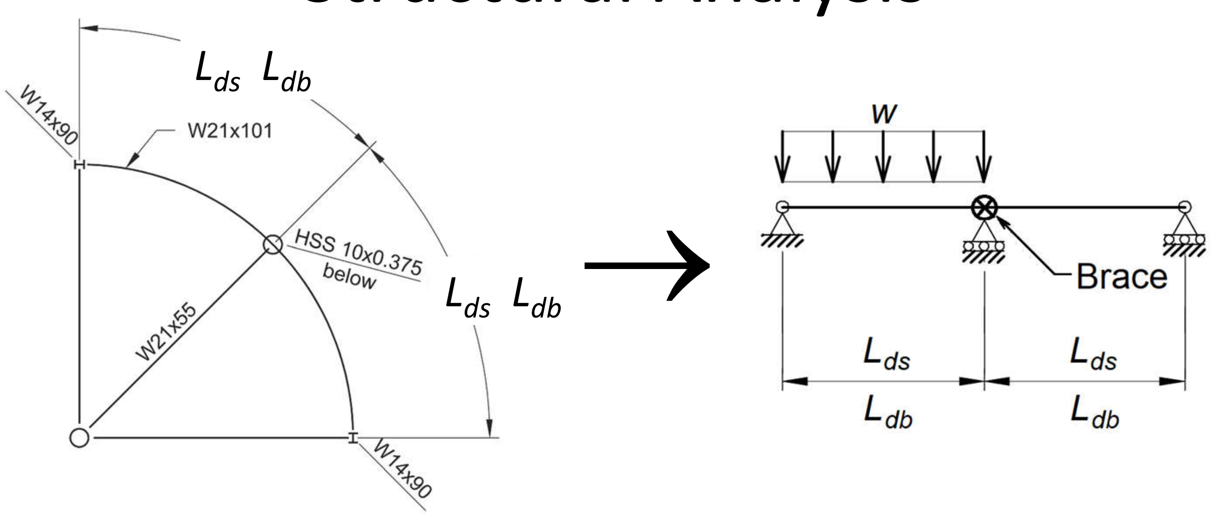
Structural Analysis



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Horizontally-Curved Members

Design Example

Structural Analysis

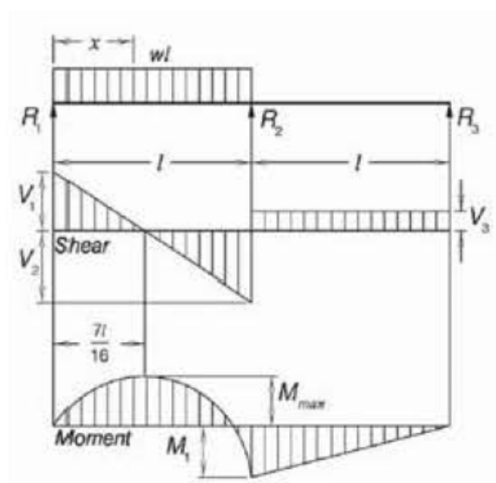
Flexural Loads

Structural Analysis

- AISC *Manual* Table 3-23 Case 29
- Max. Moment

  M_{ux} = +480 kip-in.
- Max. Shear

  V_u = 9.95 kips



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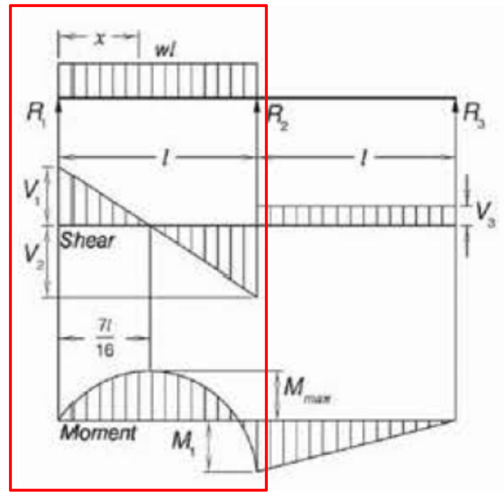
Structural Analysis

