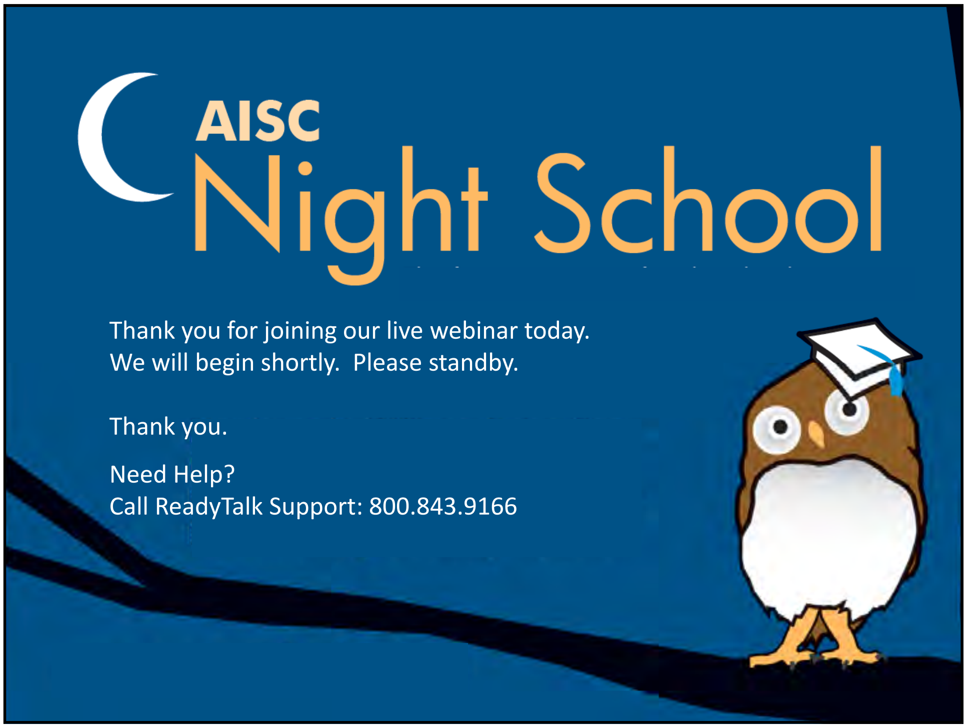

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Please standby.
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Course Description

Course Introduction and Behavior of Compression Members

June 5, 2017

This lecture will begin with a brief overview of the 8-lecture course. The behavior of compression members will then be covered. The assumptions in the solution to the Euler column problem will be used as a basis for systematically moving from the theoretical solution presented in 1757 to the modern day methods of design and analysis of compression members. Emphasis will be placed on the effects of material yielding accentuated by the presence of residual stresses, initial imperfections, and end conditions. The flexural buckling strength of members without slender elements will be covered and ultimately presented in the form of column curves.

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Learning Objectives

- List the assumptions in the Euler column solution.
- Describe how bending is produced on a column.
- Describe the aspects that are taken into account to define a column's strength in the AISC Specification's column curve.
- Explain how column end restraint affects the behavior of a column.

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Fundamentals of Stability for Steel Design

Session 1: Course Introduction and Behavior of Compression Members

June 5, 2017

Presented by
Ronald D. Ziemian, Ph.D., P.E.
Professor
Bucknell University, Lewisburg, PA

structural STEEL

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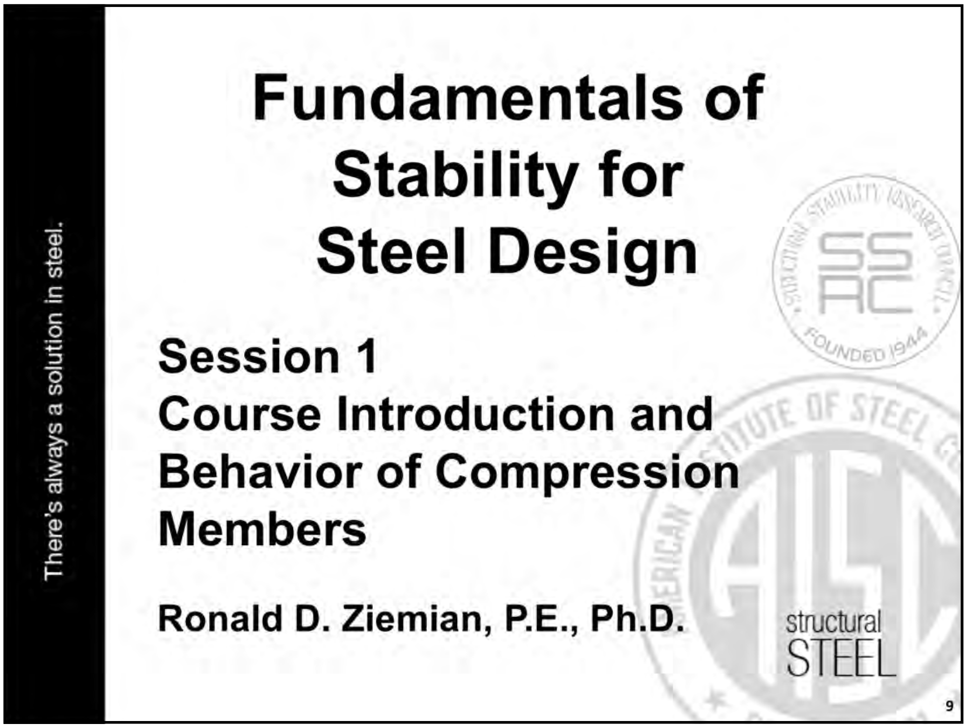

Course Overview

- Session Topics
  - Compression Members (1 & 2)
  - Flexural Members (3 & 4)
  - Systems / Beam-Columns (5 & 6)
  - Bracing (7 & 8)
- Topics in two parts
  - Behavior (1, 3, 5, 7)
  - Design (2, 4, 6, 8)
- Lectures by members of the Structural Stability Research Council (SSRC)
  - P.S. Green, T.A. Helwig, D.W. White, J.A. Yura, R.D. Ziemian
  - Great to join AISC in this effort!

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Course Overview (2)

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Course Overview (3)

- Focus of the course is on fundamentals!
- Better understanding of behavior will result in improved design
- Key Definitions
  - **Stability:** Under load, component returns to current state after applying a small disturbance such as a deflection
  - **Bifurcation (critical load)**: Theoretical point at which loading a component results in an instantaneous change from current state to significant deflection – two options: not buckled or buckled
  - **Instability:** Loading a component results in an actual transition from small deflection to significant deflection – buckling preceded by significant deflection

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Course Overview (4)

"Buckling" Design Methods:

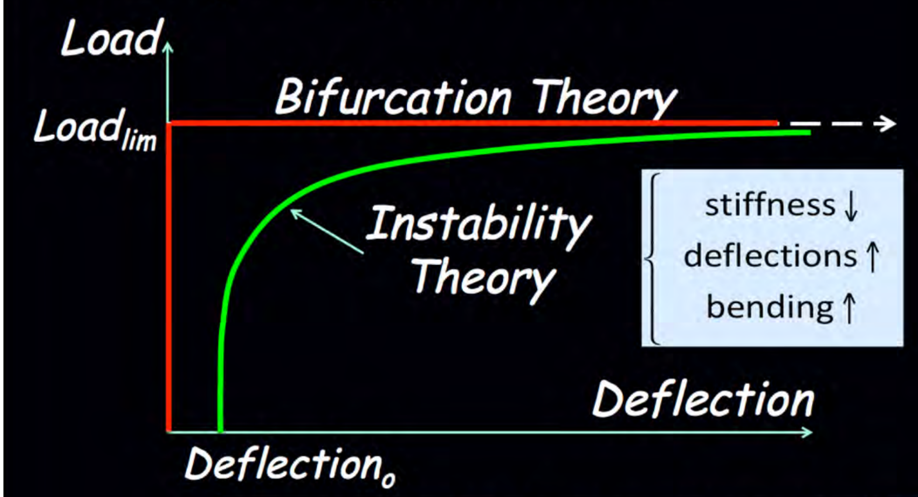

Figure applicable to system, member, and cross-section behavior

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Course Overview (5)

Let's start at the end…

ANSI/AISC 360-16
An American National Standard

Specification for Structural Steel Buildings

C1. GENERAL STABILITY REQUIREMENTS

Stability shall be provided for the structure as a whole and for each of its elements. The effects of all of the following on the stability of the structure and its elements shall be considered: (a) flexural, shear and axial member deformations, and all other component and connection deformations that contribute to the displacements of the structure; (b) second-order effects (including P-Δ and P-δ effects); (c) geometric imperfections; (d) stiffness reductions due to inelasticity, including the effect of partial yielding of the cross section which may be accentuated by the presence of residual stresses; and (e) uncertainty in system, member, and connection strength and stiffness. All load-dependent effe… sponding to LRFD load combinat…

Why these for the Big 5?

Any rational method of design for stability that considers all of the listed effects is permitted; this includes the methods identified in Sections C1.1 and C1.2.

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Fundamentals of Stability for Steel Design

There's always a solution in steel.

Session 1
Course Introduction and Behavior of Compression Members

Ronald D. Ziemian, P.E., Ph.D.

structural STEEL

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Limit States of Compression Members

- Full yielding (today)
- Instability
  - Along the member length
    - Flexural buckling (today's emphasis!)
    - Torsional buckling
    - Flexural-torsional buckling
  - At the cross section
    - local buckling

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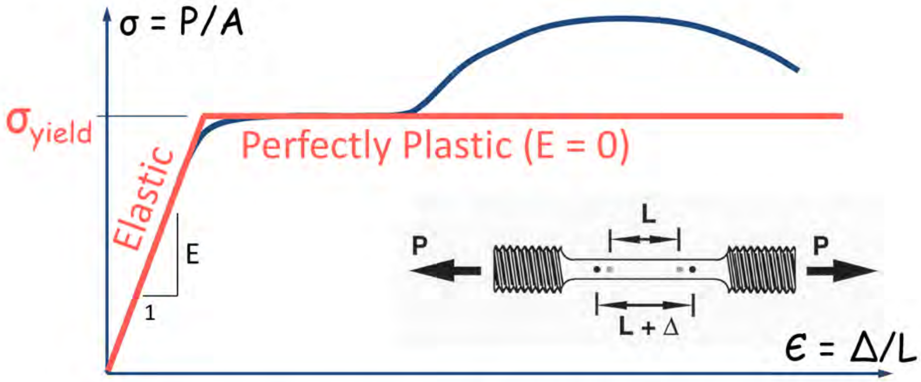
Full Yielding

- Tensile test



- Assume same response for compression
  - $\sigma_{y,compression} = \sigma_{y,tension} = \sigma_{yield}$
  - Neglect strain hardening (assume elastic-plastic)

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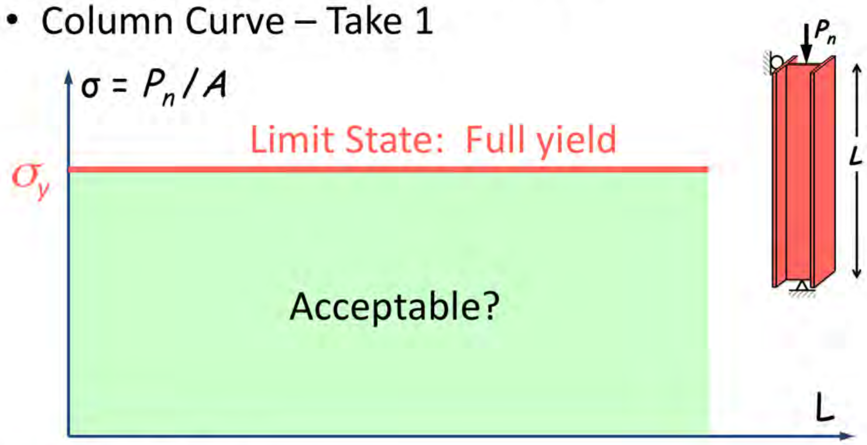
Full Yielding (2)

- Column Curve – Take 1



- What about:
  - member instability ??? (today!)
  - cross section instability (local buckling) ???

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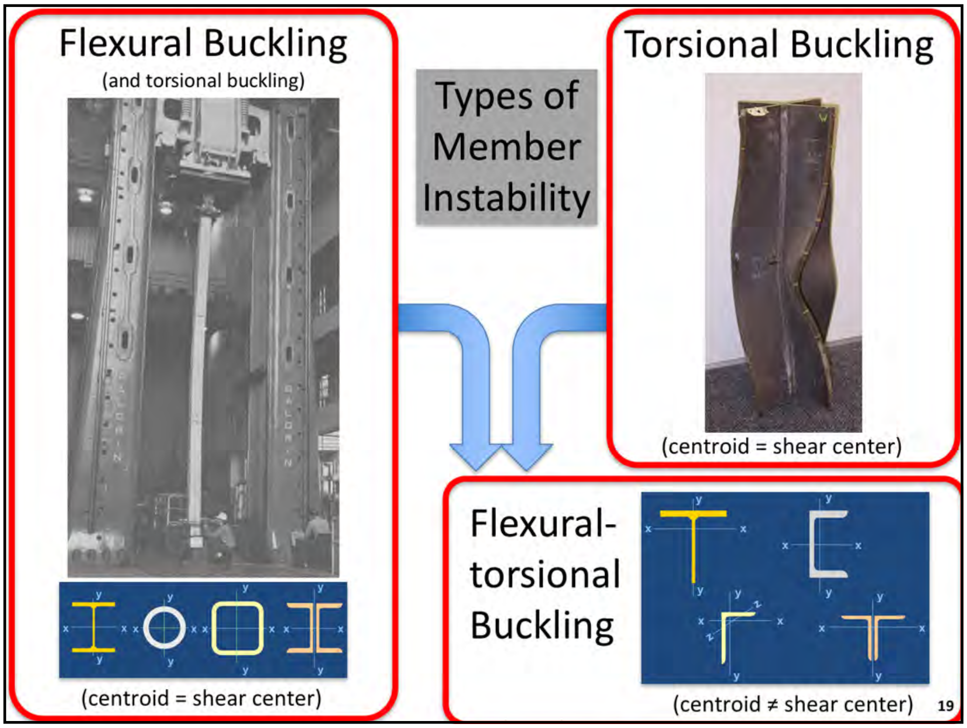

Flexural Buckling

- Euler's column
  - solution
  - assumptions
- Undoing Euler's assumptions (approaching reality)
  - bending before bifurcation
  - not fully elastic (partial yielding)
  - support conditions
- Column curves
  - AISC
  - others

