AISC Live Webinars

Thank you for joining our live webinar today.
We will begin shortly. Please standby.

Thank you.

Need Help?
Call ReadyTalk Support: 800.843.9166

Design of Curved Members
Part 2: Design of Horizontally-Curved Members
December 12, 2019

Smarter. Stronger. Steel.

AISC Live Webinars

Today's live webinar will begin shortly. Please stand by.

As a reminder, all lines have been muted. Please type any questions or comments through the chat feature on the left portion of your screen.

Today's audio will be broadcast through the internet.
Alternatively, to hear the audio through the phone, dial:

(866)-519-2796
Passcode: 418875

Smarter. Stronger. Steel.

AISC Live Webinars

Audio Options

Today's audio will be broadcast through the internet.

Alternatively, to hear the audio through the phone, dial:

(866)-519-2796
Passcode: 418875

Smarter. Stronger. Steel.

AISC Live Webinars

AIA Credit

AISC is a Registered Provider with The American Institute of Architects Continuing Education Systems (AIA/CES). Credit(s) earned on completion of this program will be reported to AIA/CES for AIA members. Certificates of Completion for both AIA members and non-AIA members are available upon request.

This program is registered with AIA/CES for continuing professional education. As such, it does not include content that may be deemed or construed to be an approval or endorsement by the AIA of any material of construction or any method or manner of handling, using, distributing, or dealing in any material or product.

Questions related to specific materials, methods, and services will be addressed at the conclusion of this presentation.

Smarter. Stronger. Steel.

AISC Live Webinars

Copyright Materials

Smarter. Stronger. Steel.

AISC Live Webinars

Course Description

Design of Horizontally-Curved Members
December 12, 2019

While curved members can be both structurally efficient and aesthetically interesting, their behavior can be much different than their straight counterparts. This two-part webinar series, presented by the author of AISC Design Guide 33 – Curved Member Design, provides guidance on how to use the AISC Specification to design curved members. This 'equivalent straight member' approach allows the use of existing commercial software for curved member design by modifying effective length factors and lateral-torsional buckling modification factors to account for the curvature. This course will provide a brief overview of the design guide and detailed design information on both vertically- and horizontally-curved members.

This session will examine the behavior and design of horizontally-curved members. The design of such members for flexural, torsional, and combined loading will be explained, as will the important issue of serviceability. The concepts will be illustrated with a design example.

Smarter. Stronger. Steel.

AISC Live Webinars

Learning Objectives

- Identify three methods for analyzing horizontally-curved members.
- Describe how to use the Isolated Flange Method for assessing the torsion on horizontally-curved members.
- Describe how to account for second-order effects when designing horizontally-curved members.
- List serviceability considerations for horizontally-curved members.

Smarter. Stronger. Steel.

Design of Curved Members

Part 2: Design of Horizontally-Curved Members
December 12, 2019

Bo Dowswell, PE, PhD
ARC International, LLC
Birmingham, AL

Smarter. Stronger. Steel.

Horizontally-Curved Members

Session Description

Session Description

- Introduction
- Structural analysis
- Flexural strength
- Torsional strength

(continued)

Photograph courtesy of the AISC Bender/Roller Committee

10

Session Description

- Combined loads
- Serviceability
- Design example

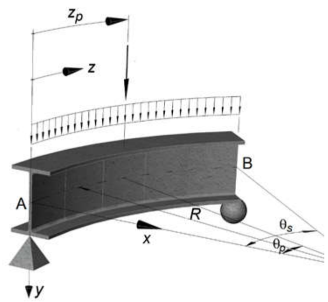
11

Horizontally-Curved Members

Introduction

Introduction

- Curved Members ≠ Straight Members
- Curved beam → Flexure + Torsion

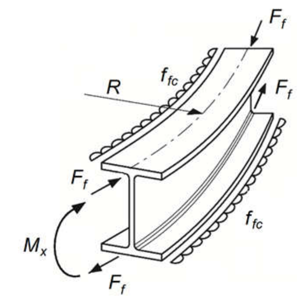


13

Introduction

Deflected Shape

- Vertical translation
- Torsional rotation

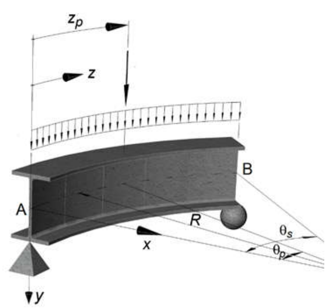


14

Introduction

Typical Behavior

- Dependent on θ_s
  - $\theta_s < 1°$: Flexure (F)
  - $1° \leq \theta_s \leq 20°$: F + T
  - $20° < \theta_s$: Torsion (T)

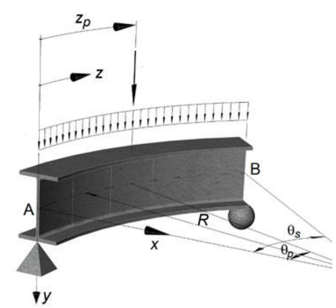


15

Introduction

Typical Limit States

- Excessive deformation
- Yielding



16

Horizontally-Curved Members

Structural Analysis

Structural Analysis

Analysis Methods

- *M*/*R* method
- Finite element models
- Other methods (Section 7.3 of Design Guide 33)
  - Theoretical Equations
  - Eccentric load method

18

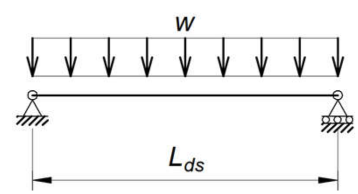
Design of Horizontally-Curved Members

Structural Analysis

M/*R* Method

M/*R* Method

- Curved beam is modeled as straight member



20

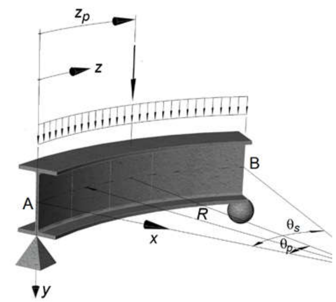
M/*R* Method

- Developed lengths
  - Span length: L_{ds}
  - Length between torsional restraints: L_{db}

