





The logo features a white crescent moon to the left of the text "AISC" in orange, with "Night School" in a larger orange font below it.

Thank you for joining our live webinar today.  
We will begin shortly. Please standby.

Thank you.

Need Help?  
Call ReadyTalk Support: 800.843.9166


A cartoon owl with brown feathers, a white belly, and large eyes, wearing a white graduation cap with a blue tassel. It is standing on a dark silhouette of a tree branch.


The logo features a white crescent moon to the left of the text "AISC" in orange, with "Night School" in a larger orange font below it.

Today's audio will be broadcast through the internet.

Alternatively, to hear the audio through the phone, dial  
800-289-0459. Passcode: 316042

---



The AISC logo is a circular seal with "AMERICAN INSTITUTE OF STEEL CONSTRUCTION" around the top, "AISC" in the center, and "FOUNDED 1921" around the bottom.

2



Today's live webinar will begin shortly.  
Please standby.

As a reminder, all lines have been muted. Please type any questions or comments through the Chat feature on the left portion of your screen.

Today's audio will be broadcast through the internet.  
Alternatively, to hear the audio through the phone, dial  
800-289-0459. Passcode: 316042



3



*AISC is a Registered Provider with The American Institute of Architects Continuing Education Systems (AIA/CES). Credit(s) earned on completion of this program will be reported to AIA/CES for AIA members. Certificates of Completion for both AIA members and non-AIA members are available upon request.*

*This program is registered with AIA/CES for continuing professional education. As such, it does not include content that may be deemed or construed to be an approval or endorsement by the AIA of any material of construction or any method or manner of handling, using, distributing, or dealing in any material or product.*

*Questions related to specific materials, methods, and services will be addressed at the conclusion of this presentation.*



4





## Copyright Materials

This presentation is protected by US and International Copyright laws. Reproduction, distribution, display and use of the presentation without written permission of AISC is prohibited.

© The American Institute of Steel Construction 2018



5



## Session Description

### **Session 4: Design Examples**

**July 9, 2018**

This session will present a design example for vertically-curved members. In addition to demonstration the design of the member, the example will address the design of the connection. Then, the session will present a design example for horizontally-curved members. In addition to demonstration the design of the member, the example will address the design of the connection.



6





## Learning Objectives

- Describes the steps in designing a vertically-curved member.
- List the limit states of the vertically-curved member connection.
- Describes the steps in designing a horizontally-curved member.
- List the limit states of the horizontally-curved member connection.



7

# Design of Curved Members

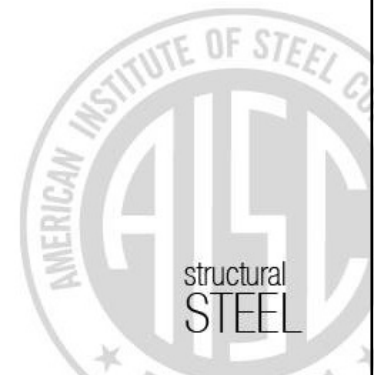
## Session 4: Design Examples

July 9, 2018



Presented by  
Bo Dowswell, P.E., Ph.D.  
ARC International, LLC  
Birmingham, AL

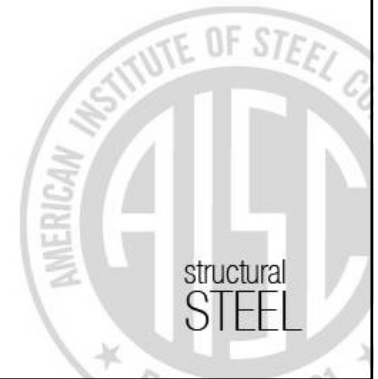
There's always a solution in steel.



There's always a solution in steel.

# Design Examples

## Session Description



## Session Description

- Design examples
  - Example 1: vertically-curved member
  - Example 2: horizontally-curved member

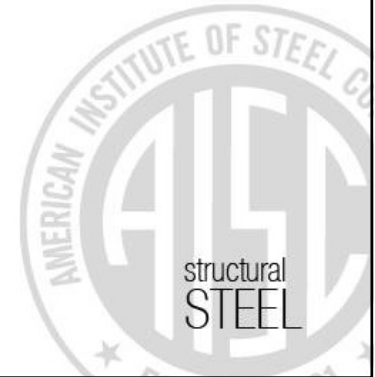


10



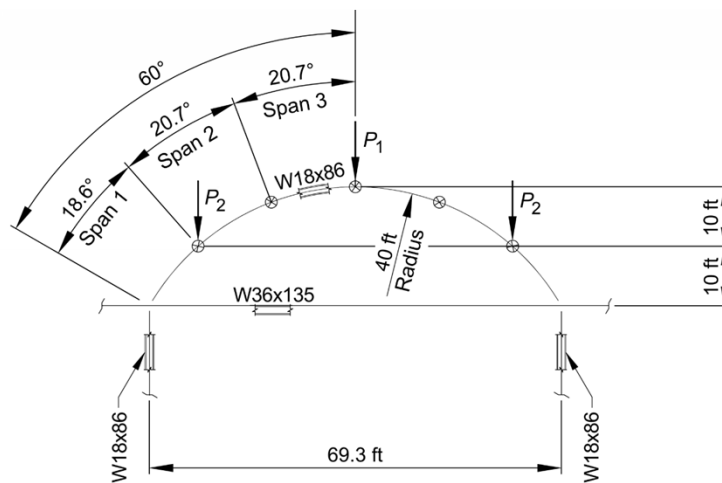
# Design Examples

## Example 1: Vertically-Curved Member



There's always a solution in steel.

### Example 1

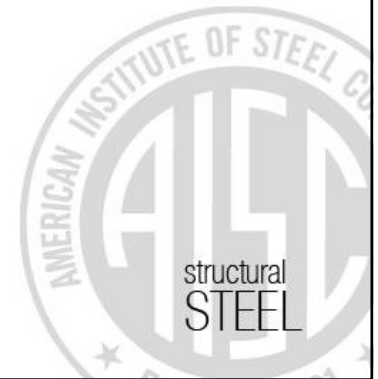


There's always a solution in steel.

# Design Examples

## Example 1

### Problem Statement



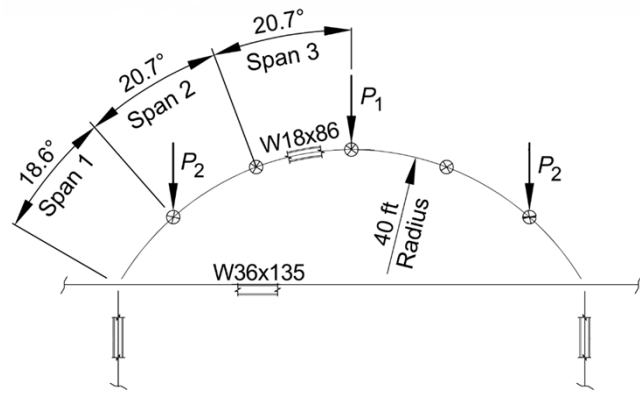
## Problem Statement

- Verify that the arch is adequate for the imposed loading
- Use LRFD



## Problem Statement

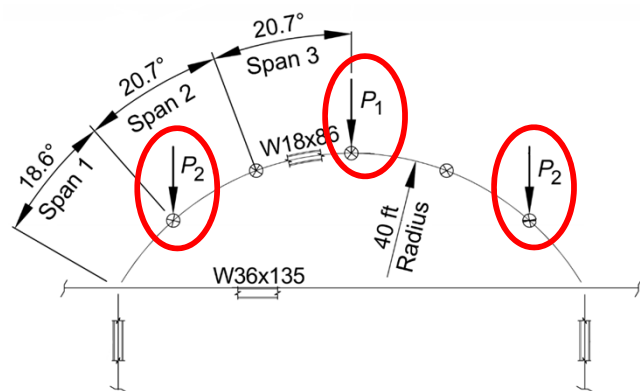
- Curved member
  - W18x86
  - ASTM A992
  - Bent the hard way
  - Circular curve



15

## Problem Statement

- The factored (LRFD) loads are
  - $P_{1u} = 120$  kips
  - $P_{2u} = 75$  kips

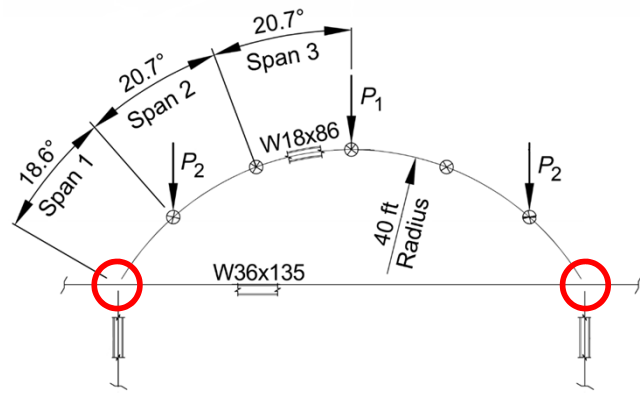


16



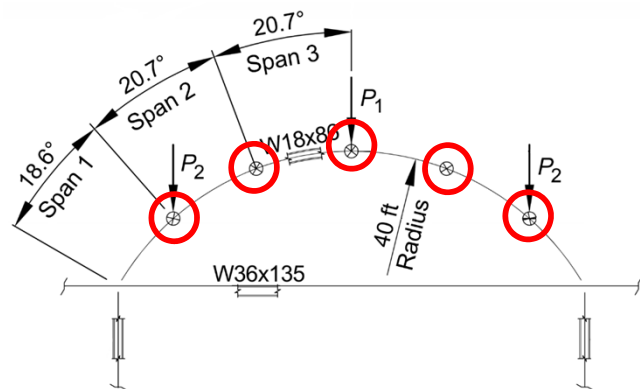
## Problem Statement

- Supports
  - Translation fixed in all directions
  - Rotation free in all directions



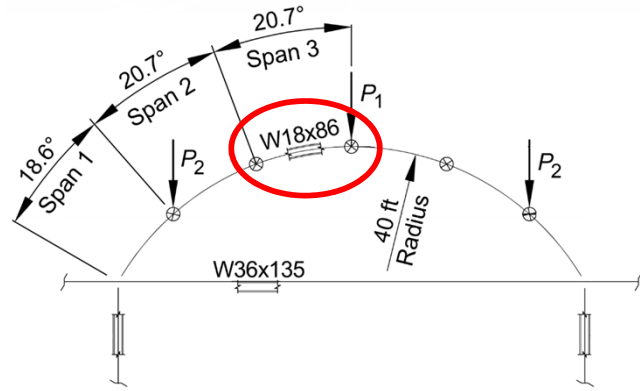
## Problem Statement

- Braces
  - Prevent out-of-plane translation
  - Prevent torsional rotation



# Problem Statement

- Assume Span 3 is critical



# Design Examples

Example 1

Properties



There's always a solution in steel.



## Properties

- Material properties of ASTM A992  
(AISC *Manual* Table 2-4)

$$F_y = 50 \text{ ksi}$$

$$F_u = 65 \text{ ksi}$$



21

## Properties

- Dimensions of W18×86  
(AISC *Manual* Table 1-1)

$$d = 18.4 \text{ in.}$$

$$t_w = 0.480 \text{ in.}$$

$$b_f = 11.1 \text{ in.}$$

$$t_f = 0.770 \text{ in.}$$

$$h_o = 17.6 \text{ in.}$$



22



## Properties

- Section properties of W18×86  
(AISC *Manual* Table 1-1)

$$I_x = 1,530 \text{ in.}^4$$

$$S_x = 166 \text{ in.}^3$$

$$r_x = 7.77 \text{ in.}$$

$$Z_x = 186 \text{ in.}^3$$

$$I_y = 175 \text{ in.}^4$$

$$r_y = 2.63 \text{ in.}$$

$$J = 4.10 \text{ in.}^4$$

$$C_w = 13,600 \text{ in.}^6$$

$$r_{ts} = 3.05 \text{ in.}$$



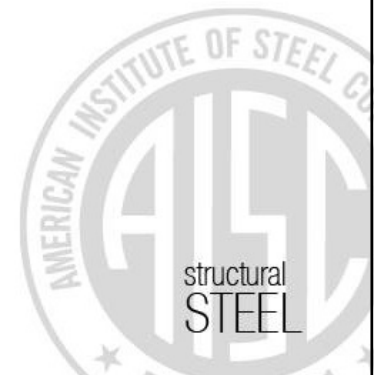
23

## Design Examples

### Example 1

### Arch Geometry

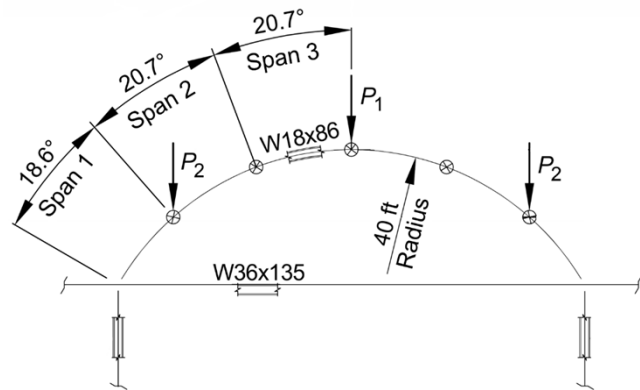
There's always a solution in steel.