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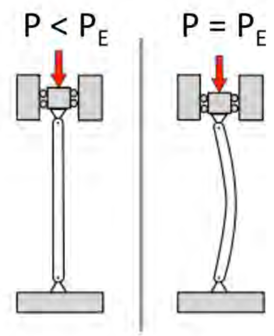
Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions!
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)



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Euler Buckling (2)

Recall: Three Keys to an Analysis

1. Equilibrium
   (balance of forces and/or moments)
2. Compatibility
   (agreement of displacements and/or rotations)
3. Constitutive Relationship
   (relate forces and/or moments to displacements and/or rotations)

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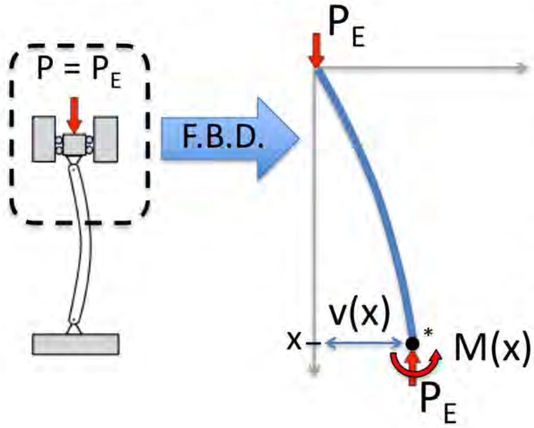
Euler Buckling (2)



Equilibrium:

$$\Sigma M_{*} = 0$$

$$M(x) + P_E v(x) = 0$$

Moment-curvature:

(constitutive relationship)

$$M(x) = EI \frac{d^2 v(x)}{dx^2}$$

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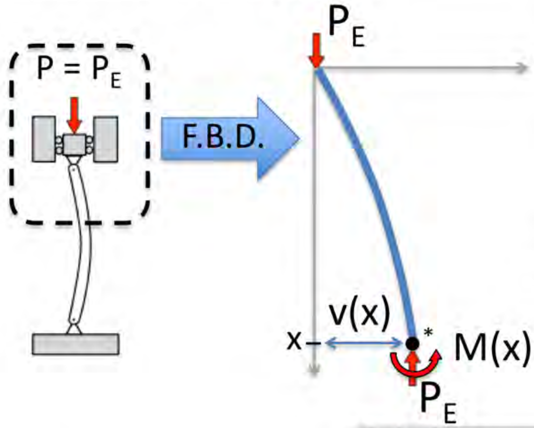
Euler Buckling (3)



Equilibrium:

$$\Sigma M_{*} = 0$$

$$M(x) + P_E v(x) = 0$$

Moment-curvature:

$$M(x) = EI \frac{d^2 v(x)}{dx^2}$$

Solution:

$$EI \frac{d^2 v}{dx^2} + P_E v = 0 \quad \Rightarrow \quad v(x) = C_1 \cos\left(\sqrt{\frac{P_E}{EI}} x\right) + C_2 \sin\left(\sqrt{\frac{P_E}{EI}} x\right)$$

wolframalpha.com
a2*y''(x)+a1*y(x)=0

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Euler Buckling (4)

$$v(x)=C_1\cos(\sqrt{\frac{P_E}{EI}}x)+C_2\sin(\sqrt{\frac{P_E}{EI}}x)$$

$P = P_E$

Compatibility:

$v(x=0)=0 \Rightarrow C_1=0 \Rightarrow v(x)=C_2\sin(\sqrt{\frac{P_E}{EI}}x)$

Boundary Conditions!

$v(x=L)=0$

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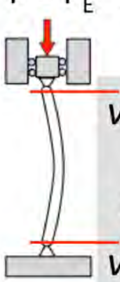
Euler Buckling (5)

$P = P_E$



Compatibility:

$v(x=0)=0 \Rightarrow v(x)=C_2\sin(\sqrt{\frac{P_E}{EI}}x)$

Boundary Conditions!

$v(x=L)=0 \Rightarrow v(x=L)=0=C_2\sin(\sqrt{\frac{P_E}{EI}}L)$

1) $C_2=0$ "trivial solution"

2) $\sin(\sqrt{\frac{P_E}{EI}}L)=0 \Rightarrow \sqrt{\frac{P_E}{EI}}L=n\pi \Rightarrow P_E=\frac{n^2\pi^2 EI}{L^2}$

$n=1,2,3,\ldots$

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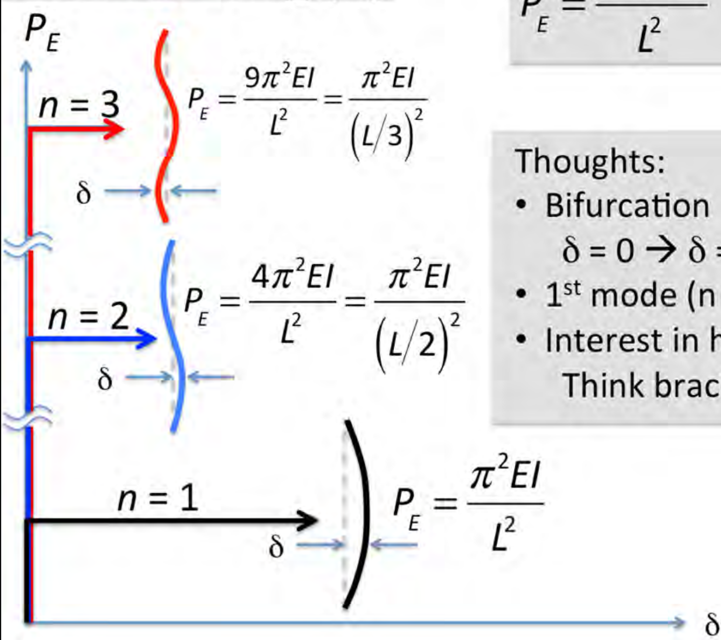
Euler Buckling (6)

$$P_E = \frac{n^2\pi^2 EI}{L^2} \quad n = 1,2,3,\ldots$$



Thoughts:
- Bifurcation
  δ = 0 → δ = unbounded
- 1st mode (n = 1) controls!
- Interest in higher modes?
  Think bracing!

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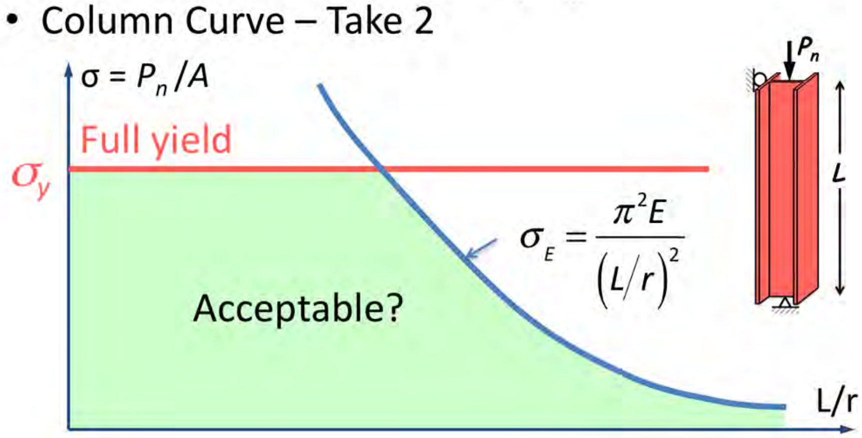
Euler Buckling (7)

- Euler Buckling Stress

$$P_E = \frac{\pi^2 EI}{L^2} \Rightarrow \sigma_E = \frac{P_E}{A} = \frac{\pi^2 E}{(L/r)^2} \quad \text{with } r = \sqrt{\frac{I}{A}}$$

- Column Curve – Take 2



- What about those assumptions?

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An interesting observation…

- Euler Buckling Stress

$$\sigma_E = \frac{\pi^2 E}{\left(L/r\right)^2} \quad \text{with } r = \sqrt{\frac{I}{A}}$$

$$\frac{\sigma_E}{E} = \frac{\pi^2}{\left(L/r\right)^2} \quad \text{and Hooke's Law } \sigma_E = E\varepsilon_E$$

- Euler Buckling Strain $\varepsilon_E = \left(\frac{\pi}{L/r}\right)^2$

A timber column and a steel column of equal dimensions (L, I , and A) and support conditions will elastically buckle at the same axial strain…WHOA!

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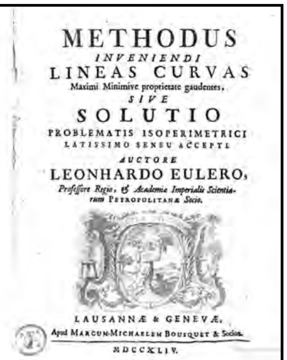
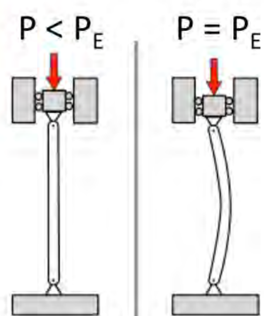
Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)


METHODUS
INVENIENDI
LINEAS CURVAS
Maximi Minimive proprietate gaudentes,
SIVE
SOLUTIO
PROBLEMATIS ISOPERIMETRICI
LATISSIMO SENSU ACCEPTI
AUCTORE
LEONHARDO EULERO,
Professore Regio, & Academiæ Imperialis Scientiarum Petropolitanæ Socio.
LAUSANNÆ & GENEVÆ,
Apud MARCUM-MICHAELEM BOUSQUET & Socios.
MDCCXLIV.



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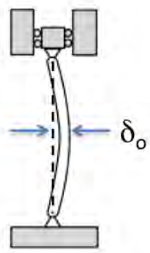
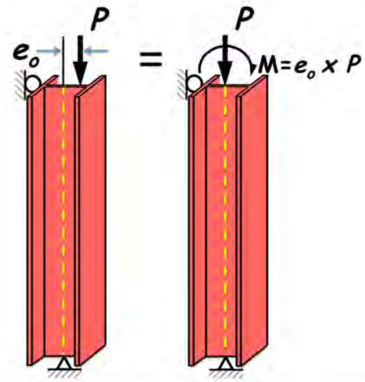
Bending

- Bending can be produced by:

1. Prior to loading, column is not perfectly straight



2. Axial load not concentrically applied (e_o is small, but not zero!)



Reality: Some combination of above exists…

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Bending (2)

Let's consider a column with initial out-of-straightness:

$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

Initial imperfection at mid-length e.g. $\delta_o = L/1000$

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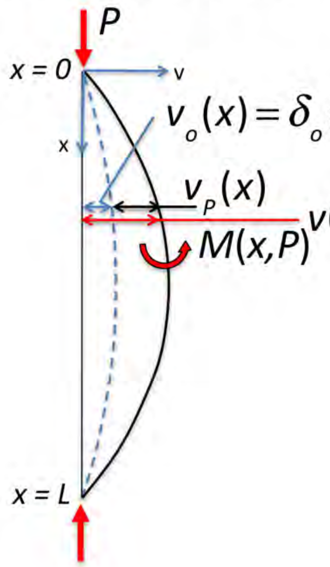
Bending (3)

Column with initial out-of-straightness:



$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

$$v_P(x)$$

$$v(x) = v_o(x) + v_P(x)$$

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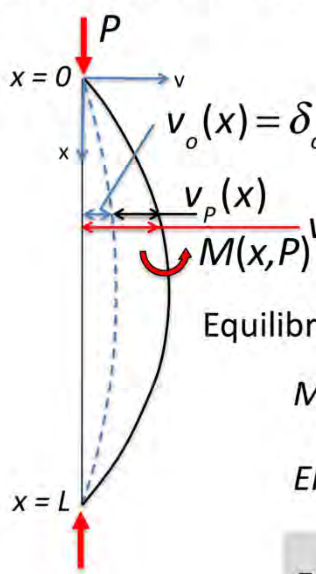
Bending (4)

Column with initial out-of-straightness:



$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

$$v_P(x)$$

$$v(x) = v_o(x) + v_P(x)$$

Equilibrium → Differential Equation:

$$M(x,P) + Pv(x) = 0$$

$$EI\frac{d^2 v_P}{dx^2} + P\left(v_o(x) + v_P(x)\right) = 0$$

$$EI\frac{d^2 v_P}{dx^2} + Pv_P(x) = -Pv_o(x) = -P\delta_o \sin\frac{\pi x}{L}$$

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Bending (5)

Column with initial out-of-straightness:

$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

$v_P(x)$

$$v(x) = v_o(x) + v_p(x)$$

wolframalpha.com
a2*y"(x)+a1*y(x)=-a1*a3*sin(a4*x)

Differential Equation → Solution with BC's

$$EI\frac{d^2 v_P}{dx^2} + P v_P(x) = -P\delta_o \sin\frac{\pi x}{L}$$

$$v_P(x) = \frac{1}{\frac{EI\pi^2}{PL^2} - 1}\delta_o \sin\frac{\pi x}{L} = \frac{1}{\frac{P_E}{P} - 1}\delta_o \sin\frac{\pi x}{L}$$

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Bending (6)

Column with initial out-of-straightness:

$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

$v_P(x)$

$$v(x) = v_o(x) + v_P(x)$$

$$v_P(x) = \frac{1}{\frac{P_E}{P} - 1}\delta_o \sin\frac{\pi x}{L}$$

$$v(x) = \delta_o \sin\frac{\pi x}{L} + \frac{1}{\frac{P_E}{P} - 1}\delta_o \sin\frac{\pi x}{L} = \left(1 + \frac{1}{\frac{P_E}{P} - 1}\right)\delta_o \sin\frac{\pi x}{L}$$

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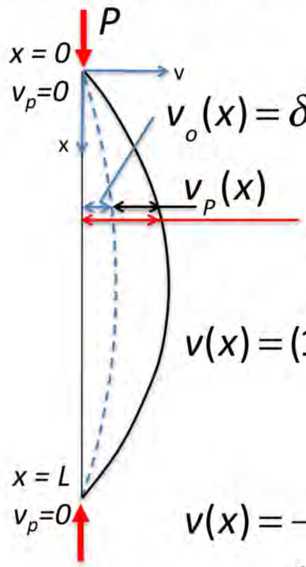
Bending (7)

Column with initial out-of-straightness:



$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

$$v(x) = v_o(x) + v_p(x)$$

$$v(x) = \left(1 + \frac{1}{\frac{P_E}{P} - 1}\right)\delta_o \sin\frac{\pi x}{L}$$

$$v(x) = \frac{1}{1 - \frac{P}{P_E}}\delta_o \sin\frac{\pi x}{L} \Rightarrow v(x) = \frac{1}{1 - \frac{P}{P_E}} v_o(x)$$

