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 Session Description

**Session 4: Design Examples**  
**July 9, 2018**

This session will present a design example for vertically-curved members. In addition to demonstration the design of the member, the example will address the design of the connection. Then, the session will present a design example for horizontally-curved members. In addition to demonstration the design of the member, the example will address the design of the connection.


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 Learning Objectives



- Describes the steps in designing a vertically-curved member.
- List the limit states of the vertically-curved member connection.
- Describes the steps in designing a horizontally-curved member.
- List the limit states of the horizontally-curved member connection.

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**Design of Curved Members**  
**Session 4: Design Examples**  
**July 9, 2018**

Presented by  
Bo Dowsell, P.E., Ph.D.  
ARC International, LLC  
Birmingham, AL



*There's always a solution in steel.*



## Design Examples

### Session Description

There's always a solution in steel.



## Session Description

- Design examples
  - Example 1: vertically-curved member
  - Example 2: horizontally-curved member



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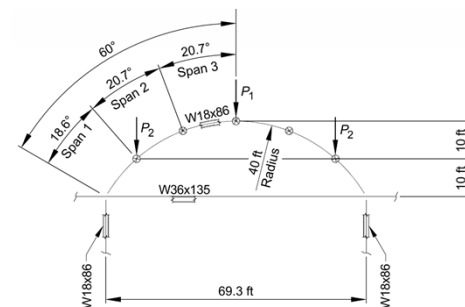
## Design Examples

### Example 1: Vertically-Curved Member

There's always a solution in steel.



## Example 1



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## Design Examples

### Example 1

#### Problem Statement



There's always a solution in steel.

## Problem Statement

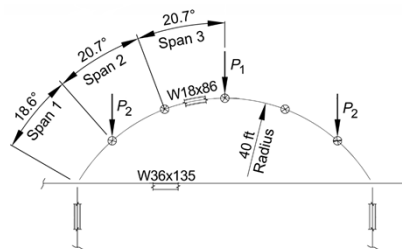
- Verify that the arch is adequate for the imposed loading
- Use LRFD



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## Problem Statement

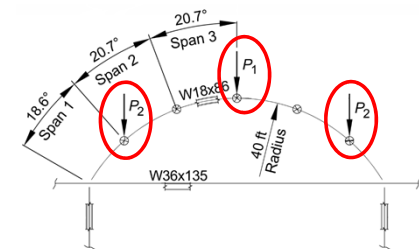
- Curved member
  - W18×86
  - ASTM A992
  - Bent the hard way
  - Circular curve



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## Problem Statement

- The factored (LRFD) loads are
  - $P_{1u} = 120$  kips
  - $P_{2u} = 75$  kips

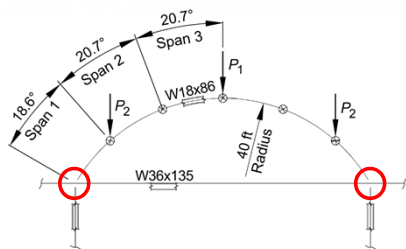


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### Problem Statement

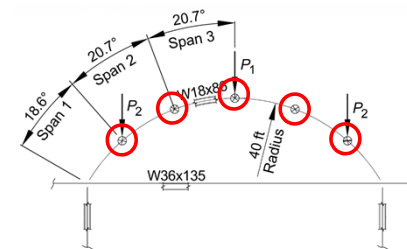
- Supports
  - Translation fixed in all directions
  - Rotation free in all directions



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### Problem Statement

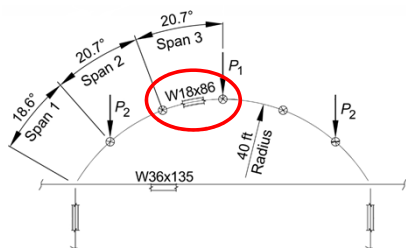
- Braces
  - Prevent out-of-plane translation
  - Prevent torsional rotation



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### Problem Statement

- Assume Span 3 is critical



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### Design Examples

Example 1

Properties

There's always a solution in steel.



## Properties

- Material properties of ASTM A992  
(AISC *Manual* Table 2-4)

$$F_y = 50 \text{ ksi}$$

$$F_u = 65 \text{ ksi}$$



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## Properties

- Dimensions of W18×86  
(AISC *Manual* Table 1-1)

$$d = 18.4 \text{ in.}$$

$$t_w = 0.480 \text{ in.}$$

$$b_f = 11.1 \text{ in.}$$

$$t_f = 0.770 \text{ in.}$$

$$h_o = 17.6 \text{ in.}$$



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## Properties

- Section properties of W18×86  
(AISC *Manual* Table 1-1)

$$I_x = 1,530 \text{ in.}^4$$

$$S_x = 166 \text{ in.}^3$$

$$r_x = 7.77 \text{ in.}$$

$$Z_x = 186 \text{ in.}^3$$

$$I_y = 175 \text{ in.}^4$$

$$r_y = 2.63 \text{ in.}$$

$$J = 4.10 \text{ in.}^4$$

$$C_w = 13,600 \text{ in.}^6$$

$$r_{ts} = 3.05 \text{ in.}$$



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## Design Examples

### Example 1

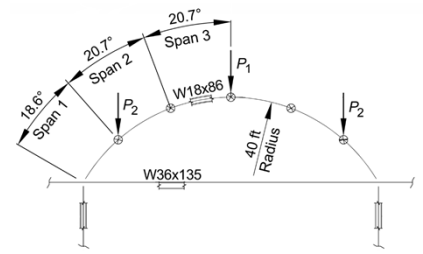
### Arch Geometry

There's always a solution in steel.



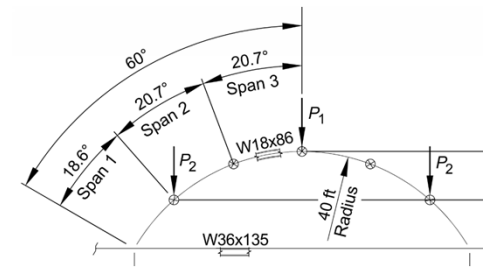
### Arch Geometry

- Centroidal radius  
 $R = (40 \text{ ft})(12 \text{ in./ft})$   
 $= 480 \text{ in.}$



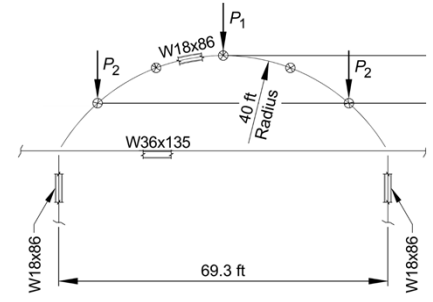
### Arch Geometry

- Arch angle  
 $\theta = (120^\circ) \left( \frac{\pi \text{ rad}}{180^\circ} \right)$   
 $= (2\pi/3) \text{ rad}$



