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Course Description

Stability of Structural Systems / Beam-Columns
July 17, 2017

This lecture will begin with a review of basic concepts related to the stability of structural systems, and specifically beam-columns. With an eye towards design, the difference between a bifurcation or critical load analysis and the loss in stiffness due to second-order effects and material yielding, as the maximum resistance of physical structures is approached, will be emphasized. The lecture will conclude with an overview of the direct analysis and effective length methods.

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Learning Objectives

- Explain the development of the AISC interaction equation.
- List the five effects that must be considered for stability per the AISC *Specification*.
- Explain second-order effects and the definitions of P-δ and P-Δ.
- Compare and contrast the direct analysis method and the effective length method.

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Fundamentals of Stability for Steel Design

Session 5: Stability of Structural Systems / Beam-Columns

July 17, 2017

Presented by
Ronald D. Ziemian, Ph.D., P.E.
Professor
Bucknell University, Lewisburg, PA

structural STEEL

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Fundamentals of Stability for Steel Design

Session 5
Course Overview and Stability of Structural Systems / Beam-Columns

Ronald D. Ziemian, P.E., Ph.D.

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Course Overview

- Session Topics
  - Compression Members (1 & 2)
  - Flexural Members (3 & 4)
  - Systems / Beam-Columns (5 & 6)
  - Bracing (7 & 8)
- Topics in two parts
  - Behavior (1, 3, 5, 7)
  - Design (2, 4, 6, 8)
- Lectures by members of the Structural Stability Research Council (SSRC)
  - P.S. Green, T.A. Helwig, D.W. White, J.A. Yura, R.D. Ziemian
  - Great to join AISC in this effort!
- Focus of the course is on fundamentals!
- Better understanding of behavior will result in improved design

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Course Overview (2)

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Course Overview (5)

Let's continue to start at the end…

ANSI/AISC 360-16
An American National Standard

Specification for Structural Steel Buildings

C1. GENERAL STABILITY REQUIREMENTS

Stability shall be provided for the structure as a whole and for each of its elements. The effects of all of the following on the stability of the structure and its elements shall be considered: (a) flexural, shear and axial member deformations, and all other component and connection deformations that contribute to the displacements of the structure; (b) second-order effects (including P-Δ and P-δ effects); (c) geometric imperfections; (d) stiffness reductions due to inelasticity, including the effect of partial yielding of the cross section which may be accentuated by the presence of residual stresses; and (e) uncertainty in system, member, and connection strength and stiffness. All load-dependent effe… sponding to LRFD load combinat…

Why these for the Big 5?

Any rational method of design for stability that considers all of the listed effects is permitted; this includes the methods identified in Sections C1.1 and C1.2.

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Fundamentals of Stability for Steel Design

Session 5
Course Overview and
Stability of Structural Systems / Beam-Columns

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Basis for Design of Systems

- Elastic Analysis (AISC Spec., Chs. A-K, Apps. 6-8)
  - Allows for no force redistribution due to yielding
  - Strength (stability) of system is indirectly assessed by assessing strength of its components
  - In other words, strength of system is assured by ensuring adequate strength of its components
- Inelastic Analysis (AISC Spec., Appendix 1)
  - Force redistribution due to yielding is accounted for in the analysis
  - System strength (stability) can be assessed directly by the analysis

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Basis for Design of Systems

- Elastic Analysis (AISC Spec., Chs. A-K, Apps. 6-8)
  - Allows for no force redistribution due to yielding
  - Strength (stability) of system is indirectly assessed by assessing strength of its components
  - In other words, strength of system is assured by ensuring adequate strength of its components

Elastic Analysis → Required Strength ≤ Design Strength ← Stability

Stability:
- Column
- Beams
- Beam-columns

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Limit States of Beam-Columns

- Full yielding (today!)
- Instability
  - Along the member length (= Beam + Column)
    - Lateral-torsional buckling (lectures on flexure)
    - Flexural buckling (lectures on compression)
    - Torsional-flexural buckling (today!)
  - At the cross section
    - local buckling

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From Section Strength to Member Strength:

Development of AISC Interaction Equation for beam-columns (sort of!)

Limit the strength to initial yield:

$$\sigma_{res} + \frac{P}{A} + \frac{M}{S} \le \sigma_y \quad \Rightarrow \quad \frac{\sigma_{res}}{\sigma_y} + \frac{P}{A\sigma_y} + \frac{M}{S\sigma_y} \le 1.0$$

Important Note!
No resistance ϕ-factors or safety Ω-factors are included in today's lecture…learn behavior based on nominal strengths

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From Section Strength to Member Strength:

Development of AISC Interaction Equation for beam-columns (sort of!)

Limit the strength to initial yield:

$$\sigma_{res} + \frac{P}{A} + \frac{M}{S} \le \sigma_y \quad \Rightarrow \quad \frac{\sigma_{res}}{\sigma_y} + \frac{P}{A\sigma_y} + \frac{M}{S\sigma_y} \le 1.0$$

Cross section strength (full yield):

$$\frac{P}{P_y} \ge 0.2 \qquad \frac{P}{P_y} + \frac{8}{9}\frac{M}{M_p} \le 1.0$$

$$\frac{P}{P_y} < 0.2 \qquad \frac{1}{2}\frac{P}{P_y} + \frac{M}{M_p} \le 1.0$$

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From Section Strength to Member Strength:

Development of AISC Interaction Equation for beam-columns (sort of!)

Limit the strength to initial yield:

$$\sigma_{res} + \frac{P}{A} + \frac{M}{S} \le \sigma_y \quad \Rightarrow \quad \frac{\sigma_{res}}{\sigma_y} + \frac{P}{A\sigma_y} + \frac{M}{S\sigma_y} \le 1.0$$

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$$\frac{P}{P_y} < 0.2 \qquad \frac{1}{2}\frac{P}{P_y} + \frac{M}{M_p} \le 1.0$$

Member strength:

$$\frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \le 1.0 \quad \text{(H1-1a)}$$

$$\frac{1}{2}\frac{P}{P_n} + \frac{M}{M_n} \le 1.0 \quad \text{(H1-1b)}$$

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Beam-Column Strengths

- Cross section strength (small *L*/*r*)
  - full yield
  - local buckling
- <u>Elastic</u> member strength (large *L*/*r*)
  - compressive (*P*) : flexural buckling (or torsional or flexural torsional)
  - flexural (M_{major}): lateral torsional buckling
  - torsional-flexural buckling ($P + M_{major}$)
- <u>Inelastic</u> member strength (intermediate *L*/*r*)
  - same possible failure modes as elastic, except reduced due to partial yielding accentuated by presence of initial imperfections (geometric and residual stresses)

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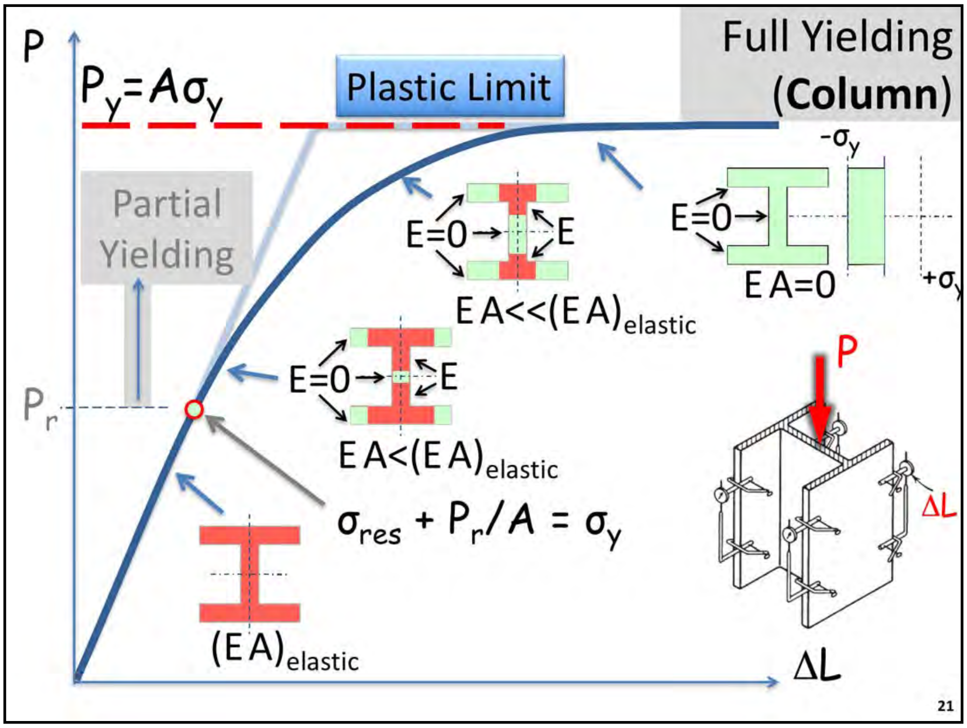

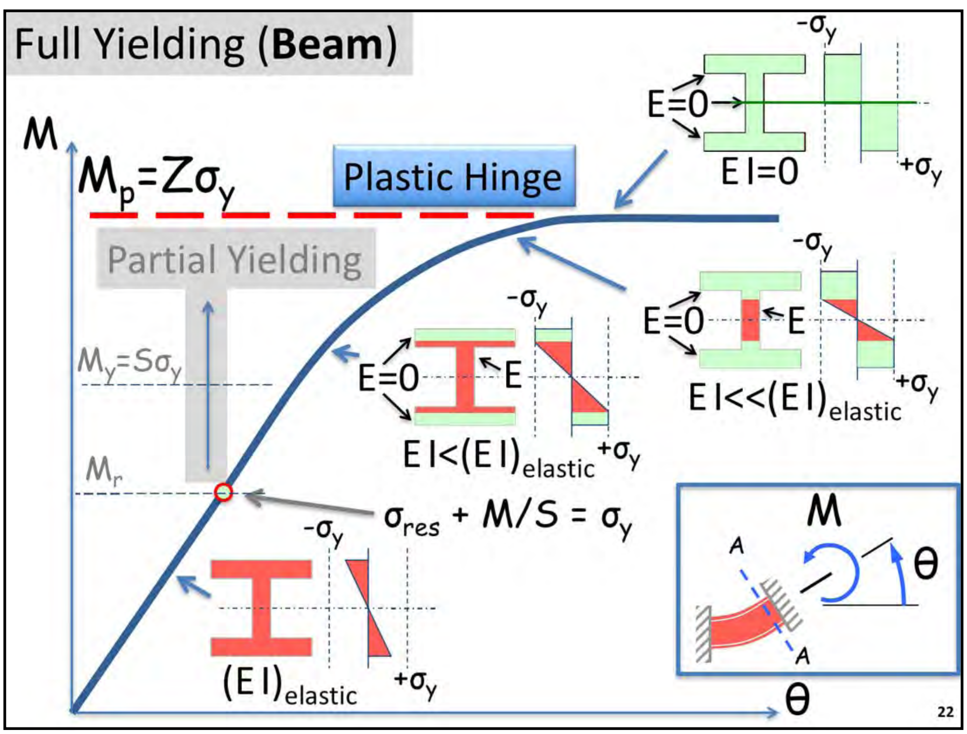

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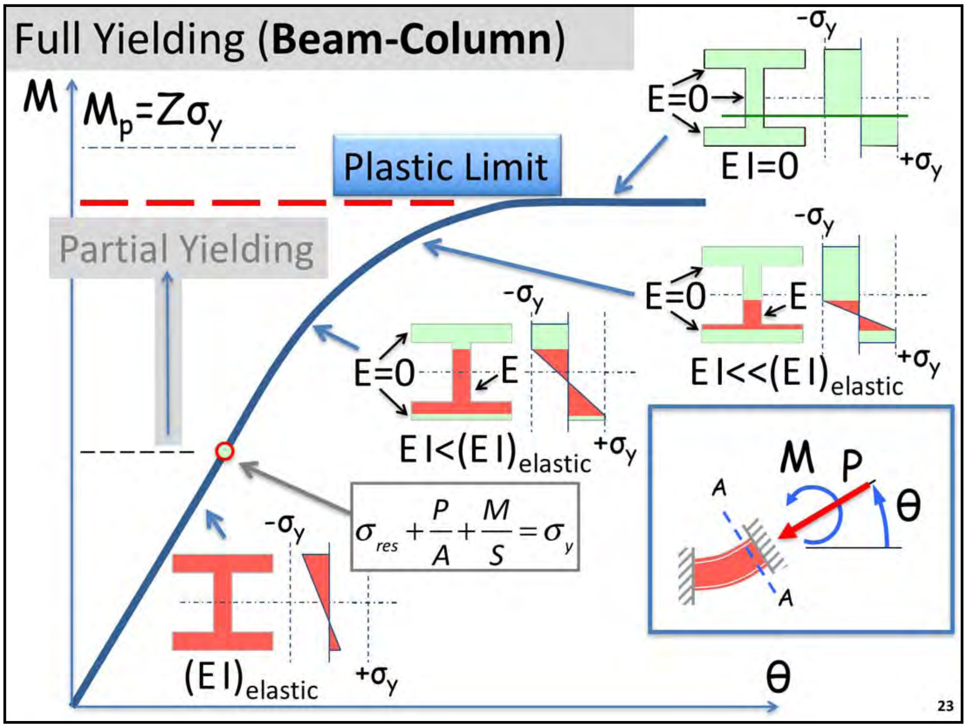