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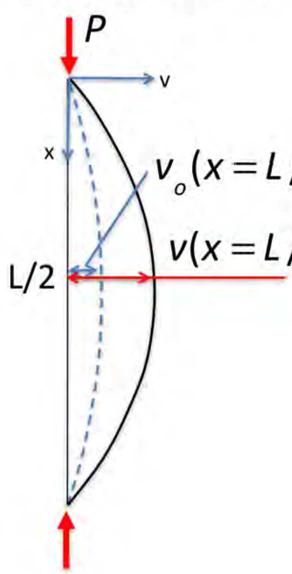
Bending (8)

Column with initial out-of-straightness:



$$v_o(x = L/2) = \delta_o$$

$$v(x = L/2) = \delta(P)$$

$$v(x) = \frac{1}{1 - \frac{P}{P_E}} v_o(x)$$

$$\delta(P) = \frac{1}{1 - \frac{P}{P_E}} \times \delta_o$$

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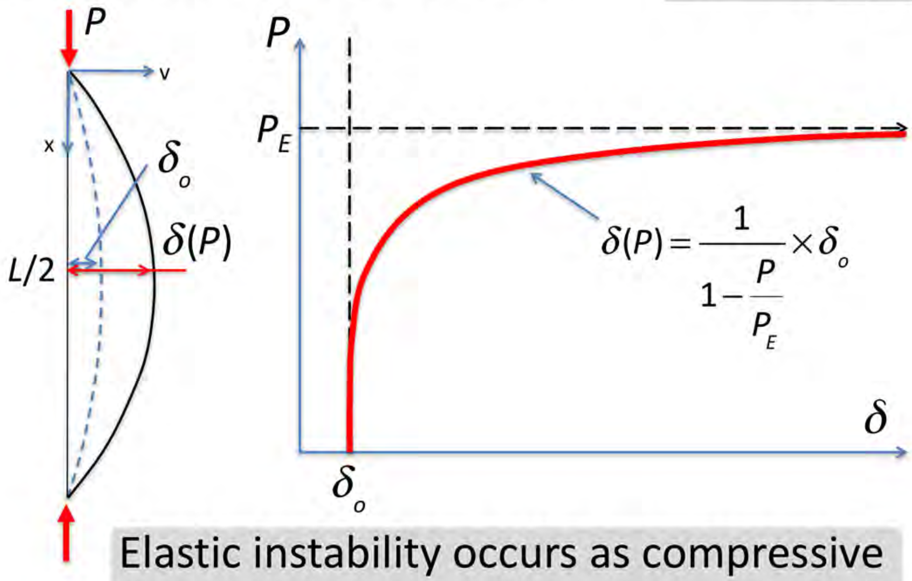
Column with initial out-of-straightness:

Bending (9)



Elastic instability occurs as compressive force *P* approaches Euler critical load P_E

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Polling Question

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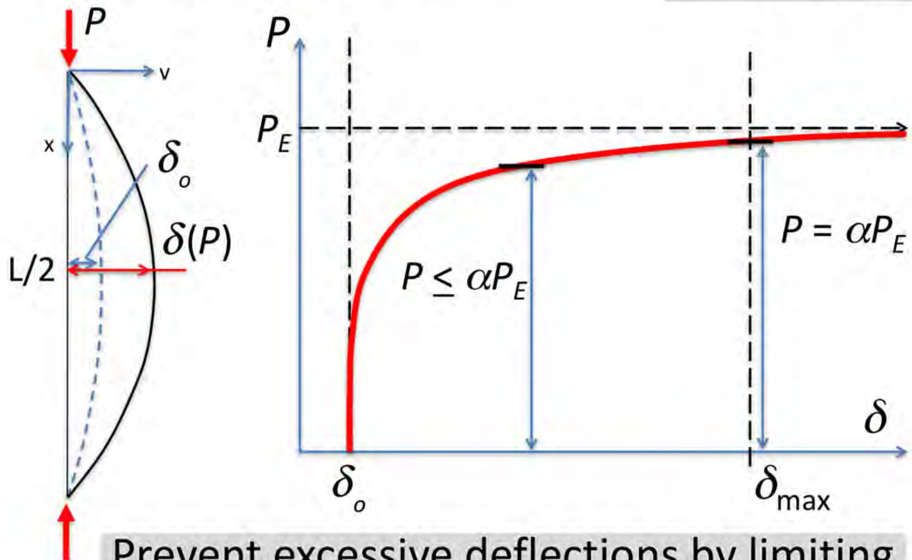
Bending (10)

Column with initial out-of-straightness:



Prevent excessive deflections by limiting P to some proportion of P_E, i.e. $P \leq \alpha P_E$

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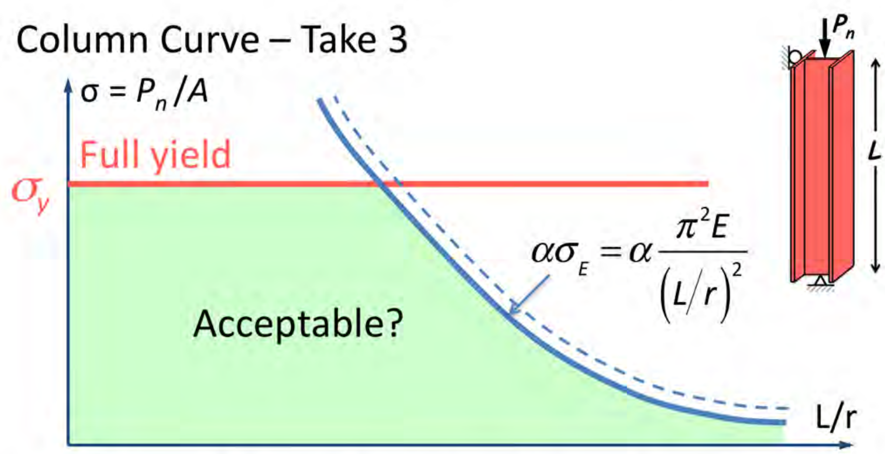
Bending (11)

- Limit elastic bending deflections

$$P \leq \alpha P_E \;\Rightarrow\; \frac{P}{A} \leq \alpha \frac{P_E}{A} \;\Rightarrow\; \sigma \leq \alpha \sigma_E$$

- Column Curve – Take 3



- Consider yielding due to bending plus axial force?

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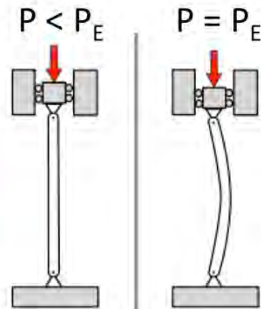
Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions!
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)



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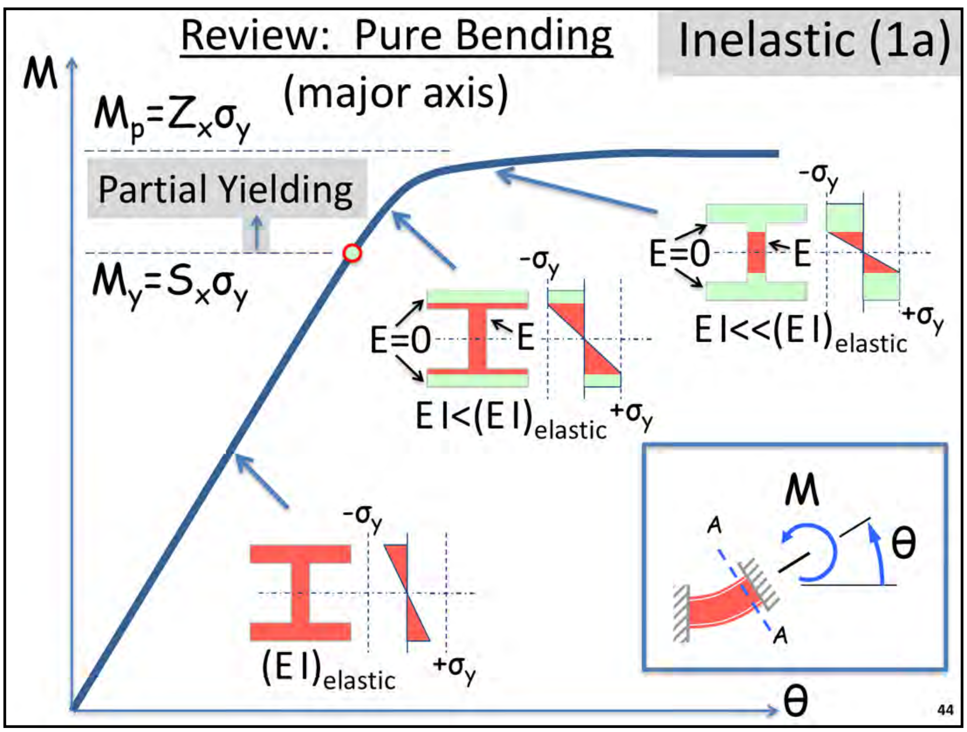