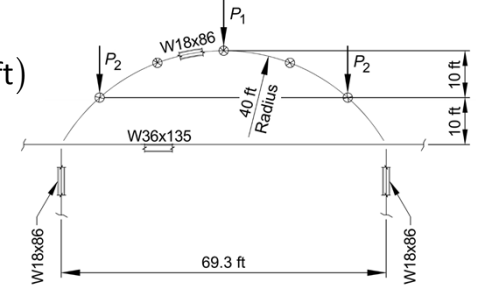
### Arch Geometry

- Span length (chord)  
 $L_s = (69.3 \text{ ft})(12 \text{ in./ft})$   
 $= 832 \text{ in.}$



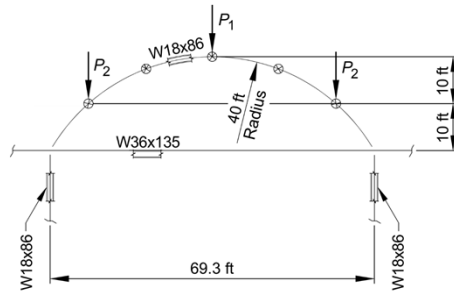
### Arch Geometry

- Rise  
 $H = (20 \text{ ft})(12 \text{ in./ft})$   
 $= 240 \text{ in.}$



## Arch Geometry

$$\frac{H}{L_s} = \frac{240 \text{ in.}}{832 \text{ in.}} = 0.288$$

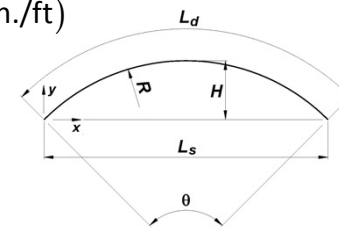


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## Arch Geometry

- Developed arc length

$$L_d = (40 \text{ ft})[(2\pi / 3)\text{rad}](12 \text{ in./ft}) = 1,010 \text{ in.}$$



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## Design Examples

### Example 1

### Structural Analysis

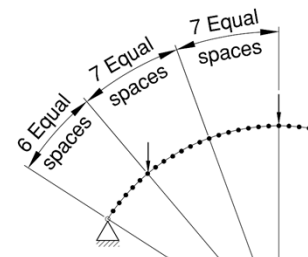


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## Structural Analysis

### Finite Element Model

- Segmented spans
  - Straight beam elements
  - $\approx 3^\circ$  arc between nodes
- First-order analysis



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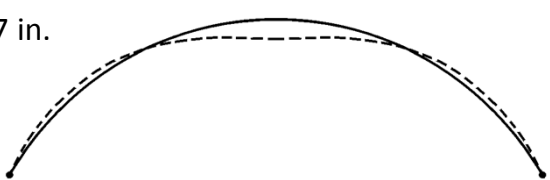



### Structural Analysis

**Deflection**

- Maximum @ apex

$\Delta_1 = 1.07$  in.




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### Structural Analysis

**Deflection**

- If  $\Delta_1$  is less than  $H/40$ , a first-order finite element analysis is sufficiently accurate

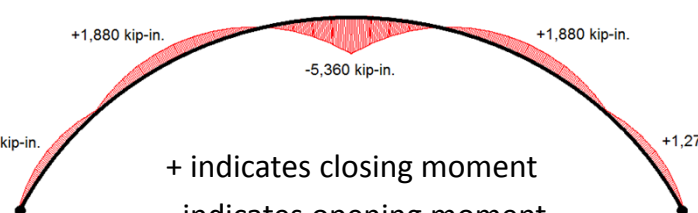
$$\frac{H}{40} = \frac{240 \text{ in.}}{40} = 6.00 \text{ in.} \quad 1.07 \text{ in.} < 6.00 \text{ in.} \quad \mathbf{o.k.}$$

Ref: King, C. and Brown, D. (2001), *Design of Curved Steel*, The Steel Construction Institute



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### Arch Geometry

**In-Plane Moment**




+ indicates closing moment  
– indicates opening moment


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### Structural Analysis

Member Loads (kip, in.)			
Location	Axial	Moment	Shear
	$P_u$	$M_{ux}$	$V_u$
Supports	182	0	25.8
Apex	118	–5,360	57.5


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## Structural Analysis

1 <sup>st</sup> -Order Member Loads (kip, in.)			
Location	Axial	Moment	Shear
	$P_u$	$M_{ux}$	$V_u$
Max./Min. @ Span 3	131	+1,380 -5,360	57.5



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## Design Examples

Example 1  
Local Buckling



There's always a solution in steel.

## Local Buckling

- Calculations are the same as for a straight member
- Axial:  $\lambda_f < \lambda_{rf}$  and  $\lambda_w < \lambda_{rw}$  → the W18×86 is non-slender
- Flexure:  $\lambda_f < \lambda_{pf}$  and  $\lambda_w < \lambda_{pw}$  → the W18×86 is compact



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## Design Examples

Example 1  
Shear Strength



There's always a solution in steel.

## Shear Strength

- AISC Manual Table 6-2

$$\phi_v V_n = 265 \text{ kips} > V_u = 57.5 \text{ kips} \quad \text{o.k.}$$



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## Design Examples

### Example 1

### Local Flange Bending

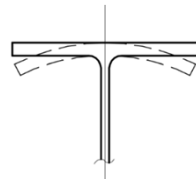


There's always a solution in steel.

## Local Flange Bending

### Simplified Method

$$k_f = \frac{9.20}{8.80 + \frac{b_f^2}{Rt_f}} \leq 1.0$$



REF: Session 2 Slide 105

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## Local Flange Bending

### Simplified Method

$$k_f = \frac{9.20}{8.80 + \frac{(11.1 \text{ in.})^2}{(480 \text{ in.})(0.770 \text{ in.})}} \leq 1.00$$

$$= 1.01 > 1.00$$



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## Local Flange Bending

### Simplified Method

- $k_f = 1.00$
- Flange bending will not occur
- The effective flexural properties are equal to the straight-member properties



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## Design Examples

### Example 1

### In-Plane Strength

There's always a solution in steel.



## In-Plane Strength

- Snap-through buckling is not critical because:
  - The supports are rigid
  - $H/L_s = 0.288 > 0.2$



REF: Session 2 Slide 52

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## In-Plane Strength

- AISC *Specification* Section E3
- Radius of gyration about the axis of curvature:  
 $r_i = r_x = 7.77$  in.



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### In-Plane Strength

- Unbraced length:  $L_d = 1,010$  in.