# Arch Geometry

- Centroidal radius

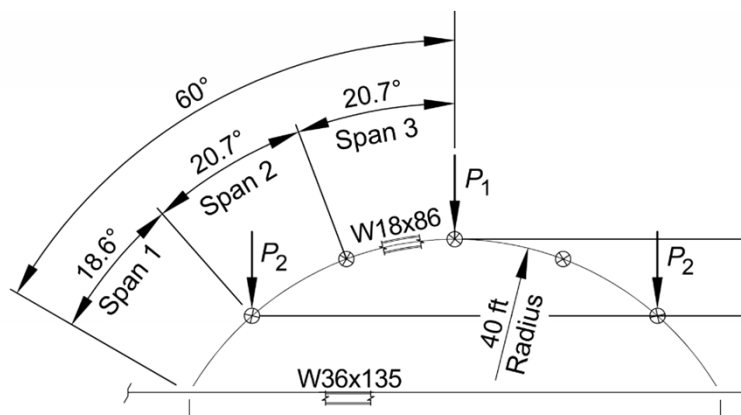
$$R = (40 \text{ ft})(12 \text{ in./ft}) = 480 \text{ in.}$$



# Arch Geometry

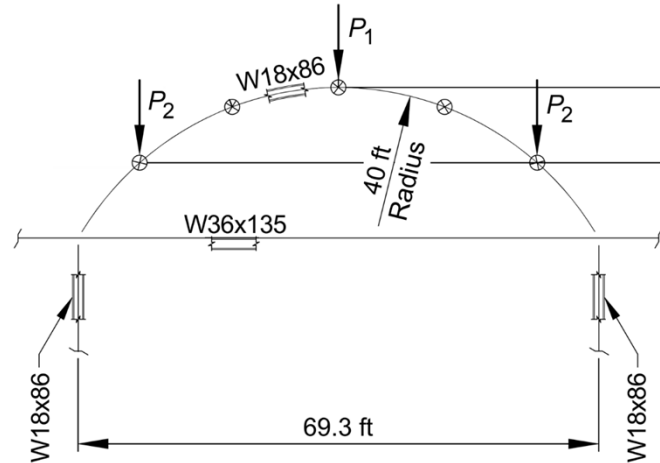
- Arch angle

$$\theta = (120^\circ) \left( \frac{\pi \text{ rad}}{180^\circ} \right) = (2\pi/3) \text{ rad}$$



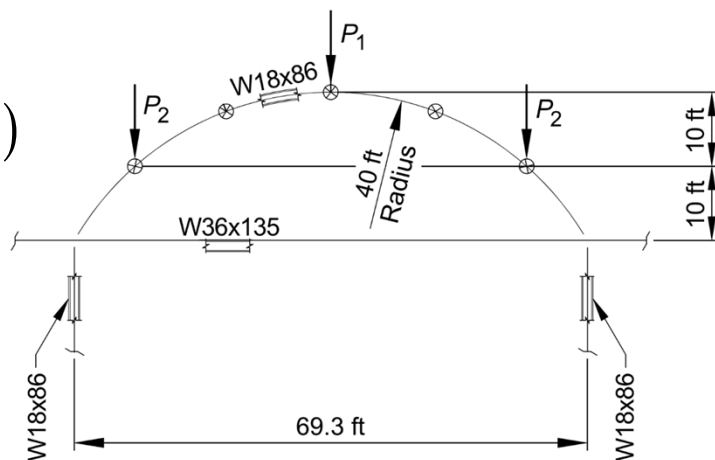
# Arch Geometry

- Span length (chord)  
 $L_s = (69.3 \text{ ft})(12 \text{ in./ft})$   
 $= 832 \text{ in.}$



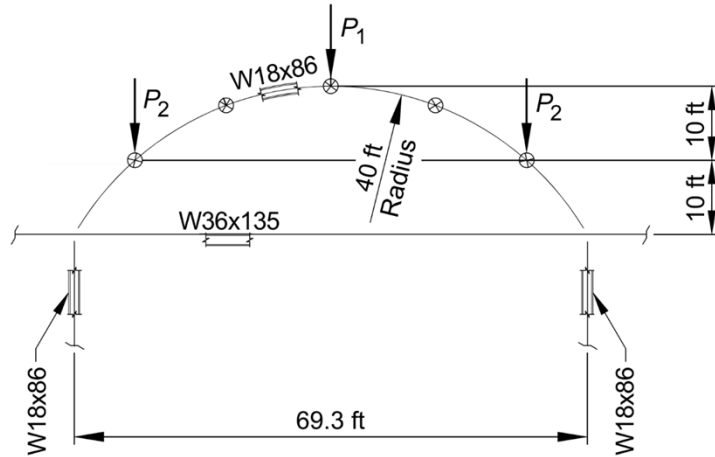
# Arch Geometry

- Rise  
 $H = (20 \text{ ft})(12 \text{ in./ft})$   
 $= 240 \text{ in.}$



# Arch Geometry

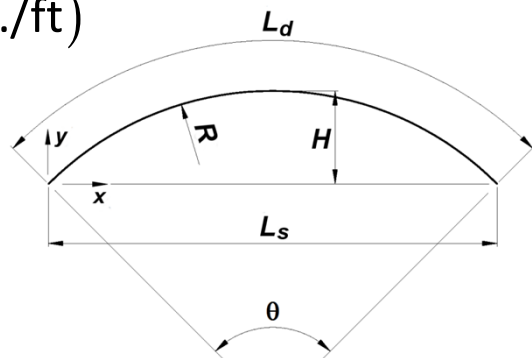
$$\frac{H}{L_s} = \frac{240 \text{ in.}}{832 \text{ in.}} = 0.288$$



# Arch Geometry

- Developed arc length

$$L_d = (40 \text{ ft}) \left[ (2\pi / 3) \text{ rad} \right] (12 \text{ in./ft}) = 1,010 \text{ in.}$$



# Design Examples

There's always a solution in steel.

## Example 1

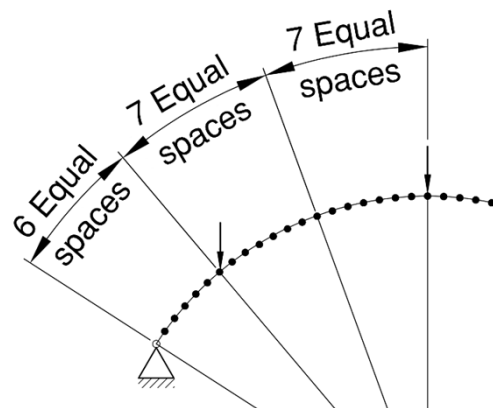
### Structural Analysis



## Structural Analysis

### Finite Element Model

- Segmented spans
  - Straight beam elements
  - $\approx 3^\circ$  arc between nodes
- First-order analysis

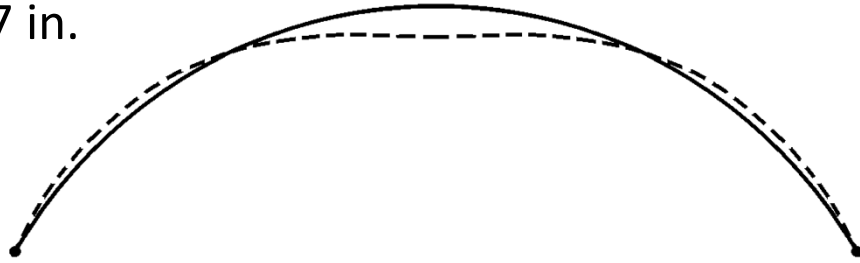


# Structural Analysis

## Deflection

- Maximum @ apex

$$\Delta_1 = 1.07 \text{ in.}$$



33

# Structural Analysis

## Deflection

- If  $\Delta_1$  is less than  $H/40$ , a first-order finite element analysis is sufficiently accurate

$$\frac{H}{40} = \frac{240 \text{ in.}}{40} = 6.00 \text{ in.} \quad 1.07 \text{ in.} < 6.00 \text{ in.} \quad \mathbf{o.k.}$$



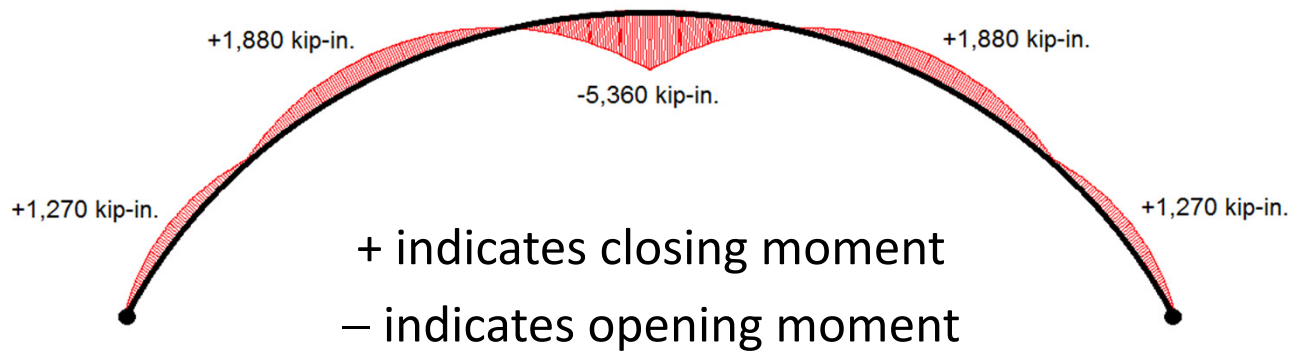
Ref: King, C. and Brown, D. (2001), *Design of Curved Steel*, The Steel Construction Institute

34



# Arch Geometry

## In-Plane Moment



# Structural Analysis

Member Loads (kip, in.)			
Location	Axial	Moment	Shear
	$P_u$	$M_{ux}$	$V_u$
Supports	182	0	25.8
Apex	118	-5,360	57.5



# Structural Analysis

1 <sup>st</sup> -Order Member Loads (kip, in.)			
Location	Axial	Moment	Shear
	$P_u$	$M_{ux}$	$V_u$
Max./Min. @ Span 3	131	+1,380 -5,360	57.5



## Design Examples

### Example 1

### Local Buckling

There's always a solution in steel.



## Local Buckling

- Calculations are the same as for a straight member
- Axial:  $\lambda_f < \lambda_{rf}$  and  $\lambda_w < \lambda_{rw}$  → the W18×86 is non-slender
- Flexure:  $\lambda_f < \lambda_{pf}$  and  $\lambda_w < \lambda_{pw}$  → the W18×86 is compact



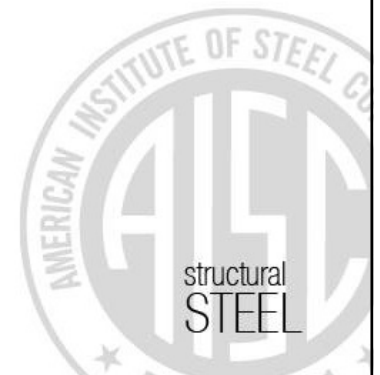
39

## Design Examples

### Example 1

### Shear Strength

There's always a solution in steel.



## Shear Strength

- AISC *Manual* Table 6-2

$$\phi_v V_n = 265 \text{ kips} > V_u = 57.5 \text{ kips} \quad \mathbf{o.k.}$$



41

## Design Examples

### Example 1

### Local Flange Bending

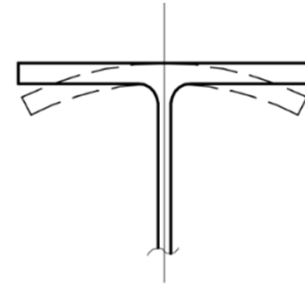
There's always a solution in steel.