21

M/*R* Method

- Developed span length, $L_{ds} = R\theta_s$

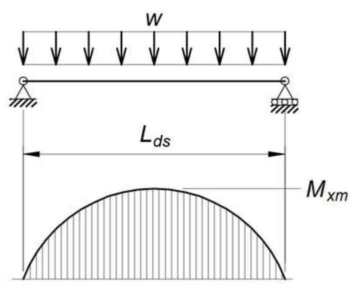


θ_s = span angle, rad

22

M/*R* Method

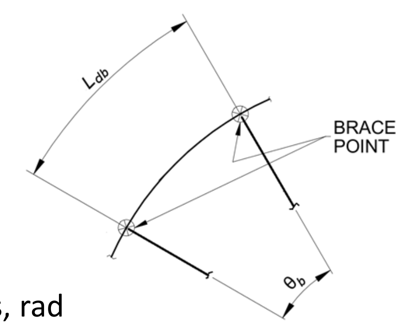
- Flexural (M_x) and shear (V) loads are calculated using L_{ds} with the flexural support conditions



23

M/*R* Method

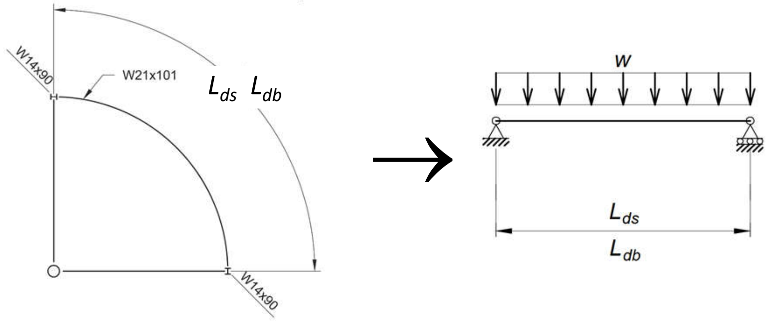
- Developed length between torsional restraints, $L_{db} = R\theta_b$



θ_b = angle between braces, rad

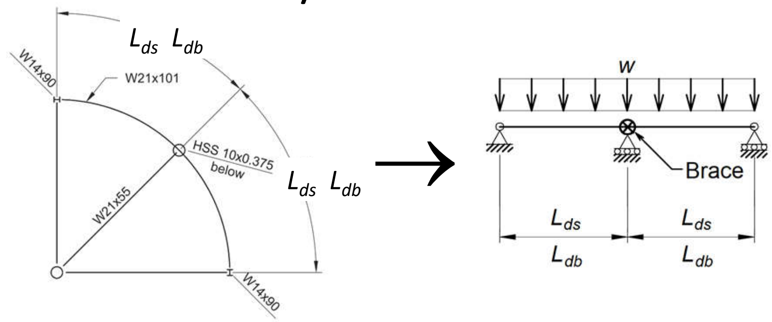
24

M/*R* Method



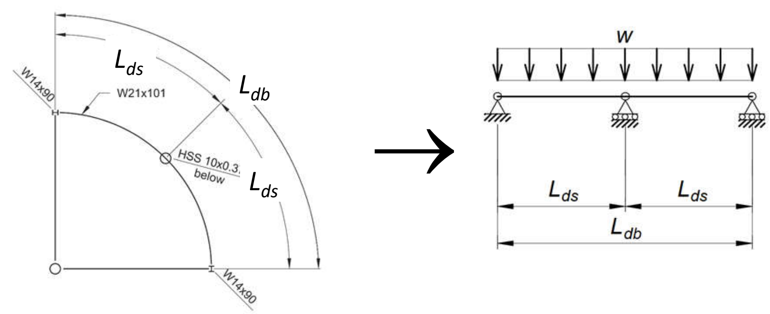
25

M/*R* Method



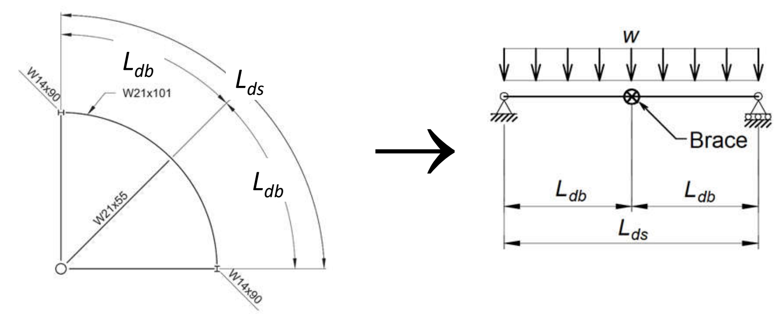
26

M/*R* Method



27

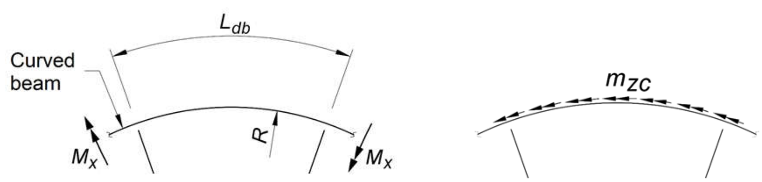
M/*R* Method



28

M/*R* Method

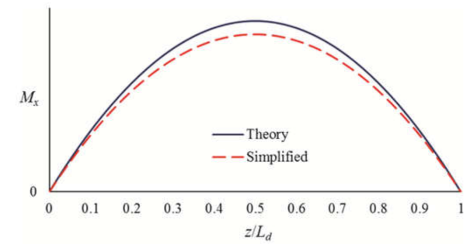
- Torsional moment per unit length: $m_{zc} = \frac{M_x}{R}$