- Isolate the loaded span



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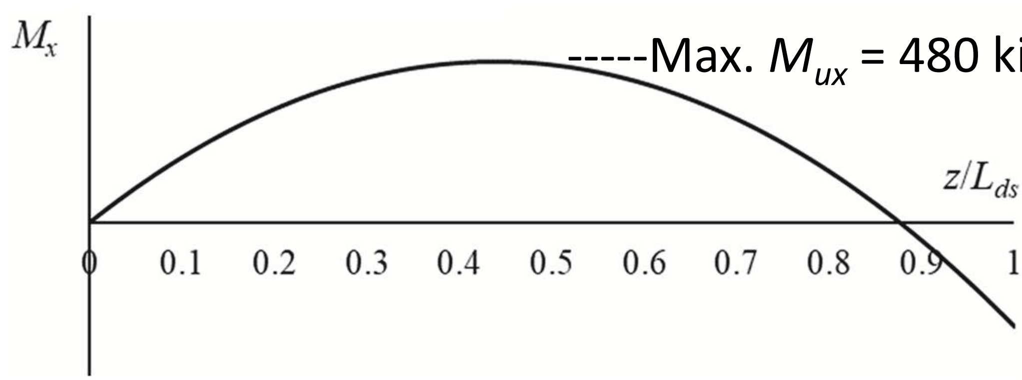
Structural Analysis

- Flexural moment at the loaded span



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Horizontally-Curved Members

Design Example

Structural Analysis

Torsional Loads

Structural Analysis

- M/R method
- Torsional moment per unit length:

$$m_{zc} = \frac{M_x}{R}$$

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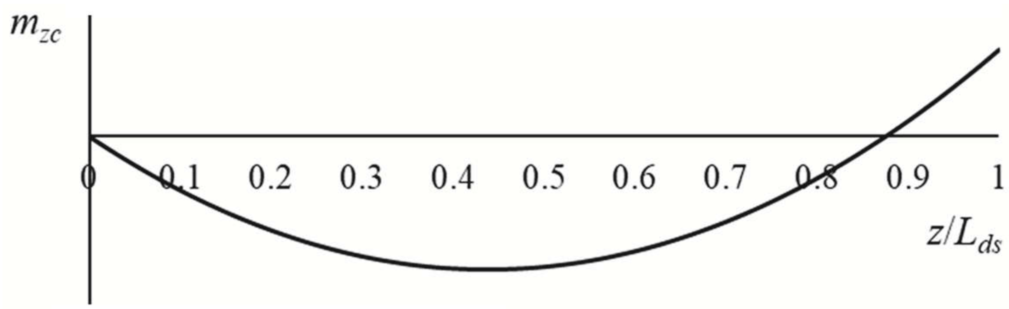
Structural Analysis

- Distributed torsion



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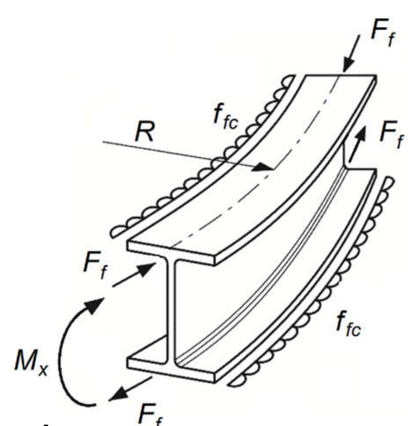
Structural Analysis

- Isolated Flange method

$$f_{fc} = \frac{m_{zc}}{h_o}$$



h_o = distance between flange centroids

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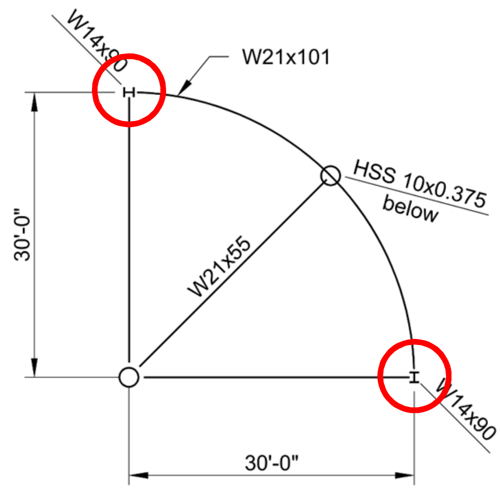
Structural Analysis