# Local Flange Bending

## Simplified Method

$$k_f = \frac{9.20}{8.80 + \frac{b_f^2}{Rt_f}} \leq 1.0$$



REF: Session 2 Slide 105

43

# Local Flange Bending

## Simplified Method

$$k_f = \frac{9.20}{8.80 + \frac{(11.1 \text{ in.})^2}{(480 \text{ in.})(0.770 \text{ in.})}} \leq 1.00$$
$$= 1.01 > 1.00$$



44

# Local Flange Bending

## Simplified Method

- $k_f = 1.00$
- Flange bending will not occur
- The effective flexural properties are equal to the straight-member properties



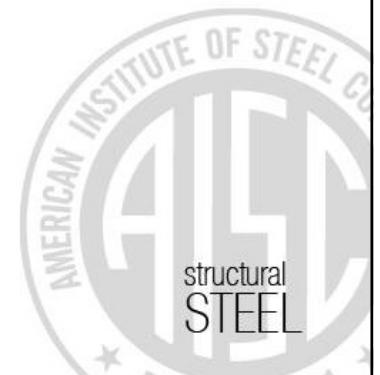
45

## Design Examples

### Example 1

### In-Plane Strength

There's always a solution in steel.



## In-Plane Strength

- Snap-through buckling is not critical because:
  - The supports are rigid
  - $H/L_s = 0.288 > 0.2$



REF: Session 2 Slide 52

47

## In-Plane Strength

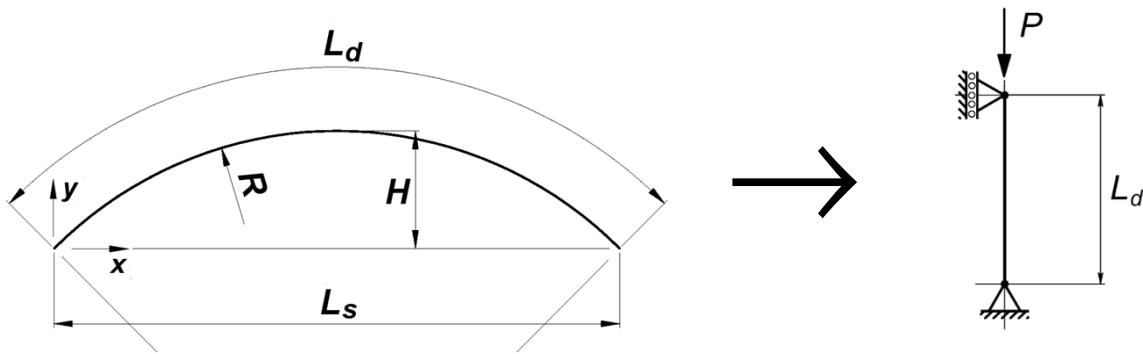
- AISC *Specification* Section E3
- Radius of gyration about the axis of curvature:  
 $r_i = r_x = 7.77$  in.



48

## In-Plane Strength

- Unbraced length:  $L_d = 1,010$  in.



## In-Plane Strength

- For a circular arch with pinned end conditions and  $H/L_s = 0.288$ ,  $K_i = 0.55$

In-plane effective length factor, $K_i$			
Arch Form	End Conditions	$H/L_s$	$K_i$
Circular	Pinned	$0.1 \leq H/L_s \leq 0.3$	0.55
		$0.3 \leq H/L_s \leq 0.5$	0.60
	Fixed	All	0.40



## In-Plane Strength

$$\begin{aligned}\frac{L_c}{r} &= \frac{K_i L_d}{r_i} \\ &= \frac{(0.55)(1,010 \text{ in.})}{7.77 \text{ in.}} \\ &= 71.5\end{aligned}$$

AISC Manual Table 4-14:  $\phi_c F_{cr} = 31.0 \text{ ksi}$



51

## In-Plane Strength

- The available strength is

$$\begin{aligned}\phi_c P_{ni} &= (31.0 \text{ ksi})(25.3 \text{ in.}^2) \\ &= 784 \text{ kips}\end{aligned}$$



52

There's always a solution in steel.

## Design Examples

### Example 1

### Out-of-Plane Strength



## Out-of-Plane Strength

- AISC *Specification* Section E3
- Moment of inertia perpendicular to the axis of curvature:  $I_o = I_y = 175 \text{ in.}^4$
- Radius of gyration perpendicular to the axis of curvature:  $r_o = r_y = 2.63 \text{ in.}$



54

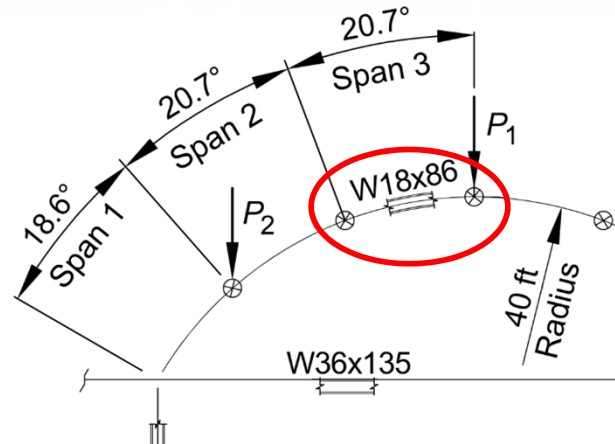


# Out-of-Plane Strength

## Span 3

$$\theta_b = (20.7^\circ) \left( \frac{\pi \text{ rad}}{180^\circ} \right)$$

$$= 0.361 \text{ rad}$$

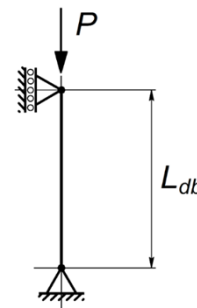


# Out-of-Plane Strength

$$L_{db} = R\theta_b$$

$$= (480 \text{ in.})(0.361 \text{ rad})$$

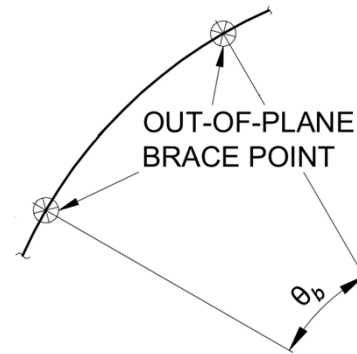
$$= 173 \text{ in.}$$



## Out-of-Plane Strength

- Effective length factor,  $K_o$ : Circular doubly-symmetric segments

$$K_o = \frac{\sqrt{1 + \frac{1}{C_o} \left( \frac{\theta_b}{\pi} \right)^2}}{1 - \left( \frac{\theta_b}{\pi} \right)^2}$$



REF: Session 2 Slide 67

57

## Out-of-Plane Strength

$$C_o = \frac{1}{I_o} \left[ \frac{GJ}{E} + C_w \left( \frac{\pi}{L_{db}} \right)^2 \right]$$

$$= \frac{1}{175 \text{ in.}^4} \left[ \frac{(11,200 \text{ ksi})(4.10 \text{ in.}^4)}{29,000 \text{ ksi}} + (13,600 \text{ in.}^6) \left( \frac{\pi}{173 \text{ in.}} \right)^2 \right]$$

$$= 0.0347$$



REF: Session 2 Slide 69

58

## Out-of-Plane Strength

$$K_o = \frac{\sqrt{1 + \frac{1}{C_o} \left( \frac{\theta_b}{\pi} \right)^2}}{1 - \left( \frac{\theta_b}{\pi} \right)^2} = \frac{\sqrt{1 + \frac{1}{0.0347} \left( \frac{0.361 \text{ rad}}{\pi} \right)^2}}{1 - \left( \frac{0.361 \text{ rad}}{\pi} \right)^2} = 1.19$$



59

## Out-of-Plane Strength

$$\begin{aligned} \frac{L_c}{r} &= \frac{K_o L_{db}}{r_o} \\ &= \frac{(1.19)(173 \text{ in.})}{2.63 \text{ in.}} \\ &= 78.3 \end{aligned}$$

AISC Manual Table 4-14:  $\phi_c F_{cr} = 28.7 \text{ ksi}$



60

## Out-of-Plane Strength

- The available strength is

$$\begin{aligned}\phi_c P_{no} &= (28.7 \text{ ksi})(25.3 \text{ in.}^2) \\ &= 726 \text{ kips}\end{aligned}$$



61

## Design Examples

### Example 1

### Second-Order Effects

There's always a solution in steel.



## Second-Order Effects

- Amplified first-order analysis
- Second-order moment:  $M_{ux2} = B_i M_{ux}$

$$B_i = \frac{1}{1 - \alpha \frac{P_u}{P_{ei}}}$$

$P_{ei}$  = elastic critical load for in-plane buckling

$\alpha = 1.00$  (LRFD)



REF: Session 2 Slide 87

63

## Second-Order Effects

$$\begin{aligned} F_e &= \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} \\ &= \frac{\pi^2 (29,000 \text{ ksi})}{(71.5)^2} \\ &= 56.0 \text{ ksi} \end{aligned}$$

$$\begin{aligned} P_{ei} &= F_e A_g \\ &= (56.0 \text{ ksi})(25.3 \text{ in.}^2) \\ &= 1,420 \text{ kips} \end{aligned}$$



64

## Second-Order Effects

$$B_i = \frac{1}{1 - \alpha \frac{P_u}{P_{ei}}} = \frac{1}{1 - (1.0) \left( \frac{182 \text{ kips}}{1,420 \text{ kips}} \right)} = 1.15$$



65

## Second-Order Effects

2 <sup>nd</sup> -Order Moments (kip-in.)	
Location	$M_{ux2}$
Max./Min. @	+1,590
Span 3	-6,160



66

There's always a solution in steel.

# Design Examples

## Example 1

### Flexural Strength



## Flexural Strength

- AISC *Specification* Section F2
- Moment of inertia perpendicular to the axis of curvature:  $I_o = I_y = 175 \text{ in.}^4$



68



# Flexural Strength

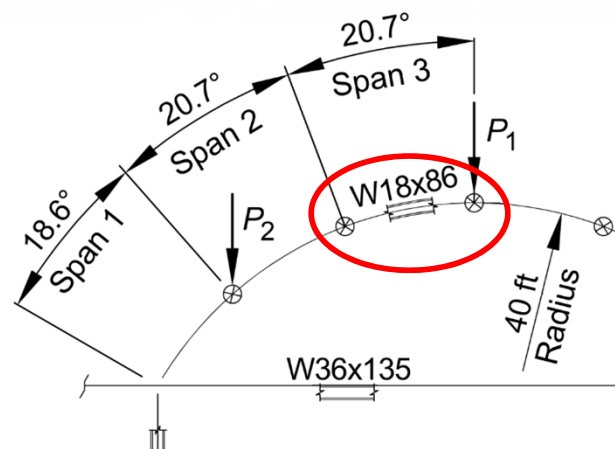
$$\begin{aligned} M_p &= F_y Z_x \\ &= (50 \text{ ksi})(186 \text{ in.}^3) \\ &= 9,300 \text{ kip-in.} \end{aligned}$$



# Flexural Strength

## Span 3

$$L_{db} = 173 \text{ in.}$$



## Flexural Strength

$M_{es}$  = elastic lateral-torsional buckling moment of the equivalent straight member subjected to uniform moment with a length equal to  $L_{db}$

$$M_{es} = \frac{\pi}{L_{db}} \sqrt{EI_o GJ + \left(\frac{\pi E}{L_{db}}\right)^2 I_o C_w} = 17,200 \text{ kip-in.}$$



REF: Session 2 Slide 78

71

## Flexural Strength

$$\begin{aligned} C_y &= EI_o \\ &= (29,000 \text{ ksi})(175 \text{ in.}^4) \\ &= 5,080,000 \text{ kip-in.}^2 \end{aligned}$$



REF: Session 2 Slide 80

72

## Flexural Strength

$$\begin{aligned} C_z &= GJ + \frac{\pi^2 EC_w}{L_{db}^2} \\ &= (11,200 \text{ ksi})(4.10 \text{ in.}^4) + \frac{\pi^2 (29,000 \text{ ksi})(13,600 \text{ in.}^6)}{(173 \text{ in.})^2} \\ &= 176,000 \text{ kip-in.}^2 \end{aligned}$$