29

M/*R* Method

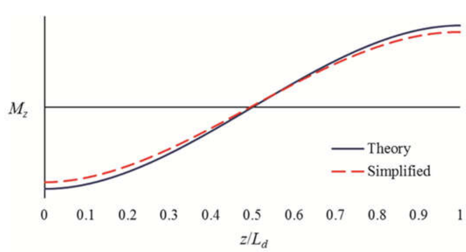
Moment Diagram (Uniform Load)



30

M/*R* Method

Torsion Diagram (Uniform Load)



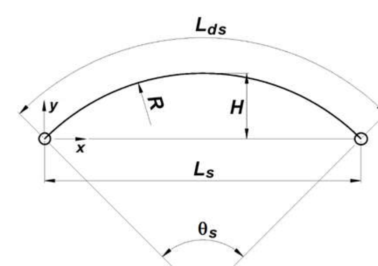
31

M/*R* Method

Corrected Moments

- Flexure: $M_{xc} = CM_x$
- Torsion: $M_{zc} = CM_z$

$$C = 1 - \frac{\theta_s}{30} + \frac{\theta_s^2}{6.2}$$



θ_s = span angle, rad

32

Horizontally-Curved Members

Structural Analysis

Finite Element Models

Finite Element Models

- Curved members are usually modeled by segmenting a series of straight elements

34

Finite Element Models

- 2-D Models: cross section is a single beam element
- 3-D Models: cross section is comprised of multiple elements (beam, plate, solid)

35

Finite Element Models

2-D Models

- Basic beam finite element formulation
  - Used in most commercial finite element programs
  - Only St Venant stiffness
  - No warping stiffness
  - Over-estimates the rotation of open sections

36

Finite Element Models

2-D Models

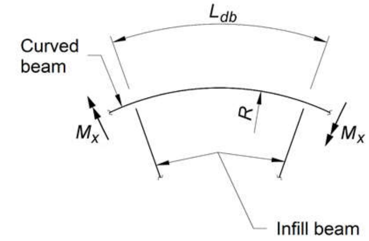
- The additional stiffness from warping can be approximated with an equivalent torsion constant, J_e

37

Finite Element Models

2-D Models

- Warping fixed at both ends of the span



38

Finite Element Models

2-D Models

$$J_e = \frac{J}{1 - \frac{\sinh\gamma}{\gamma} + \frac{(\cosh\gamma - 1)^2}{\gamma \sinh\gamma}}$$

39

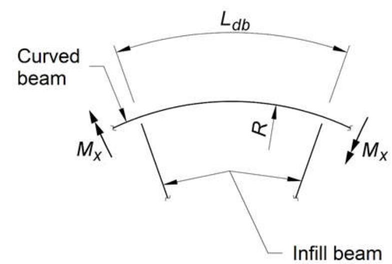
Finite Element Models

2-D Models

$$\gamma = L_{db}\sqrt{\frac{GJ}{EC_w}}$$

C_w = warping constant

J = torsional constant



40

Finite Element Models

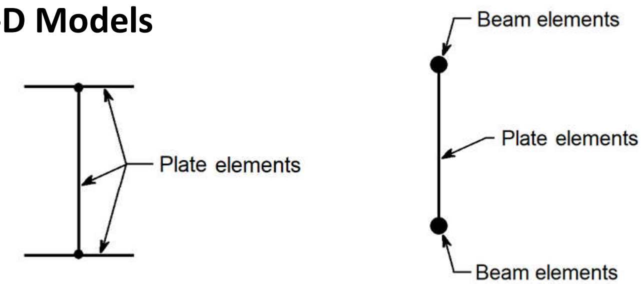
3-D Models

- Cross section is comprised of multiple elements

41

Finite Element Models

3-D Models



42

Horizontally-Curved Members

Flexural Strength

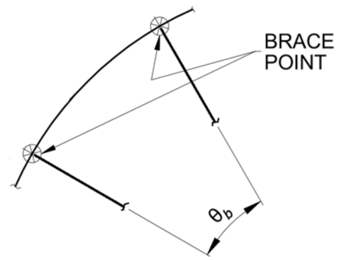
Flexural Strength

- Design as a straight beam
- AISC *Specification* Chapter F
- Unbraced length, $L_b \rightarrow L_{db}$
- Lateral-torsional buckling modification factor, $C_b \rightarrow C_{bo}$

44

Flexural Strength

$$C_{bo} = C_{bs}\left[1-\left(\frac{\theta_b}{\pi}\right)^2\right]^2$$



θ_b = angle between braces, rad

C_{bs} = C_b for an equivalent straight member

45

Horizontally-Curved Members

Torsional Strength

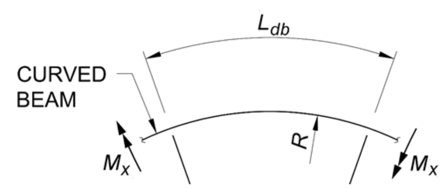
Torsional Strength

- Design as a straight beam
- Properly account for end conditions
  - Warping fixed
  - Warping free

47

Torsional Strength

- Member length = developed length between torsional restraints, L_{db}



48

Torsional Strength

- Analysis Methods
  - Isolated flange method
  - Elastic method (Section 7.5.1 of Design Guide 33)