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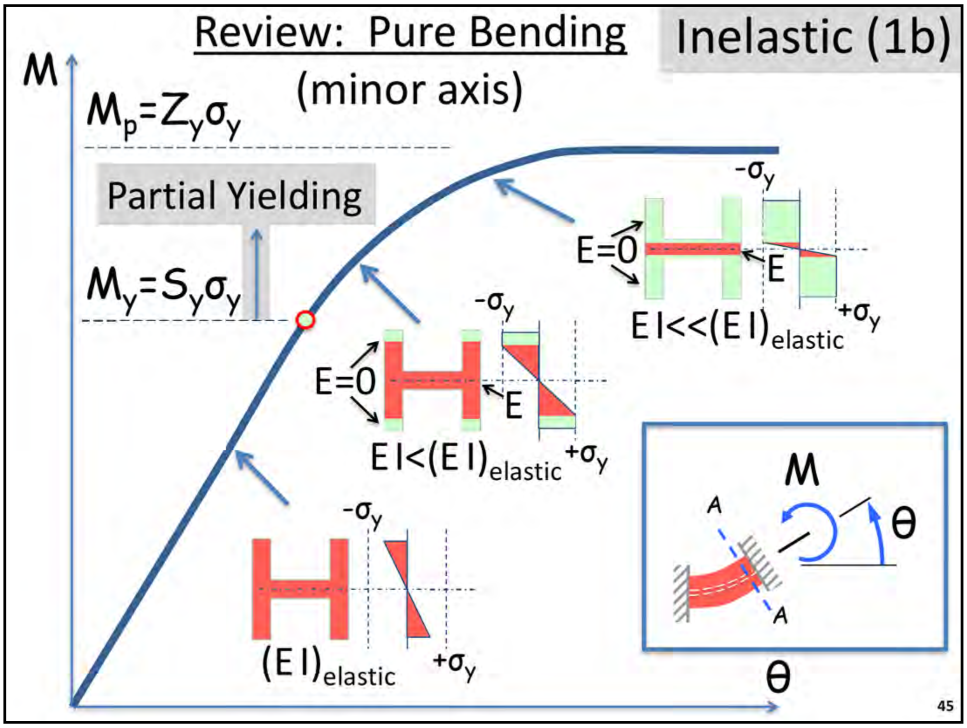
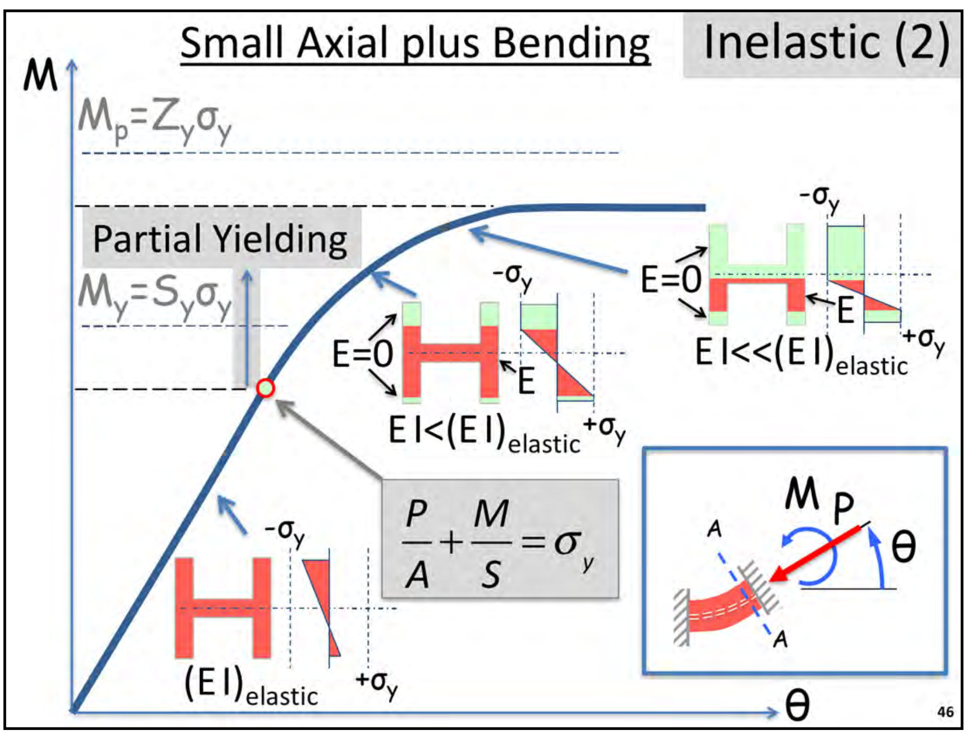
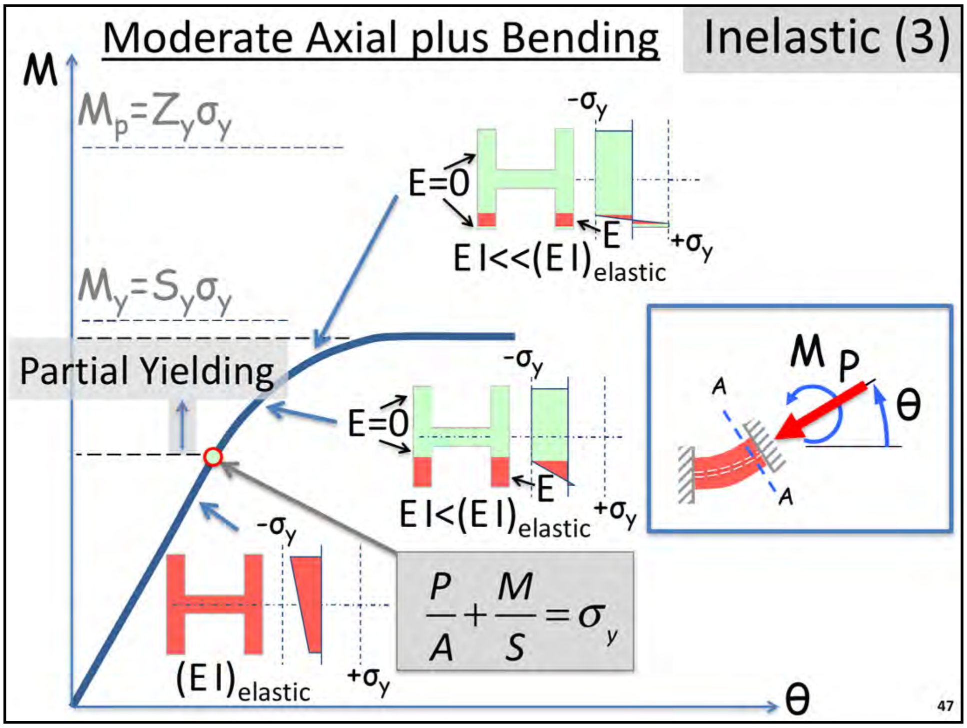
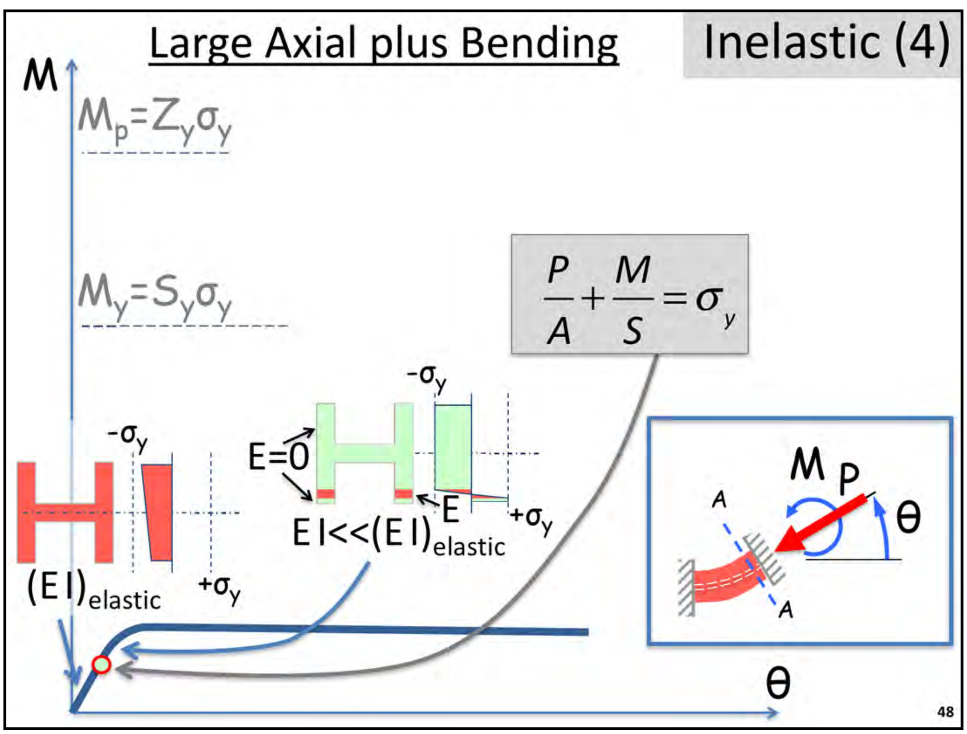
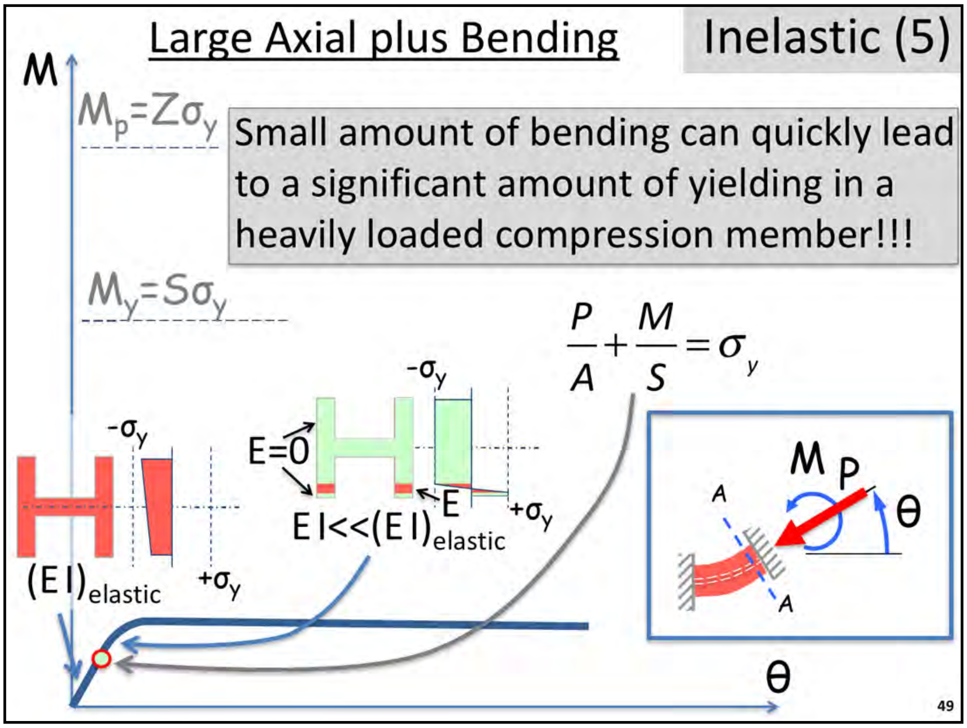

Review: Pure Bending
(minor axis)
Inelastic (1b)
M
$M_p=Z_y\sigma_y$
Partial Yielding
$M_y=S_y\sigma_y$
$-\sigma_y$
E=0
E
$EI<<(EI)_{elastic}$
$+\sigma_y$
$EI<(EI)_{elastic}$
$(EI)_{elastic}$
A
θ
45


Small Axial plus Bending
Inelastic (2)
M
$M_p=Z_y\sigma_y$
Partial Yielding
$M_y=S_y\sigma_y$
$-\sigma_y$
E=0
E
$EI<<(EI)_{elastic}$
$+\sigma_y$
$EI<(EI)_{elastic}$
$\frac{P}{A}+\frac{M}{S}=\sigma_y$
$(EI)_{elastic}$
M P
A
θ
46

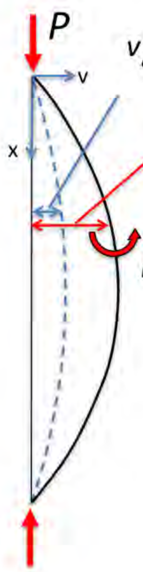

Moderate Axial plus Bending
Inelastic (3)
M
$M_p=Z_y\sigma_y$
$M_y=S_y\sigma_y$
Partial Yielding
E=0
E
$-\sigma_y$
$+\sigma_y$
$EI<<(EI)_{elastic}$
$EI<(EI)_{elastic}$
$(EI)_{elastic}$
$\frac{P}{A}+\frac{M}{S}=\sigma_y$
M P
θ
A
A
θ
47


Large Axial plus Bending
Inelastic (4)
M
$M_p=Z_y\sigma_y$
$M_y=S_y\sigma_y$
$\frac{P}{A}+\frac{M}{S}=\sigma_y$
E=0
E
$-\sigma_y$
$+\sigma_y$
$EI<<(EI)_{elastic}$
$(EI)_{elastic}$
M P
θ
A
A
θ
48



Inelastic (6)

Let's pause and have a closer look at elastic bending:



$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

$$v(x) = \frac{1}{1-\frac{P}{P_E}} v_o(x)$$

$M(x,P)$

Equilibrium:

$$M(x,P) + Pv(x) = 0$$

$$M(x,P) = -Pv(x)$$

$$M(x,P) = -P\frac{1}{1-\frac{P}{P_E}}\delta_o \sin\frac{\pi x}{L}$$

$$M(x,P) = \frac{1}{1-\frac{P}{P_E}}\cdot\left(-P\delta_o \sin\frac{\pi x}{L}\right)$$

Note: amplification factor to account for 2^{nd}-order effects

$$M(x,P) = \frac{1}{1-\frac{P}{P_E}}\cdot M(x,P)^{1st-order}$$

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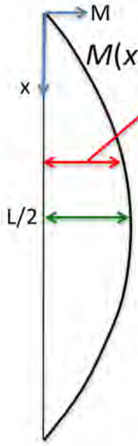
Inelastic (7)

Closer look at that bending:

Elastic *M*-diagram:



$$M(x,P)=\frac{-P}{1-\frac{P}{P_E}}\delta_o \sin\frac{\pi x}{L}$$

$$M(\frac{L}{2},P)=\frac{-P}{1-\frac{P}{P_E}}\delta_o$$

All is good…as long as all is elastic, i.e. no yielding!

$$\left|\frac{P}{A}\right|+\left|\frac{M(x,P)}{S}\right|<\sigma_y$$

But, yielding will occur when

$$\left|\frac{P}{A}\right|+\left|\frac{M(L/2,P)}{S}\right|=\sigma_y$$

or, an axial load *P* that satisfies:

$$\frac{P}{A}+\frac{1}{\left(1-\frac{P}{P_E}\right)}\frac{P\delta_o}{S}=\sigma_y$$

Note: relatively simple equation to compute axial force that produces first yield (excludes σ_{res})

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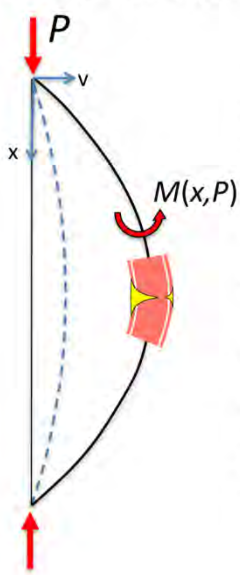
Inelastic (8)

And, once yielding occurs (ouch!):



1. Yielded portion loses stiffness, $EI\downarrow$
2. Increases in deflection, $v(x)\uparrow$
3. Increases moment, $M(x)=P\cdot v(x)\uparrow$
4. Resulting in more yielding…

5. If equilibrium, apply more *P*
6. Repeat above steps 1 to 4
7. Apply more *P* repeating steps 1 to 6 until instability!

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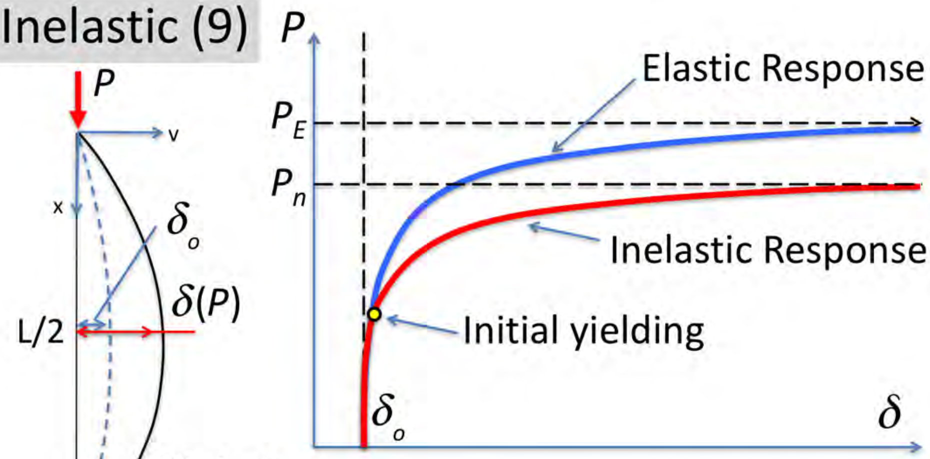
Inelastic (9)



Notes:

1. Inelastic instability occurs below the Euler critical load, i.e. $P_n < P_E$
2. The smaller the column slenderness L/r, the further P_n is below P_E ($L/r \downarrow \; P_E \Uparrow$)

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Inelastic (10)

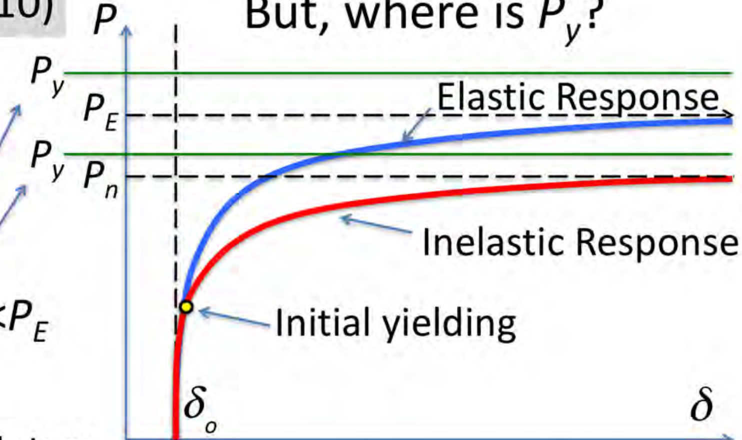
But, where is P_y?

Only two options:

$P_y \geq P_E$

-or-

$P_n \leq P_y < P_E$



Notes:

1. Because of bending, P_n cannot exceed P_y
2. The larger the column slenderness L/r, the more bending, and the further P_n is below P_y

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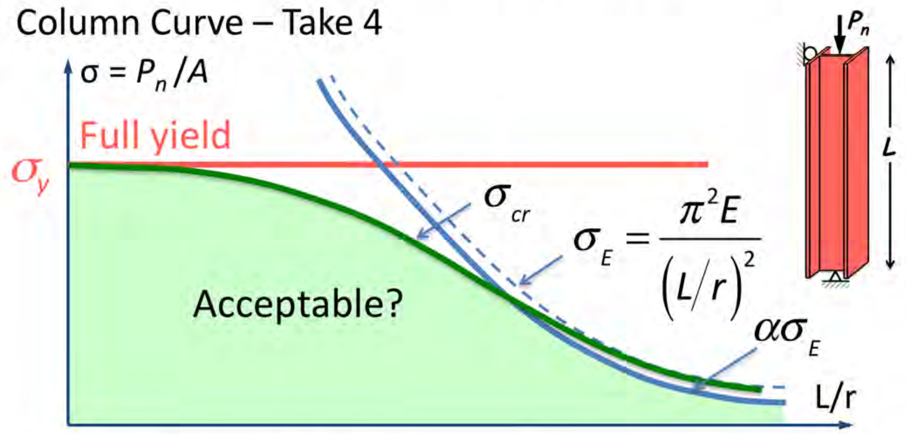
Inelastic (11)

- Axial plus bending may cause yielding

$$\sigma_{cr} = \frac{P_n}{A} \qquad L/r \rightarrow 0,\ \sigma_{cr} = \sigma_y$$

$$L/r \uparrow,\ \sigma_{cr} < \sigma_y \text{ and } \sigma_{cr} < \sigma_E$$

- Column Curve – Take 4



- What about residual stresses?