REF: Session 2 Slide 80

73

## Flexural Strength

$$\begin{aligned} C_a &= \frac{C_y + C_z}{2RM_{es}} \\ &= \frac{5,080,000 \text{ kip-in.}^2 + 176,000 \text{ kip-in.}^2}{(2)(480 \text{ in.})(17,200 \text{ kip-in.})} \\ &= 0.318 \end{aligned}$$

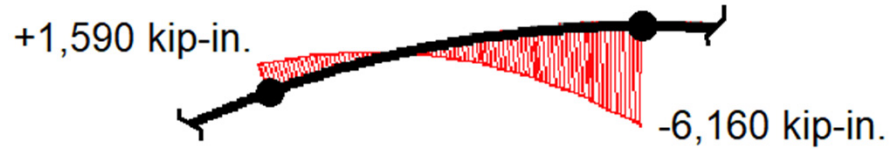


REF: Session 2 Slide 80

74

# Flexural Strength

- Span 3



$$C_{bs} = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} = 2.38$$



REF: Session 2 Slide 77

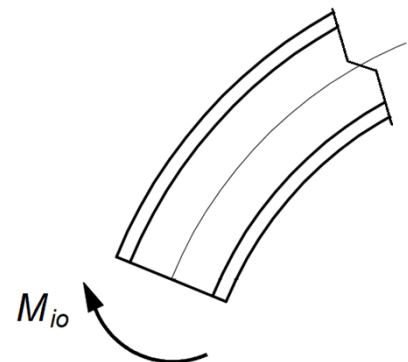
75

# Flexural Strength

Opening moments → negative root

$$C_{bi} = C_{bs} \left( \sqrt{1 + C_a^2 - \frac{C_y C_z}{R^2 M_{es}^2}} - C_a \right)$$

$$= 1.75$$



REF: Session 2 Slide 76

76

## Flexural Strength

- AISC *Specification* Section F2
  - $L_b = L_{db} = 173 \text{ in.} = 14.4 \text{ ft}$
  - $C_b = C_{bi} = 1.75$
- AISC *Manual* Table 3-6
  - $L_p = 9.29 \text{ ft}$
  - $L_r = 28.6 \text{ ft}$



77

## Flexural Strength

- $L_p < L_b < L_r \rightarrow$  Use AISC *Spec.* Eq. F2-2

$$M_n = C_b \left[ M_p - (M_p - 0.7F_y S_x) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p$$
$$= 14,700 \text{ kip-in.} > 9,300 \text{ kip-in.}$$



78

## Flexural Strength

- The available strength is

$$\begin{aligned}\phi_b M_n &= 0.90(9,300 \text{ kip-in.}) \\ &= 8,370 \text{ kip-in.}\end{aligned}$$



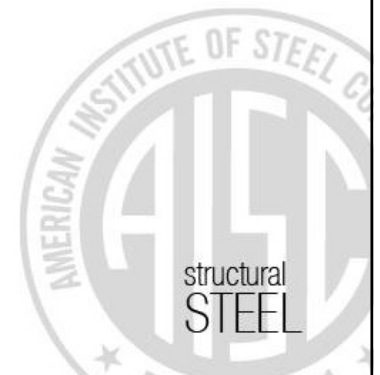
79

## Design Examples

### Example 1

### Combined Loading

There's always a solution in steel.



## Combined Loading

- AISC *Specification* Section H1
- In-plane buckling
- Out-of-plane buckling



81

## Combined Loading

### In-Plane Buckling

- The largest flexural load ratio is at the apex
- The largest axial load ratio is at the supports

$$\frac{P_u}{\phi_c P_{ni}} = \frac{182 \text{ kips}}{784 \text{ kips}} = 0.232$$



82



## Combined Loading

$0.232 > 0.2 \rightarrow$  AISC *Specification* Equation H1-1a

$$\frac{P_u}{\phi_c P_{ni}} + \frac{8}{9} \left( \frac{M_u}{\phi_b M_n} \right) \leq 1.0$$

$$0.232 + \left( \frac{8}{9} \right) \left( \frac{6,160 \text{ kip-in.}}{8,370 \text{ kip-in.}} \right) \leq 1.0$$

$0.886 < 1.0$  **o.k.**



83

## Combined Loading

### Out-of-Plane Buckling (Span 3)

- Largest flexural load in the span
- Largest axial load in the span

$$\frac{P_u}{\phi_c P_{no}} = \frac{131 \text{ kips}}{726 \text{ kips}} = 0.180$$



84

## Combined Loading

$0.180 < 0.2 \rightarrow$  AISC *Specification* Equation H1-1b

$$\frac{P_u}{2\phi_c P_{no}} + \frac{M_u}{\phi_b M_n} \leq 1.0$$
$$\frac{0.180}{2} + \frac{6,160 \text{ kip-in.}}{8,370 \text{ kip-in.}} \leq 1.0$$

$0.826 < 1.0$  **o.k.**



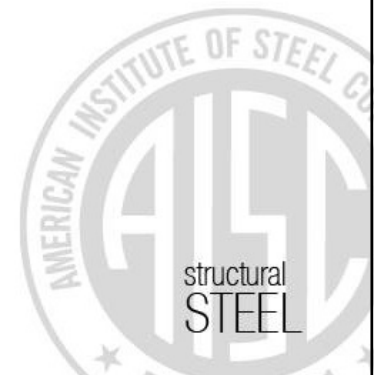
85

## Design Examples

### Example 1

### End Connections

There's always a solution in steel.



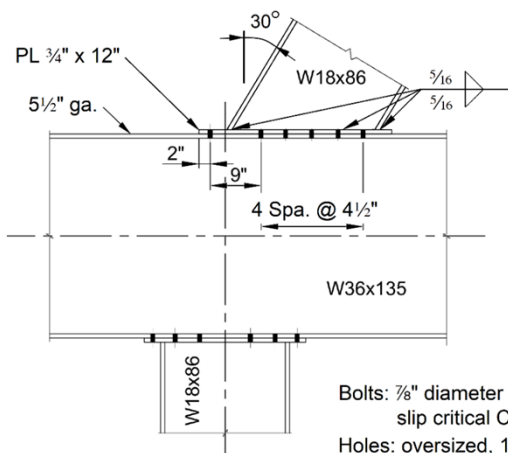
## End Connections

- Support reactions from FE model
  - Vertical:  $R_{uy} = 140$  kips
  - Horizontal:  $R_{ux} = 117$  kips



87

## End Connections

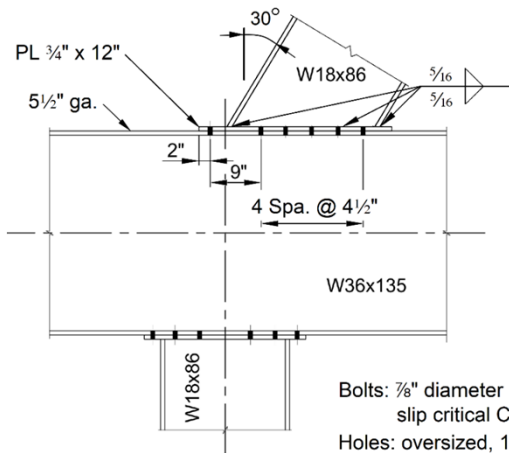


To satisfy the boundary conditions assumed in design, out-of-plane translation must be restrained.



88

# End Connections

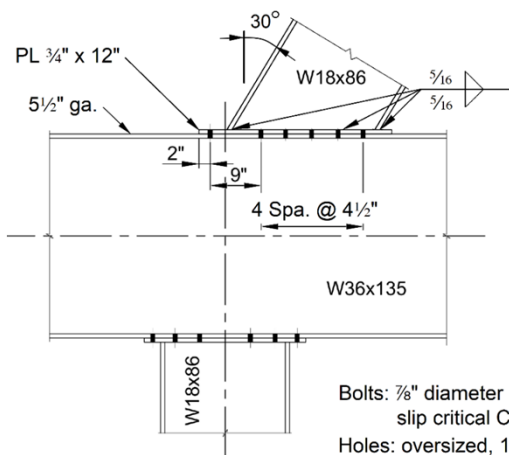


W36 web local compression strength > 140 kips (AISC Specification Section J10)

Bolts: 7/8" diameter Group A, slip critical Class A  
 Holes: oversized, 1 1/16" diameter



# End Connections



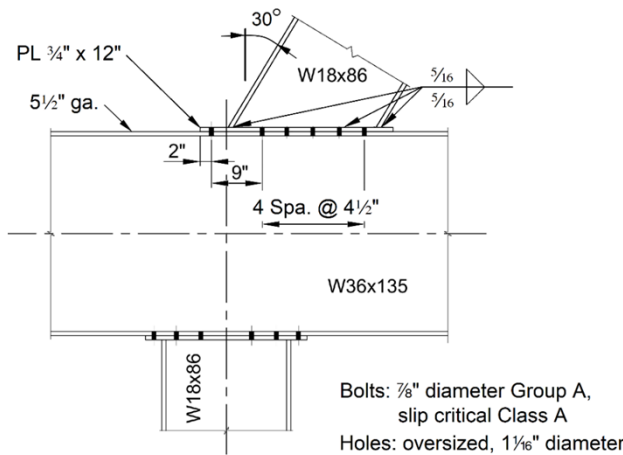
Slip critical joints increase horizontal rigidity by limiting bolt slip

Bolt slip load > 117 kips

Bolts: 7/8" diameter Group A, slip critical Class A  
 Holes: oversized, 1 1/16" diameter



# End Connections



AISC COSP chord length  
tolerance:  $\pm 1/8$  in.

Slip critical bolts allow the  
use of oversized holes for  
easier field fit-up



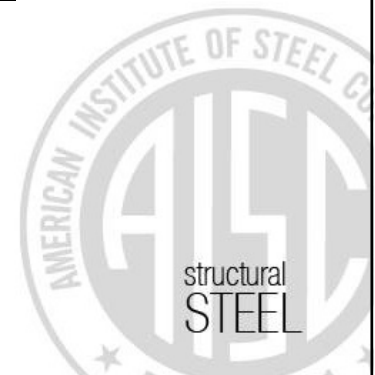
REF: Session 2 Slide 23

91

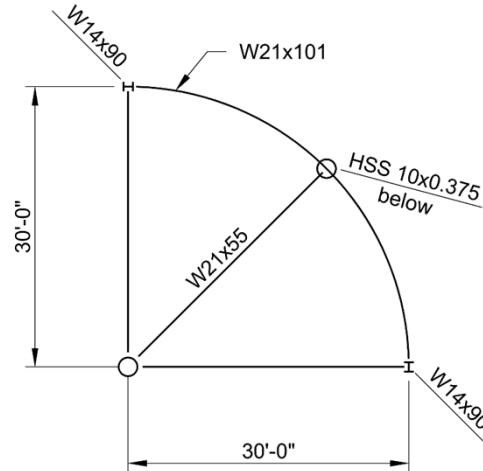
## Design Examples

### Example 2: Horizontally-Curved Member

There's always a solution in steel.



## Example 2



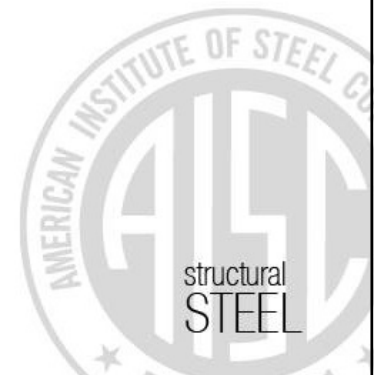
93

## Design Examples

### Example 2

### Problem Statement

There's always a solution in steel.