49

Horizontally-Curved Members

Torsional Strength

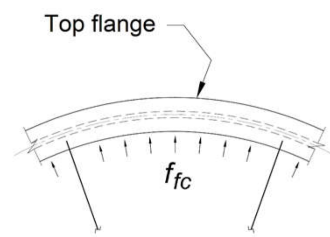
Isolated Flange Method

Isolated Flange Method

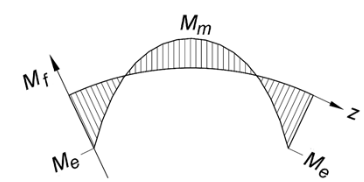
- I-shaped members
- Flanges are modeled as independent rectangular beams

51

Isolated Flange Method



Radial Load

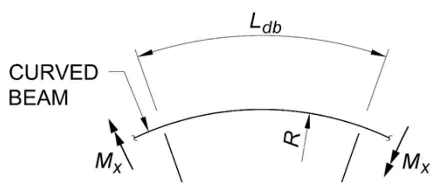


Moment Diagram

52

Isolated Flange Method

- Length of the isolated flange = developed (arc) length between torsional restraints, L_{db}

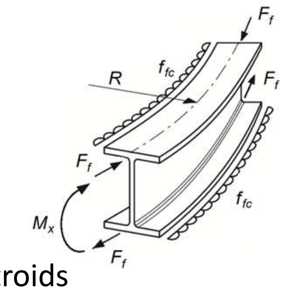


Isolated Flange Method

- Radial load, f_{fc}, is applied in the horizontal plane

$$f_{fc} = \frac{m_{zc}}{h_o} = \frac{M_x}{Rh_o}$$

h_o = distance between flange centroids

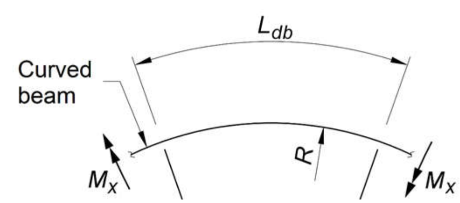


Isolated Flange Method

- Flexural boundary conditions of the isolated flange are based on the warping boundary conditions of the curved member

Isolated Flange Method

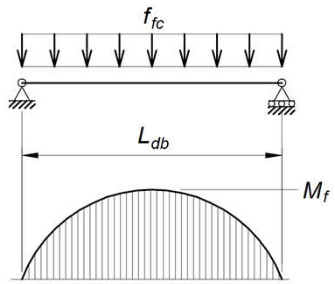
Equal End Moments



Isolated Flange Method

Equal End Moments

- Moment diagram for the isolated flange
- Warping free

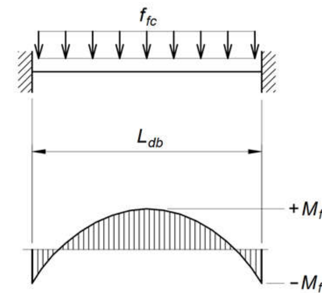


57

Isolated Flange Method

Equal End Moments

- Moment diagram for the isolated flange
- Warping fixed



58

Isolated Flange Method

- Nominal flexural strength of the isolated flange: $M_{nw} = F_y Z_f$

$$Z_f = \frac{t_f b_f^2}{4}$$

b_f = flange width

t_f = flange thickness

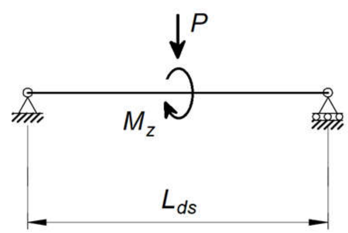
59

Horizontally-Curved Members

Combined Loads

Combined Loads

Flexure-torsion Interaction



61

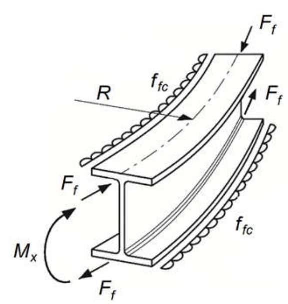
Horizontally-Curved Members

Combined Loads

Second-Order Effects

Second-Order Effects

- Increase torsional moments and torsional rotations
- Isolated compression flange is analogous to a beam-column



63

Second-Order Effects

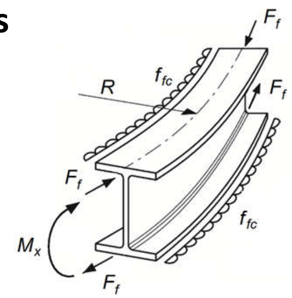
- Rigorous second-order analysis
- Amplified first-order analysis

64

Second-Order Effects

Amplified First-Order Analysis

- Open sections subjected to torsion + strong-axis flexure



65

Second-Order Effects

Amplified First-Order Analysis

- Second-order torsional rotation: $\theta_2 = B_o\theta_1$
- Second-order torsional moment: $M_{rz} = B_oM_z$

M_z = first-order torsional moment
θ_1 = first-order torsional rotation

66

Second-Order Effects

Amplified First-Order Analysis

$$B_o = \frac{0.85}{1-\alpha\dfrac{M_{ro}}{M_{eo}}} \geq 1.0$$

M_{eo} = elastic lateral-torsional buckling moment
M_{ro} = required strong-axis flexural moment (M_{rx})
α = 1.00 (LRFD); 1.60 (ASD)

67

Horizontally-Curved Members

Combined Loads

Member Strength

Member Strength

I-Shaped Members

- Interaction method is based on the analysis method
  - Isolated flange method
  - Elastic method (Section 7.6.2 of Design Guide 33)
  - FE model (Section 7.6.2 of Design Guide 33)

Member Strength

I-Shaped Members: Isolated Flange Method

$$\frac{M_{ro}}{M_{co}} + \frac{8}{9}\frac{M_{rw}}{M_{cw}} \leq 1.0$$

M_{co} M_{cw} = available flexural strengths: member, flange

M_{ro} M_{rw} = required flexural strength: member, flange

Horizontally-Curved Members

Serviceability

structural STEEL

Serviceability

- Large deformations at ultimate strength
- Member sizes are usually controlled by serviceability