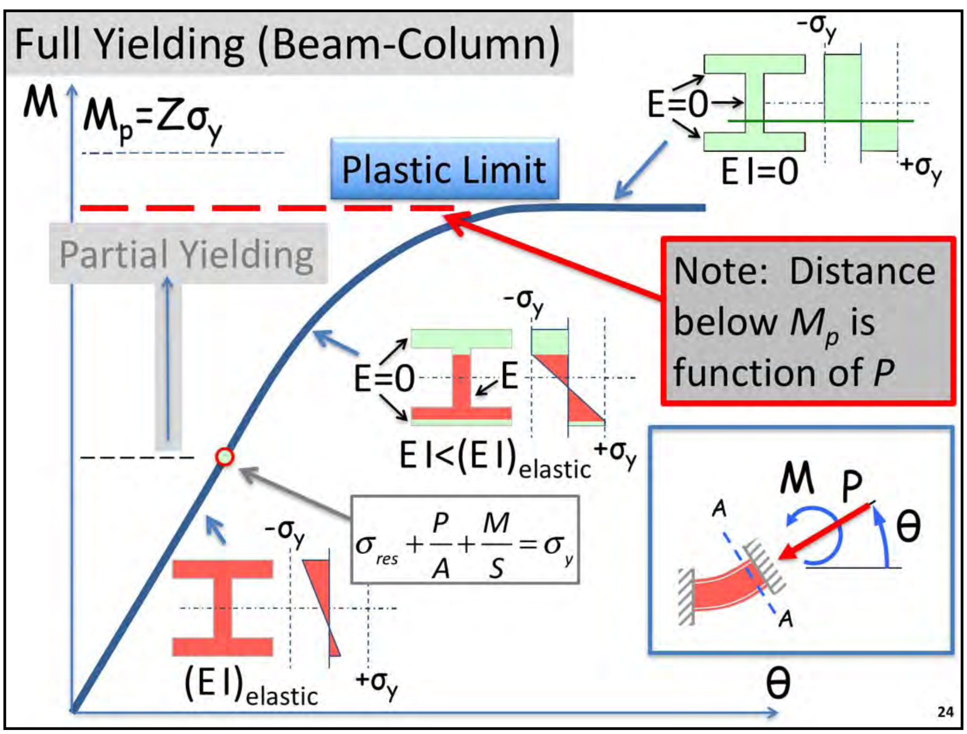

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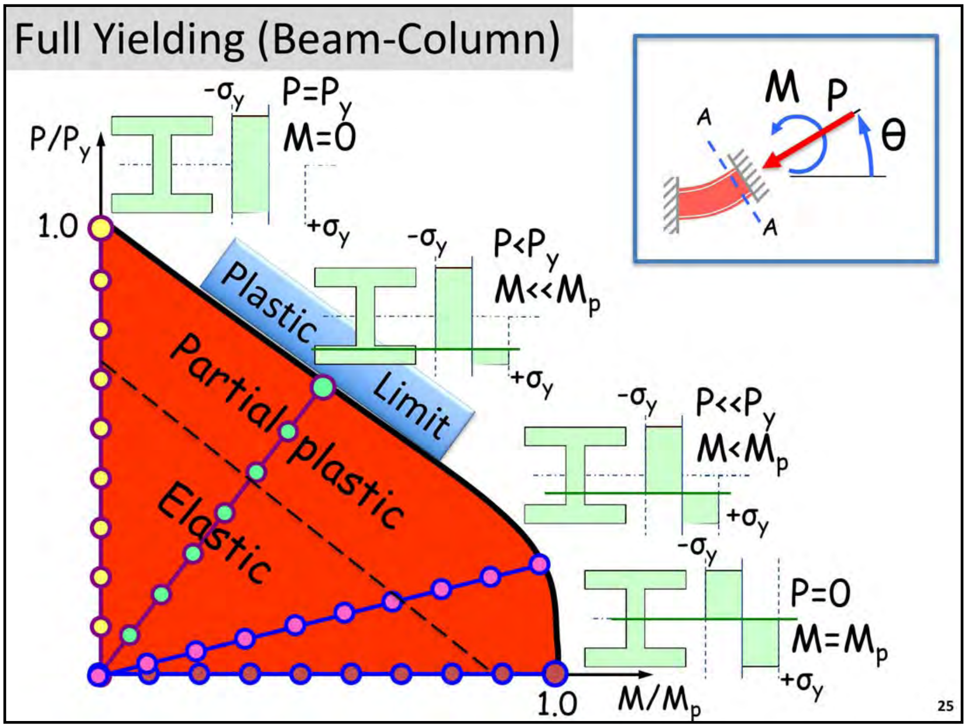
Full Yielding (Beam-Column)
P/Py
1.0
-σy
P=Py
M=0
+σy
P<Py
M<<Mp
Plastic
Limit
Partial plastic
Elastic
P<<Py
M<Mp
P=0
M=Mp
M
P
θ
A
1.0
M/Mp
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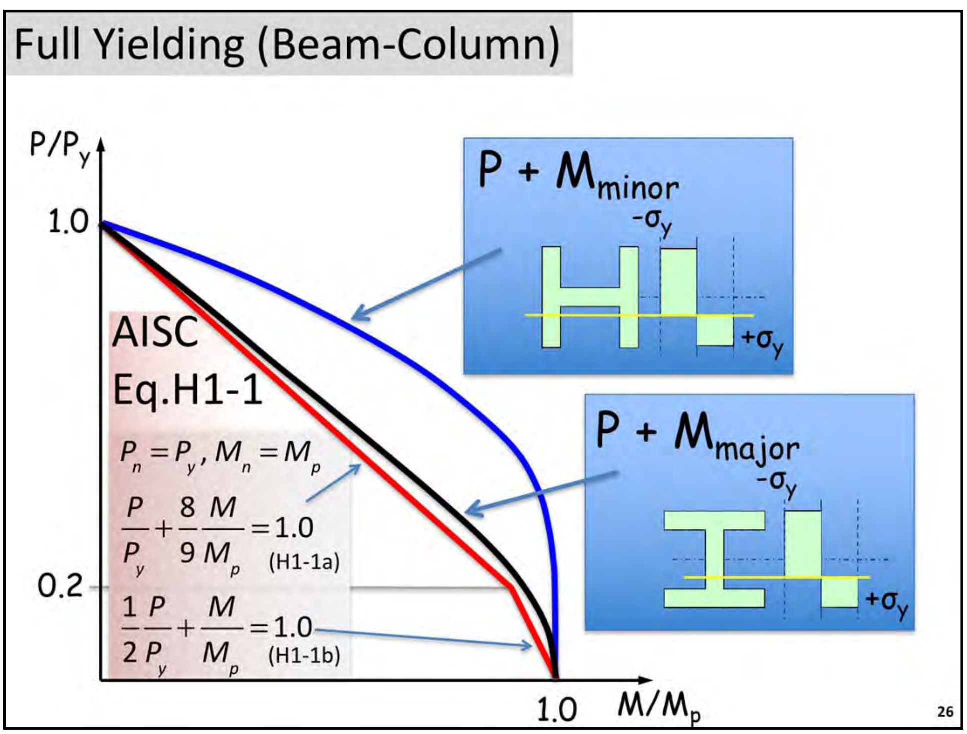
Full Yielding (Beam-Column)
P/Py
1.0
0.2
AISC
Eq.H1-1
$P_n = P_y, M_n = M_p$
$\frac{P}{P_y} + \frac{8}{9}\frac{M}{M_p} = 1.0$ (H1-1a)
$\frac{1}{2}\frac{P}{P_y} + \frac{M}{M_p} = 1.0$ (H1-1b)
P + Mminor
-σy
+σy
P + Mmajor
1.0
M/Mp
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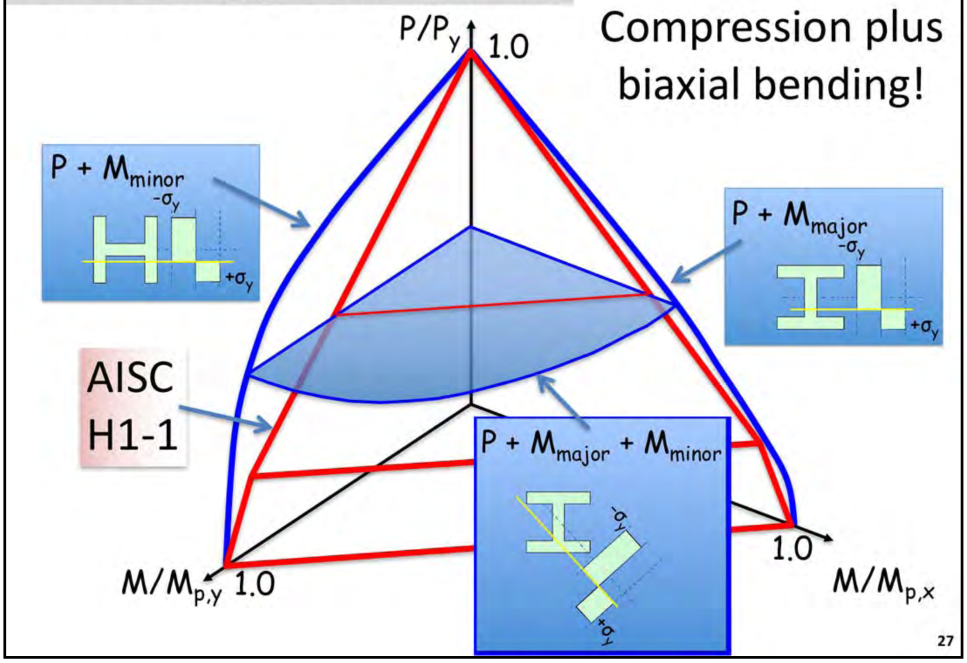
Full Yielding (Beam-Column)



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Beam-Column Strengths

- Cross section strength (small *L/r*)
  - full yield
  - local buckling
- <u>Elastic</u> member strength (large *L/r*)
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  - flexural (M_{major}): lateral torsional buckling
  - torsional-flexural buckling ($P + M_{major}$)
- <u>Inelastic</u> member strength (intermediate *L/r*)
  - same possible failure modes as elastic, except reduced due to partial yielding accentuated by presence of initial imperfections (geometric and residual stresses)

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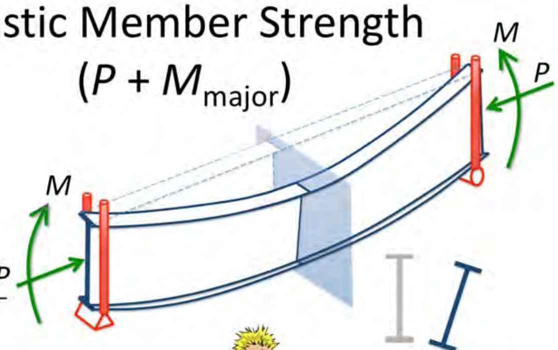
Elastic Member Strength ($P + M_{major}$)

$P \neq 0$ and $M = 0$
(compression slides)

$$P_E = \pi^2 EI_y / L_b^2$$

$P = 0$ and $M \neq 0$
(flexure slides)

$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$



$P \neq 0$ and $M \neq 0$ (Beam-Column)