## Problem Statement

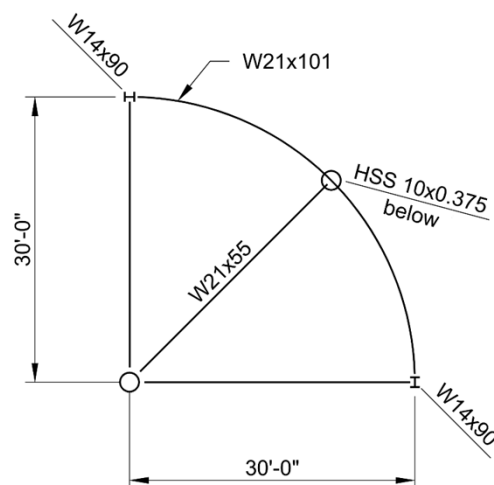
- Verify that the horizontally-curved beam is adequate for the imposed loading
- Use LRFD



95

## Problem Statement

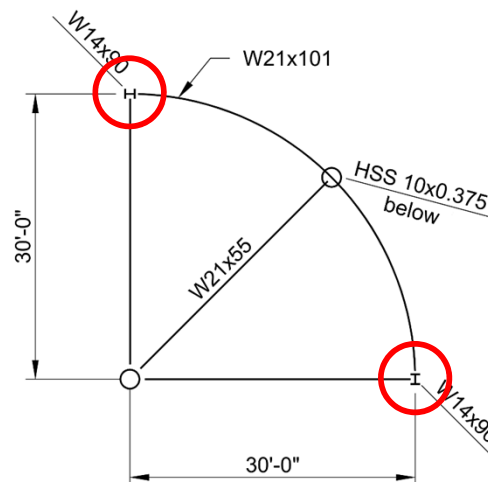
- Curved member
  - W21×101
  - ASTM A992
  - Bent the easy way
  - Circular curve



96

## Problem Statement

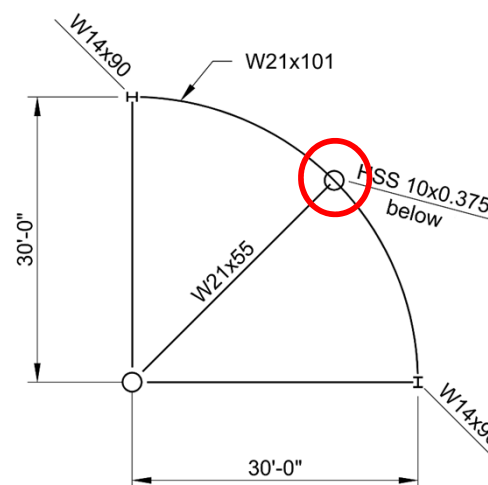
- End connections
  - Torsional rotation is restrained
  - No warping restraint
  - No flexural restraint



97

## Problem Statement

- Connections @ the HSS10
  - Continuous for flexure
  - Torsion is restrained by the W21 beam
  - Continuous for warping



98

## Problem Statement

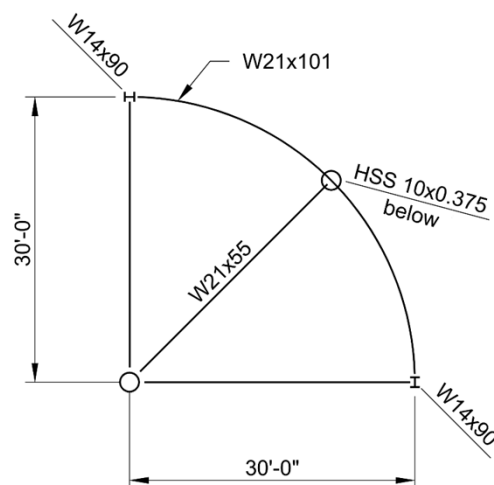
- The factored uniformly distributed load along the member circumference including the beam self weight is  $w_u = 0.750$  kip/ft



99

## Problem Statement

- Assume the critical condition is for patch loading (one span loaded)



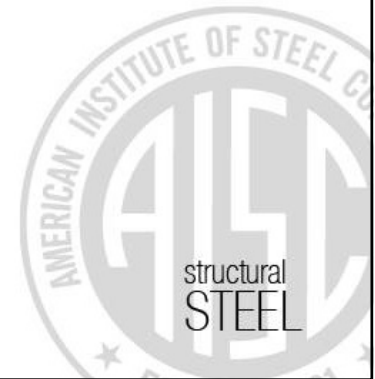
100

There's always a solution in steel.

## Design Examples

### Example 2

### Properties



## Properties

- Material properties of ASTM A992  
(AISC *Manual* Table 2-4)

$$F_y = 50 \text{ ksi}$$

$$F_u = 65 \text{ ksi}$$



102



## Properties

- Dimensions of W21×101  
(AISC *Manual* Table 1-1)

$$d = 21.4 \text{ in.}$$

$$t_w = 0.500 \text{ in.}$$

$$b_f = 12.3 \text{ in.}$$

$$t_f = 0.800 \text{ in.}$$

$$h_o = 20.6 \text{ in.}$$



103

## Properties

- Section properties of W21×101  
(AISC *Manual* Table 1-1)

$$I_x = 2,420 \text{ in.}^4$$

$$S_x = 227 \text{ in.}^3$$

$$Z_x = 253 \text{ in.}^3$$

$$J = 5.21 \text{ in.}^4$$

$$C_w = 26,200 \text{ in.}^6$$

$$r_{ts} = 3.35 \text{ in.}$$



104

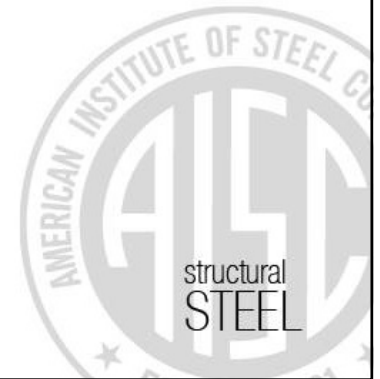


# Design Examples

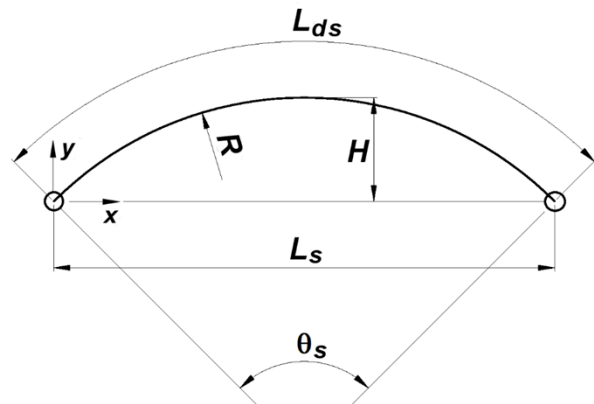
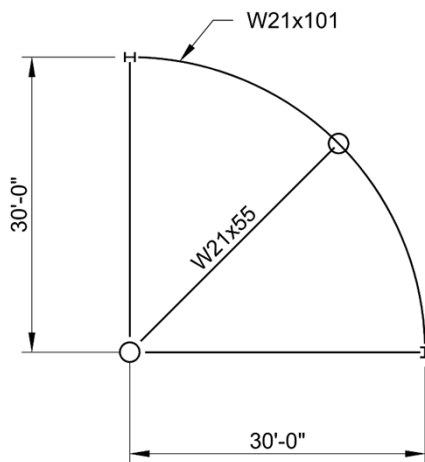
There's always a solution in steel.

## Example 2

### Beam Geometry



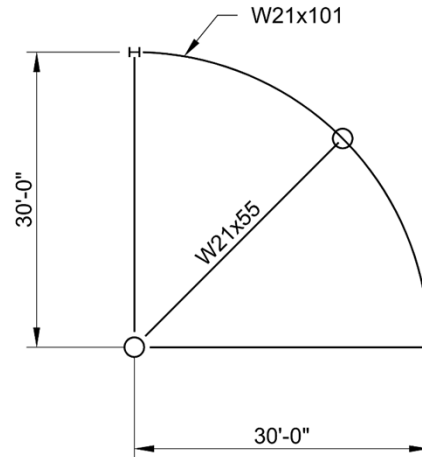
### Beam Geometry



## Beam Geometry

- Centroidal radius

$$R = (30 \text{ ft})(12 \text{ in./ft}) \\ = 360 \text{ in.}$$

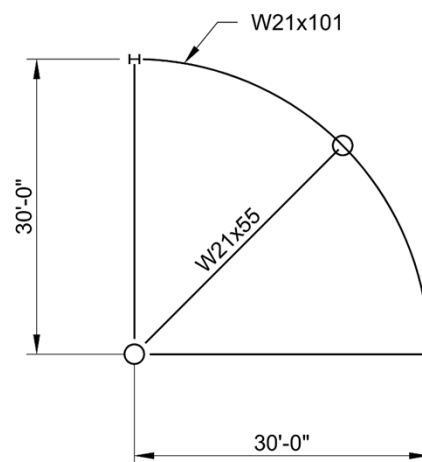


107

## Beam Geometry

- Span angle

$$\theta_s = (45^\circ) \left( \frac{\pi \text{ rad}}{180^\circ} \right) \\ = \pi / 4 \text{ rad}$$

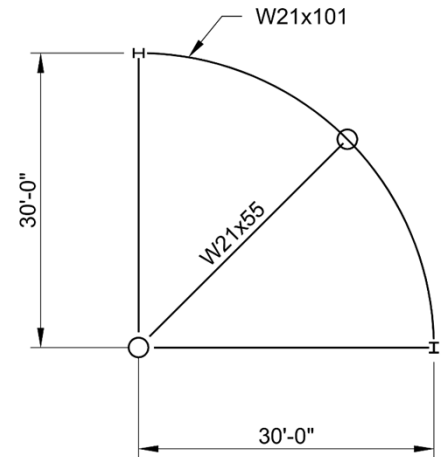


108

## Beam Geometry

- Developed span length

$$L_{ds} = (30 \text{ ft})(\pi / 4 \text{ rad})(12 \text{ in./ft})$$
$$= 283 \text{ in.}$$

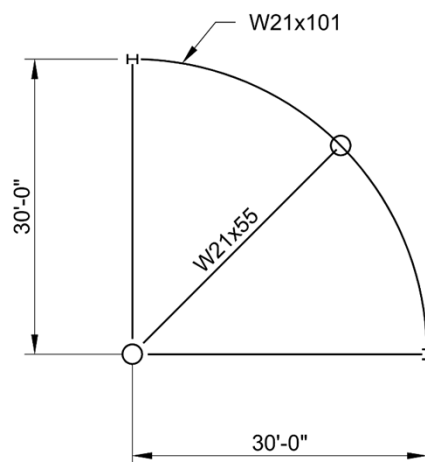


109

## Beam Geometry

- Angle between torsional restraints

$$\theta_b = (45^\circ) \left( \frac{\pi \text{ rad}}{180^\circ} \right)$$
$$= \pi / 4 \text{ rad}$$

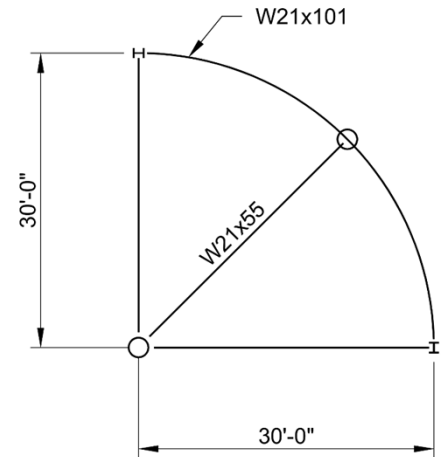


110

## Beam Geometry

- Developed length between braces

$$\begin{aligned} L_{db} &= (30 \text{ ft})(\pi / 4 \text{ rad}) \\ &= 23.6 \text{ ft} \\ &= 283 \text{ in.} \end{aligned}$$



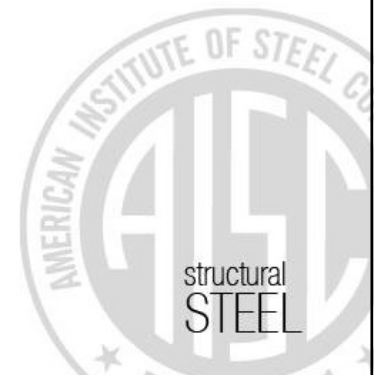
111

## Design Examples

### Example 2

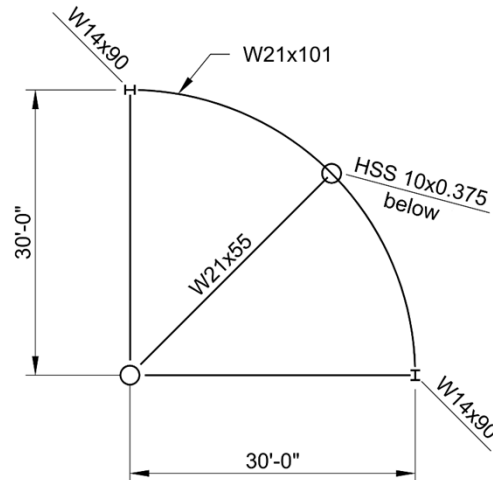
### Structural Analysis

There's always a solution in steel.