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### In-Plane Strength

- For a circular arch with pinned end conditions and  $H/L_s = 0.288$ ,  $K_i = 0.55$

In-plane effective length factor, $K_i$			
Arch Form	End Conditions	$H/L_s$	$K_i$
Circular	Pinned	$0.1 \leq H/L_s \leq 0.3$	0.55
		$0.3 \leq H/L_s \leq 0.5$	0.60
	Fixed	All	0.40

REF: Session 2 Slide 59

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### In-Plane Strength

$$\frac{L_c}{r} = \frac{K_i L_d}{r_i}$$

$$= \frac{(0.55)(1,010 \text{ in.})}{7.77 \text{ in.}}$$

$$= 71.5$$

AISC Manual Table 4-14:  $\phi_c F_{cr} = 31.0$  ksi

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### In-Plane Strength

- The available strength is

$$\phi_c P_{ni} = (31.0 \text{ ksi})(25.3 \text{ in.}^2)$$

$$= 784 \text{ kips}$$

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## Design Examples

### Example 1

### Out-of-Plane Strength

There's always a solution in steel.



## Out-of-Plane Strength

- AISC *Specification* Section E3
- Moment of inertia perpendicular to the axis of curvature:  $I_o = I_y = 175 \text{ in.}^4$
- Radius of gyration perpendicular to the axis of curvature:  $r_o = r_y = 2.63 \text{ in.}$

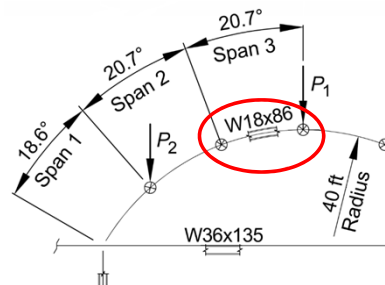


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## Out-of-Plane Strength

### Span 3

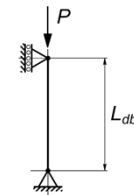
$$\theta_b = (20.7^\circ) \left( \frac{\pi \text{ rad}}{180^\circ} \right) = 0.361 \text{ rad}$$



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## Out-of-Plane Strength

$$\begin{aligned} L_{db} &= R\theta_b \\ &= (480 \text{ in.})(0.361 \text{ rad}) \\ &= 173 \text{ in.} \end{aligned}$$

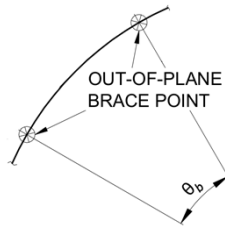


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## Out-of-Plane Strength

- Effective length factor,  $K_o$ : Circular doubly-symmetric segments

$$K_o = \frac{\sqrt{1 + \frac{1}{C_o} \left( \frac{\theta_b}{\pi} \right)^2}}{1 - \left( \frac{\theta_b}{\pi} \right)^2}$$



REF: Session 2 Slide 67



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## Out-of-Plane Strength

$$\begin{aligned} C_o &= \frac{1}{I_o} \left[ \frac{GJ}{E} + C_w \left( \frac{\pi}{L_{db}} \right)^2 \right] \\ &= \frac{1}{175 \text{ in.}^4} \left[ \frac{(11,200 \text{ ksi})(4.10 \text{ in.}^4)}{29,000 \text{ ksi}} + (13,600 \text{ in.}^6) \left( \frac{\pi}{173 \text{ in.}} \right)^2 \right] \\ &= 0.0347 \end{aligned}$$

REF: Session 2 Slide 69



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## Out-of-Plane Strength

$$K_o = \frac{\sqrt{1 + \frac{1}{C_o} \left( \frac{\theta_b}{\pi} \right)^2}}{1 - \left( \frac{\theta_b}{\pi} \right)^2} = \frac{\sqrt{1 + \frac{1}{0.0347} \left( \frac{0.361 \text{ rad}}{\pi} \right)^2}}{1 - \left( \frac{0.361 \text{ rad}}{\pi} \right)^2} = 1.19$$



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## Out-of-Plane Strength

$$\begin{aligned} \frac{L_c}{r} &= \frac{K_o L_{db}}{r_o} \\ &= \frac{(1.19)(173 \text{ in.})}{2.63 \text{ in.}} \\ &= 78.3 \end{aligned}$$

AISC Manual Table 4-14:  $\phi_c F_{cr} = 28.7 \text{ ksi}$



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## Out-of-Plane Strength

- The available strength is

$$\begin{aligned}\phi_c P_{no} &= (28.7 \text{ ksi})(25.3 \text{ in.}^2) \\ &= 726 \text{ kips}\end{aligned}$$



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## Design Examples

### Example 1

### Second-Order Effects



There's always a solution in steel.

## Second-Order Effects

- Amplified first-order analysis
- Second-order moment:  $M_{ux2} = B_1 M_{ux}$

$$B_1 = \frac{1}{1 - \alpha \frac{P_u}{P_{ei}}}$$

$P_{ei}$  = elastic critical load for in-plane buckling

$\alpha = 1.00$  (LRFD)



REF: Session 2 Slide 87

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## Second-Order Effects

$$\begin{aligned}F_e &= \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} \\ &= \frac{\pi^2 (29,000 \text{ ksi})}{(71.5)^2} \\ &= 56.0 \text{ ksi}\end{aligned}$$

$$\begin{aligned}P_{ei} &= F_e A_g \\ &= (56.0 \text{ ksi})(25.3 \text{ in.}^2) \\ &= 1,420 \text{ kips}\end{aligned}$$



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## Second-Order Effects

$$B_i = \frac{1}{1 - \alpha \frac{P_u}{P_{ei}}} = \frac{1}{1 - (1.0) \left( \frac{182 \text{ kips}}{1,420 \text{ kips}} \right)} = 1.15$$



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## Second-Order Effects

2 <sup>nd</sup> -Order Moments (kip-in.)	
Location	$M_{ux2}$
Max./Min. @ Span 3	+1,590 -6,160



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## Design Examples

### Example 1

### Flexural Strength



There's always a solution in steel.