- Warping is not restrained at the end connections → use pinned ends at the isolated flange



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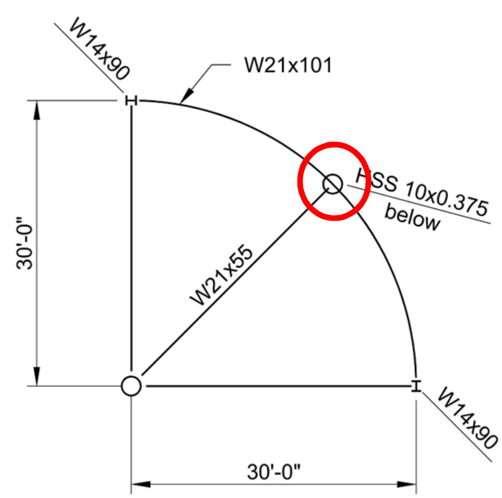
Structural Analysis

- Warping is continuous at the HSS10 → the isolated flange is continuous for flexure



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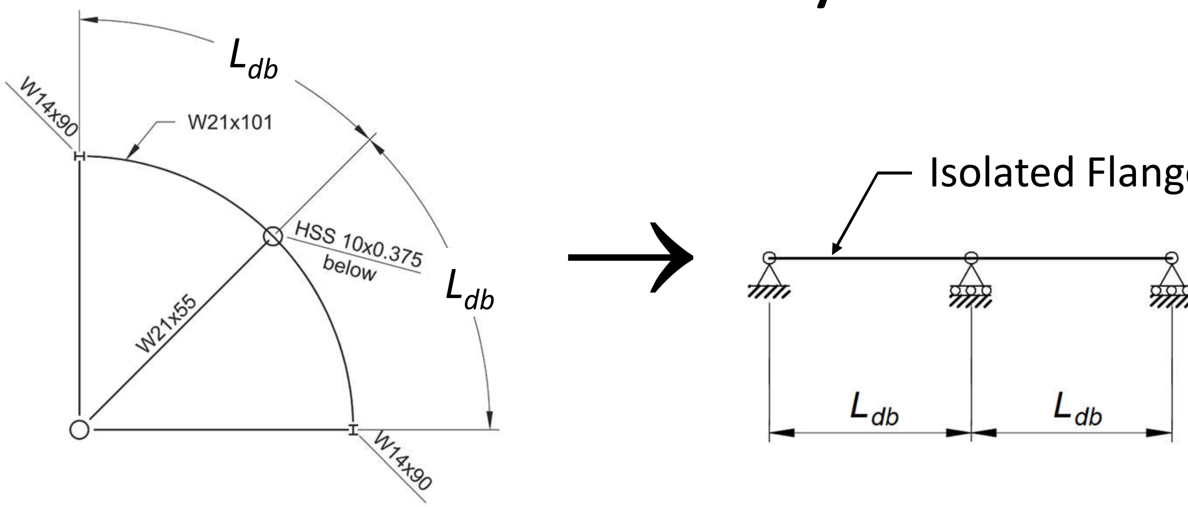
Structural Analysis



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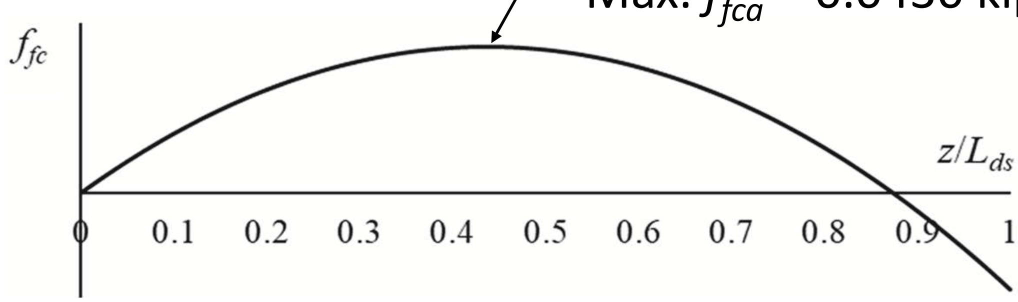
Structural Analysis