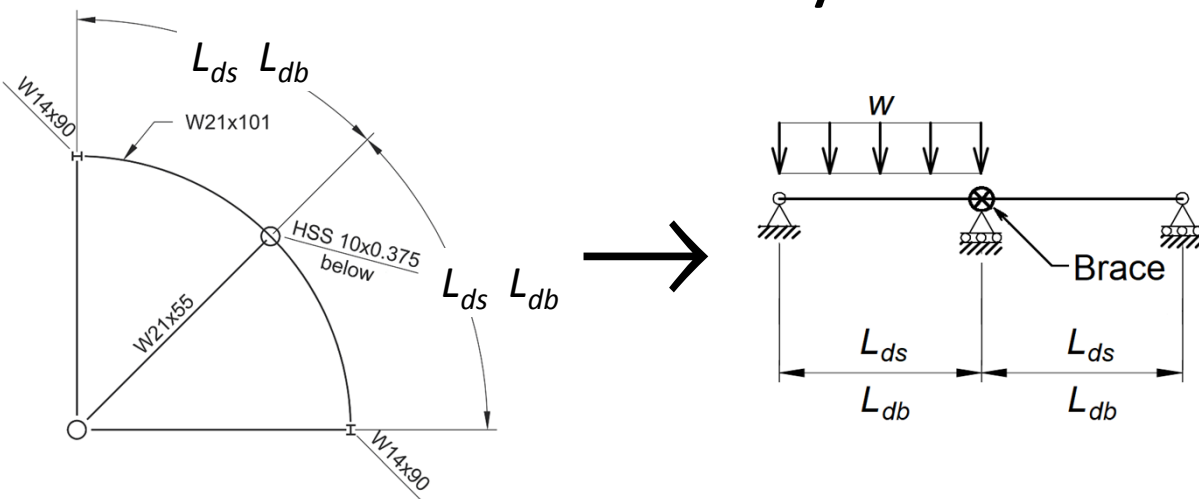
# Structural Analysis

- At the ends
  - No warping restraint
  - No flexural restraint
- At the HSS10
  - Continuous for flexure
  - Continuous for warping



113

# Structural Analysis



114

# Design Examples

There's always a solution in steel.

## Example 2 Structural Analysis Flexural Loads



# Structural Analysis

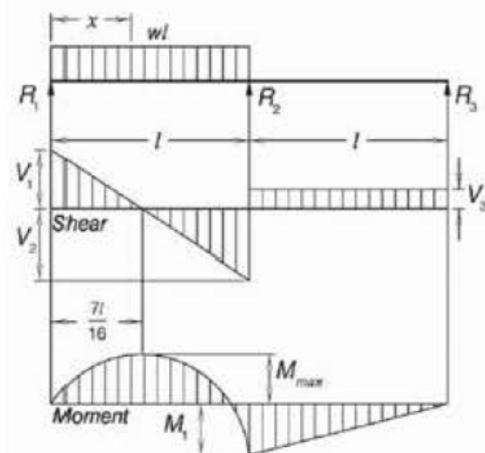
- AISC *Manual* Table 3-23  
Case 29

- Max. Moment

$$M_{ux} = +480 \text{ kip-in.}$$

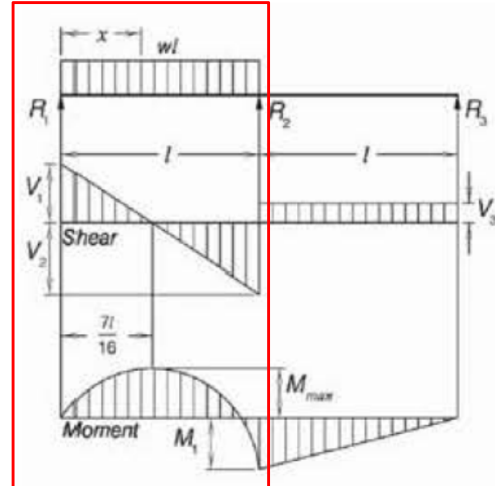
- Max. Shear

$$V_u = 9.95 \text{ kips}$$



# Structural Analysis

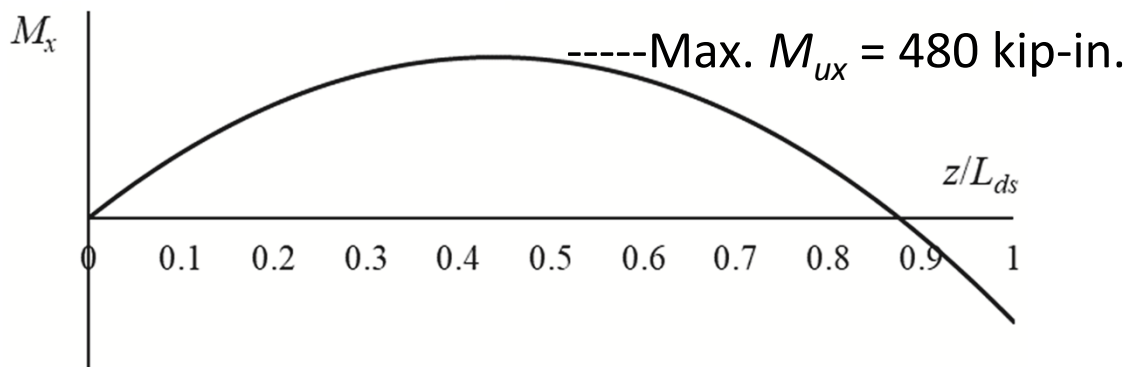
- Isolate the loaded span



117

# Structural Analysis

- Flexural moment at the loaded span

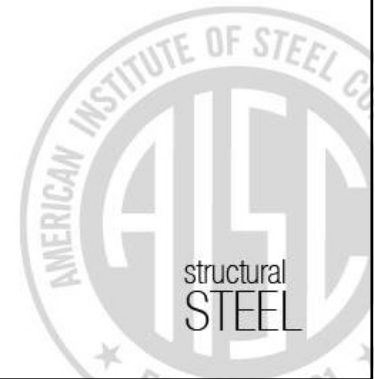


118

There's always a solution in steel.

# Design Examples

## Example 2 Structural Analysis Torsional Loads



## Structural Analysis

- $M/R$  method
- Torsional moment per unit length:

$$m_{zc} = \frac{M_x}{R}$$



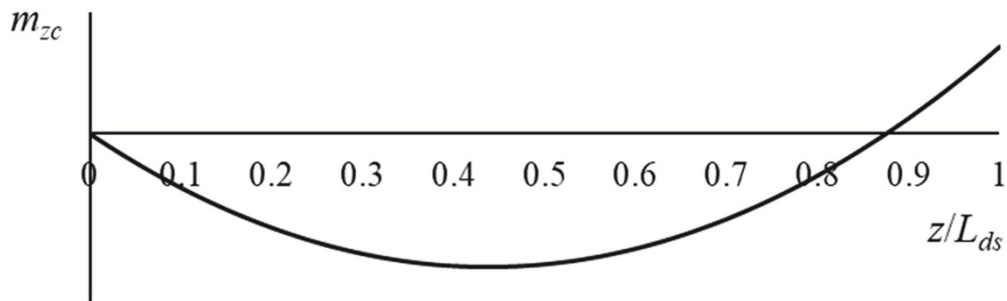
REF: Session 3 Slide 44

120



# Structural Analysis

- Distributed torsion



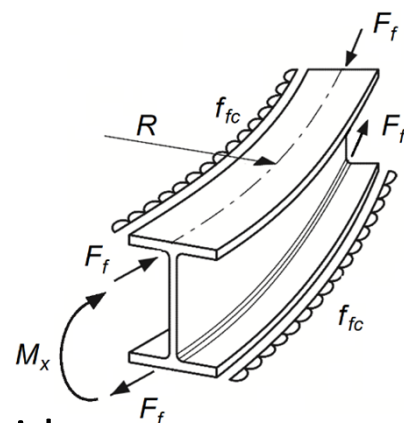
121

# Structural Analysis

- Isolated Flange method

$$f_{fc} = \frac{m_{zc}}{h_o}$$

$h_o$  = distance between flange centroids

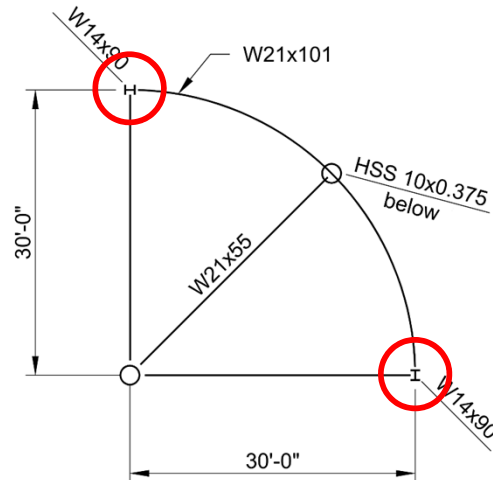


REF: Session 3 Slide 82

122

## Structural Analysis

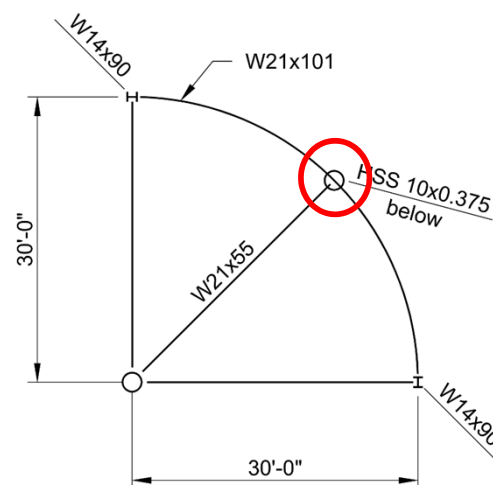
- Warping is not restrained at the end connections  
→ use pinned ends at the isolated flange



123

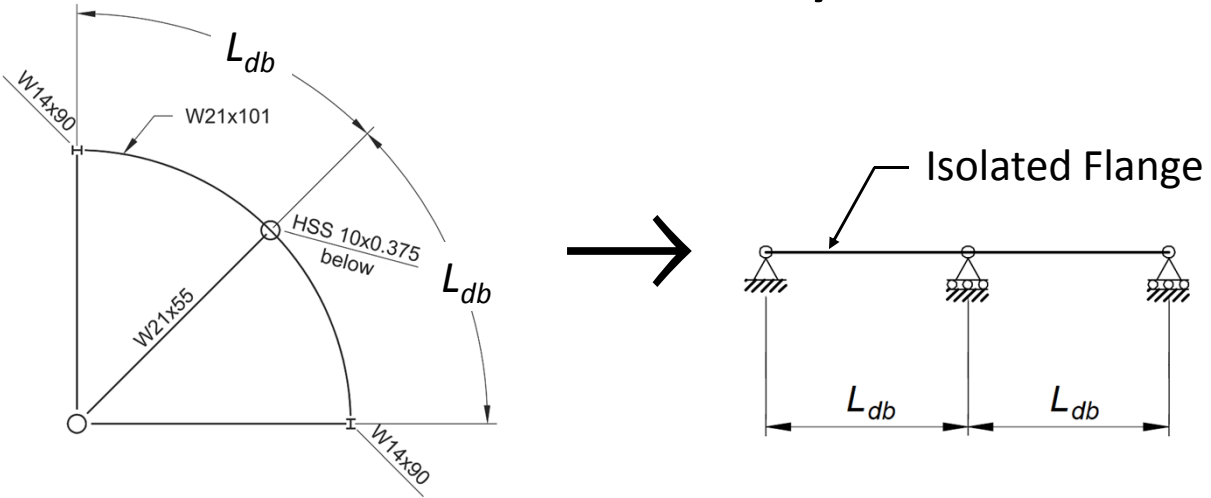
## Structural Analysis

- Warping is continuous at the HSS10 → the isolated flange is continuous for flexure