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Residual Stresses

- Occurs in structural shapes
  - Uneven cooling of hot-rolled shape after rolling
  - Welding of plates for fabricated or built-up shapes
  - Cold bending during fabrication
- Magnitude and distribution of residual stresses depend on the cross-sectional shape and dimensions
- Residual stresses are usually independent of steel yield strength (despite $0.3F_y$ or $0.5F_y$ often included in design equations)
- Thermal residual stresses occur in rolled wide flange shapes because locations with larger surface area (e.g., flange tips) cool well before locations with smaller surface area (flange-to-web intersections)

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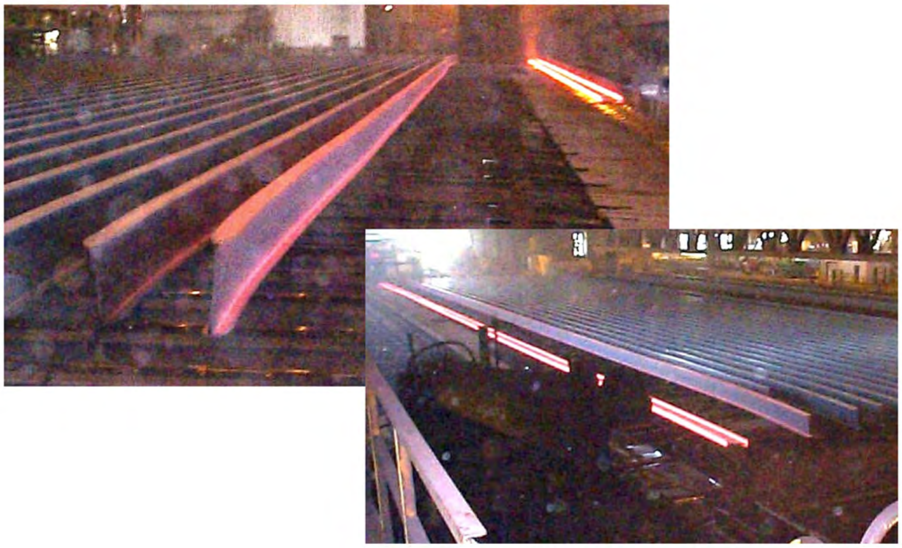
Residual Stresses (2)



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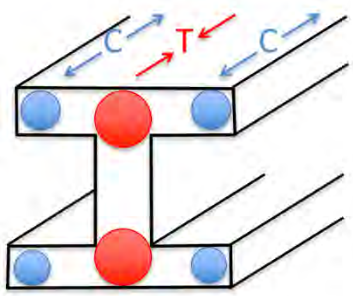
Residual Stresses (3)



1. Entire section hot and starts to cool…lengthwise contraction with $E_o << E$
2. Flange tips (surface area!) cool relatively faster than flange-web intersection (smaller surface) area, $E_{fl} \approx E$
3. Flange-web intersection (smaller surface area) now cools and wants to contract, but flange tips are already set and do not want to contract.
4. Result – locations to cool last end up in tension and equilibrium requires locations that cooled first to end up in compression.

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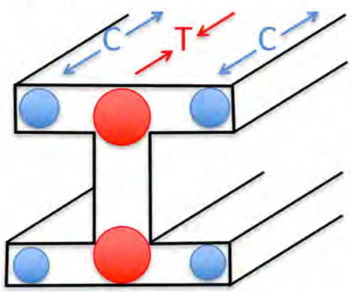
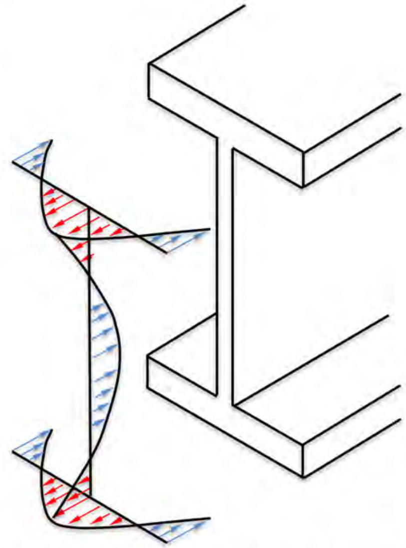
Residual Stresses (4)

From previous slide





Closer to actual distribution

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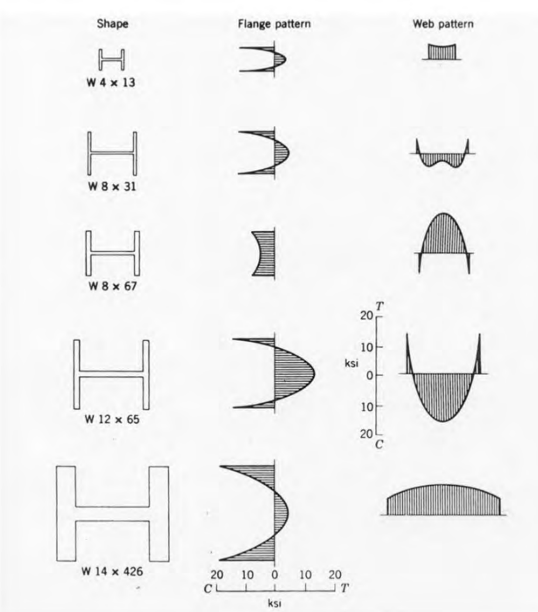
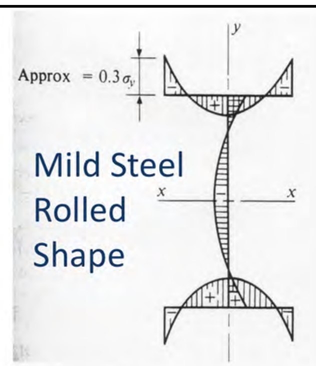
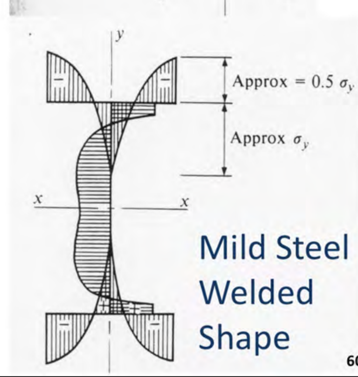
Residual Stresses (5)







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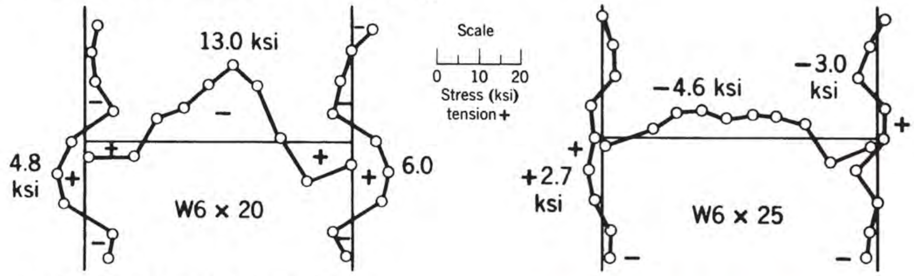
Residual Stresses (6)



Rotary Straightening

1. Standard practice at rolling mills
2. Sections up to W24x370
3. Reduces residual stresses in the flange tips
4. Result: Actual residual stress pattern in as-delivered rolled sections is quite variable

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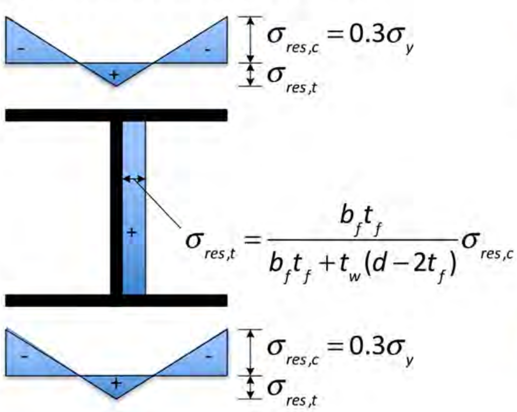
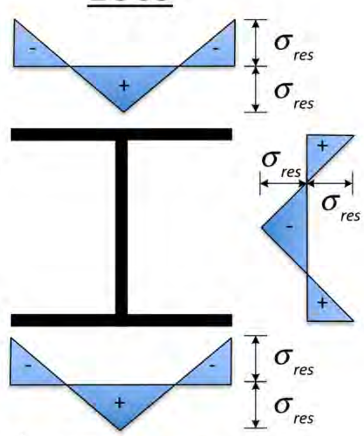
Residual Stresses (7)

Residual Stress patterns often used in prior computational studies:

Galambos and Ketter



ECCS



$$d/b_f \le 1.2 \Rightarrow \sigma_{res} = 0.5\sigma_y$$
$$d/b_f > 1.2 \Rightarrow \sigma_{res} = 0.3\sigma_y$$

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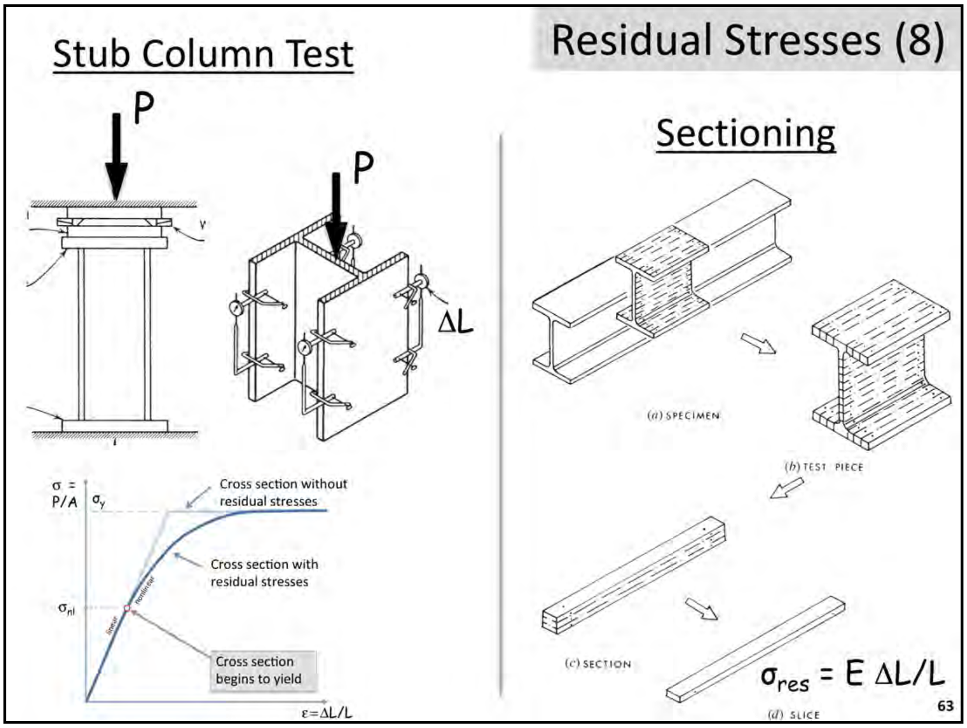
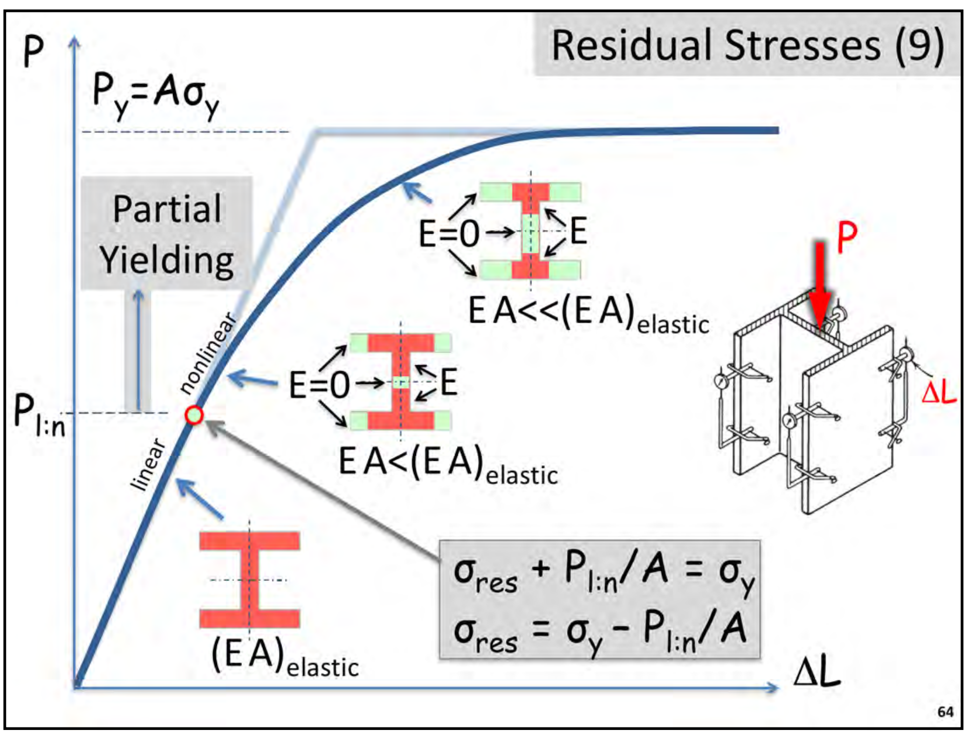
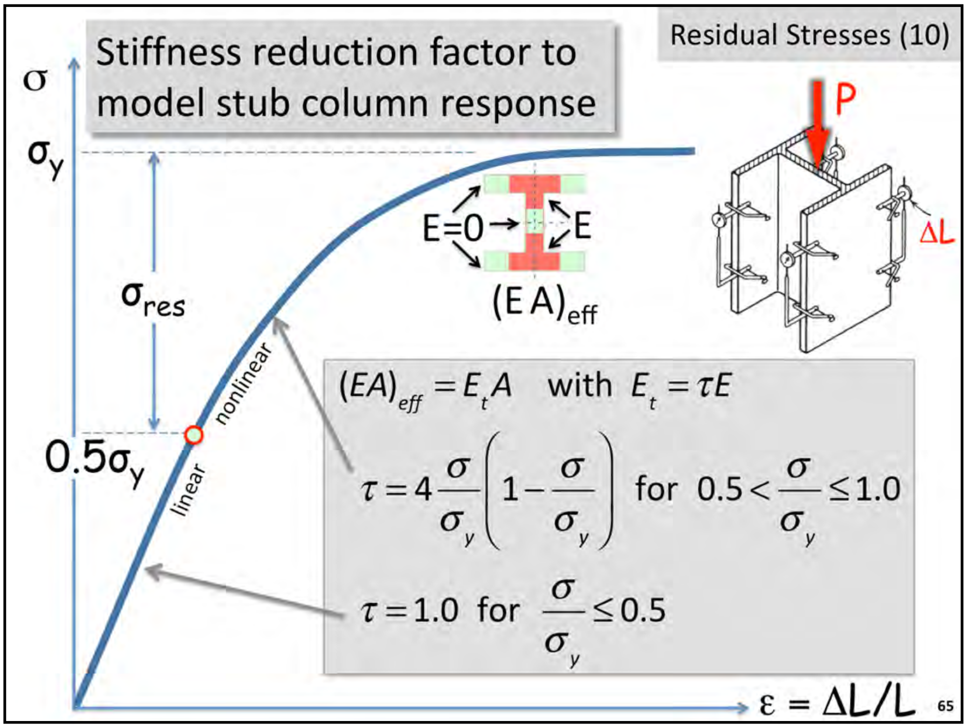