## Flexural Strength

- AISC *Specification* Section F2
- Moment of inertia perpendicular to the axis of curvature:  $I_o = I_y = 175 \text{ in.}^4$



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## Flexural Strength

$$\begin{aligned} M_p &= F_y Z_x \\ &= (50 \text{ ksi})(186 \text{ in.}^3) \\ &= 9,300 \text{ kip-in.} \end{aligned}$$

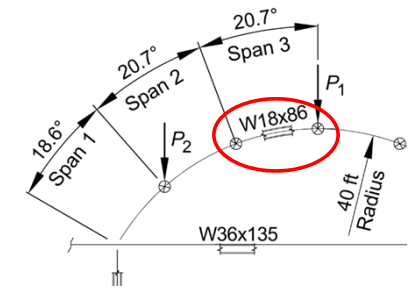


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## Flexural Strength

**Span 3**

$$L_{db} = 173 \text{ in.}$$



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## Flexural Strength

$M_{es}$  = elastic lateral-torsional buckling moment of the equivalent straight member subjected to uniform moment with a length equal to  $L_{db}$

$$M_{es} = \frac{\pi}{L_{db}} \sqrt{EI_o GJ + \left(\frac{\pi E}{L_{db}}\right)^2 I_o C_w} = 17,200 \text{ kip-in.}$$



REF: Session 2 Slide 78

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## Flexural Strength

$$\begin{aligned} C_y &= EI_o \\ &= (29,000 \text{ ksi})(175 \text{ in.}^4) \\ &= 5,080,000 \text{ kip-in.}^2 \end{aligned}$$



REF: Session 2 Slide 80

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## Flexural Strength

$$C_z = GJ + \frac{\pi^2 EC_w}{L_{db}^2}$$

$$= (11,200 \text{ ksi})(4.10 \text{ in.}^4) + \frac{\pi^2 (29,000 \text{ ksi})(13,600 \text{ in.}^6)}{(173 \text{ in.})^2}$$

$$= 176,000 \text{ kip-in.}^2$$



REF: Session 2 Slide 80

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## Flexural Strength

$$C_a = \frac{C_y + C_z}{2RM_{es}}$$

$$= \frac{5,080,000 \text{ kip-in.}^2 + 176,000 \text{ kip-in.}^2}{(2)(480 \text{ in.})(17,200 \text{ kip-in.})}$$

$$= 0.318$$

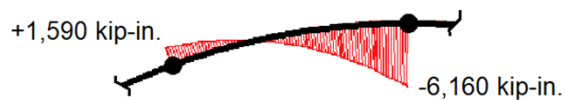


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## Flexural Strength

- Span 3



$$C_{bs} = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} = 2.38$$



REF: Session 2 Slide 77

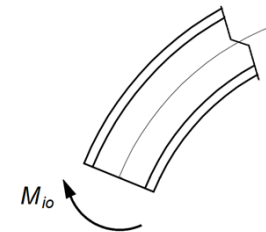
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## Flexural Strength

Opening moments → negative root

$$C_{bi} = C_{bs} \left( \sqrt{1 + C_a^2 - \frac{C_y C_z}{R^2 M_{es}^2}} - C_a \right)$$

$$= 1.75$$



REF: Session 2 Slide 76

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## Flexural Strength

- AISC *Specification* Section F2
  - $L_b = L_{db} = 173 \text{ in.} = 14.4 \text{ ft}$
  - $C_b = C_{bi} = 1.75$
- AISC *Manual* Table 3-6
  - $L_p = 9.29 \text{ ft}$
  - $L_r = 28.6 \text{ ft}$



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## Flexural Strength

- $L_p < L_b < L_r \rightarrow$  Use AISC *Spec.* Eq. F2-2

$$M_n = C_b \left[ M_p - (M_p - 0.7F_y S_x) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p$$
$$= 14,700 \text{ kip-in.} > 9,300 \text{ kip-in.}$$



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## Flexural Strength

- The available strength is
$$\phi_b M_n = 0.90(9,300 \text{ kip-in.})$$
$$= 8,370 \text{ kip-in.}$$



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## Design Examples

### Example 1

### Combined Loading

There's always a solution in steel.



## Combined Loading

- AISC *Specification* Section H1
- In-plane buckling
- Out-of-plane buckling



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## Combined Loading

### In-Plane Buckling

- The largest flexural load ratio is at the apex
- The largest axial load ratio is at the supports

$$\frac{P_u}{\phi_c P_{ni}} = \frac{182 \text{ kips}}{784 \text{ kips}} = 0.232$$



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## Combined Loading

$0.232 > 0.2 \rightarrow$  AISC *Specification* Equation H1-1a

$$\frac{P_u}{\phi_c P_{ni}} + \frac{8}{9} \left( \frac{M_u}{\phi_b M_n} \right) \leq 1.0$$

$$0.232 + \left( \frac{8}{9} \right) \left( \frac{6,160 \text{ kip-in.}}{8,370 \text{ kip-in.}} \right) \leq 1.0$$

$0.886 < 1.0$  **o.k.**



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## Combined Loading

### Out-of-Plane Buckling (Span 3)

- Largest flexural load in the span
- Largest axial load in the span

$$\frac{P_u}{\phi_c P_{no}} = \frac{131 \text{ kips}}{726 \text{ kips}} = 0.180$$



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## Combined Loading

$0.180 < 0.2 \rightarrow$  AISC *Specification* Equation H1-1b

$$\frac{P_u}{2\phi_c P_{no}} + \frac{M_u}{\phi_b M_n} \leq 1.0$$

$$\frac{0.180}{2} + \frac{6,160 \text{ kip-in.}}{8,370 \text{ kip-in.}} \leq 1.0$$

$$0.826 < 1.0 \quad \text{o.k.}$$



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## Design Examples

### Example 1

### End Connections

There's always a solution in steel.



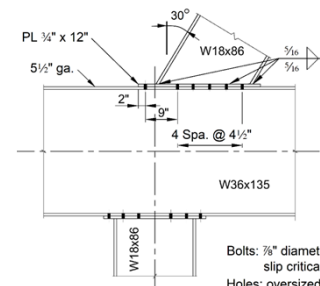
## End Connections

- Support reactions from FE model
  - Vertical:  $R_{uy} = 140$  kips
  - Horizontal:  $R_{ux} = 117$  kips



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## End Connections



To satisfy the boundary conditions assumed in design, out-of-plane translation must be restrained.

Bolts:  $\frac{7}{8}$ " diameter Group A, slip critical Class A  
Holes: oversized,  $1\frac{1}{8}$ " diameter




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### End Connections

**W36 web local compression strength > 140 kips (AISC Specification Section J10)**

Bolts: 7/8" diameter Group A, slip critical Class A  
Holes: oversized, 1 1/8" diameter




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### End Connections

**Slip critical joints increase horizontal rigidity by limiting bolt slip**

**Bolt slip load > 117 kips**

Bolts: 7/8" diameter Group A, slip critical Class A  
Holes: oversized, 1 1/8" diameter



90


### End Connections

**AISC COSP chord length tolerance:  $\pm 1/8$  in.**

**Slip critical bolts allow the use of oversized holes for easier field fit-up**

Bolts: 7/8" diameter Group A, slip critical Class A  
Holes: oversized, 1 1/8" diameter


REF: Session 2 Slide 23



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### Design Examples


**Example 2: Horizontally-Curved Member**



There's always a solution in steel.

### Example 2

W14x90  
W21x101  
30'-0"  
W21x85  
HSS 10x0.375 below  
W14x90  
30'-0"




93

## Design Examples

### Example 2


### Problem Statement



There's always a solution in steel.