124

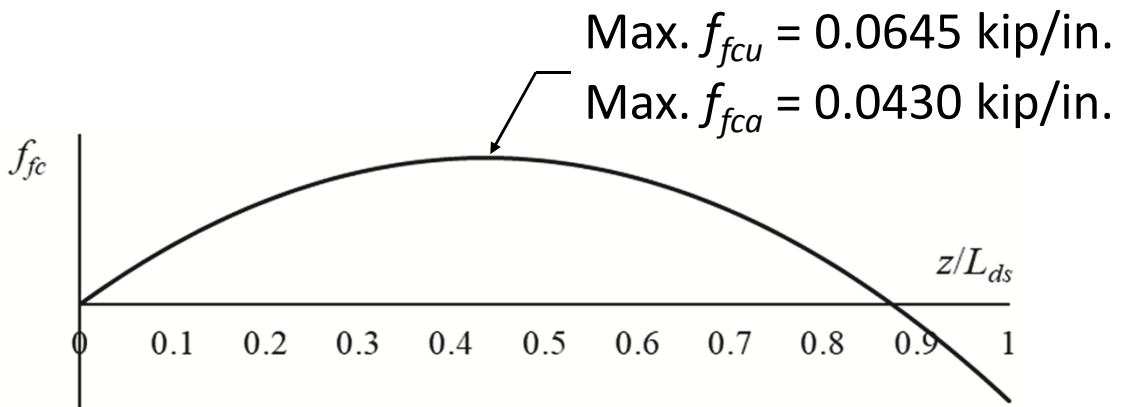
# Structural Analysis



125

# Structural Analysis

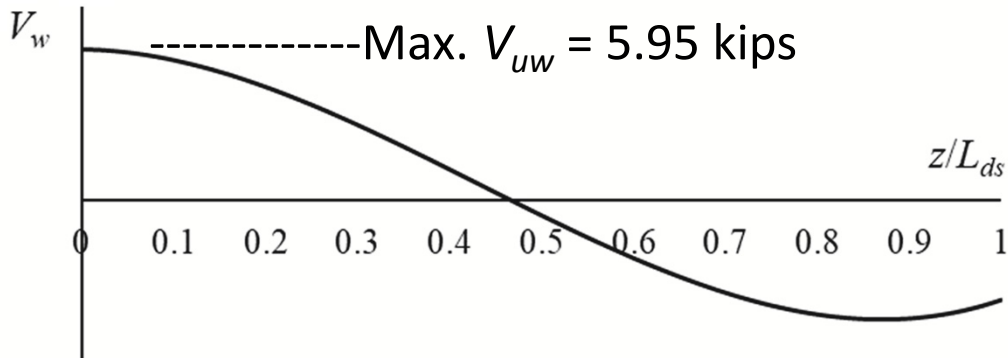
- Distributed flange force



126

# Structural Analysis

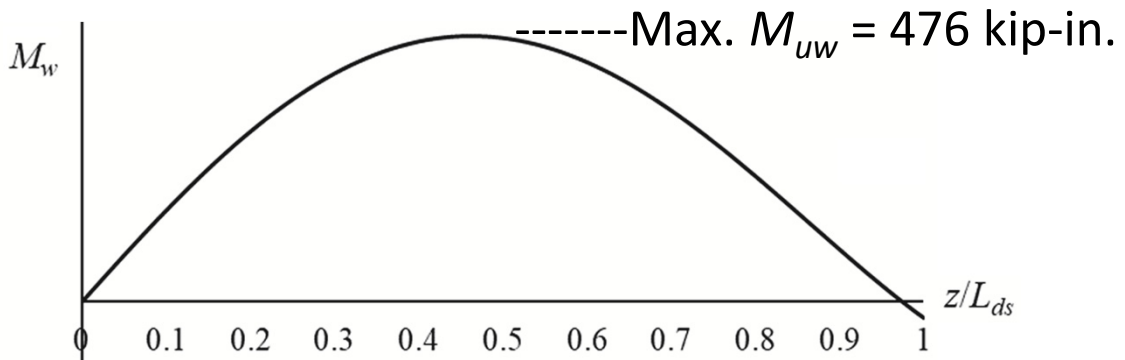
- Horizontal shear



127

# Structural Analysis

- Warping moment

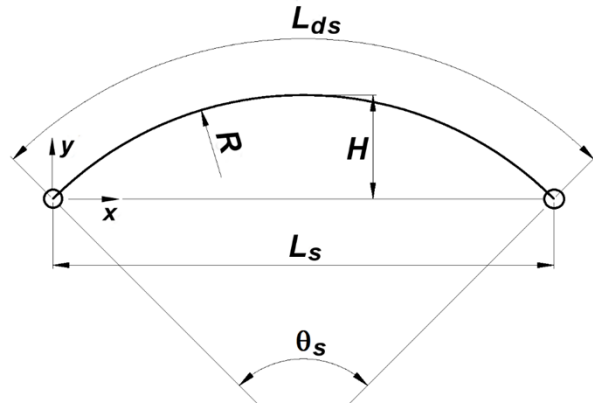


128

## Structural Analysis

### Corrected Moments

$$\begin{aligned}
 C &= 1 - \frac{\theta_s}{30} + \frac{\theta_s^2}{6.2} \\
 &= 1 - \frac{\pi/4}{30} + \frac{(\pi/4)^2}{6.2} \\
 &= 1.07
 \end{aligned}$$



REF: Session 3 Slide 47

129

## Structural Analysis

### Corrected Moments

- Flexure:

$$M_{u_{xc}} = CM_{ux} = (1.07)(480 \text{ kip-in.}) = 514 \text{ kip-in.}$$

- Warping:

$$M_{u_{wc}} = CM_{uw} = (1.07)(476 \text{ kip-in.}) = 509 \text{ kip-in.}$$



130

There's always a solution in steel.

## Design Examples

### Example 2

### Shear Strength



## Shear Strength

- AISC *Manual* Table 6-2

$$\phi_v V_n = 321 \text{ kips} > V_u = 9.95 \text{ kips} \quad \mathbf{o.k.}$$



132

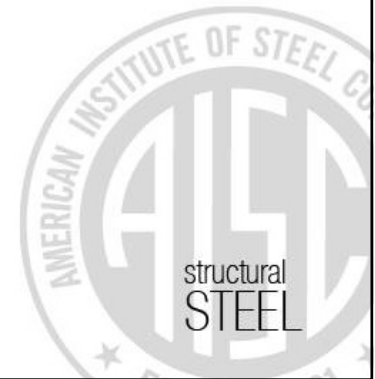


There's always a solution in steel.

## Design Examples

### Example 2

### Local Buckling



## Local Buckling

- Calculations are the same as for a straight member
- Flexure:  $\lambda_f < \lambda_{pf}$  and  $\lambda_w < \lambda_{pw}$  → the W21×101 is compact



134



There's always a solution in steel.

# Design Examples

## Example 2

### Flexural Strength



## Flexural Strength

- Design as a straight beam
- AISC *Specification* Chapter F



136



## Flexural Strength

Use  $C_{bs} = 1.0$

$$C_{bo} = C_{bs} \left[ 1 - \left( \frac{\theta_b}{\pi} \right)^2 \right]^2$$
$$= (1.0) \left[ 1 - \left( \frac{\pi/4}{\pi} \right)^2 \right]^2 = 0.879$$



REF: Session 3 Slide 68

137

## Flexural Strength

- *AISC Specification* Section F2
  - $L_b = L_{db} = 23.6$  ft
  - $C_b = C_{bo} = 0.879$
- *AISC Manual* Table 3-6
  - $L_p = 10.2$  ft
  - $L_r = 30.1$  ft



138

## Flexural Strength

$$\begin{aligned}M_p &= F_y Z_x \\ &= (50 \text{ ksi})(253 \text{ in.}^3) \\ &= 12,700 \text{ kip-in.}\end{aligned}$$



139

## Flexural Strength

- $L_p < L_b < L_r \rightarrow$  Use AISC *Spec.* Eq. F2-2

$$\begin{aligned}M_n &= C_b \left[ M_p - (M_p - 0.7F_y S_x) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p \\ &= 8,350 \text{ kip-in.} < 12,700 \text{ kip-in.}\end{aligned}$$



140

## Flexural Strength

- The available strength is

$$\begin{aligned}\phi_b M_n &= 0.90(8,350 \text{ kip-in.}) \\ &= 7,520 \text{ kip-in.}\end{aligned}$$



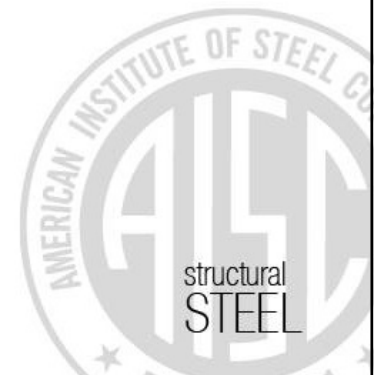
141

## Design Examples

### Example 1

### Second-Order Effects

There's always a solution in steel.



## Second-Order Effects

- Amplified First-Order Analysis
- Second-order warping moment:  $M_{uw} = B_o M_{uwc}$

$$B_o = \frac{0.85}{1 - \alpha \frac{M_{uxc}}{M_{eo}}} \geq 1.0$$

$M_{eo}$  = elastic lateral-torsional buckling moment

$\alpha$  = 1.00 (LRFD)



REF: Session 3 Slide 95

143

## Second-Order Effects

$$F_{cr} = \frac{C_{bo} \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

= 44.9 ksi



144

## Second-Order Effects

$$\begin{aligned}M_{eo} &= F_{cr} S_x \\ &= (44.9 \text{ ksi})(227 \text{ in.}^3) \\ &= 10,200 \text{ kip-in.}\end{aligned}$$



145

## Second-Order Effects

$$\begin{aligned}B_o &= \frac{0.85}{1 - (1.0) \left( \frac{514 \text{ kip-in.}}{10,200 \text{ kip-in.}} \right)} \geq 1.0 \\ &= 0.895 < 1.0 \\ \rightarrow B_o &= 1.0\end{aligned}$$



146

## Second-Order Effects

$$M_{uw} = (1.0)(509 \text{ kip-in.}) \\ = 509 \text{ kip-in.}$$



147

## Design Examples

### Example 2

### Warping Strength

There's always a solution in steel.



## Warping Strength

- The isolated flange plastic modulus is

$$\begin{aligned} Z_f &= \frac{t_f b_f^2}{4} \\ &= \frac{(0.800 \text{ in.})(12.3 \text{ in.})^2}{4} \\ &= 30.3 \text{ in.}^3 \end{aligned}$$



REF: Session 3 Slide 87

149

## Warping Strength

- The nominal flexural strength of the isolated flange is

$$\begin{aligned} M_{nw} &= F_y Z_f \\ &= (50 \text{ ksi})(30.3 \text{ in.}^3) \\ &= 1,520 \text{ kip-in.} \end{aligned}$$



150

## Warping Strength

- The available flexural strength of the isolated flange is

$$\begin{aligned}M_{cw} &= \phi_b M_{nw} \\ &= 0.90(1,520 \text{ kip-in.}) \\ &= 1,370 \text{ kip-in.}\end{aligned}$$



151

## Design Examples

### Example 2

### Combined Loading

There's always a solution in steel.



## Combined Loading

- Flexural moment + flange warping moment

$$\frac{M_{uxc}}{\phi_b M_n} + \frac{8}{9} \frac{M_{uw}}{\phi M_{nw}} \leq 1.0$$

$$\frac{514 \text{ kip-in.}}{7,520 \text{ kip-in.}} + \left(\frac{8}{9}\right) \left(\frac{509 \text{ kip-in.}}{1,370 \text{ kip-in.}}\right) \leq 1.0$$

$$0.399 < 1.00 \quad \mathbf{o.k.}$$



REF: Session 3 Slide 100

153

## Design Examples

There's always a solution in steel.

### Example 1

### Serviceability



## Serviceability

- The torsional rotation can be estimated using the horizontal deflection of the isolated flange
- Maximum second-order distributed flange force under service loads:  $f_{fc} = 0.0430$  kip/in.