For a given M, P_{cr} is smallest root of:

$$(P_E - P_{cr})(P_t - P_{cr}) = AM^2 / I_\rho$$

with:

$$I_\rho = I_x + I_y$$

$$P_t = A(GJ + \pi^2 EC_w) / L_b^2$$

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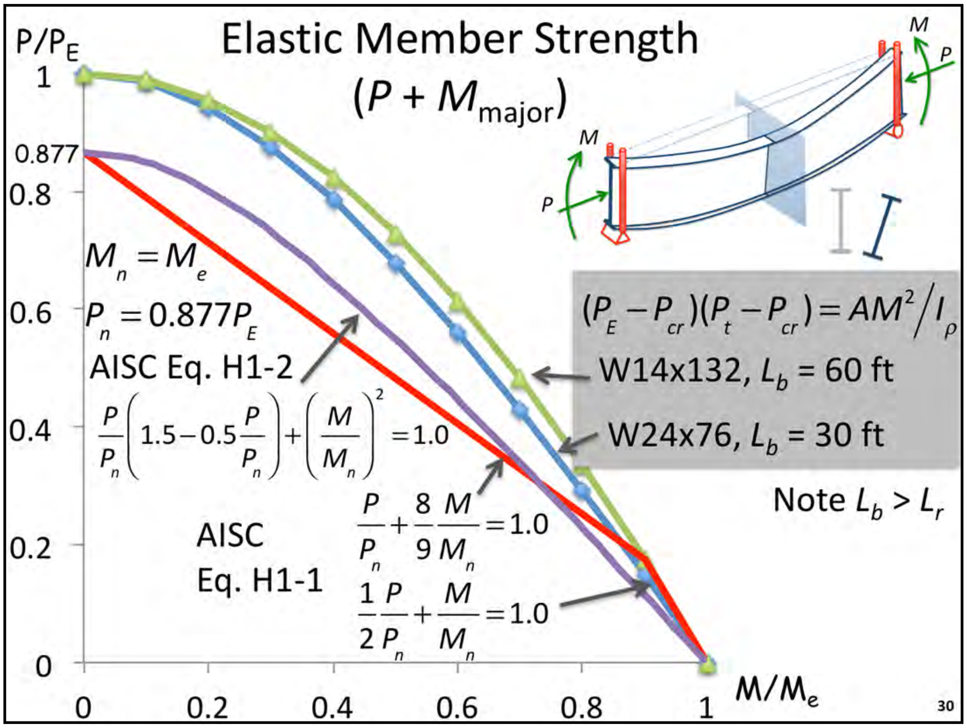
Elastic Member Strength ($P + M_{major}$)



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Beam-Column Strengths

- Cross section strength (small L/r)
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  - local buckling
- Elastic member strength (large L/r)
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- Inelastic member strength (intermediate L/r)
  - same possible failure modes as elastic, except reduced due to partial yielding accentuated by presence of initial imperfections (geometric and residual stresses)

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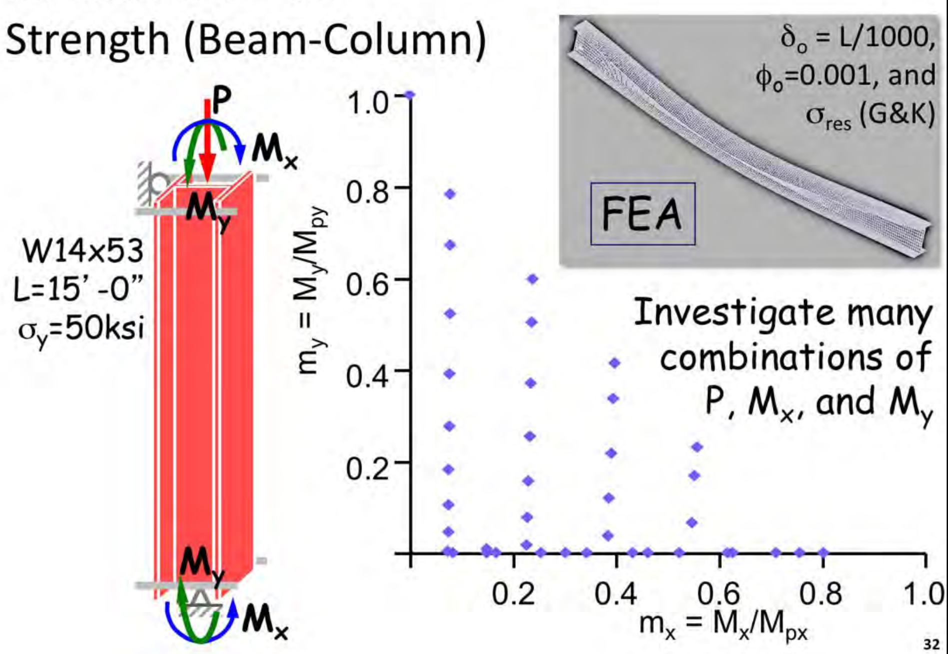
Inelastic Member Strength (Beam-Column)



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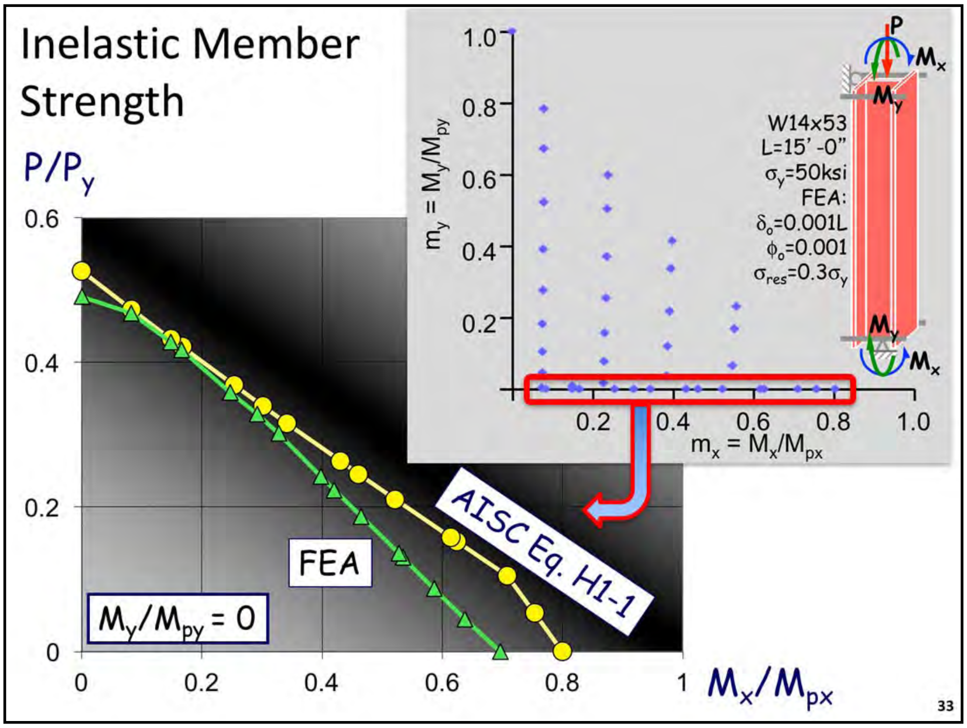

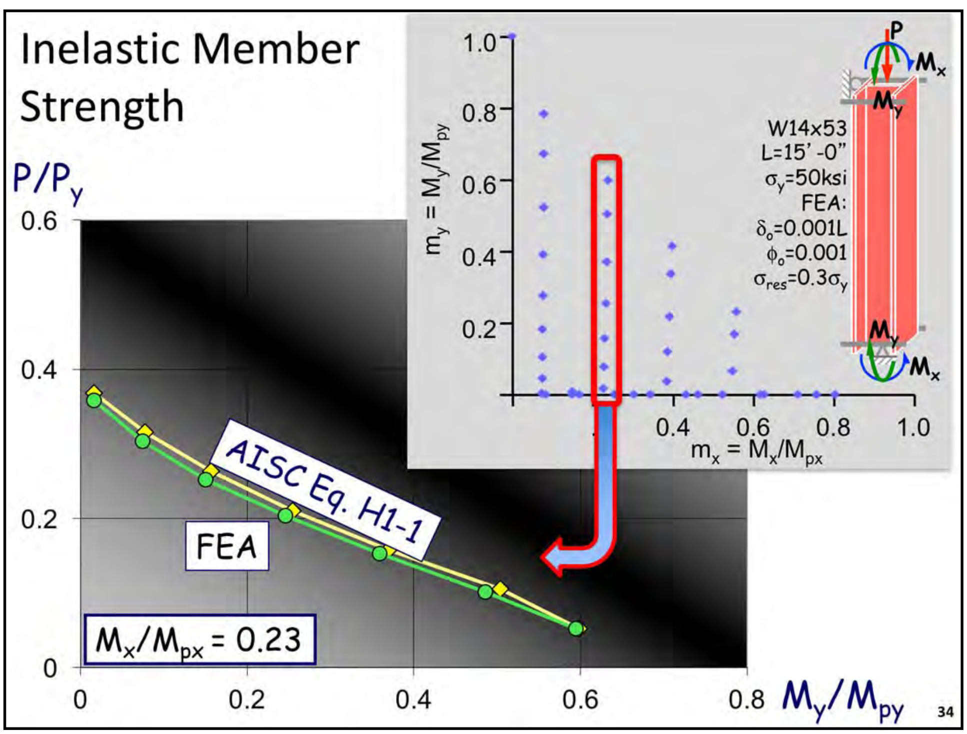

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AISC Interaction Eq. (H1-1) for Member Strength

P and *M* are required strengths from the structural analysis and must account for effects that may impact the stability of the system and its elements

$$\text{For } \frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$$

$$\text{For } \frac{P}{P_n} < 0.2, \quad \frac{1}{2}\frac{P}{P_n} + \frac{M}{M_n} \leq 1.0$$

M_n = flexural strength (earlier lectures)

P_n = Compressive strength (earlier lectures)

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Polling Question #1

The interaction equation (Eq. H1-1) appearing in the AISC Specification

a. only accounts for the combined effects of axial force and major axis flexure
b. may be used to represent the inelastic strength of a beam-column
c. includes the effects of shear force
d. does not include second-order effects
e. should only be used for checking cross-section strength

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Analysis Essentials to obtain Required Strengths

For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact the stability of the system and its components (AISC, Ch C – *Design for Stability*):

- flexural, shear, axial deformations, etc.
- second-order effects (both *P*-Δ and *P*-δ)
- geometric imperfections
- stiffness reductions due to inelasticity

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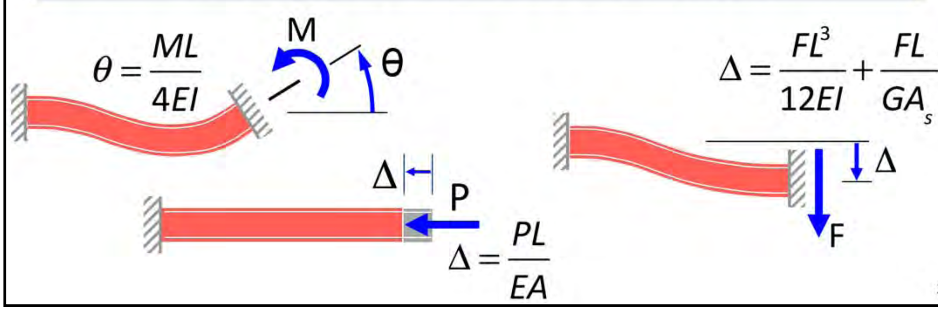
Analysis Essentials to obtain Required Strengths

For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact stability of system and its elements (AISC, Ch C – *Design for Stability*):

- flexural, shear, axial deformations, etc.



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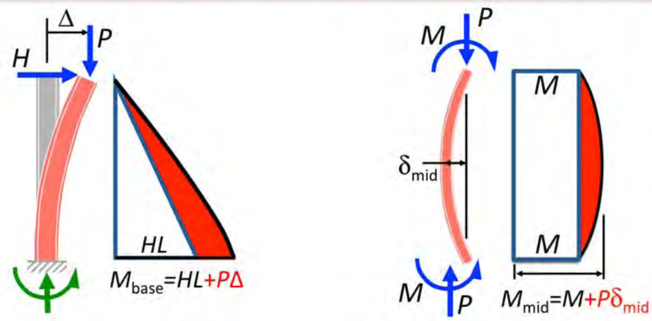
Analysis Essentials to obtain Required Strengths

For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact stability of system and its elements (AISC, Ch C – *Design for Stability*):

- second-order effects (both P-Δ and P-δ)



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Including Second-Order Effects

- Analysis options for :
  - Rigorous computational analysis (recommended!)
    - loads applied incrementally/iteratively
    - geometric stiffness matrix
    - updating geometry after each increment of loading
  - B_1 and B_2 amplification factors
    - $M = B_1 M_{nt} + B_2 M_{lt}$
- Approximate indicators of their significance
  - $P\delta$ provided by B_1
  - $P\Delta$ provided by B_2

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Axial plus <u>uniform</u> bending: P-δ (1)

Derive B_1 factor…more of an FYI!