### Problem Statement

- Verify that the horizontally-curved beam is adequate for the imposed loading
- Use LRFD




95

### Problem Statement

- Curved member
  - W21×101
  - ASTM A992
  - Bent the easy way
  - Circular curve

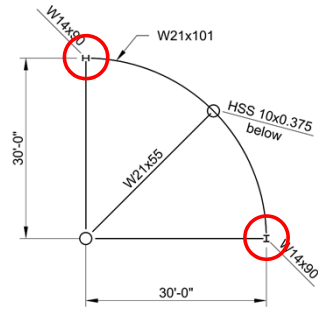
W14x90  
W21x101  
30'-0"  
W21x85  
HSS 10x0.375 below  
W14x90  
30'-0"



96

### Problem Statement

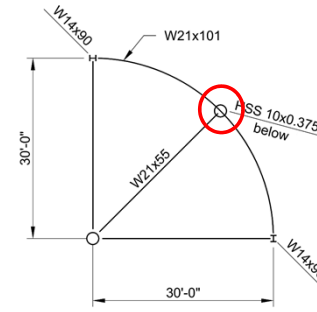
- End connections
  - Torsional rotation is restrained
  - No warping restraint
  - No flexural restraint



97

### Problem Statement

- Connections @ the HSS10
  - Continuous for flexure
  - Torsion is restrained by the W21 beam
  - Continuous for warping



98

### Problem Statement

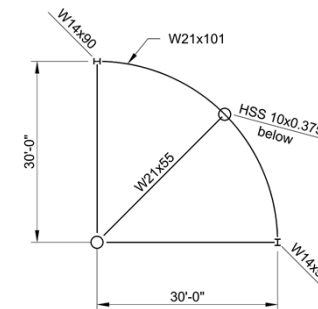
- The factored uniformly distributed load along the member circumference including the beam self weight is  $w_u = 0.750$  kip/ft



99

### Problem Statement

- Assume the critical condition is for patch loading (one span loaded)



100

There's always a solution in steel.

## Design Examples

### Example 2

### Properties



## Properties

- Material properties of ASTM A992  
(AISC *Manual* Table 2-4)

$$F_y = 50 \text{ ksi}$$

$$F_u = 65 \text{ ksi}$$



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## Properties

- Dimensions of W21×101  
(AISC *Manual* Table 1-1)

$$d = 21.4 \text{ in.} \quad t_w = 0.500 \text{ in.}$$

$$b_f = 12.3 \text{ in.} \quad t_f = 0.800 \text{ in.}$$

$$h_o = 20.6 \text{ in.}$$



103

## Properties

- Section properties of W21×101  
(AISC *Manual* Table 1-1)

$$I_x = 2,420 \text{ in.}^4 \quad S_x = 227 \text{ in.}^3$$

$$Z_x = 253 \text{ in.}^3$$

$$J = 5.21 \text{ in.}^4$$

$$r_{ts} = 3.35 \text{ in.}$$

$$C_w = 26,200 \text{ in.}^6$$



104




There's always a solution in steel.

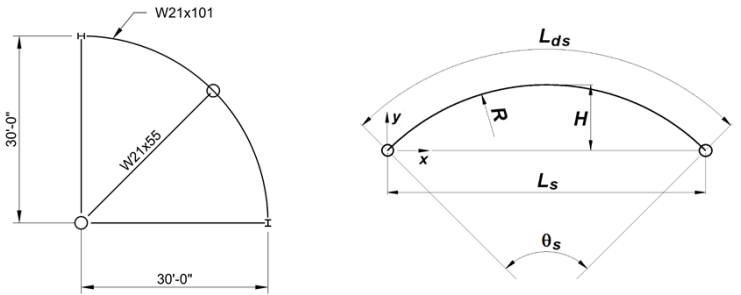

## Design Examples

### Example 2

### Beam Geometry



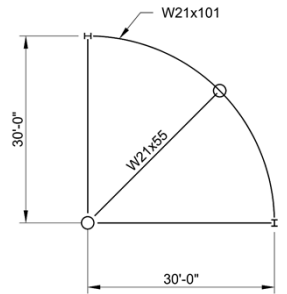

## Beam Geometry

106

## Beam Geometry

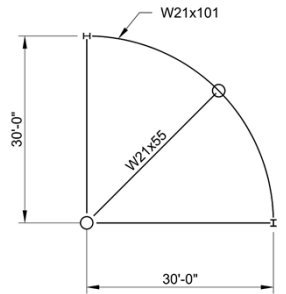

- Centroidal radius  
 $R = (30 \text{ ft})(12 \text{ in./ft})$   
 $= 360 \text{ in.}$

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## Beam Geometry

- Span angle  
 $\theta_s = (45^\circ) \left( \frac{\pi \text{ rad}}{180^\circ} \right)$   
 $= \pi / 4 \text{ rad}$

108

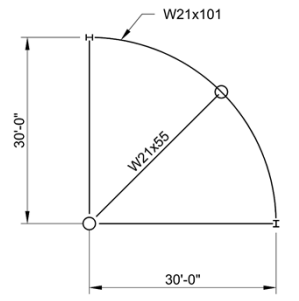


## Beam Geometry

- Developed span length

$$L_{ds} = (30 \text{ ft})(\pi / 4 \text{ rad})(12 \text{ in./ft})$$

$$= 283 \text{ in.}$$



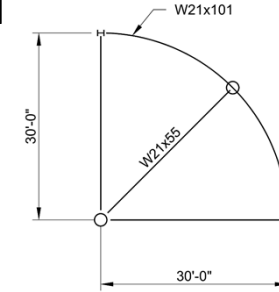
109

## Beam Geometry

- Angle between torsional restraints

$$\theta_b = (45^\circ) \left( \frac{\pi \text{ rad}}{180^\circ} \right)$$

$$= \pi / 4 \text{ rad}$$



110

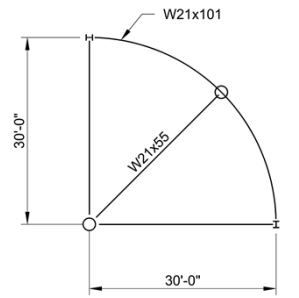
## Beam Geometry

- Developed length between braces

$$L_{db} = (30 \text{ ft})(\pi / 4 \text{ rad})$$

$$= 23.6 \text{ ft}$$

$$= 283 \text{ in.}$$



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## Design Examples

### Example 2

### Structural Analysis

There's always a solution in steel.





### Structural Analysis

- Isolate the loaded span

117

### Structural Analysis

- Flexural moment at the loaded span

118

There's always a solution in steel.