- Distributed flange force

Max. f_{fcu} = 0.0645 kip/in.
Max. f_{fca} = 0.0430 kip/in.



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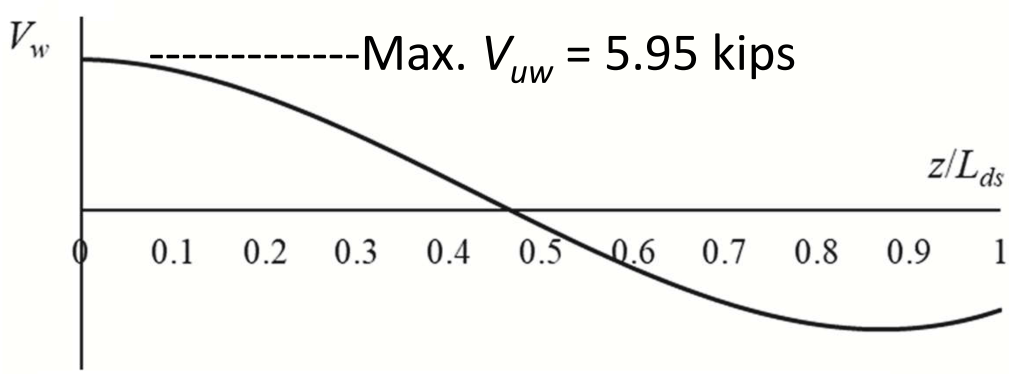
Structural Analysis

- Horizontal shear



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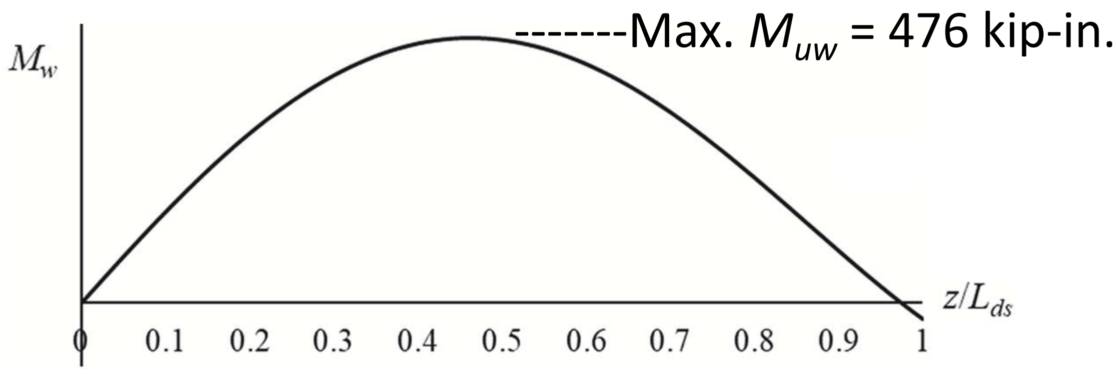
Structural Analysis

- Warping moment



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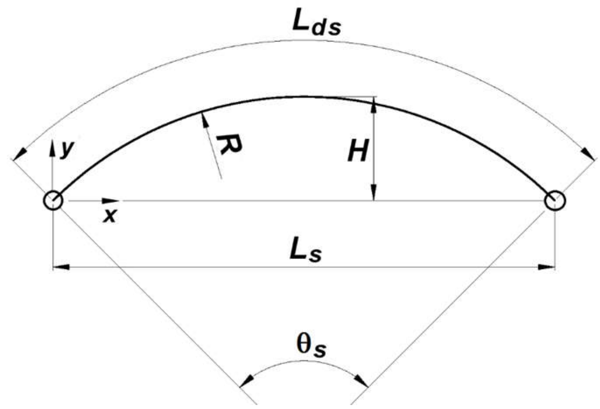
Structural Analysis