Serviceability

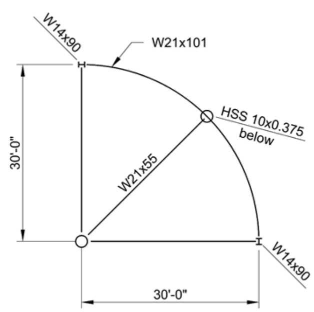
- Torsional rotation limits
  - Not in building codes
  - Some judgment may be required to define appropriate limits
  - Maintain geometry of the structure
  - Prevent damage to nonstructural elements

73

Horizontally-Curved Members

Design Example

Design Example



75

Horizontally-Curved Members

Design Example

Problem Statement

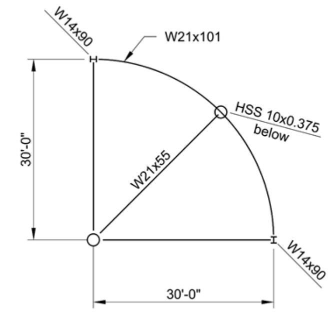
Problem Statement

- Verify that the horizontally-curved beam is adequate for the imposed loading
- Use LRFD

77

Problem Statement

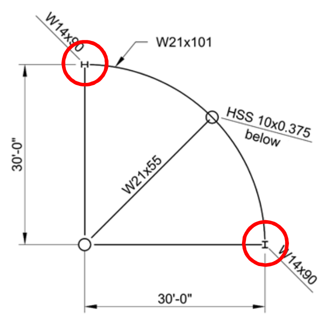
- Curved member
  - W21×101
  - ASTM A992
  - Bent the easy way
  - Circular curve



78

Problem Statement

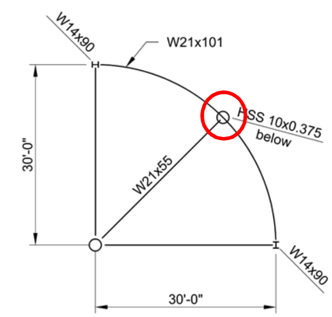
- End connections
  - Torsional rotation is restrained
  - No warping restraint
  - No flexural restraint



79

Problem Statement

- Connections @ the HSS10
  - Continuous for flexure
  - Torsion is restrained by the W21 beam
  - Continuous for warping



80

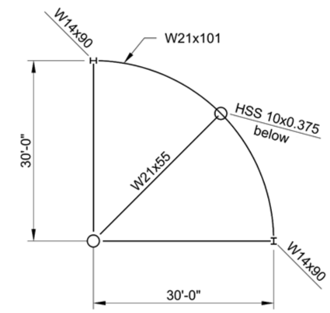
Problem Statement

- The factored uniformly distributed load along the member circumference including the beam self weight is w_u = 0.750 kip/ft

81

Problem Statement

- Assume the critical condition is for patch loading (one span loaded)



82

Horizontally-Curved Members

Design Example

Properties

structural
STEEL

Properties

- Material properties of ASTM A992 (AISC *Manual* Table 2-4)

  F_y = 50 ksi

  F_u = 65 ksi

84

Properties

- Dimensions of W21×101 (AISC *Manual* Table 1-1)

d = 21.4 in. t_w = 0.500 in.
b_f = 12.3 in. t_f = 0.800 in.
h_o = 20.6 in.

85

Properties

- Section properties of W21×101 (AISC *Manual* Table 1-1)

I_x = 2,420 in.^{4} S_x = 227 in.^{3}
Z_x = 253 in.^{3}
J = 5.21 in.^{4} C_w= 26,200 in.^{6}
r_{ts} = 3.35 in.

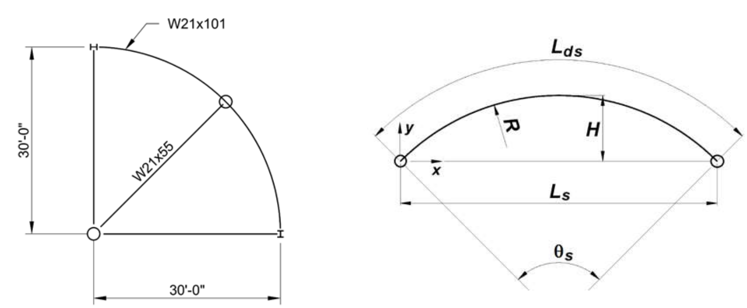
86

Horizontally-Curved Members

Design Example

Beam Geometry

Beam Geometry

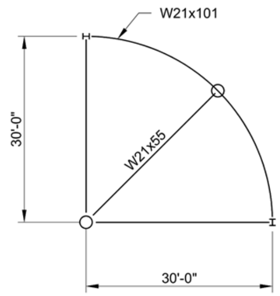


88

Beam Geometry

- Centroidal radius

$$R = (30\text{ ft})(12\text{ in./ft})$$
$$= 360\text{ in.}$$

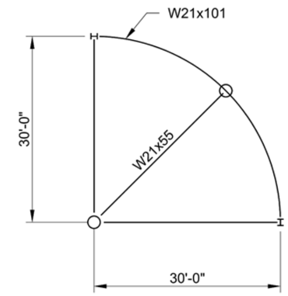


89

Beam Geometry

- Span angle

$$\theta_s = (45^\circ)\left(\frac{\pi\text{ rad}}{180^\circ}\right)$$
$$= \pi / 4\text{ rad}$$