<u>Recall: Three Keys to an Analysis</u>

1. Equilibrium
   (balance of forces and/or moments)
2. Compatibility
   (agreement of displacements and/or rotations)
3. Constitutive Relationship
   (relate forces and/or moments to displacements and/or rotations)

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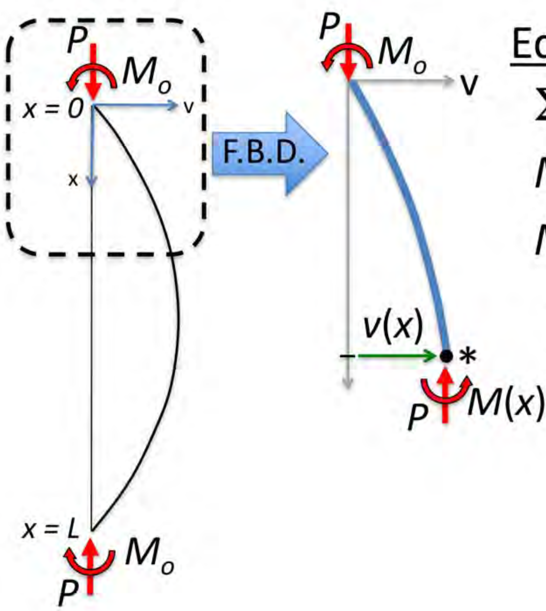
Axial plus <u>uniform</u> bending: P-δ (1)



<u>Equilibrium</u>:

$$\Sigma M_{*} = 0$$

$$M(x) + M_o + Pv(x) = 0$$

$$M(x) + Pv(x) = -M_o$$

<u>Moment-curvature</u>:
(constitutive relationship)

$$M(x) = EI\frac{d^2v(x)}{dx^2}$$

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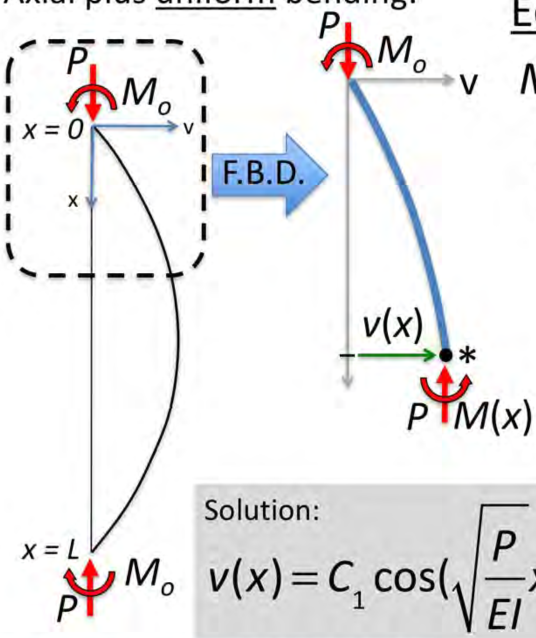
Axial plus uniform bending:

P-δ (2)



Equilibrium:

$$M(x) + Pv(x) = -M_o$$

Moment-curvature:
(constitutive relationship)

$$M(x) = EI\frac{d^2v(x)}{dx^2}$$

$$EI\frac{d^2v}{dx^2} + Pv = -M_o$$

Solution:

$$v(x) = C_1\cos\left(\sqrt{\frac{P}{EI}}x\right) + C_2\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{M_o}{P}$$

wolframalpha.com: a2*y"(x)+a1*y(x)=a3

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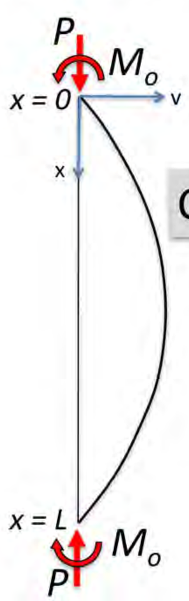
Axial plus uniform bending:

P-δ (3)



$$v(x) = C_1\cos\left(\sqrt{\frac{P}{EI}}x\right) + C_2\sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{M_o}{P}$$

Compatibility: Boundary Conditions!

$$v(x=0) = 0 \quad \Rightarrow \quad C_1 = M_o/P$$

$$v(x=L) = 0 \quad \Rightarrow \quad C_2 = \frac{M_o}{P}\frac{1-\cos\left(\sqrt{\frac{P}{EI}}L\right)}{\sin\left(\sqrt{\frac{P}{EI}}L\right)}$$

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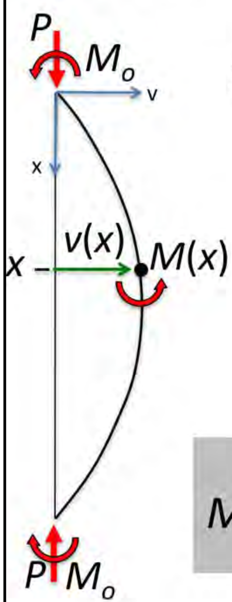
Axial plus uniform bending:

P-δ (4)



$$v(x) = C_1 \cos\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{M_o}{P}$$

Moment-curvature:

$$M(x) = EI\frac{d^2v(x)}{dx^2}$$

$$M(x) = -C_1 P\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2 P\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

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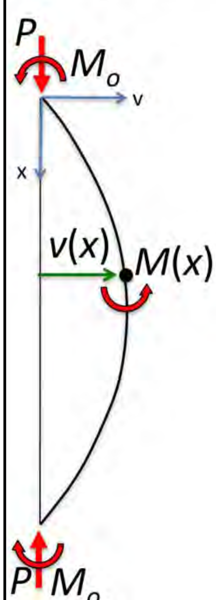
Axial plus uniform bending:

P-δ (5)



$$M(x) = -C_1 P\cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2 P\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

Substituting C_1 and C_2 from previous slide and letting

$$\beta = \left(1 - \cos\left(\sqrt{\frac{P}{EI}}L\right)\right) \Big/ \sin\left(\sqrt{\frac{P}{EI}}L\right)$$

$$M(x) = -M_o\left(\cos\left(\sqrt{\frac{P}{EI}}x\right) + \beta\sin\left(\sqrt{\frac{P}{EI}}x\right)\right)$$

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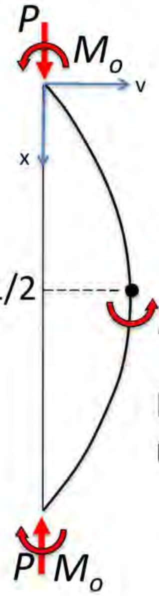
Axial plus uniform bending:

P-δ (6)



$$M(x) = -M_o \left(\cos\left(\sqrt{\frac{P}{EI}}x\right) + \beta \sin\left(\sqrt{\frac{P}{EI}}x\right) \right)$$

with $\beta = \left(1 - \cos\left(\sqrt{\frac{P}{EI}}L\right)\right) \Big/ \sin\left(\sqrt{\frac{P}{EI}}L\right)$

Let's monitor the moment at some point, say the mid-span (which should be the largest)

$$M\left(x = \frac{L}{2}\right) = -M_o \left(\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \beta \sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) \right)$$

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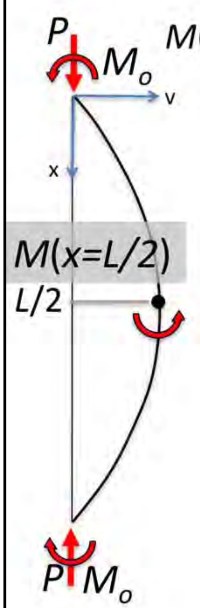
Axial plus uniform bending:

P-δ (7)



$$M\left(x = \frac{L}{2}\right) = -M_o \left(\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \beta \sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) \right)$$

with $\beta = \left(1 - \cos\left(\sqrt{\frac{P}{EI}}L\right)\right) \Big/ \sin\left(\sqrt{\frac{P}{EI}}L\right)$

Letting:

$$u = \sqrt{\frac{P}{EI}}\frac{L}{2}$$

$$M\left(x = \frac{L}{2}\right) = -M_o \left(\cos(u) + \beta \sin(u) \right)$$

with $\beta = \left(1 - \cos(2u)\right) / \sin(2u)$

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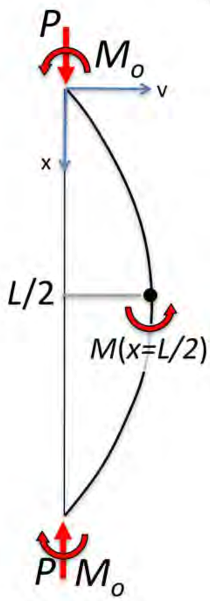
Axial plus uniform bending: P-δ (8)

$$M(x=\frac{L}{2}) = -M_o\left(\cos(u)+\beta\sin(u)\right)$$



with $\beta = \left(1-\cos(2u)\right)/\sin(2u)$

Employing trig. identities

$$\cos(2u) = 1-2\sin^2(u)$$
$$\sin(2u) = 2\sin(u)\cos(u)$$

after simplifying

$$M(x=\frac{L}{2}) = -M_o\left(\frac{1}{\cos(u)}\right) \quad \text{with} \quad u=\sqrt{\frac{P}{EI}}\frac{L}{2}$$

$$M(x=\frac{L}{2}) = -M_o\left(\frac{1}{\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)}\right)$$

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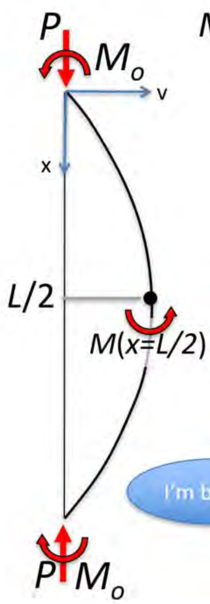
Axial plus uniform bending: P-δ (9)



$$M(x=\frac{L}{2}) = -M_o\left(\frac{1}{\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)}\right)$$

BUT!!! As $\sqrt{\frac{P}{EI}}\frac{L}{2} \to \frac{\pi}{2}$, $\quad \frac{1}{\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)} \to \infty$

Noting:

$$\sqrt{\frac{P}{EI}}\frac{L}{2} = \frac{\pi}{2} \Rightarrow P = \frac{\pi^2 EI}{L^2} = P_E \quad \text{and} \quad \sqrt{\frac{P}{EI}}\frac{L}{2} = \frac{\pi}{2}\sqrt{\frac{P}{P_E}}$$

$$M(x=\frac{L}{2}) = -M_o\left(\frac{1}{\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)}\right)$$

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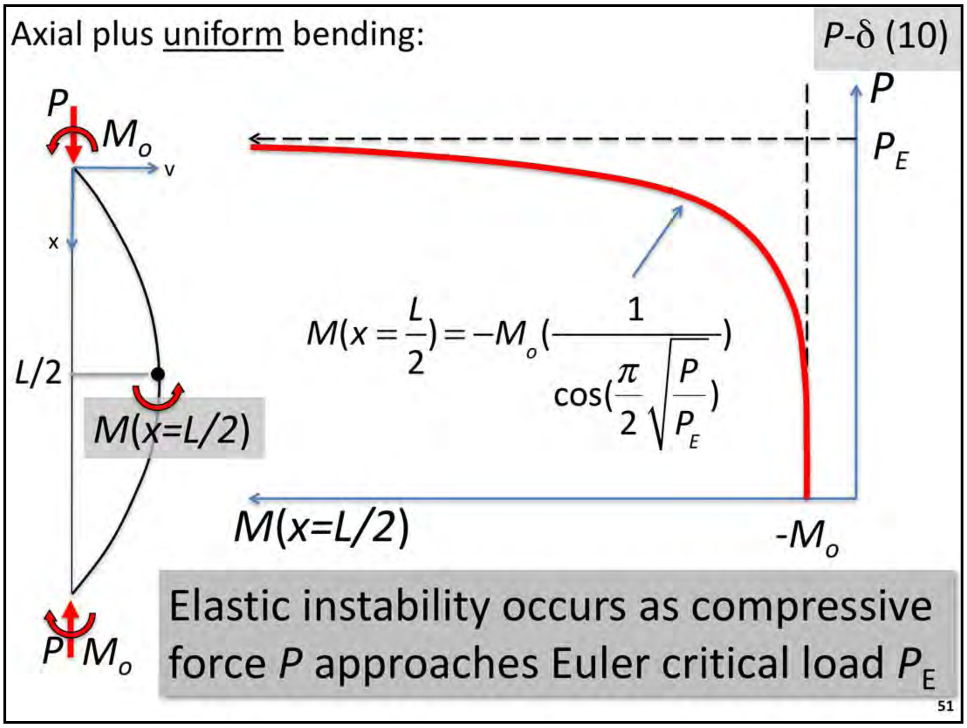