## Design Examples

### Example 2

### Structural Analysis

### Torsional Loads

119

### Structural Analysis

- M/R method
- Torsional moment per unit length:

$$m_{zc} = \frac{M_x}{R}$$

REF: Session 3 Slide 44

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### Structural Analysis

- Distributed torsion

$m_{zc}$

$z/L_{ds}$



121


### Structural Analysis

- Isolated Flange method

$$f_{fc} = \frac{m_{zc}}{h_o}$$

$h_o = \text{distance between flange centroids}$


REF: Session 3 Slide 82



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### Structural Analysis


- Warping is not restrained at the end connections  
→ use pinned ends at the isolated flange



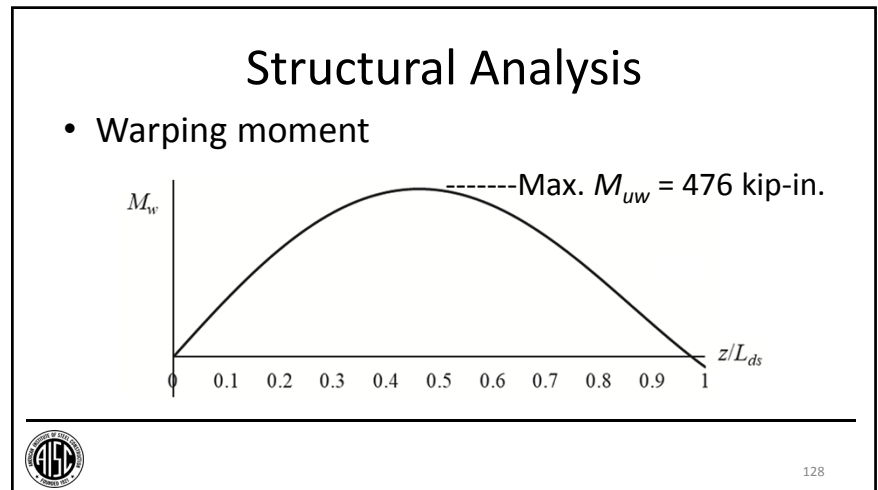
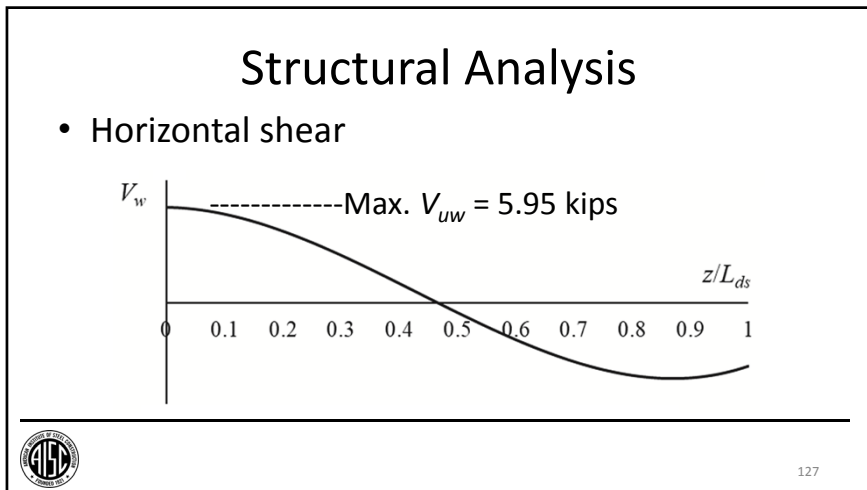
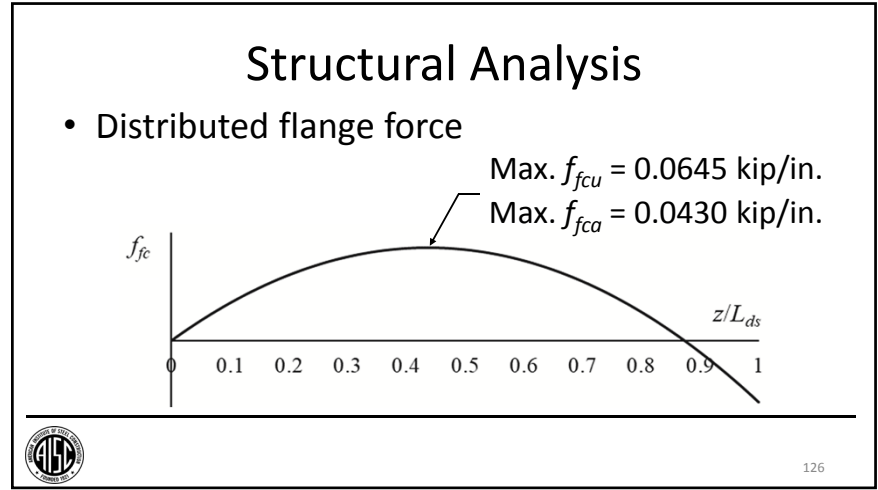
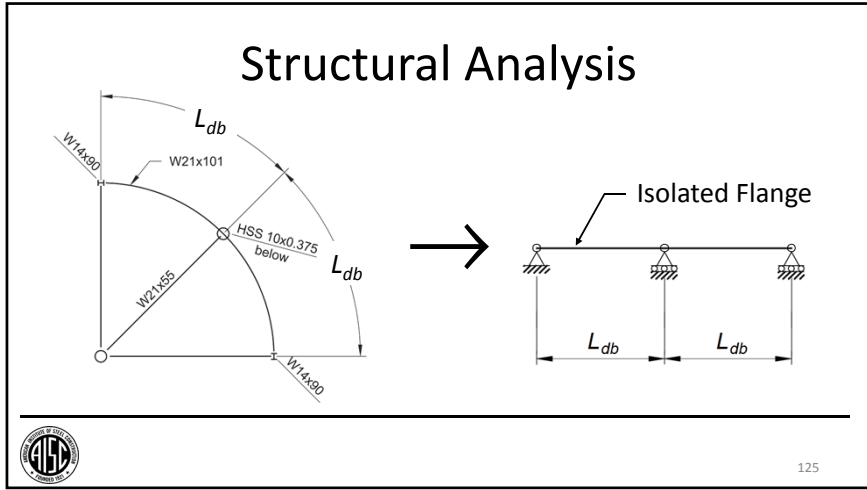
123

### Structural Analysis

- Warping is continuous at the HSS10 → the isolated flange is continuous for flexure



124

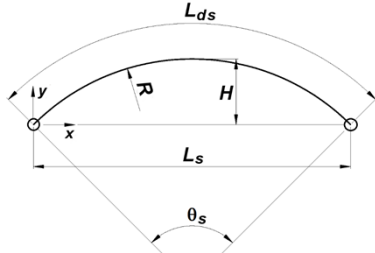


## Structural Analysis

### Corrected Moments

$$C = 1 - \frac{\theta_s}{30} + \frac{\theta_s^2}{6.2}$$

$$= 1 - \frac{\pi/4}{30} + \frac{(\pi/4)^2}{6.2}$$

$$= 1.07$$


REF: Session 3 Slide 47

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## Structural Analysis

### Corrected Moments

- Flexure:  
 $M_{u_{xc}} = CM_{ux} = (1.07)(480 \text{ kip-in.}) = 514 \text{ kip-in.}$
- Warping:  
 $M_{u_{wc}} = CM_{uw} = (1.07)(476 \text{ kip-in.}) = 509 \text{ kip-in.}$


130

There's always a solution in steel.