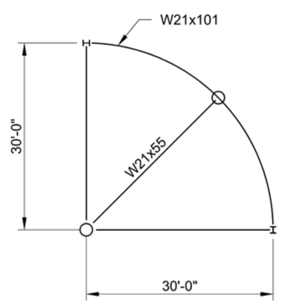


90

Beam Geometry

- Developed span length

$$L_{ds} = (30\text{ ft})(\pi / 4\text{ rad})(12\text{ in./ft})$$
$$= 283\text{ in.}$$

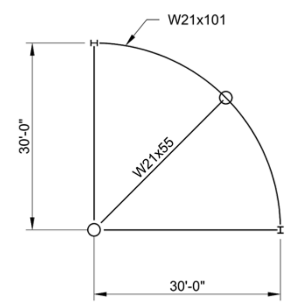


91

Beam Geometry

- Angle between torsional restraints

$$\theta_b = (45^\circ)\left(\frac{\pi\text{ rad}}{180^\circ}\right)$$
$$= \pi / 4\text{ rad}$$

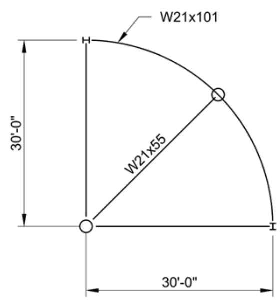


92

Beam Geometry

- Developed length between braces

$$L_{db} = (30\text{ ft})(\pi / 4\text{ rad})$$
$$= 23.6\text{ ft}$$
$$= 283\text{ in.}$$



93

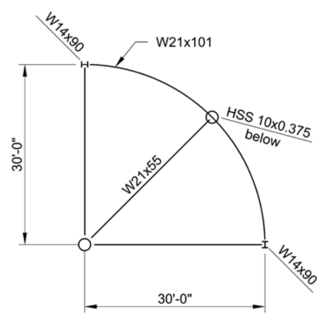
Horizontally-Curved Members

Design Example

Structural Analysis

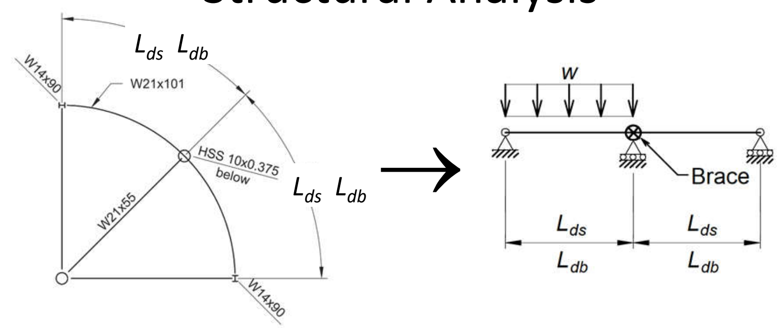
Structural Analysis

- At the ends
  - No warping restraint
  - No flexural restraint
- At the HSS10
  - Continuous for flexure
  - Continuous for warping



95

Structural Analysis



96

Horizontally-Curved Members

Design Example

Structural Analysis

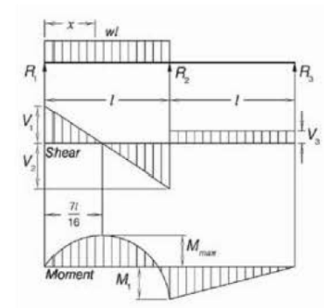
Flexural Loads

Structural Analysis

- AISC *Manual* Table 3-23 Case 29
- Max. Moment

  M_{ux} = +480 kip-in.
- Max. Shear

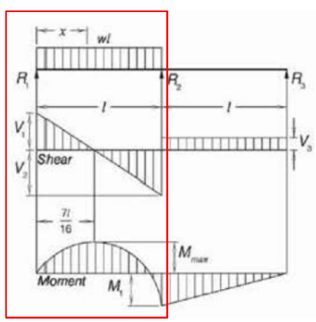
  V_u = 9.95 kips



98

Structural Analysis

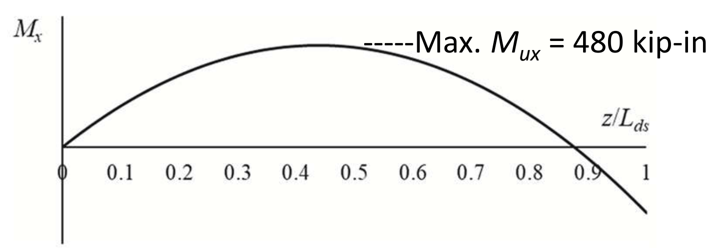
- Isolate the loaded span



99

Structural Analysis

- Flexural moment at the loaded span



100

Horizontally-Curved Members

Design Example

Structural Analysis

Torsional Loads

Structural Analysis

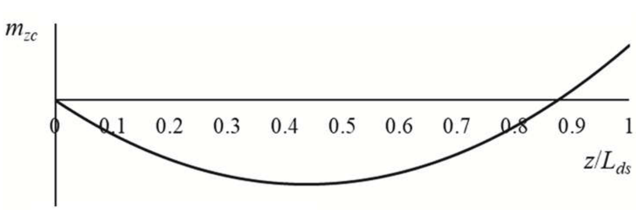
- *M*/*R* method
- Torsional moment per unit length:

$$m_{zc} = \frac{M_x}{R}$$

102

Structural Analysis

- Distributed torsion

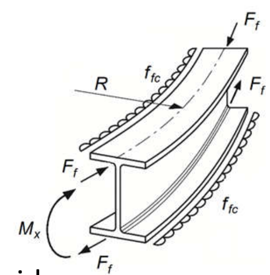


103

Structural Analysis

- Isolated Flange method

$$f_{fc} = \frac{m_{zc}}{h_o}$$

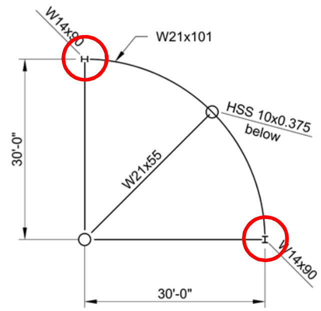


h_o = distance between flange centroids

104

Structural Analysis

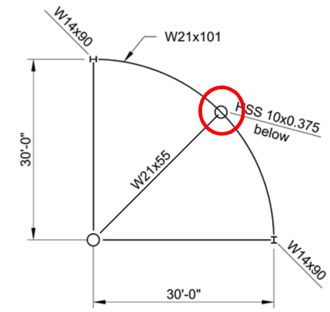
- Warping is not restrained at the end connections → use pinned ends at the isolated flange