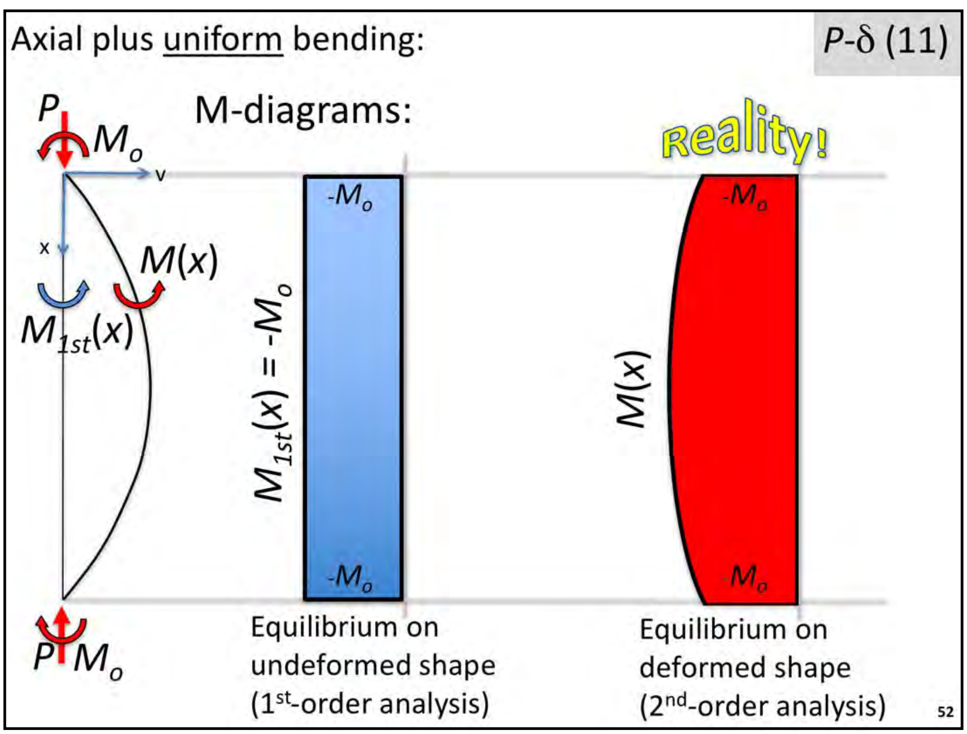

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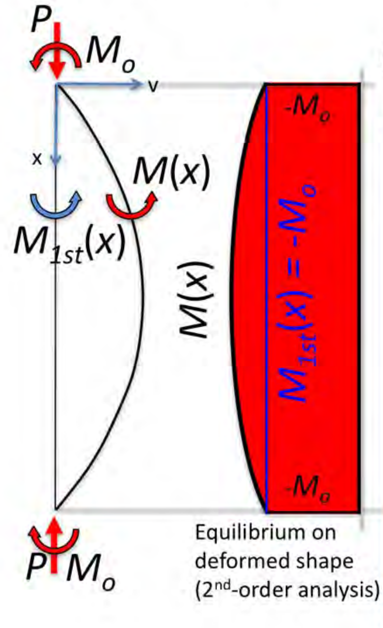
Axial plus uniform bending: P-δ (12)



$$M(x) = -M_o\left(\cos\left(\sqrt{\frac{P}{EI}}x\right) + \beta\sin\left(\sqrt{\frac{P}{EI}}x\right)\right)$$

In terms of a <u>moment amplification factor</u>, B_1:

$$M(x) = B_1 M_{1st}(x)$$

with

$$B_1 = \cos\left(\sqrt{\frac{P}{EI}}x\right) + \beta\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

and

$$\beta = \left(1 - \cos\left(\sqrt{\frac{P}{EI}}L\right)\right) \Big/ \sin\left(\sqrt{\frac{P}{EI}}L\right)$$

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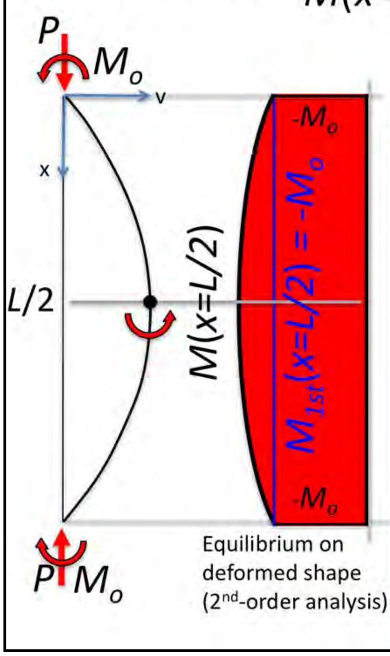
At mid-span: P-δ (13)

$$M\left(x = \frac{L}{2}\right) = -M_o\left(\frac{1}{\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)}\right)$$



In terms of a <u>moment amplification factor</u>, B_1:

$$M\left(x = \frac{L}{2}\right) = B_1 M_{1st}\left(x = \frac{L}{2}\right)$$

with

$$B_1 = \frac{1}{\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)}$$

-or-

$$M^{max} = B_1 M_{1st}^{max}$$

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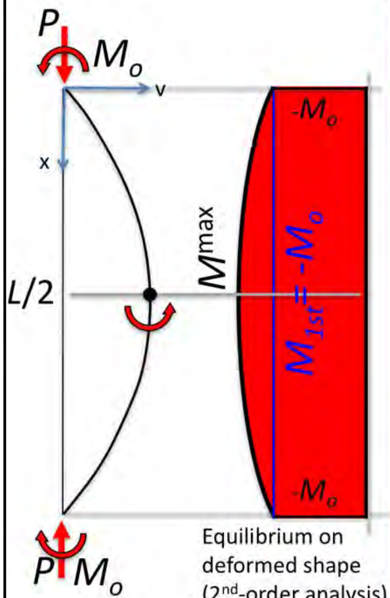
Axial plus uniform bending:

P-δ (14)



Equilibrium on deformed shape (2nd-order analysis)

In terms of a moment amplification factor, B_1:

$$M^{max} = B_1 M_{1st}^{max}$$

with

$$B_1 = \frac{1}{\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)}$$

which is often approximated by

$$B_1 = \frac{1}{1 - \frac{P}{P_E}}$$

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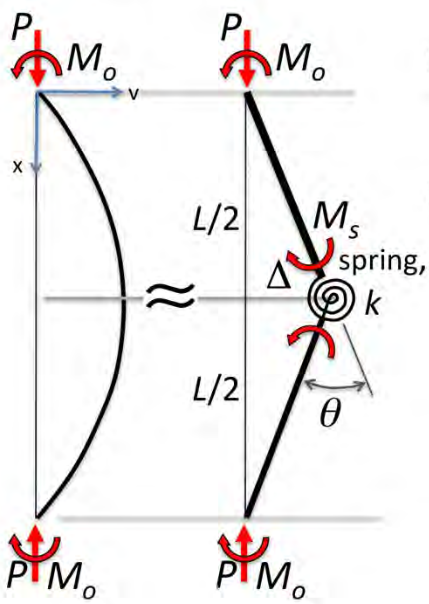
Approximate B_1:

P-δ (15)



Spring: $M_s = k\theta = \frac{4k}{L}\Delta$

Equilibrium w/ M_o=0, provides P_{cr}

$$M_s = P_{cr}\Delta \;\Rightarrow\; P_{cr} = \frac{4k}{L}$$

Equilibrium w/ M_o≠0,

$$M_s = M_o + P\Delta$$

$$\text{w/ } \Delta = \frac{L}{4k}M_s = \frac{1}{P_{cr}}M_s$$

$$M_s = M_o + \frac{P}{P_{cr}}M_s$$

$$M_s = \left(\frac{1}{1 - \frac{P}{P_{cr}}}\right)M_o = B_1 M_o$$

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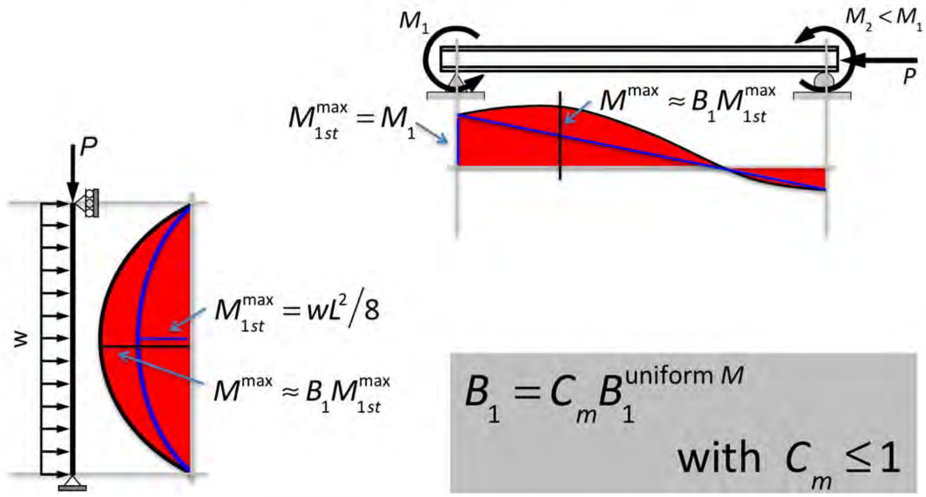
P-δ (16)

What about cases where the M(x) is not uniform?



$$B_1 = C_m B_1^{\text{uniform } M}$$

with $C_m \le 1$

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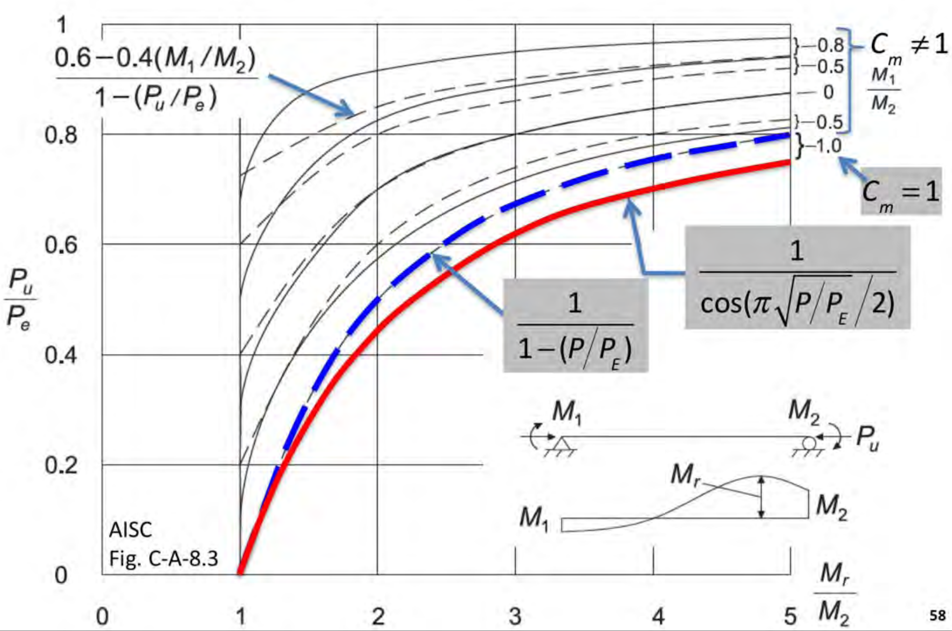
P-δ (17)

Comparison of Exact and Approximate B_1's



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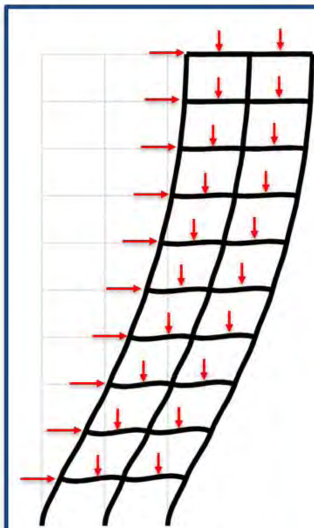
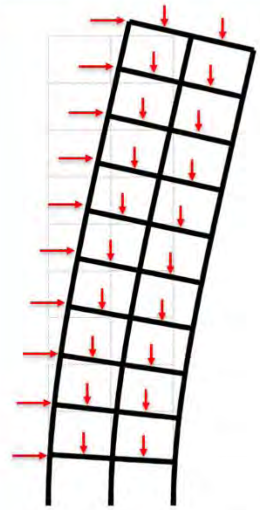
Second-order Effects: P-Δ

P-Δ (1)

Lateral displacement due to:



Story Shear



Frame Bending

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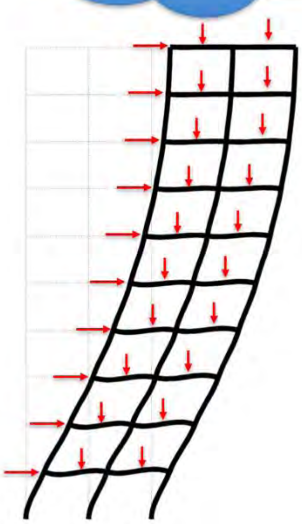
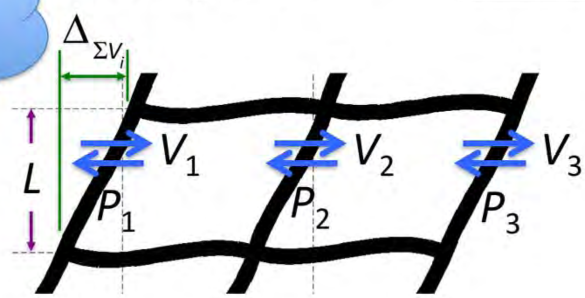
Story Shear:

P-Δ (2)

$$\frac{1}{1-\frac{P}{P_E}} \Rightarrow \frac{1}{1-\frac{\Sigma P}{\Sigma P_E}}$$





$$B_2 = \frac{1}{1-\frac{P_{story}}{P_{e,\,story}}}$$

with:

$$P_{story} = \sum_{i=1}^{3} P_i \quad \text{and} \quad P_{e,\,story} = R_M \frac{\Sigma V_i}{\Delta_{\Sigma V_i}} L$$

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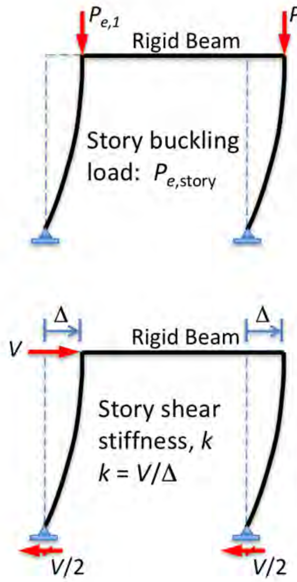
Simple example relating story buckling load to story shear stiffness:



$$P_{e,\,story} = \sum_{i=1}^{\#\text{ of cols}} P_{e,i}$$

$$P_{e,\,story} = 2 \times \frac{\pi^2 EI}{(2.0L)^2} = \frac{\pi^2}{2}\left(\frac{EI}{L^3}\right)L$$

$$\Delta = \frac{(V/2)L^3}{3EI}$$

$$\frac{V}{\Delta} = 6\frac{EI}{L^3}$$

$$P_{e,\,story} = \frac{\pi^2}{2}\left(\frac{1}{6}\frac{V}{\Delta}\right)L$$

$$P_{e,\,story} = 0.82\frac{V}{\Delta}L$$

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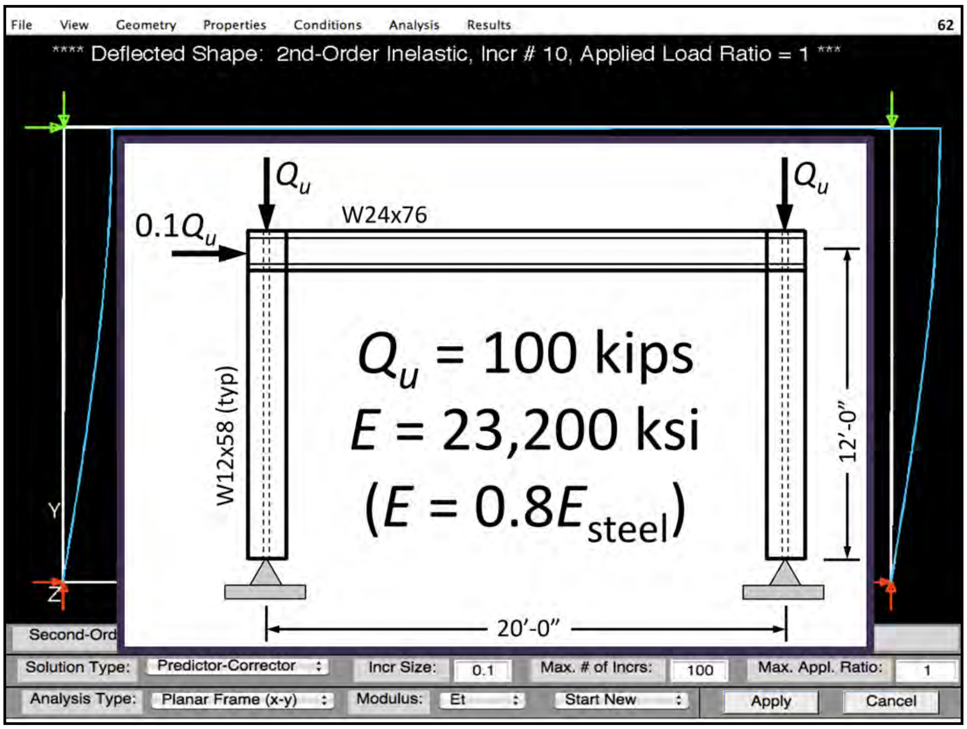

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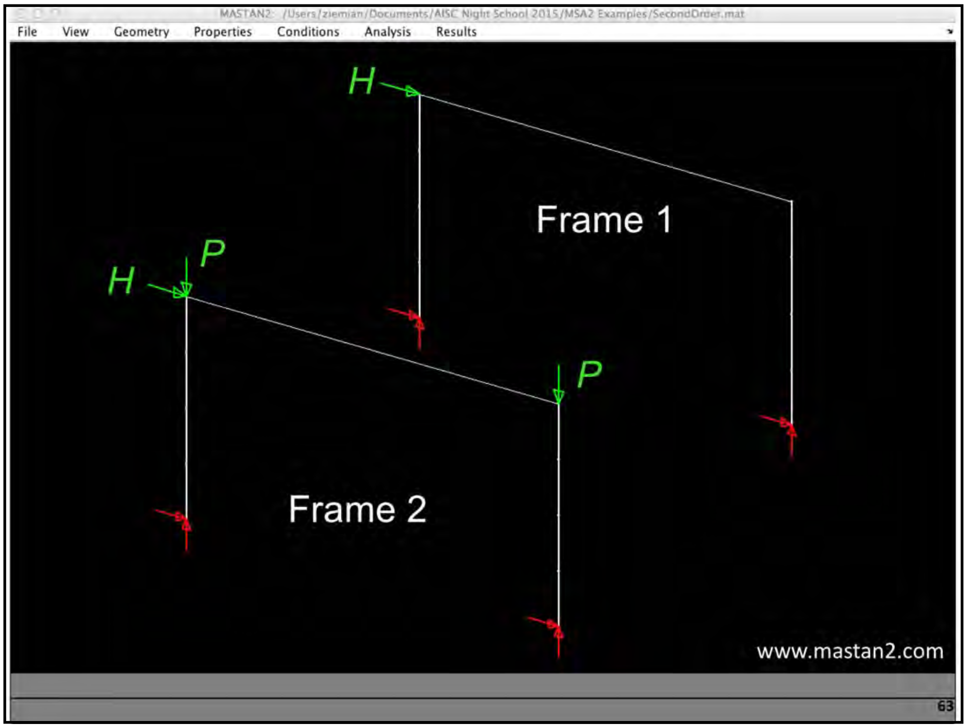
H
H
P
P
Frame 1
Frame 2
www.mastan2.com
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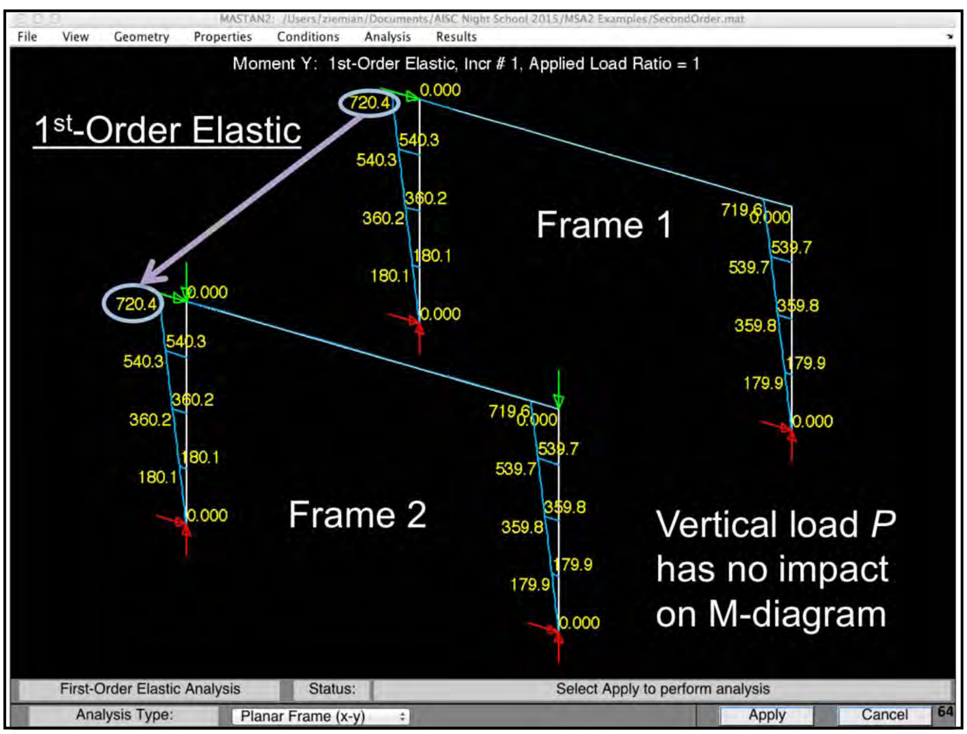
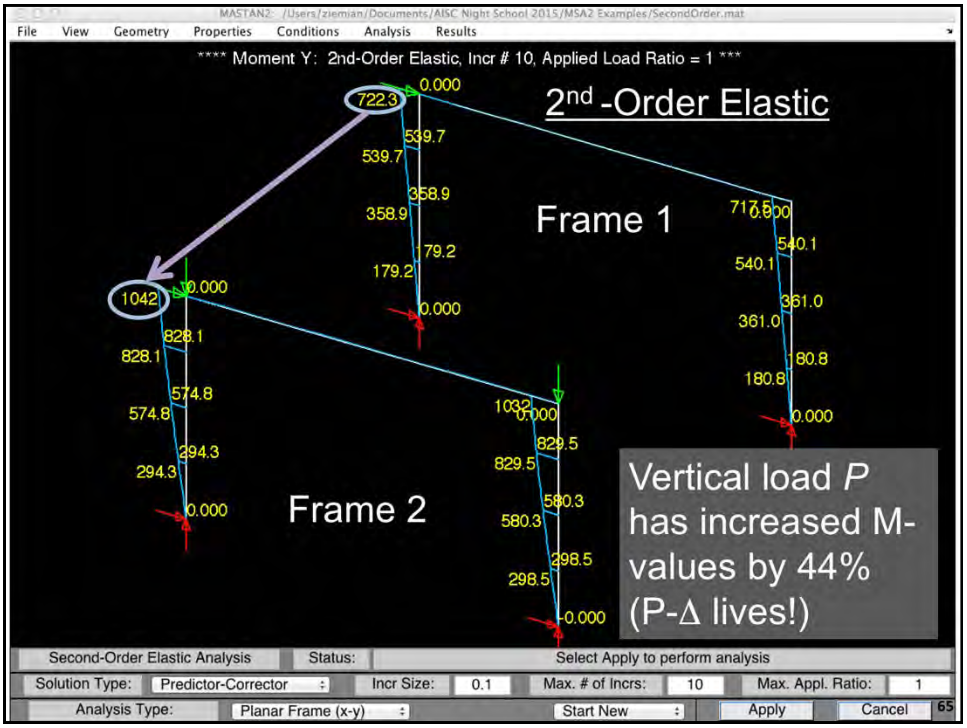

Moment Y: 1st-Order Elastic, Incr # 1, Applied Load Ratio = 1
1st-Order Elastic
720.4
720.4
Frame 1
Frame 2
Vertical load P has no impact on M-diagram
First-Order Elastic Analysis
Status:
Select Apply to perform analysis
Analysis Type:
Planar Frame (x-y)
Apply
Cancel
64



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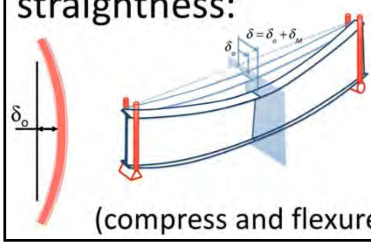
Analysis Essentials to obtain Required Strengths

For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact stability of system and its elements (AISC, Ch C – *Design for Stability*):