Corrected Moments

$$C = 1 - \frac{\theta_s}{30} + \frac{\theta_s^2}{6.2}$$

$$= 1 - \frac{\pi/4}{30} + \frac{(\pi/4)^2}{6.2}$$

$$= 1.07$$



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Structural Analysis

Corrected Moments

- Flexure:

$$M_{uxc} = CM_{ux} = (1.07)(480 \text{ kip-in.}) = 514 \text{ kip-in.}$$

- Warping:

$$M_{uwc} = CM_{uw} = (1.07)(476 \text{ kip-in.}) = 509 \text{ kip-in.}$$

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Horizontally-Curved Members

Design Example

Shear Strength

Shear Strength

- AISC *Manual* Table 6-2

$\phi_v V_n = 321 \text{ kips} > V_u = 9.95 \text{ kips}$ **o.k.**

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Horizontally-Curved Members

Design Example

Local Buckling

Local Buckling

- Calculations are the same as for a straight member
- Flexure: $\lambda_f < \lambda_{pf}$ and $\lambda_w < \lambda_{pw} \rightarrow$ the W21×101 is compact

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Horizontally-Curved Members

Design Example

Flexural Strength

Flexural Strength

- Design as a straight beam
- AISC *Specification* Chapter F

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Flexural Strength

Use C_{bs} = 1.0

$$C_{bo} = C_{bs}\left[1-\left(\frac{\theta_b}{\pi}\right)^2\right]^2$$

$$=(1.0)\left[1-\left(\frac{\pi/4}{\pi}\right)^2\right]^2 = 0.879$$

Flexural Strength

- AISC *Specification* Section F2
  - $L_b = L_{db}$ = 23.6 ft
  - $C_b = C_{bo}$ = 0.879
- AISC *Manual* Table 3-6
  - L_p = 10.2 ft
  - L_r = 30.1 ft

Flexural Strength

$$M_p = F_y Z_x$$
$$= (50 \text{ ksi})(253 \text{ in.}^3)$$
$$= 12{,}700 \text{ kip-in.}$$

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Flexural Strength

- $L_p < L_b < L_r \rightarrow$ Use AISC *Spec.* Eq. F2-2

$$M_n = C_b \left[M_p - (M_p - 0.7F_y S_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$
$$= 8{,}350 \text{ kip-in.} < 12{,}700 \text{ kip-in.}$$

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Flexural Strength

- The available strength is

$$\phi_b M_n = 0.90(8{,}350 \text{ kip-in.}) = 7{,}520 \text{ kip-in.}$$

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Horizontally-Curved Members

Design Example

Second-Order Effects

structural STEEL

Second-Order Effects

- Amplified First-Order Analysis
- Second-order warping moment: $M_{uw} = B_o M_{uwc}$

$$B_o = \frac{0.85}{1 - \alpha \dfrac{M_{uxc}}{M_{eo}}} \geq 1.0$$

M_{eo} = elastic lateral-torsional buckling moment

α = 1.00 (LRFD)

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Second-Order Effects

$$F_{cr} = \frac{C_{bo}\pi^2 E}{\left(\dfrac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

$$= 44.9 \text{ ksi}$$

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Second-Order Effects

$$M_{eo} = F_{cr} S_x$$
$$= (44.9 \text{ ksi})(227 \text{ in.}^3)$$
$$= 10{,}200 \text{ kip-in.}$$

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Second-Order Effects

$$B_o = \frac{0.85}{1 - (1.0)\left(\dfrac{514 \text{ kip-in.}}{10{,}200 \text{ kip-in.}}\right)} \geq 1.0$$
$$= 0.895 < 1.0$$