Residual Stresses (8)
Stub Column Test
P
P
ΔL
σ = P/A
σ_y
Cross section without residual stresses
Cross section with residual stresses
σ_nl
Cross section begins to yield
ε=ΔL/L
Sectioning
(a) SPECIMEN
(b) TEST PIECE
(c) SECTION
(d) SLICE
σ_res = E ΔL/L
63


Residual Stresses (9)
P
P_y=Aσ_y
Partial Yielding
E=0
E
EA<<(EA)_elastic
P
nonlinear
E=0
E
EA<(EA)_elastic
ΔL
P_l:n
linear
σ_res + P_l:n/A = σ_y
σ_res = σ_y − P_l:n/A
(EA)_elastic
ΔL
64



Residual Stresses (11)

- Euler -to- Inelastic Buckling Stress

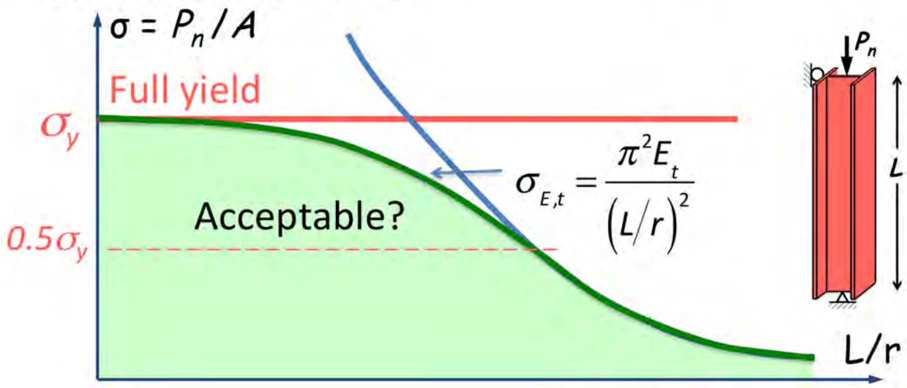
$$\sigma_E = \frac{\pi^2 E}{(L/r)^2} \Rightarrow \sigma_{E,t} = \frac{\pi^2 E_t}{(L/r)^2}$$

$E_t = \tau E$

$\tau = 4\frac{\sigma}{\sigma_y}\left(1-\frac{\sigma}{\sigma_y}\right)$ for $0.5 < \frac{\sigma}{\sigma_y} \le 1.0$

$\tau = 1.0$ for $\frac{\sigma}{\sigma_y} \le 0.5$

- Column Curve – Take 5



- But wait! What about bending?

66

Residual Stresses (12)

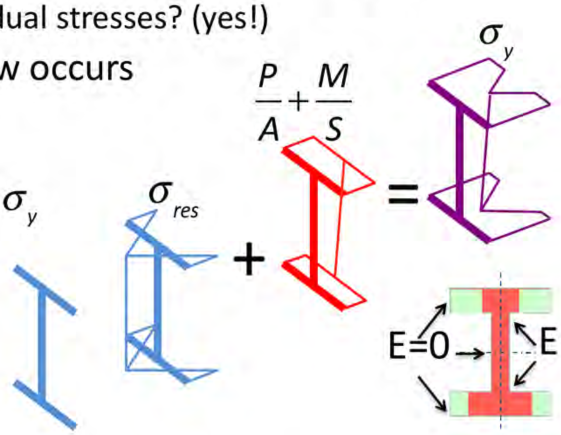
- Compression members include
  - Bending without residual stresses? (no!)
  - No bending with residual stress? (no!)
  - Bending with residual stresses? (yes!)

1950-70's
British Standard
AISC

- Partial yielding now occurs sooner when:

$$\sigma_{res} + \left(\frac{P}{A} + \frac{M}{S}\right) = \sigma_y$$

Note: *M* is due to initial imperfection and/or non-concentric loading



- Partial yielding = loss of flexural stiffness, $EI << EI_{elastic}$

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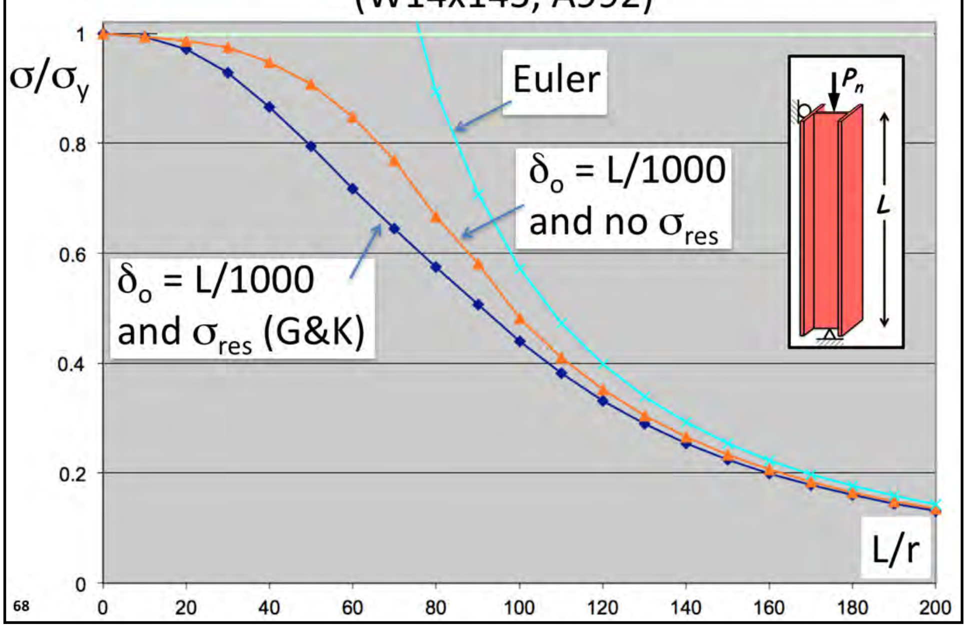
Minor Axis Compressive Strength by FEA
(W14x145, A992)



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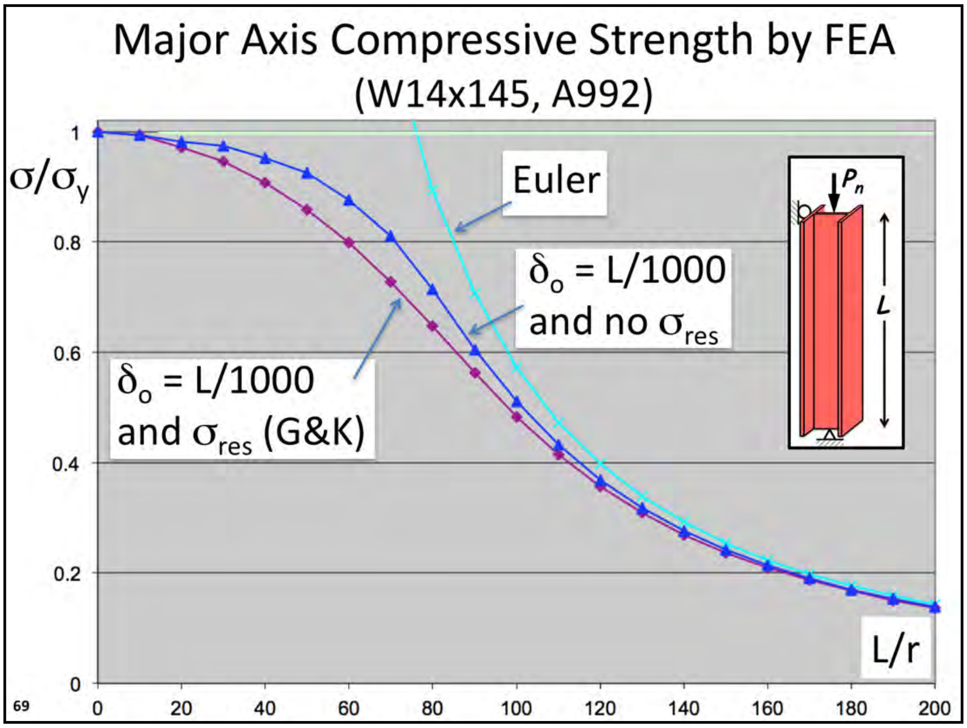
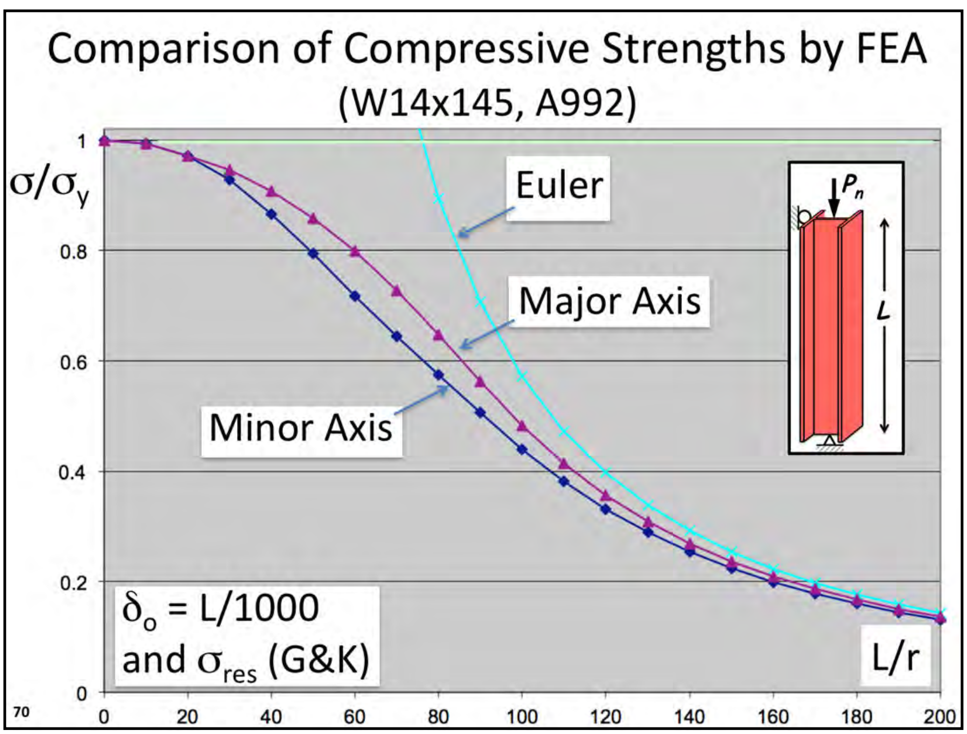

Major Axis Compressive Strength by FEA
(W14x145, A992)
σ/σy
Euler
δo = L/1000
and no σres
δo = L/1000
and σres (G&K)
Pn
L
L/r
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
120
140
160
180
200
69


Comparison of Compressive Strengths by FEA
(W14x145, A992)
σ/σy
Euler
Major Axis
Minor Axis
δo = L/1000
and σres (G&K)
Pn
L
L/r
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
120
140
160
180
200
70

Compressive Strength Curves

- Key observations from FEA
  - Strength reduced for initial imperfection and further reduced for residual stresses
  - All curves approach Euler, but are slightly below
  - Partial yielding accentuated by residual stresses impact minor axis strength more than major axis strength
  - Different strength curves for major and minor axis bending
- Additional thoughts
  - Strength curves for W-shapes are function of dimensions, and thus will vary depending on W-shape
  - Other shapes (e.g., HSS, C's, and built-up shapes) will also have different compressive strength curves

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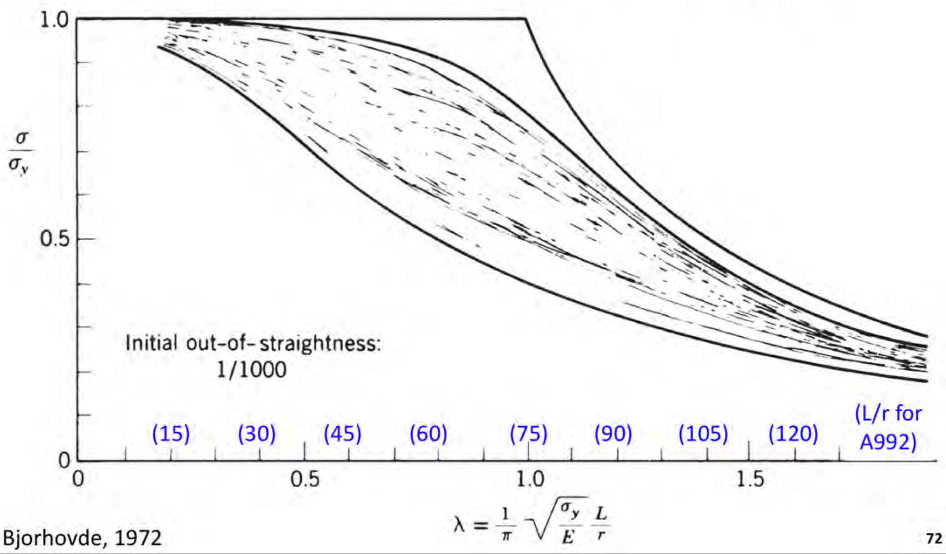
Maximum Compressive Strength Curves for Many Different Column Types