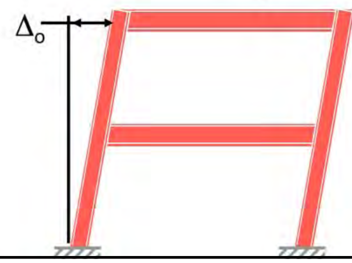
- geometric imperfections

Member out-of-straightness:



(compress and flexure slides)

Frame out-of-plumb:



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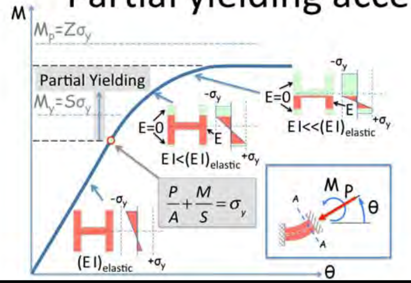
Analysis Essentials to obtain Required Strengths

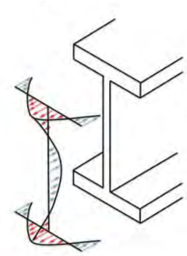
For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact stability of system and its elements (AISC, Ch C – *Design for Stability*):

- stiffness reductions due to inelasticity

Partial yielding accentuated by residual stresses:





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Analysis Essentials to obtain Required Strengths

For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact the stability of the system and its elements (AISC, Ch C – *Design for Stability*):

- flexural, shear, axial deformations, etc. ✔
- second-order effects (both *P*-Δ and *P*-δ) ✔
- geometric imperfections ✔
- stiffness reductions due to inelasticity ✔

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Methods for Designing for Stability

- Any rational method that considers all of the effects
- Methods appearing in AISC Specification
  - Based on 2nd-order Elastic Analysis (today!)
    - Effective Length Method
      - Established in early 1960's
      - Appendix 7
    - Direct Analysis Method
      - Established in 2005
      - Ch. C
  - Based on 2nd-order Inelastic Analysis
    - Appendix 1
  - Based on 1st-Order Elastic Analysis (not today!)
    - Direct Analysis Method concept (Appendix 7)

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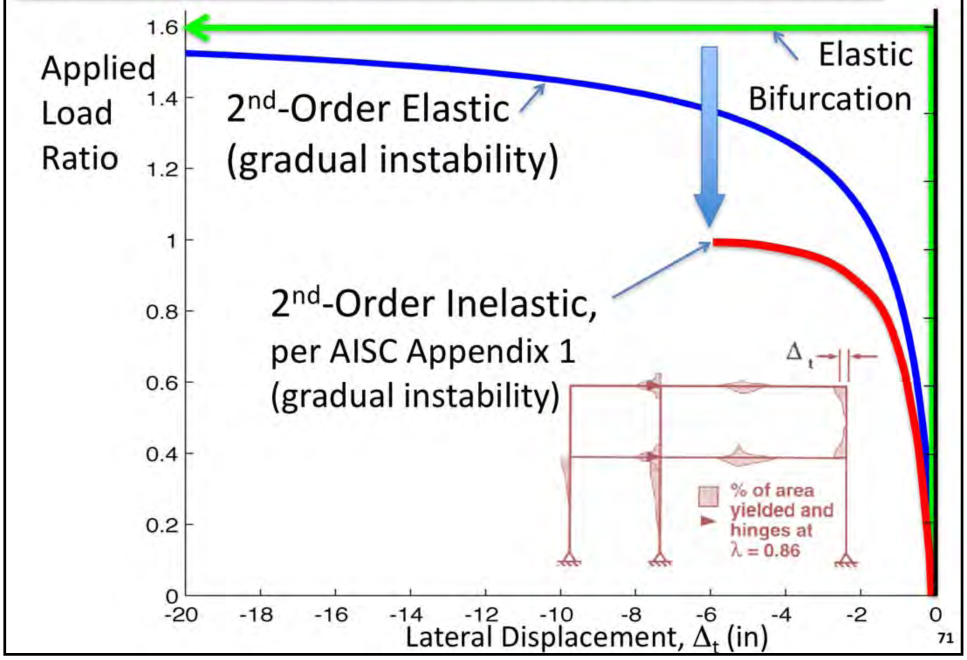
Predicting "actual" system stability with elastic analysis:



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Methods for Designing for Stability

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    - Appendix 1
  - Based on 1st-Order Elastic Analysis (not today!)
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Effective Length Method

2nd-order elastic analysis

$$\text{For } \frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$$

P_n based on effective length KL, with $KL \geq L$:

comparing KL/r to $4.71\sqrt{E/\sigma_y}$

$$\sigma_{cr} = 0.658^{\frac{\sigma_y}{\sigma_e}} \sigma_y$$

or

$$\sigma_{cr} = 0.877\sigma_e$$

$$\sigma_e = \frac{\pi^2 E}{\left(KL/r\right)^2}$$

Effects on system stability:

- flexural, shear, axial deformations, etc.
- second-order effects
- geometric imperfections
- stiffness reductions due to inelasticity

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Effective Length Method (2)

- Does the K-factor really account for those system stability effects?
  - Indirectly, through its conservatism…
- Common methods for computing *K*-factors
  - Alignment charts or modifications based on undoing inherent assumptions
    - be very careful!
  - Buckling or critical load analyses (eigenvalue)
    - $KL = \pi\sqrt{EI/P}$
    - be very careful!
- How confident are you in your *K*-factors???

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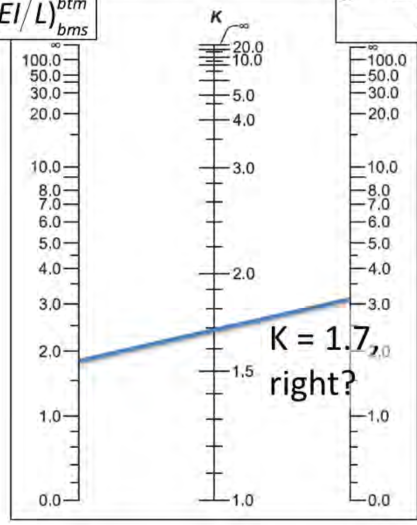
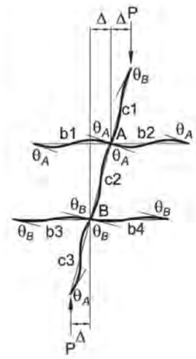
Effective Length Method (3)

Alignment Charts

$$G^{btm} = \frac{\Sigma(EI/L)^{btm}_{cols}}{\Sigma(EI/L)^{btm}_{bms}} \qquad G^{top} = \frac{\Sigma(EI/L)^{top}_{cols}}{\Sigma(EI/L)^{top}_{bms}}$$





So easy…so much fun…so addictive…BE CAREFUL!

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Effective Length Method (4)

The most common method for determining K is through use of the *alignment charts*, which are shown in Figure C-A-7.1 for frames with sidesway inhibited and Figure C-A-7.2 for frames with sidesway uninhibited (Kavanagh, 1962). These charts are based on assumptions of idealized conditions, which seldom exist in real structures, as follows:

(1) Behavior is purely elastic.
(2) All members have constant cross section.
(3) All joints are rigid.
(4) For columns in frames with sidesway inhibited, rotations at opposite ends of the restraining beams are equal in magnitude and opposite in direction, producing single curvature bending.
(5) For columns in frames with sidesway uninhibited, rotations at opposite ends of the restraining beams are equal in magnitude and direction, producing reverse curvature bending.
(6) The stiffness parameter $L\sqrt{P/EI}$ of all columns is equal.
(7) Joint restraint is distributed to the column above and below the joint in proportion to EI/L for the two columns.
(8) All columns buckle simultaneously.
(9) No significant axial compression force exists in the girders.

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Effective Length Method (5)

- Excellent resources for improving odds of computing accurate *K*'s
  - ASCE Task Committee on Effective Length (1997), *Effective Length and Notional Load Approaches for Assessing Frame Stability: Implications for American Steel Design*, American Society of Civil Engineers, NY.
  - Ziemian, R.D. (ed.) (2010), *Guide to Stability Design Criteria for Metal Structures*, 6th Ed., John Wiley & Sons, Inc., Hoboken, NJ.

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Methods for Designing for Stability

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  - Based on 1st-Order Elastic Analysis (not today!)
    - Direct Analysis Method concept (Appendix 7)

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Direct Analysis Method

2^{nd}-order "elastic" analysis

$$\text{For } \frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$$

P_n based on effective length *KL*, with ***KL* = *L***

Effects on system stability:

- flexural, shear, axial deformations, etc.
- second-order effects
- geometric imperfections
- stiffness reductions due to inelasticity

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Direct Analysis Method (2)

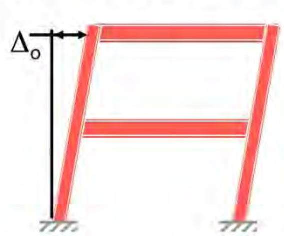
2^{nd}-order "elastic" analysis

$$\text{For } \frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$$

Effects on system stability:

- geometric imperfections

1. Member out-of-straightness accounted for in column curve
2. Frame out-of-plumb



Direct Modeling: distort computational model to include maximum amount considered in design (e.g. AISC *Code of Standard Practice*, $\Delta_o \leq H/500$)

Notional Loads: represent effect of imperfection by equivalent lateral load of $N_i = 0.002Y_i$ for level *i*

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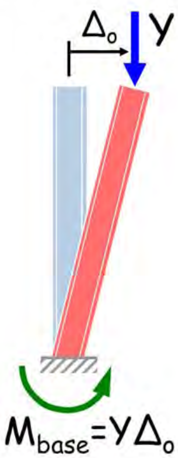
Wait! Where did that equivalent notional load come from?

Direct Modeling:



Require equivalent base moments:

$$M_{base} = Y\Delta_o = Nh$$

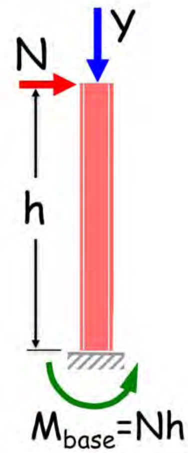
$$\text{or } N = Y\Delta_o/h$$

$$\text{with } \Delta_o = h/500$$

$$N = Y(h/500)/h$$

$$N = 0.002Y$$

Notional Load:



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Direct Analysis Method (3)

2nd-order "elastic" analysis

$$\text{For } \frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$$

Effects on system stability:
- stiffness reductions due to inelasticity

Loss in stiffness due to partial yielding accentuated by presence of residual stresses

From extensive studies calibrating DM to advanced FEA results (0.8 is applied to all stiffness contributing to system stability)

$$(EI)_{analysis} = 0.8\tau(EI)$$

Think back to compression slides...

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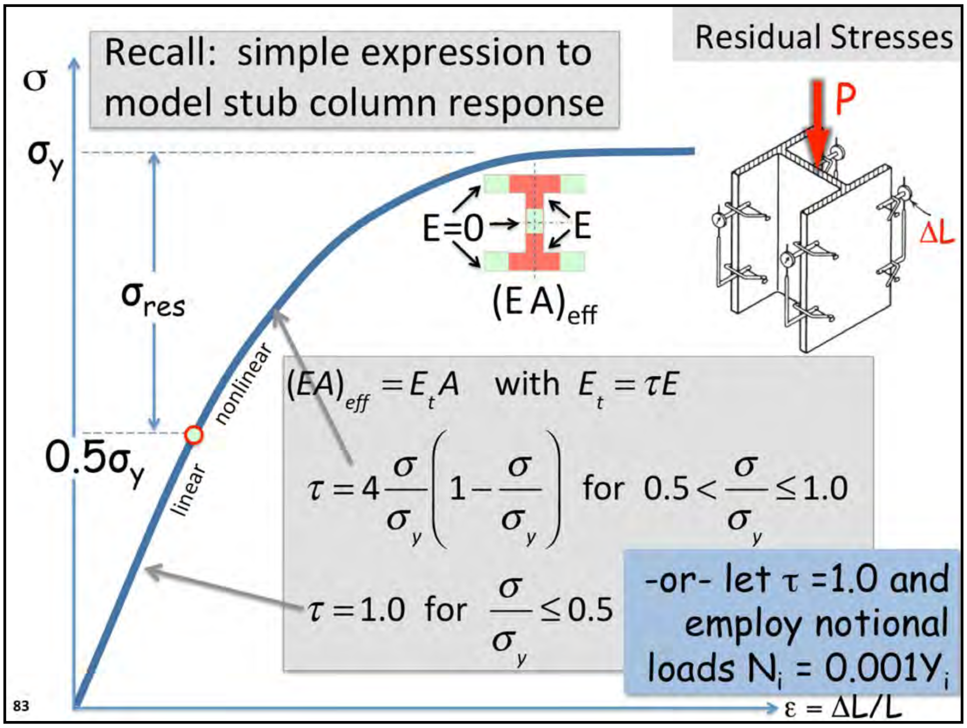

Direct Analysis Method

2nd-order "elastic" analysis

$$\text{For } \frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$$

P_n based on effective length KL, with $\boldsymbol{KL = L}$

Effects on system stability:

- flexural, shear, axial deformations, etc. ✔
- second-order effects ✔
- geometric imperfections ✔
- stiffness reductions due to inelasticity ✔

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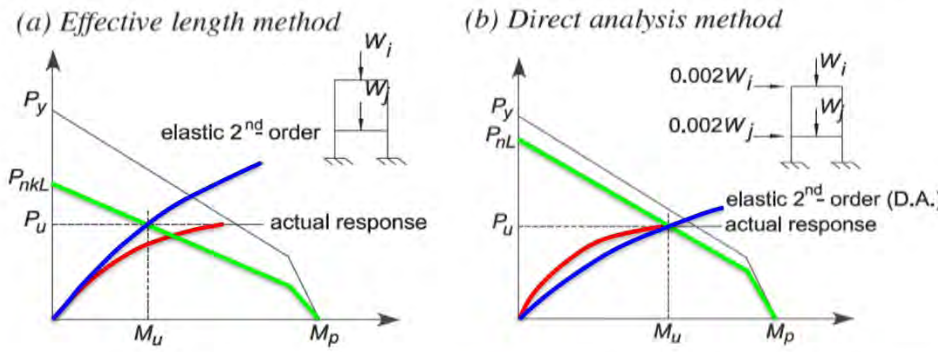
How do ELM and DM designs compare?