105

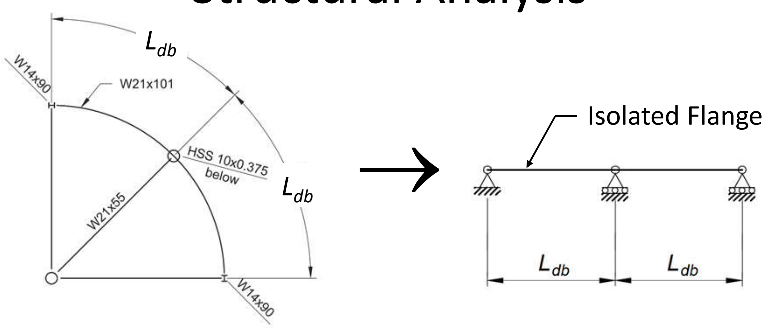
Structural Analysis

- Warping is continuous at the HSS10 → the isolated flange is continuous for flexure



106

Structural Analysis

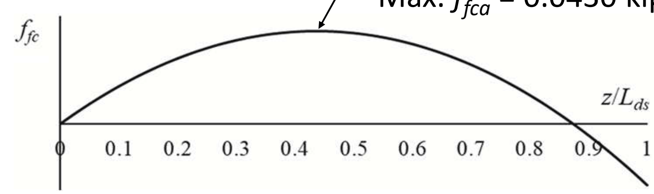


107

Structural Analysis

- Distributed flange force

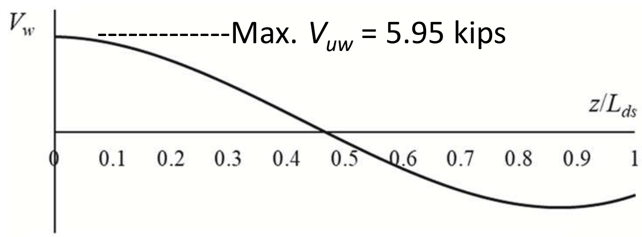
Max. f_{fcu} = 0.0645 kip/in.
Max. f_{fca} = 0.0430 kip/in.



108

Structural Analysis

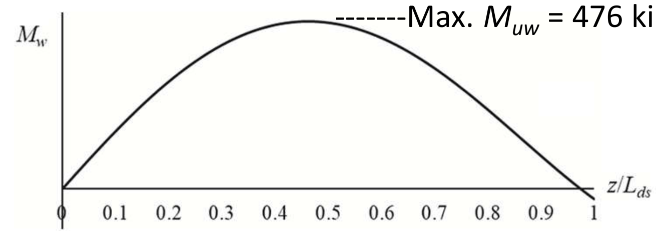
- Horizontal shear



109

Structural Analysis

- Warping moment



110

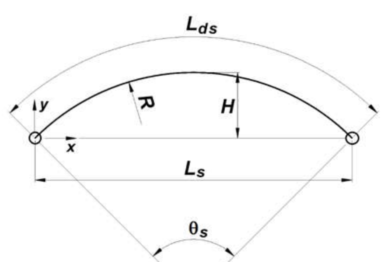
Structural Analysis

Corrected Moments

$$C = 1 - \frac{\theta_s}{30} + \frac{\theta_s^2}{6.2}$$

$$= 1 - \frac{\pi/4}{30} + \frac{(\pi/4)^2}{6.2}$$

$$= 1.07$$



111

Structural Analysis

Corrected Moments

- Flexure:

$$M_{uxc} = CM_{ux} = (1.07)(480 \text{ kip-in.}) = 514 \text{ kip-in.}$$

- Warping:

$$M_{uwc} = CM_{uw} = (1.07)(476 \text{ kip-in.}) = 509 \text{ kip-in.}$$

112

Horizontally-Curved Members

Design Example

Shear Strength

Shear Strength

- AISC *Manual* Table 6-2

$\phi_v V_n = 321 \text{ kips} > V_u = 9.95 \text{ kips}$ **o.k.**

114

Horizontally-Curved Members

Design Example

Local Buckling

Local Buckling

- Calculations are the same as for a straight member
- Flexure: $\lambda_f < \lambda_{pf}$ and $\lambda_w < \lambda_{pw} \rightarrow$ the W21×101 is compact

116

Horizontally-Curved Members

Design Example

Flexural Strength

structural STEEL

Flexural Strength

- Design as a straight beam
- AISC *Specification* Chapter F

118

Flexural Strength

Use C_{bs} = 1.0

$$C_{bo} = C_{bs}\left[1-\left(\frac{\theta_b}{\pi}\right)^2\right]^2$$

$$=(1.0)\left[1-\left(\frac{\pi/4}{\pi}\right)^2\right]^2 = 0.879$$

119

Flexural Strength

- AISC *Specification* Section F2
  - $L_b = L_{db}$ = 23.6 ft
  - $C_b = C_{bo}$ = 0.879
- AISC *Manual* Table 3-6
  - L_p = 10.2 ft
  - L_r = 30.1 ft

120

Flexural Strength

$$M_p = F_y Z_x$$
$$= (50 \text{ ksi})(253 \text{ in.}^3)$$
$$= 12{,}700 \text{ kip-in.}$$