## Design Examples

### Example 2

### Shear Strength



132

## Shear Strength

- AISC Manual Table 6-2  
 $\phi_v V_n = 321 \text{ kips} > V_u = 9.95 \text{ kips} \quad \mathbf{o.k.}$

132

There's always a solution in steel.

## Design Examples

### Example 2 Local Buckling



## Local Buckling

- Calculations are the same as for a straight member
- Flexure:  $\lambda_f < \lambda_{pf}$  and  $\lambda_w < \lambda_{pw}$  → the W21×101 is compact



134

There's always a solution in steel.

## Design Examples

### Example 2 Flexural Strength



## Flexural Strength

- Design as a straight beam
- AISC *Specification* Chapter F



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## Flexural Strength

Use  $C_{bs} = 1.0$

$$C_{bo} = C_{bs} \left[ 1 - \left( \frac{\theta_b}{\pi} \right)^2 \right]^2$$

$$= (1.0) \left[ 1 - \left( \frac{\pi/4}{\pi} \right)^2 \right]^2 = 0.879$$



REF: Session 3 Slide 68

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## Flexural Strength

- AISC *Specification* Section F2
  - $L_b = L_{db} = 23.6$  ft
  - $C_b = C_{bo} = 0.879$
- AISC *Manual* Table 3-6
  - $L_p = 10.2$  ft
  - $L_r = 30.1$  ft



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## Flexural Strength

$$M_p = F_y Z_x$$

$$= (50 \text{ ksi})(253 \text{ in.}^3)$$

$$= 12,700 \text{ kip-in.}$$



139

## Flexural Strength

- $L_p < L_b < L_r \rightarrow$  Use AISC *Spec.* Eq. F2-2

$$M_n = C_b \left[ M_p - (M_p - 0.7F_y S_x) \left( \frac{L_b - L_p}{L_r - L_p} \right) \right] \leq M_p$$

$$= 8,350 \text{ kip-in.} < 12,700 \text{ kip-in.}$$



140

## Flexural Strength

- The available strength is

$$\begin{aligned}\phi_b M_n &= 0.90(8,350 \text{ kip-in.}) \\ &= 7,520 \text{ kip-in.}\end{aligned}$$



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## Design Examples

### Example 1

### Second-Order Effects



There's always a solution in steel.

## Second-Order Effects

- Amplified First-Order Analysis
- Second-order warping moment:  $M_{uw} = B_o M_{uwc}$

$$B_o = \frac{0.85}{1 - \alpha \frac{M_{uxc}}{M_{eo}}} \geq 1.0$$

$M_{eo}$  = elastic lateral-torsional buckling moment

$\alpha = 1.00$  (LRFD)



REF: Session 3 Slide 95

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## Second-Order Effects

$$F_{cr} = \frac{C_{bo} \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

$= 44.9 \text{ ksi}$



144

## Second-Order Effects

$$\begin{aligned}M_{e0} &= F_{cr} S_x \\ &= (44.9 \text{ ksi})(227 \text{ in.}^3) \\ &= 10,200 \text{ kip-in.}\end{aligned}$$



145

## Second-Order Effects

$$\begin{aligned}B_o &= \frac{0.85}{1 - (1.0) \left( \frac{514 \text{ kip-in.}}{10,200 \text{ kip-in.}} \right)} \geq 1.0 \\ &= 0.895 < 1.0 \\ \rightarrow B_o &= 1.0\end{aligned}$$



146

## Second-Order Effects

$$\begin{aligned}M_{uw} &= (1.0)(509 \text{ kip-in.}) \\ &= 509 \text{ kip-in.}\end{aligned}$$



147

## Design Examples

### Example 2

### Warping Strength

There's always a solution in steel.



## Warping Strength

- The isolated flange plastic modulus is

$$\begin{aligned} Z_f &= \frac{t_f b_f^2}{4} \\ &= \frac{(0.800 \text{ in.})(12.3 \text{ in.})^2}{4} \\ &= 30.3 \text{ in.}^3 \end{aligned}$$



REF: Session 3 Slide 87

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## Warping Strength

- The nominal flexural strength of the isolated flange is

$$\begin{aligned} M_{nw} &= F_y Z_f \\ &= (50 \text{ ksi})(30.3 \text{ in.}^3) \\ &= 1,520 \text{ kip-in.} \end{aligned}$$



150

## Warping Strength

- The available flexural strength of the isolated flange is

$$\begin{aligned} M_{CW} &= \phi_b M_{nw} \\ &= 0.90(1,520 \text{ kip-in.}) \\ &= 1,370 \text{ kip-in.} \end{aligned}$$



151

## Design Examples

### Example 2

### Combined Loading

There's always a solution in steel.



## Combined Loading

- Flexural moment + flange warping moment

$$\frac{M_{uxc}}{\phi_b M_n} + \frac{8 M_{uw}}{9 \phi M_{nw}} \leq 1.0$$

$$\frac{514 \text{ kip-in.}}{7,520 \text{ kip-in.}} + \left(\frac{8}{9}\right) \left(\frac{509 \text{ kip-in.}}{1,370 \text{ kip-in.}}\right) \leq 1.0$$

$$0.399 < 1.00 \quad \mathbf{o.k.}$$



REF: Session 3 Slide 100

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## Design Examples

### Example 1

### Serviceability

There's always a solution in steel.



## Serviceability

- The torsional rotation can be estimated using the horizontal deflection of the isolated flange
- Maximum second-order distributed flange force under service loads:  $f_{fc} = 0.0430 \text{ kip/in.}$