Bjorhovde, 1972

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Column Curves for Design

- AISC employs a single curve "fit" to experimental and analytical data. Other codes use multiple curves.
- Background to AISC curve:
  - Hall. D. H. (1981), "Proposed Steel Column Design Criteria." ASCE J. Struct. Div.. Vol. 107. No. ST4.
  - Tide, R.H.R. (1985), "Reasonable Column Design Equations," Proc. SSRC Annual Tech. Session.
  - Tide, R.H.R. (2001), "A Technical Note: Derivation of the LRFD Column Design Equations," Engineering Journal, AISC, Vol. 38, No. 3, 3rd Quarter

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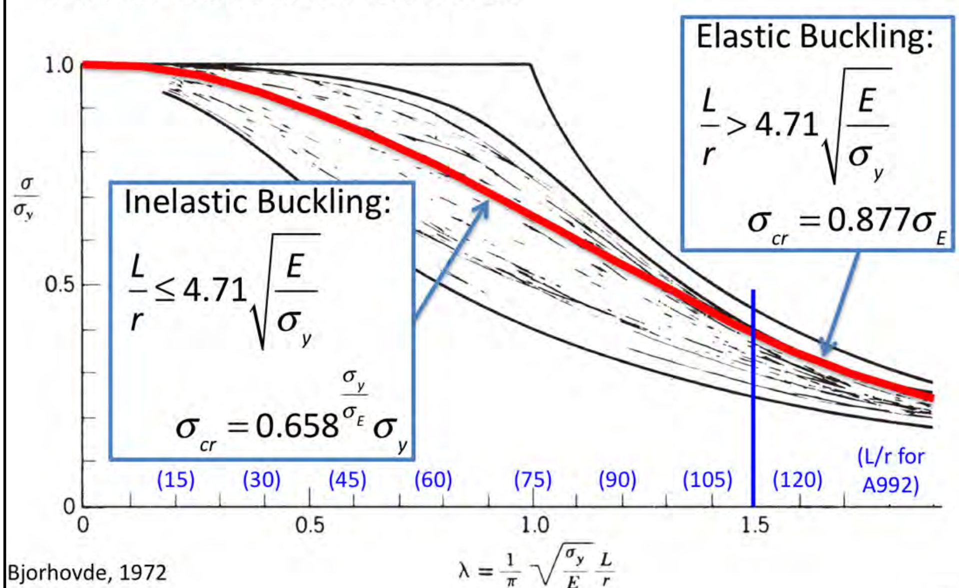
AISC Column Curve:



Bjorhovde, 1972

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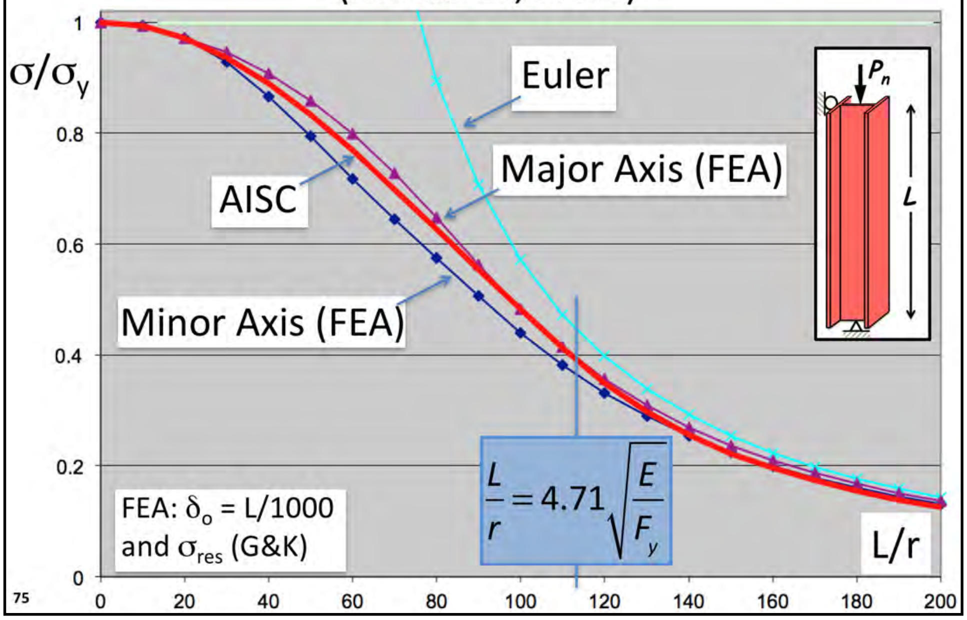
Comparison of Compressive Strengths

(W14x145, A992)



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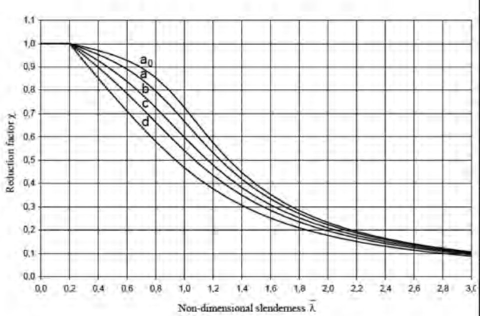
Multiple Column Curves:

Eurocode 3



For discussion and key references see Ziemian, R.D. (ed.) (2010), *Guide to Stability Design Criteria for Metal Structures*, 6th Ed., John Wiley & Sons, Inc., Hoboken, NJ.

Cross section		Limits	Buckling about axis	Buckling curve S 235, S 275, S 355, S 420	Buckling curve S 460
Rolled sections	$h/b > 1{,}2$	$t_f \le 40$ mm	y – y z – z	a b	a_0 a_0
		40 mm < $t_f \le 100$	y – y z – z	b c	a a
	$h/b \le 1{,}2$	$t_f \le 100$ mm	y – y z – z	b c	a a
		$t_f > 100$ mm	y – y z – z	d d	c c
Welded I-sections		$t_f \le 40$ mm	y – y z – z	b c	b c
		$t_f > 40$ mm	y – y z – z	c d	c d
Hollow sections		hot finished	any	a	a_0
		cold formed	any	c	c
Welded box sections		generally (except as below)	any	b	b
		thick welds: $a > 0{,}5t_f$ $b/t_f < 30$ $h/t_w < 30$	any	c	c
U-, T- and solid sections			any	c	c
L-sections			any	b	b

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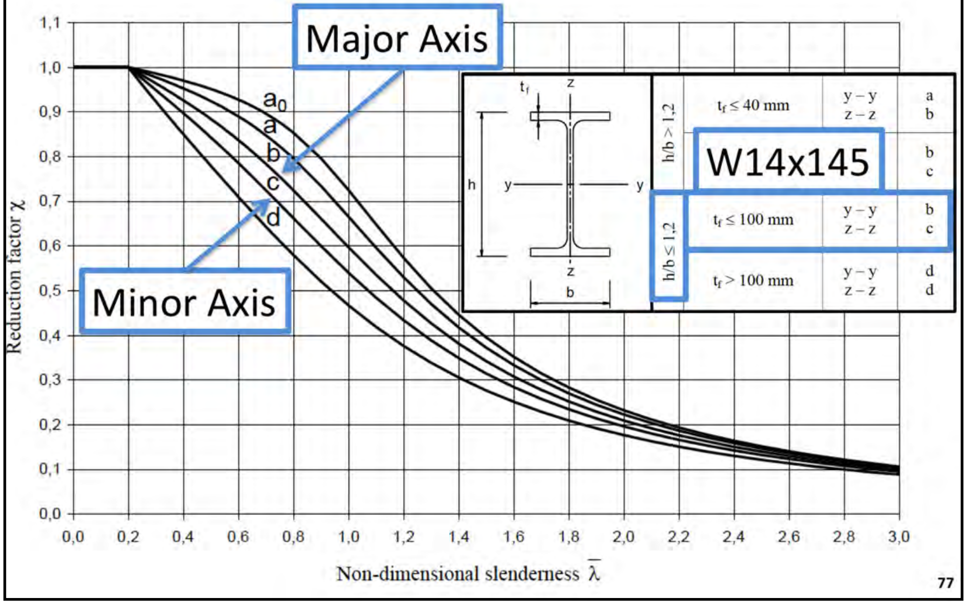
Multiple Column Curves: Eurocode 3



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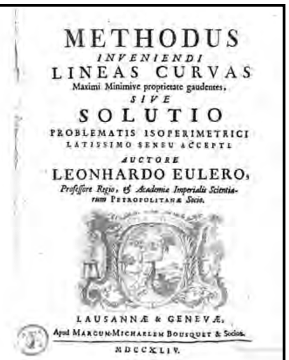
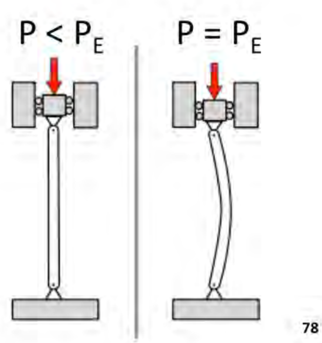
Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)





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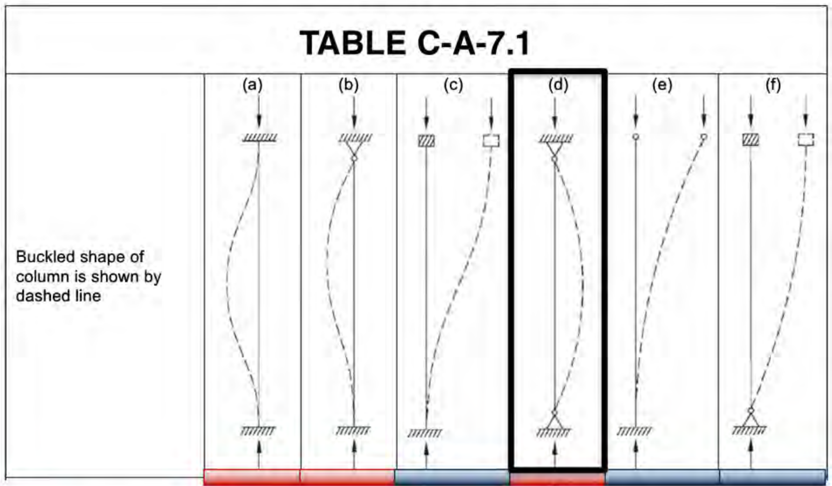
Support Conditions



No-sway (braced)

Sway (unbraced)

Euler Buckling

What about the others?

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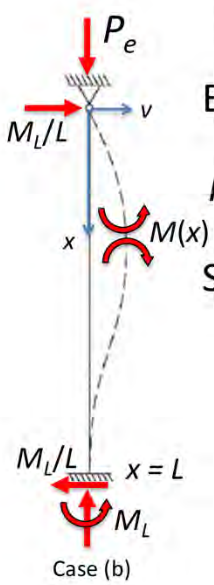
Support Conditions (2)



Equilibrium → Constitutive → Diff. Eq.

$$M(x) + P_e v(x) = \frac{M_L}{L}x \quad \Rightarrow \quad EI\frac{d^2v}{dx^2} + P_e v = \frac{M_L x}{L}$$

Solution:

$$v(x) = C_1 \cos\left(\sqrt{\frac{P_e}{EI}}x\right) + C_2 \sin\left(\sqrt{\frac{P_e}{EI}}x\right) + \frac{M_L x}{P_e L}$$

wolframalpha.com
a2*y"(x)+a1*y(x)=a3*x

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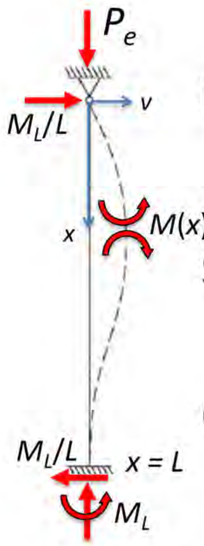
Support Conditions (3)



Case (b)

Equilibrium → Constitutive → Diff. Eq.

$$M(x) + P_e v(x) = \frac{M_L}{L}x \;\Rightarrow\; EI\frac{d^2v}{dx^2} + P_e v = \frac{M_L x}{L}$$

Solution:

$$v(x) = C_1 \cos\left(\sqrt{\frac{P_e}{EI}}x\right) + C_2 \sin\left(\sqrt{\frac{P_e}{EI}}x\right) + \frac{M_L x}{P_e L}$$

Compatibility (Boundary Conditions):

$$v(x=0) = 0,\; v(x=L) = 0,\; v'(x=L) = 0$$

$$P_e = \frac{\pi^2 EI}{(0.70L)^2} \;\Rightarrow\; \sigma_e = \frac{P_e}{A} = \frac{\pi^2 E}{(KL/r)^2} \text{ with } K = 0.70$$

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Support Conditions (4)

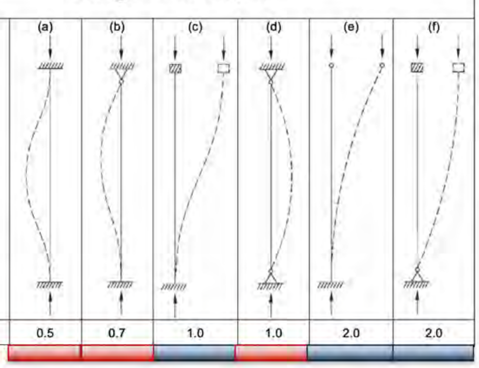

TABLE C-A-7.1
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0

No-sway (braced)

Sway (unbraced)

Elastic Buckling Stress:

$$\sigma_e = \frac{\pi^2 E}{(KL/r)^2}$$

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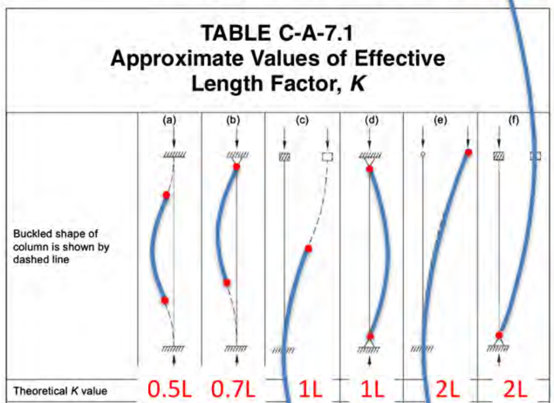
Support Conditions (5)



Elastic Buckling Stress:

$$\sigma_e = \frac{\pi^2 E}{\left(KL/r\right)^2}$$

Notes on "effective length" *KL*:

- Find the Euler column?!
- Distance between inflection points (M=0)

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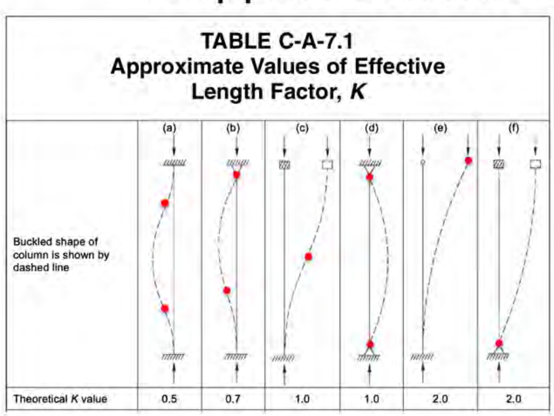
Support Conditions (6)



Elastic Buckling Stress:

$$\sigma_e = \frac{\pi^2 E}{\left(KL/r\right)^2}$$

Notes on "effective length" *KL*:

- Distance between inflection points (M=0)
- Function of degree of column end-restraint
- Degree of column end-restraint can be difficult to compute accurately in real structures (hmmm…)

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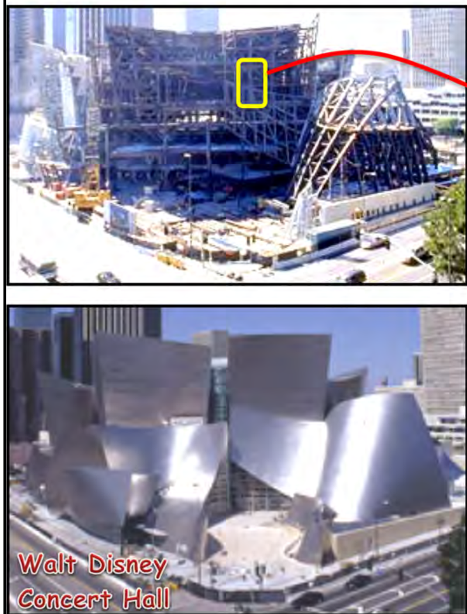
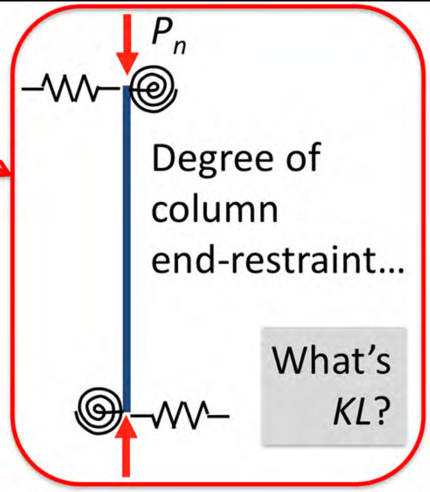
Support Conditions (7)





Possible solutions:

- Diff. Eq./Eigenvalue FEA
- Alignment charts (careful!)
- Direct Analysis Method!

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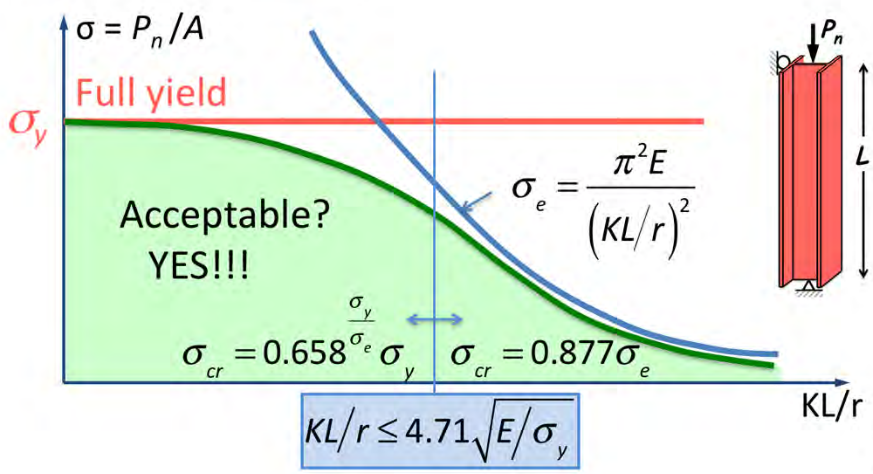
Support Conditions (4)

- Degree of column end restraint accounted for by use of "effective length" KL (i.e., $\sigma_E \rightarrow \sigma_e$)
- AISC Column Curve – Final Take!



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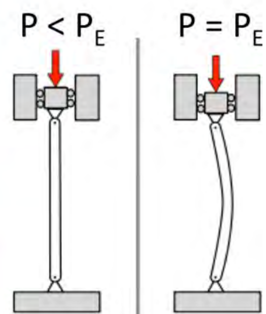
Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions!
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)

Undone

Undone

Undone



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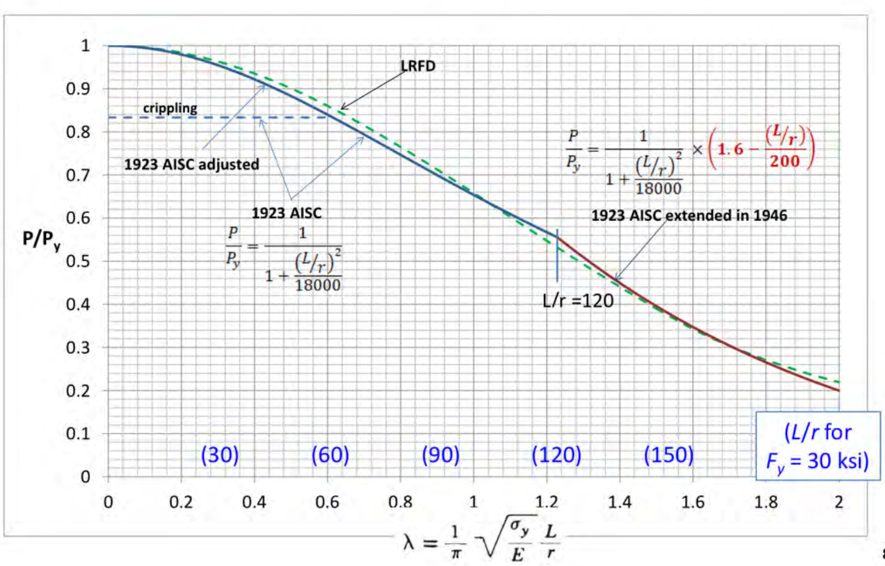
Another interesting observation...



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Polling Question

In regard to defining a column's strength, which of the following is false?

a. In using a single strength curve, the AISC Specification does not account for potential differences in major and minor axis flexural buckling strengths of I-shaped members.
b. The AISC Specification's column curve accounts for an initial imperfection, such as out-of-straightness.
c. The AISC Specification's column curve accounts for partial yielding accentuated by the presence of residual stresses.
d. The AISC Specification's column curve accounts for various end support conditions.
e. The AISC Specification's column curve provides an exact prediction for all I-shaped sections appearing in the AISC manual.

Summary – Compression

- Course introduction and stability concepts
- Limit states of compression members with focus on flexural buckling
- Euler Buckling → Maximum Compressive Strength Column Curve
- Column curve accounts for:
  - full yielding
  - bending due to initial imperfection (out-of-straightness)
  - partial yielding accentuated by presence of residual stresses
  - degree of end restraint

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Summary – Compression (cont.)

- AISC and other column curves
- Other ideas introduced, including
  - moment amplification factor (2nd-order effects)
  - stiffness reduction τ-factor
  - complexity in computing K-factors…

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Up Next…

- Session 2: June 12 –
  Design of Compression Members
  by P.S. Green, PE, PhD
- Initially, an overview of flexural, torsional, and flexural-torsional resistance of individual column members will be provided. Emphasis then will be placed on defining and assessing the AISC LRFD and ASD strengths of various structural shapes, including wide flange, round and square HSS, cruciform, equal and unequal single and double leg angles, WT, channel, and built-up shapes.

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Individual Webinar Registrants

CEU/PDH Certificates

Within 2 business days…

- You will receive an email on how to report attendance from: registration@aisc.org.
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!

Individual Webinar Registrants

CEU/PDH Certificates

Within 2 business days…

- New reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.

8-Session Registrants

CEU/PDH Certificates

One certificate will be issued at the conclusion of all 8 sessions.

8-Session Registrants

Access to the quiz: Information for accessing the quiz will be emailed to you by Wednesday. It will contain a link to access the quiz. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG

Quiz and Attendance records: Posted Tuesday mornings.
www.aisc.org/nightschool - click on Current Course Details.

Reasons for quiz:

- EEU – must take all quizzes and final to receive EEU
- CEUs/PDHS – If you watch a recorded session you must take quiz for CEUs/PDHs.
- REINFORCEMENT – Reinforce what you learned tonight. Get more out of the course.

NOTE: If you attend the live presentation, you do not have to take the quizzes to receive CEUs/PDHs.

8-Session Registrants

Access to the recording: Information for accessing the recording will be emailed to you by this Wednesday. The recording will be available for three weeks. For 8-session registrants only. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG.

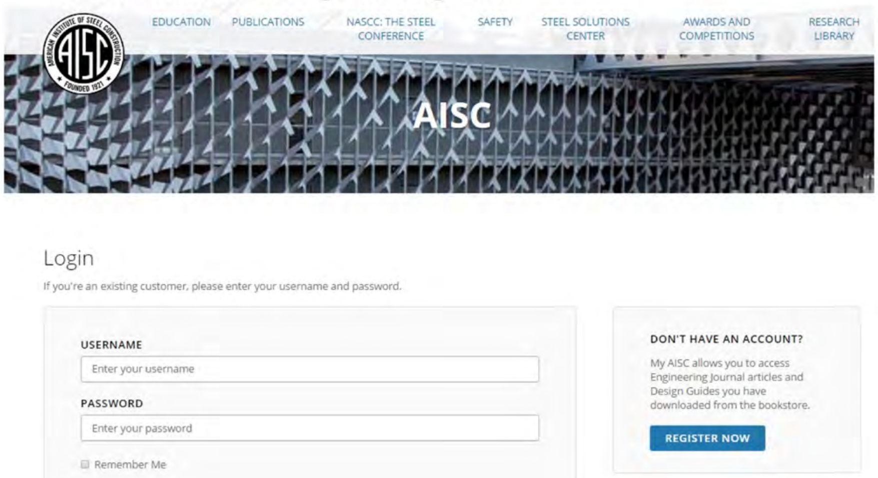
CEUs/PDHS – If you watch a recorded session you must take AND PASS the quiz for CEUs/PDHs.

Night School Resources for 8-session package Registrants

Find all your handouts, quizzes and quiz scores, recording access, and attendance information all in one place!

Night School Resources for 8-session package Registrants

Go to www.aisc.org and sign in.



Night School Resources for 8-session package Registrants

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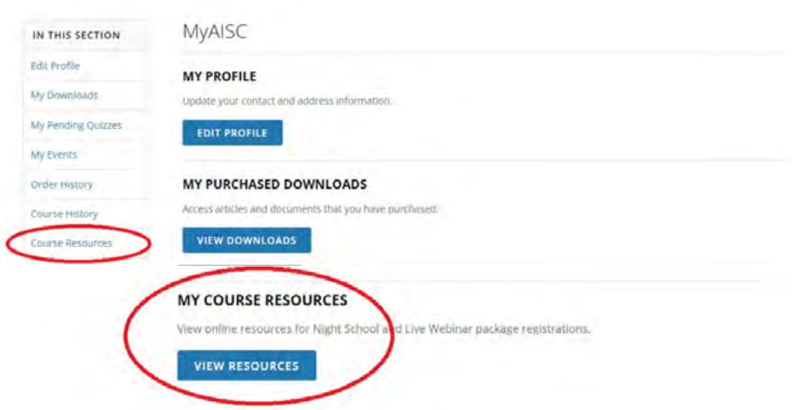

Night School Resources for 8-session package Registrants

Course Resources

Event	Start Date
NS 13 8-Session Package-Night School 13 - Design of Industrial Buildings	1/30/2017 7:00:00 PM
NS 14 8-Session Package-Night School 14 - Fundamentals of Stability	6/5/2017 7:00:00 PM

Night School Resources for 8-session package Registrants

Night School 13: Design of Industrial Buildings

8-SESSION PACKAGE RESOURCES

Event	Date	Handouts	Video	Quiz	Attendance
NS13 - Design Criteria	1/30/2017 7:00:00 PM	Handouts	View Passcode: NS13DSN	Pass Score: 80	Pending
NS13 - Economic Considerations	2/6/2017 7:00:00 PM	Handouts	Available 02/08/2017 5pm EST	Available 02/08/2017 5pm EST	Pending
NS13 - Lateral Load Systems and Details	2/13/2017 7:00:00 PM	Handouts	Available 02/15/2017 5pm EST	Available 02/15/2017 5pm EST	Pending
NS13 - Preliminary Design Procedures	2/27/2017 7:00:00 PM	Handouts	Available 03/01/2017 5pm EST	Available 03/01/2017 5pm EST	Pending
NS13 - Crane Girder Design and Frame Analysis	3/6/2017 7:00:00 PM	Handouts	Available 03/08/2017 5pm EST	Available 03/08/2017 5pm EST	Pending
NS13 - Frame Member and Connection Design	3/13/2017 7:00:00 PM	Handouts	Available 03/15/2017 5pm EST	Available 03/15/2017 5pm EST	Pending
NS13 - Transfer Crane Girder & Longitudinal Bldg Bracing Dsn	3/27/2017 7:00:00 PM	Handouts	Available 03/29/2017 5pm EST	Available 03/29/2017 5pm EST	Pending
NS13 - Building Envelope and Bracing Design	4/3/2017 7:00:00 PM	Handouts	Available 04/05/2017 5pm EST	Available 04/05/2017 5pm EST	Pending
NS13 - Final Exam	4/10/2017 7:00:00 PM			Available 04/12/2017 5pm EST	

Night School Resources for 8-session package Registrants

- Weekly "quiz and recording" email.
- Weekly updates of the master Quiz and Attendance record found at www.aisc.org/nightschool. Scroll down to Quiz and Attendance records.
  - Updated on Tuesday mornings.

Night School Resources for 8-session package Registrants

- Webinar connection information:
  - Found in your registration confirmation/receipt.
  - Reminder email sent out Monday mornings.
- Link to handouts also found here.


There's always a solution in steel.
Thank You
Please give us your feedback!
Survey at conclusion of webinar.
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