121

Flexural Strength

- $L_p < L_b < L_r \rightarrow$ Use AISC *Spec.* Eq. F2-2

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_y S_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$
$$= 8{,}350 \text{ kip-in.} < 12{,}700 \text{ kip-in.}$$

122

Flexural Strength

- The available strength is

$$\phi_b M_n = 0.90(8{,}350 \text{ kip-in.})$$
$$= 7{,}520 \text{ kip-in.}$$

123

Horizontally-Curved Members

Design Example

Second-Order Effects

Second-Order Effects

- Amplified First-Order Analysis
- Second-order warping moment: $M_{uw} = B_o M_{uwc}$

$$B_o = \frac{0.85}{1 - \alpha \dfrac{M_{uxc}}{M_{eo}}} \geq 1.0$$

M_{eo} = elastic lateral-torsional buckling moment

α = 1.00 (LRFD)

125

Second-Order Effects

$$F_{cr} = \frac{C_{bo}\pi^2 E}{\left(\dfrac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

$$= 44.9 \text{ ksi}$$

126

Second-Order Effects

$$M_{eo} = F_{cr} S_x$$

$$= (44.9 \text{ ksi})(227 \text{ in.}^3)$$

$$= 10{,}200 \text{ kip-in.}$$

127

Second-Order Effects

$$B_o = \frac{0.85}{1 - (1.0)\left(\dfrac{514 \text{ kip-in.}}{10{,}200 \text{ kip-in.}}\right)} \geq 1.0$$

$$= 0.895 < 1.0$$

$\rightarrow B_o = 1.0$

128

Second-Order Effects

$$M_{uw} = (1.0)(509 \text{ kip-in.})$$
$$= 509 \text{ kip-in.}$$

129

Horizontally-Curved Members

Design Example

Warping Strength

structural STEEL

Warping Strength

- The isolated flange plastic modulus is

$$Z_f = \frac{t_f b_f^2}{4}$$
$$= \frac{(0.800 \text{ in.})(12.3 \text{ in.})^2}{4}$$
$$= 30.3 \text{ in.}^3$$

131

Warping Strength

- The nominal flexural strength of the isolated flange is

$$M_{nw} = F_y Z_f$$
$$= (50 \text{ ksi})(30.3 \text{ in.}^3)$$
$$= 1{,}520 \text{ kip-in.}$$

132

Warping Strength

- The available flexural strength of the isolated flange is

$$M_{cw} = \phi_b M_{nw}$$
$$= 0.90(1{,}520 \text{ kip-in.})$$
$$= 1{,}370 \text{ kip-in.}$$

133

Horizontally-Curved Members

Design Example

Combined Loading

Combined Loading

- Flexural moment + flange warping moment

$$\frac{M_{uxc}}{\phi_b M_n} + \frac{8}{9}\frac{M_{uw}}{\phi M_{nw}} \le 1.0$$

$$\frac{514 \text{ kip-in.}}{7{,}520 \text{ kip-in.}} + \left(\frac{8}{9}\right)\left(\frac{509 \text{ kip-in.}}{1{,}370 \text{ kip-in.}}\right) \le 1.0$$

$$0.399 < 1.00 \quad \textbf{o.k.}$$

135

Horizontally-Curved Members

Design Example

Serviceability

Serviceability

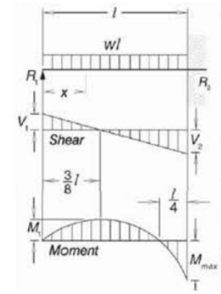
- The torsional rotation can be estimated using the horizontal deflection of the isolated flange
- Maximum second-order distributed flange force under service loads: f_{fc} = 0.0430 kip/in.

137

Serviceability

- AISC *Manual* Table 3-23 Case 12

$$\Delta_{max} = \frac{f_{fc} L_{ds}^4}{185 E I_f}$$



138

Serviceability

- The isolated flange moment of inertia is

$$I_f = \frac{t_f b_f^3}{12} = \frac{(0.800 \text{ in.})(12.3 \text{ in.})^3}{12}$$
$$= 124 \text{ in.}^4$$

139

Serviceability

$$\Delta_{max} = \frac{f_{fc} L_{ds}^4}{185 E I_f} = \frac{(0.0430 \text{ kip/in.})(283 \text{ in.})^4}{(185)(29{,}000 \text{ ksi})(124 \text{ in.}^4)}$$
$$= 0.415 \text{ in.}$$

140

Serviceability

- The 1st-order torsional rotation is

$$\theta_1 = \tan^{-1}\left(\frac{2\Delta_{max}}{h_o}\right) = \tan^{-1}\left[\frac{(2)(0.415\text{ in.})}{20.6\text{ in.}}\right]$$
$$= 2.31°$$

h_o = distance between flange centroids

141

Serviceability

- The 2nd-order torsional rotation is

$$\theta_2 = B_o\theta_1$$
$$= (1.00)(2.31°)$$
$$= 2.31°$$

142

Combined Loading

Design Summary

- The W21×101 strength is adequate
- See Design Guide Example 8.2 for further calculations
  - Connections

143

Conclusions

Horizontally-Curved Members

- Design as an equivalent straight member
- Subjected to both flexural and torsional loads
- Flexural strength is calculated with C_{bo}
- Member sizes are usually controlled by serviceability

144

Question time

structural STEEL

AISC | Questions?

Smarter. Stronger. Steel.

CEU / PDH Certificates

- You will receive an email on how to report attendance from: registration@aisc.org.
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!

Smarter. Stronger. Steel.

CEU / PDH Certificates

- Reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.

Smarter. Stronger. Steel.


AISC | Thank you
Smarter.
Stronger.
Steel.