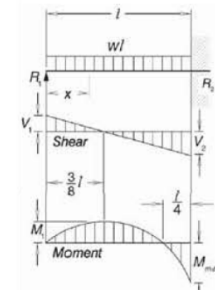


155

## Serviceability

- AISC *Manual* Table 3-23  
Case 12

$$\Delta_{max} = \frac{f_{fc} L_{ds}^4}{185 E I_f}$$



156

## Serviceability

- The isolated flange moment of inertia is

$$I_f = \frac{t_f b_f^3}{12} = \frac{(0.800 \text{ in.})(12.3 \text{ in.})^3}{12}$$

$$= 124 \text{ in.}^4$$



157

## Serviceability

$$\Delta_{max} = \frac{f_{fc} L_{ds}^4}{185 E I_f} = \frac{(0.0430 \text{ kip/in.})(283 \text{ in.})^4}{(185)(29,000 \text{ ksi})(124 \text{ in.}^4)}$$

$$= 0.415 \text{ in.}$$



158

## Serviceability

- The 1<sup>st</sup>-order torsional rotation is

$$\theta_1 = \tan^{-1} \left( \frac{2\Delta_{max}}{h_o} \right) = \tan^{-1} \left[ \frac{(2)(0.415 \text{ in.})}{20.6 \text{ in.}} \right]$$

$$= 2.31^\circ$$

$h_o$  = distance between flange centroids



159

## Serviceability

- The 2<sup>nd</sup>-order torsional rotation is

$$\theta_2 = B_o \theta_1$$

$$= (1.00)(2.31^\circ)$$

$$= 2.31^\circ$$



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## Design Examples

### Example 2 Connections

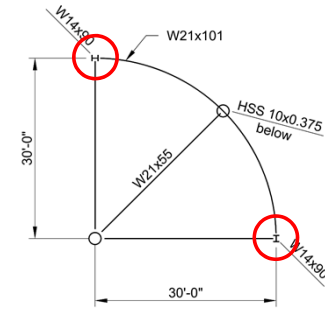


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## Problem Statement

### End Connections

- Torsional rotation is restrained
- No warping restraint
- No flexural restraint



162

## Connections

- Support reactions
  - Vertical reaction:  $R_u = 7.74$  kips
  - Horizontal shear at flanges:  $V_{uw} = 5.95$  kips



163

## Connections

- Torsional moment

$$\begin{aligned} M_{uz} &= V_{uw} h_o \\ &= (5.95 \text{ kips})(20.6 \text{ in.}) \\ &= 123 \text{ kip-in.} \end{aligned}$$

$h_o$  = distance between flange centroids



164

### Connections

Shim as required

Typical each flange

W21x101


4 Spas @ 3"

PL 3/8" x 14" x 1'-11"

8" ga.

End-plate connections are efficient in transferring torsion

Flange welds transfer  $V_{uw}$



165

### Connections

Shim as required

Typical each flange


W21x101

4 Spas @ 3"

PL 3/8" x 14" x 1'-11"

8" ga.

A wide column gage provides efficient torsional resistance



166

### Connections

Shim as required

Typical each flange


W21x101

4 Spas @ 3"

PL 3/8" x 14" x 1'-11"

8" ga.

Shims are used to ensure proper fit-up




167

### Problem Statement

**Connections @ the HSS10**

- Torsion is restrained by the W21 beam



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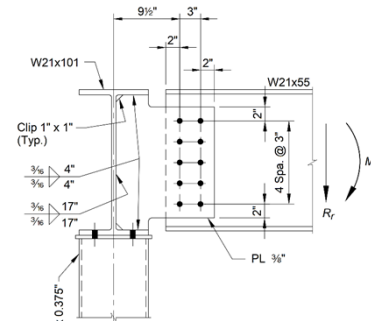
## Connections

- Support reactions
  - Vertical reaction @ W21x55:  $R_u = 30$  kips
  - Torsion @ W21x101:  $M_{uz} = 80.5$  kip-in.



169

## Connections



The torsional reaction from the curved beam is resisted by the single-plate connection



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## Question time

There's always a solution in steel.



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- You will receive an email on how to report attendance from: [registration@aisc.org](mailto:registration@aisc.org).
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One certificate will be issued at the conclusion of all 8 sessions.



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Access to the quiz: Information for accessing the quiz will be emailed to you by Wednesday. It will contain a link to access the quiz. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG

Quiz and Attendance records: Posted Tuesday mornings.  
[www.aisc.org/nightschool](http://www.aisc.org/nightschool) - click on Current Course Details.

Reasons for quiz:

- EEU – must take all quizzes and final to receive EEU
- CEUs/PDHS – If you watch a recorded session you must take quiz for CEUs/PDHS.
- REINFORCEMENT – Reinforce what you learned tonight. Get more out of the course.

NOTE: If you attend the live presentation, you do not have to take the quizzes to receive CEUs/PDHS.



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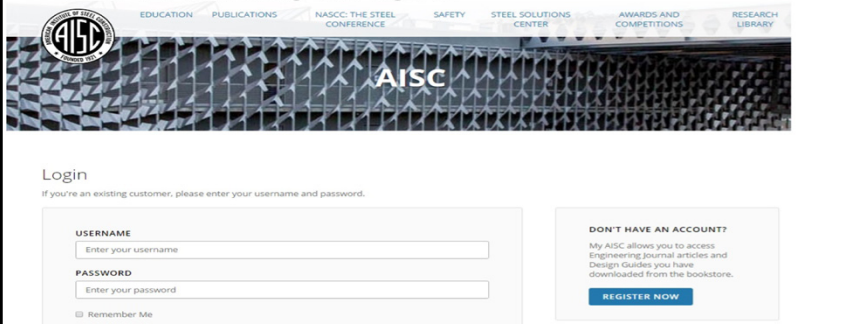
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NS13 - Design Criteria	1/30/2017 7:00:00 PM	<a href="#">Handouts</a>	<a href="#">Video</a>	Pass Score: 80	Pending
NS13 - Economic Considerations	2/6/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 02/08/2017 5pm EST	Available 02/08/2017 5pm EST	Pending
NS13 - Lateral Load Systems and Details	2/13/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 02/15/2017 5pm EST	Available 02/15/2017 5pm EST	Pending
NS13 - Preliminary Design Procedures	2/27/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 03/05/2017 5pm EST	Available 03/05/2017 5pm EST	Pending
NS13 - Crane Girder Design and Frame Analysis	3/6/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 03/08/2017 5pm EST	Available 03/08/2017 5pm EST	Pending
NS13 - Frame Member and Connection Design	3/13/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 03/15/2017 5pm EST	Available 03/15/2017 5pm EST	Pending
NS13 - Transfer Crane Girder & Longitudinal Rig Bracing Dn	3/27/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 03/29/2017 5pm EST	Available 03/29/2017 5pm EST	Pending
NS13 - Building Envelope and Bracing Design	4/3/2017 7:00:00 PM	<a href="#">Handouts</a>	Available 04/05/2017 5pm EST	Available 04/05/2017 5pm EST	Pending
NS13 - Final Exam	4/10/2017 7:00:00 PM			Available 04/12/2017 5pm EST	



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- Weekly “quiz and recording” email.
- Weekly updates of the master Quiz and Attendance record found at [www.aisc.org/nightschool](http://www.aisc.org/nightschool). Scroll down to Quiz and Attendance records.
  - Updated on Tuesday mornings.



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- Webinar connection information:
  - Found in your registration confirmation/receipt.
  - Reminder email sent out Monday mornings.
- Link to handouts also found here.



# Thank You

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*Survey at conclusion of webinar.*



There's always a solution in steel.