<u>Trade-off</u>:

$$M^{DM:\ \Delta_o, 0.8\tau E} >> M^{ELM:\ no\ \Delta_o, E}$$

$$\frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$$

$$P_n^{ELM:\ K>1} << P_n^{DM:\ K=1}$$

In many cases, ELM and DM provide similar designs

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COMMENTARY
on the Specification for Structural Steel Buildings (ANSI/AISC 360-10)

page **16.1**-273

TABLE C-C1.1
Comparison of Basic Stability Requirements with Specific Provisions

Basic Requirement in Section C1		Provision in Direct Analysis Method (DM)	Provision in Effective Length Method (ELM)
(1) Consider all deformations		C2.1(1). Consider all deformations	Same as DM (by reference to C2.1)
(2) Consider second-order effects (both P-Δ and P-δ)		C2.1(2). Consider second-order effects (P-Δ and P-δ)**	Same as DM (by reference to C2.1)
(3) Consider geometric imperfections *This includes joint-position imperfections* (which affect structure response) and member imperfections (which affect structure response and member strength)*	Effect of joint-position imperfections* on structure response	C2.2a. Direct modeling or C2.2b. Notional loads	Same as DM, second option only (by reference to C2.2b)
	Effect of member imperfections on structure response	Included in the stiffness reduction specified in C2.3	All these effects are considered by using KL from a sidesway buckling analysis in the member strength check. Note that the only difference between DM and ELM is that: • DM uses reduced stiffness in the analysis; $KL = L$ in the member strength check • ELM uses full stiffness in the analysis; KL from sidesway buckling analysis in the member strength check for frame members
	Effect of member imperfections on member strength	Included in member strength formulas, with $KL = L$	
(4) Consider stiffness reduction due to inelasticity *This affects structure response and member strength*	Effect of stiffness reduction on structure response	Included in the stiffness reduction specified in C2.3	
	Effect of stiffness reduction on member strength	Included in member strength formulas, with $KL = L$	
(5) Consider uncertainty in strength and stiffness *This affects structure response and member strength*	Effect of stiffness/strength uncertainty on structure response	Included in the stiffness reduction specified in C2.3	
	Effect of stiffness/strength uncertainty on member strength	Included in member strength formulas, with $KL = L$	

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Polling Question #2

Which of the following is true?

a. designs by the direct analysis method and the effective length method are usually quite similar
b. the direct analysis method should only be used to design braced frames
c. the effective length method should only be used to design moment frames
d. the effective length method requires a factor of 0.8 be applied to all stiffnesses contributing to the stability of the structural system
e. the direct analysis method requires member out-of-straightness be explicitly modeled when computing required strengths.

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Summary – Systems & B-C's

- Basis for Design of Systems
  - Elastic Analysis (AISC Spec., Chs. A-K, Apps. 6-8)
    - strength of system is assured by ensuring adequate strength of its components
  - Inelastic Analysis (AISC Spec., Appendix 1)
    - System strength (stability) can be assessed directly by the analysis
- Stability of members
  - Compression (earlier lectures)
  - Flexural (earlier lectures)
  - Combined compression and flexure
    - Behavior of beam-columns (today!)
    - Design

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Summary – Systems & B-C's (cont.)

- Behavior of Beam-Columns
  - full yield (interaction surface)
  - member instability
    - from flexural buckling -to- lateral torsional buckling, including torsional-flexural buckling
  - AISC Interaction Eqs.
- Factors impacting system stability
  - flexural, shear, axial deformations
  - second-order effects
    - rigorous analysis vs. amplification factors B_1 and B_2
  - geometric imperfections
  - stiffness reductions due to inelasticity

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Summary – Systems & B-C's (cont.)

- Design Systems for Stability
  - elastic analysis vs. inelastic analysis
- Elastic analysis
  - Effective length method ($KL > L$)
  - Direct analysis method ($KL = L$)
  - Discussion on comparison
- Finally, if you still do not have confidence that your structural system is stable, then you can always…

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Up Next…

July 24, 2017 – Behavior and Design of Beam-Columns

by D.W. White, PhD

This session builds on Lecture 5, starting with an overview of the fundamental stability behavior and key technical background to the AISC beam-column strength Eqs. H1-1a & H1-1b. Several design examples are presented highlighting (1) the importance of overall lateral stiffness in beam-column design and (2) the efficient application of Eqs. H1-1 in the context of the AISC Direct Analysis Method, using Table 6-1 of the AISC 14th Edition Manual. The discussion then focuses on the background and use of AISC Eq. H1-2 to account more realistically for the out-of-plane strength of I-section members loaded in axial compression and major-axis bending. A more advanced application of Table 6-1 based on Eq. H1-2 is presented. The session closes with an explanation and application of the C_b modifier provided in AISC Section H1.2, accounting for the beneficial effects of axial tension on I-section member LTB strength.

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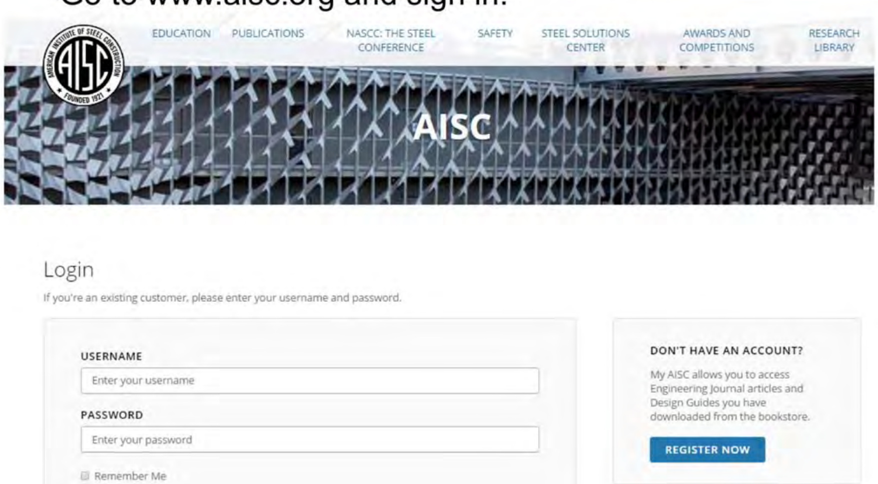

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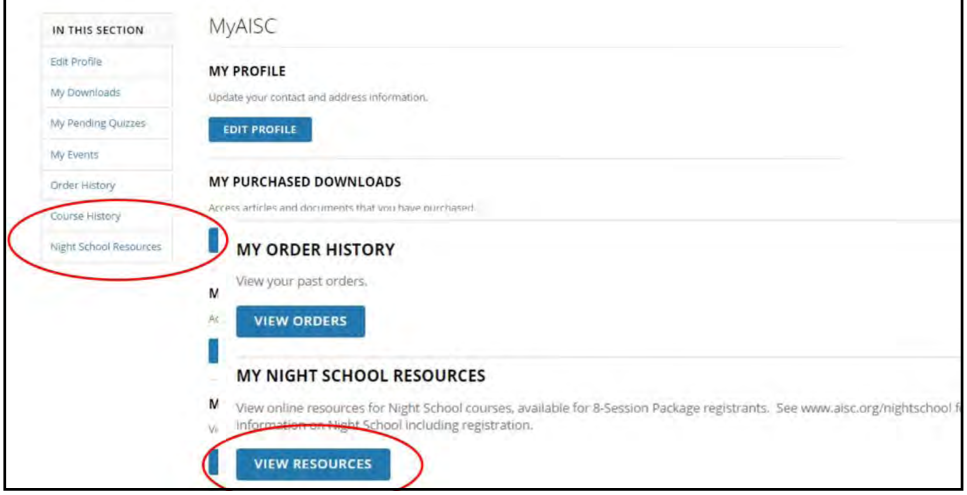

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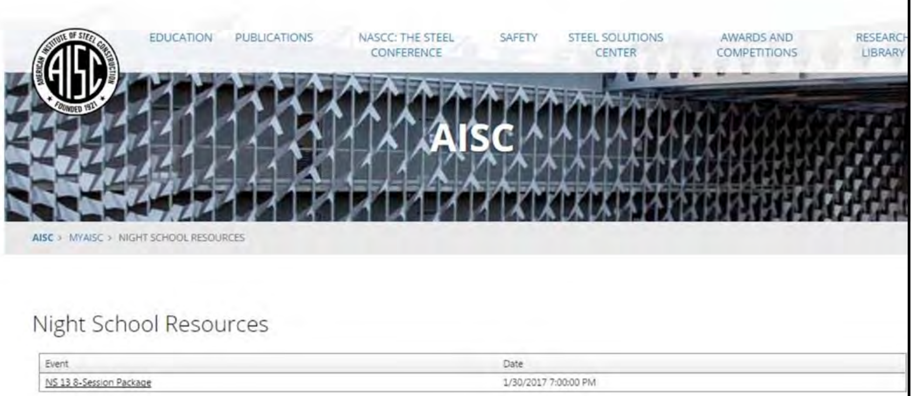

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Night School 13: Design of Industrial Buildings

8-SESSION PACKAGE RESOURCES

Event	Date	Handouts	Video	Quiz	Attendance
NS13 - Design Criteria	1/30/2017 7:00:00 PM	Handouts	View Passcode: NS13DSN	Pass Score: 80	Pending
NS13 - Economic Considerations	2/6/2017 7:00:00 PM	Handouts	Available 02/08/2017 5pm EST	Available 02/08/2017 5pm EST	Pending
NS13 - Lateral Load Systems and Details	2/13/2017 7:00:00 PM	Handouts	Available 02/15/2017 5pm EST	Available 02/15/2017 5pm EST	Pending
NS13 - Preliminary Design Procedures	2/27/2017 7:00:00 PM	Handouts	Available 03/01/2017 5pm EST	Available 03/01/2017 5pm EST	Pending
NS13 - Crane Girder Design and Frame Analysis	3/6/2017 7:00:00 PM	Handouts	Available 03/08/2017 5pm EST	Available 03/08/2017 5pm EST	Pending
NS13 - Frame Member and Connection Design	3/13/2017 7:00:00 PM	Handouts	Available 03/15/2017 5pm EST	Available 03/15/2017 5pm EST	Pending
NS13 - Transfer Crane Girder & Longitudinal Bldg Bracing Dsn	3/27/2017 7:00:00 PM	Handouts	Available 03/29/2017 5pm EST	Available 03/29/2017 5pm EST	Pending
NS13 - Building Envelope and Bracing Design	4/3/2017 7:00:00 PM	Handouts	Available 04/05/2017 5pm EST	Available 04/05/2017 5pm EST	Pending
NS13 - Final Exam	4/10/2017 7:00:00 PM			Available 04/12/2017 5pm EST	

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- Weekly "quiz and recording" email.
- Weekly updates of the master Quiz and Attendance record found at www.aisc.org/nightschool. Scroll down to Quiz and Attendance records.
    - Updated on Tuesday mornings.

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There's always a solution in steel.

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