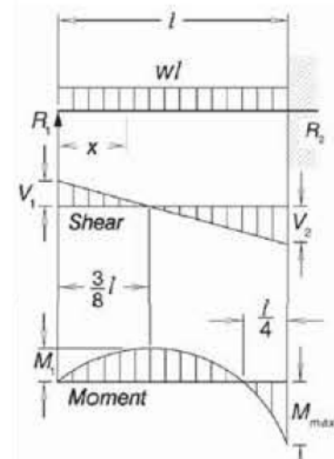


155

## Serviceability

- AISC *Manual* Table 3-23  
Case 12

$$\Delta_{max} = \frac{f_{fc} L_{ds}^4}{185 E I_f}$$



156

## Serviceability

- The isolated flange moment of inertia is

$$I_f = \frac{t_f b_f^3}{12} = \frac{(0.800 \text{ in.})(12.3 \text{ in.})^3}{12}$$
$$= 124 \text{ in.}^4$$



157

## Serviceability

$$\Delta_{max} = \frac{f_{fc} L_{ds}^4}{185 E I_f} = \frac{(0.0430 \text{ kip/in.})(283 \text{ in.})^4}{(185)(29,000 \text{ ksi})(124 \text{ in.}^4)}$$
$$= 0.415 \text{ in.}$$



158

## Serviceability

- The 1<sup>st</sup>-order torsional rotation is

$$\theta_1 = \tan^{-1} \left( \frac{2\Delta_{max}}{h_o} \right) = \tan^{-1} \left[ \frac{(2)(0.415 \text{ in.})}{20.6 \text{ in.}} \right]$$
$$= 2.31^\circ$$

$h_o$  = distance between flange centroids

---



159

## Serviceability

- The 2<sup>nd</sup>-order torsional rotation is

$$\theta_2 = B_o \theta_1$$
$$= (1.00)(2.31^\circ)$$
$$= 2.31^\circ$$



160

# Design Examples

There's always a solution in steel.

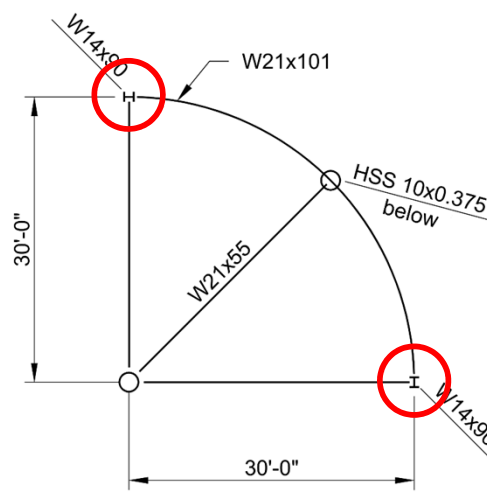
## Example 2 Connections



# Problem Statement

## End Connections

- Torsional rotation is restrained
- No warping restraint
- No flexural restraint



## Connections

- Support reactions
  - Vertical reaction:  $R_u = 7.74$  kips
  - Horizontal shear at flanges:  $V_{uw} = 5.95$  kips



163

## Connections

- Torsional moment

$$\begin{aligned}M_{uz} &= V_{uw} h_o \\ &= (5.95 \text{ kips})(20.6 \text{ in.}) \\ &= 123 \text{ kip-in.}\end{aligned}$$

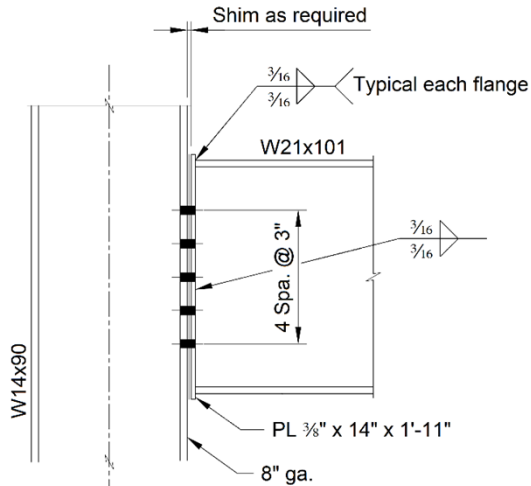
$h_o$  = distance between flange centroids

---



164

# Connections



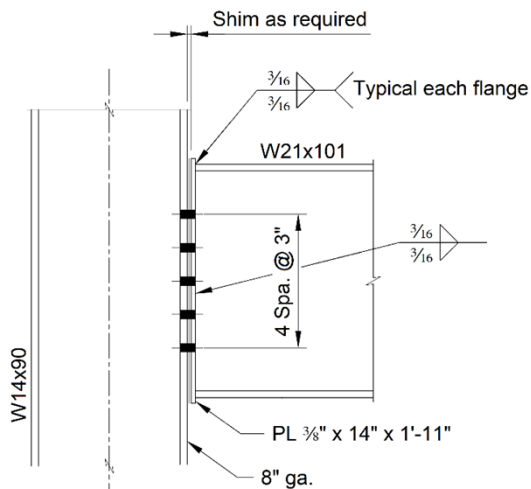
End-plate connections are efficient in transferring torsion

Flange welds transfer  $V_{uw}$



165

# Connections

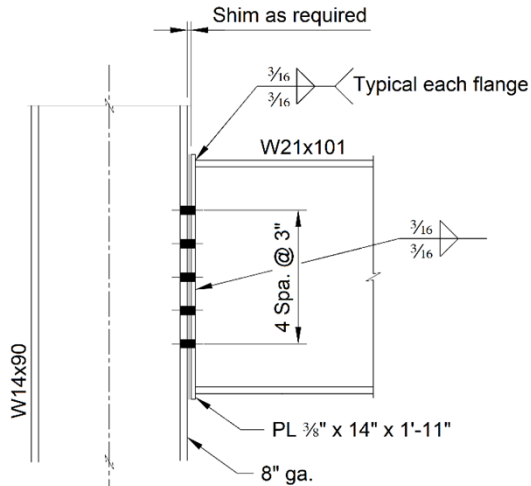


A wide column gage provides efficient torsional resistance



166

# Connections



Shims are used to ensure proper fit-up

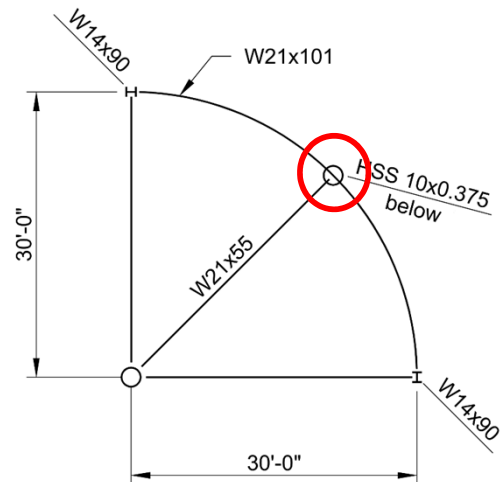


167

# Problem Statement

## Connections @ the HSS10

- Torsion is restrained by the W21 beam



168

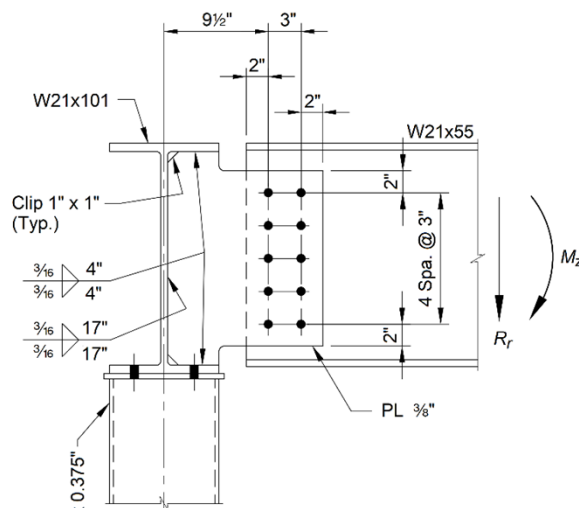
# Connections

- Support reactions
  - Vertical reaction @ W21x55:  $R_u = 30$  kips
  - Torsion @ W21x101:  $M_{uz} = 80.5$  kip-in.



169

# Connections



The torsional reaction from the curved beam is resisted by the single-plate connection



170

There's always a solution in steel.

# Question time



## Individual Webinar Registrants

---

### CEU/PDH Certificates

Within 2 business days...

- You will receive an email on how to report attendance from: [registration@aisc.org](mailto:registration@aisc.org).
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!



## Individual Webinar Registrants

---

### CEU/PDH Certificates

Within 2 business days...

- New reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.



## 8-Session Registrants

---

### CEU/PDH Certificates

One certificate will be issued at the conclusion of  
all 8 sessions.



## 8-Session Registrants

---

Access to the quiz: Information for accessing the quiz will be emailed to you by Wednesday. It will contain a link to access the quiz. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG

Quiz and Attendance records: Posted Tuesday mornings.  
[www.aisc.org/nightschool](http://www.aisc.org/nightschool) - click on Current Course Details.

Reasons for quiz:

- EEU – must take all quizzes and final to receive EEU
- CEUs/PDHS – If you watch a recorded session you must take quiz for CEUs/PDHS.
- REINFORCEMENT – Reinforce what you learned tonight. Get more out of the course.

NOTE: If you attend the live presentation, you do not have to take the quizzes to receive CEUs/PDHS.



## 8-Session Registrants

---

**Access to the recording:** Information for accessing the recording will be emailed to you by this Wednesday. The recording will be available for three weeks. For 8-session registrants only. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG.

**CEUs/PDHS** – If you watch a recorded session you must take AND PASS the quiz for CEUs/PDHS.



## Night School Resources for 8-session package Registrants

Find all your handouts, quizzes and quiz scores, recording access, and attendance information all in one place!



## Night School Resources for 8-session package Registrants

Go to [www.aisc.org](http://www.aisc.org) and sign in.



### Login

If you're an existing customer, please enter your username and password.

#### USERNAME

#### PASSWORD

Remember Me

#### DON'T HAVE AN ACCOUNT?

My AISC allows you to access Engineering Journal articles and Design Guides you have downloaded from the bookstore.

[REGISTER NOW](#)



# Night School Resources for 8-session package Registrants

Go to [www.aisc.org](http://www.aisc.org) and sign in.

**IN THIS SECTION**

- Edit Profile
- My Downloads
- My Pending Quizzes
- My Events
- Order History
- Course History
- Course Resources**

**MyAISC**

**MY PROFILE**  
Update your contact and address information.  
[EDIT PROFILE](#)

**MY PURCHASED DOWNLOADS**  
Access articles and documents that you have purchased.  
[VIEW DOWNLOADS](#)

**MY COURSE RESOURCES**  
View online resources for Night School and Live Webinar package registrations.  
[VIEW RESOURCES](#)

# Night School Resources for 8-session package Registrants



## Night School 13: Design of Industrial Buildings

### 8-SESSION PACKAGE RESOURCES

Event	Date	Handouts	Video	Quiz	Attendance
NS13 - Design Criteria	1/30/2017 7:00:00 PM	<a href="#">Handouts</a>	<a href="#">View</a> Passcode: NS13DSN	Pass Score: 80	Pending
NS13 - Economic Considerations	2/6/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 02/08/2017 5pm EST	Available 02/08/2017 5pm EST	Pending
NS13 - Lateral Load Systems and Details	2/13/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 02/15/2017 5pm EST	Available 02/15/2017 5pm EST	Pending
NS13 - Preliminary Design Procedures	2/27/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 03/01/2017 5pm EST	Available 03/01/2017 5pm EST	Pending
NS13 - Crane Girder Design and Frame Analysis	3/6/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 03/08/2017 5pm EST	Available 03/08/2017 5pm EST	Pending
NS13 - Frame Member and Connection Design	3/13/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 03/15/2017 5pm EST	Available 03/15/2017 5pm EST	Pending
NS13 - Transfer Crane Girder & Longitudinal Bldg Bracing Dsn	3/27/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 03/29/2017 5pm EST	Available 03/29/2017 5pm EST	Pending
NS13 - Building Envelope and Bracing Design	4/3/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 04/05/2017 5pm EST	Available 04/05/2017 5pm EST	Pending
NS13 - Final Exam	4/10/2017 7:00:00 PM			Available 04/12/2017 5pm EST	

## Night School Resources for 8-session package Registrants

---

- Weekly “quiz and recording” email.
- Weekly updates of the master Quiz and Attendance record found at [www.aisc.org/nightschool](http://www.aisc.org/nightschool). Scroll down to Quiz and Attendance records.
  - Updated on Tuesday mornings.



## Night School Resources for 8-session package Registrants

---

- Webinar connection information:
  - Found in your registration confirmation/receipt.
  - Reminder email sent out Monday mornings.
- Link to handouts also found here.



There's always a solution in steel.

# Thank You

Please give us your feedback!  
*Survey at conclusion of webinar.*