$\rightarrow B_o = 1.0$

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Second-Order Effects

$$M_{uw} = (1.0)(509 \text{ kip-in.})$$
$$= 509 \text{ kip-in.}$$

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Horizontally-Curved Members

Design Example

Warping Strength

structural STEEL

Warping Strength

- The isolated flange plastic modulus is

$$Z_f = \frac{t_f b_f^2}{4}$$
$$= \frac{(0.800 \text{ in.})(12.3 \text{ in.})^2}{4}$$
$$= 30.3 \text{ in.}^3$$

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Warping Strength

- The nominal flexural strength of the isolated flange is

$$M_{nw} = F_y Z_f$$
$$= (50 \text{ ksi})(30.3 \text{ in.}^3)$$
$$= 1{,}520 \text{ kip-in.}$$

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Warping Strength

- The available flexural strength of the isolated flange is

$$M_{cw} = \phi_b M_{nw}$$
$$= 0.90(1{,}520 \text{ kip-in.})$$
$$= 1{,}370 \text{ kip-in.}$$

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Horizontally-Curved Members

Design Example

Combined Loading

Combined Loading

- Flexural moment + flange warping moment

$$\frac{M_{uxc}}{\phi_b M_n} + \frac{8}{9}\frac{M_{uw}}{\phi M_{nw}} \le 1.0$$

$$\frac{514 \text{ kip-in.}}{7{,}520 \text{ kip-in.}} + \left(\frac{8}{9}\right)\left(\frac{509 \text{ kip-in.}}{1{,}370 \text{ kip-in.}}\right) \le 1.0$$

$$0.399 < 1.00 \quad \textbf{o.k.}$$

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Horizontally-Curved Members

Design Example

Serviceability

Serviceability

- The torsional rotation can be estimated using the horizontal deflection of the isolated flange
- Maximum second-order distributed flange force under service loads: f_{fc} = 0.0430 kip/in.

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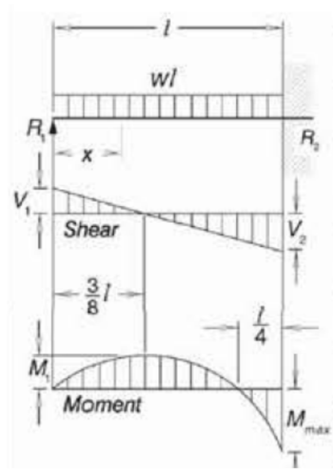
Serviceability

- AISC *Manual* Table 3-23 Case 12

$$\Delta_{max} = \frac{f_{fc} L_{ds}^4}{185 E I_f}$$



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Serviceability

- The isolated flange moment of inertia is

$$I_f = \frac{t_f b_f^3}{12} = \frac{(0.800\text{ in.})(12.3\text{ in.})^3}{12}$$

$$= 124\text{ in.}^4$$

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Serviceability

$$\Delta_{max} = \frac{f_{fc} L_{ds}^4}{185 E I_f} = \frac{(0.0430\text{ kip/in.})(283\text{ in.})^4}{(185)(29{,}000\text{ ksi})(124\text{ in.}^4)}$$

$$= 0.415\text{ in.}$$

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Serviceability

- The 1st-order torsional rotation is

$$\theta_1 = \tan^{-1}\left(\frac{2\Delta_{max}}{h_o}\right) = \tan^{-1}\left[\frac{(2)(0.415\text{ in.})}{20.6\text{ in.}}\right]$$
$$= 2.31^\circ$$

h_o = distance between flange centroids

Serviceability

- The 2nd-order torsional rotation is

$$\theta_2 = B_o\theta_1$$
$$= (1.00)(2.31^\circ)$$
$$= 2.31^\circ$$

Combined Loading

Design Summary

- The W21×101 strength is adequate
- See Design Guide Example 8.2 for further calculations
  - Connections

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Conclusions

Horizontally-Curved Members

- Design as an equivalent straight member
- Subjected to both flexural and torsional loads
- Flexural strength is calculated with C_{bo}
- Member sizes are usually controlled by serviceability

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Question time

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CEU / PDH Certificates

- You will receive an email on how to report attendance from: registration@aisc.org.
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!

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- Reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.

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