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Classical Methods of Structural Analysis

Louis F. Geschwindner

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Session Description

20.3 Deflections by Virtual Work
June 24, 2019

This lesson will address calculating deflections by virtual work. The process for applying the principle of virtual work will be developed. Calculating deflections due to axial forces, shear and flexure will be covered as well as the relationship between flexural and shear deflections. The influence of temperature change on the structure will be introduced. Approaches for writing moment equations will be also reviewed.

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Learning Objectives:

- Develop the process for applying the principle of virtual work.
- Investigate deflections due to axial and shear forces, as well as due to flexure.
- Formulate the different approaches for writing moment equations.
- Explain the relationship between flexural and shear deflections in steel W-shapes for uniform load.

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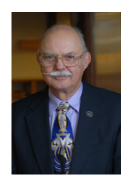
Classical Methods of Structural Analysis

Session 3: Deflections by Virtual Work
June 24, 2019



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Senior Consultant, Providence Engineering
State College, Pennsylvania

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Classical Methods of Structural Analysis:
How we did it before computers

Night School 20
Lesson 3
Deflections by Virtual Work

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9

Lesson 3
Deflections by Virtual Work

- Develop the process for applying the principle of virtual work.
- Investigate deflections due to axial forces.
- Consider the influence of temperature change.
- Investigate deflections and rotations due to flexure.
- Highlight approaches for writing moment equations.
- Investigate deflections due to shear.
- Study the relationship between flexural and shear deflections in steel W-shapes for uniform load.
- Consider the influence of combined axial and flexural stresses.

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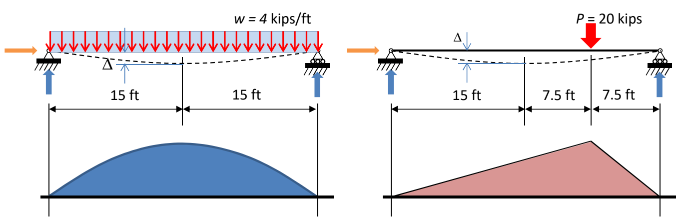
10

Deflections for Other Loads

The previous lesson illustrated situations where the deflection was desired at a point other than directly under the single load on the structure.



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11

Virtual Work

- Principle of Virtual Work: *If a deformable structure, in equilibrium and sustaining a given load or system of loads, is subjected to a virtual deformation as the result of some additional action, the* <u>*external virtual work*</u> *of the given load or system of loads is equal to the* <u>*internal virtual work*</u> *of the stresses caused by the given load or system of loads.*

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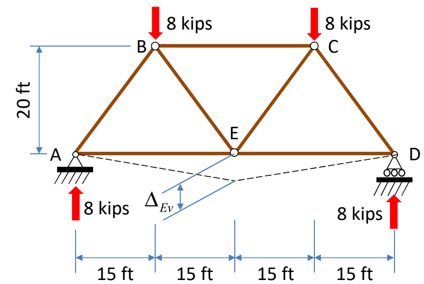
12

Virtual Work



Using the given truss as the basis, develop the method of virtual work for calculating the deflection of the lower node, E, due to the given loading.

Since the truss is determinate and is in equilibrium, the forces in the members can be determined.

Member	Force , S, (kips)
AB	-10.0
BC	-6.0
CD	-10.0
AE	+6.0
ED	+6.0
BE	0
CE	0

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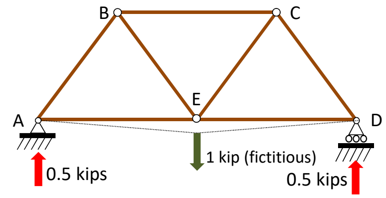
13

Virtual Work

The Process:



This is the VIRTUAL system.

1. To the unloaded structure, apply a unit load at the point and in the direction of the desired deflection. This is an imaginary or fictitious force called a "virtual" force. (The structure will displace under this virtual load but we are not interested in this displacement.)
2. The structure is in equilibrium under this virtual force so it constitutes the "given load or system of loads" in our statement of the principle of virtual work.
3. The member forces, u, can be determined for this virtual loading.

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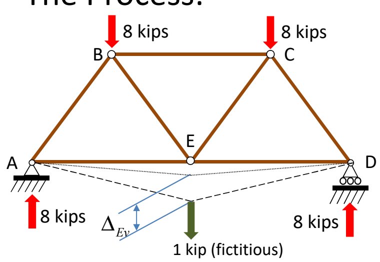
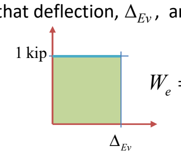
14

Virtual Work

The Process:



4. Apply the real loads to the structure with the virtual load in place.
5. Under the action of the real loads, the structure will deflect.
6. The virtual load will move through that deflection, Δ_{Ev}, and do work:



$$W_e = (1)(\Delta_{Ev})$$

7. The member forces due to the real load system, S, can be determined from equilibrium. See earlier slide 13 for results of that analysis.

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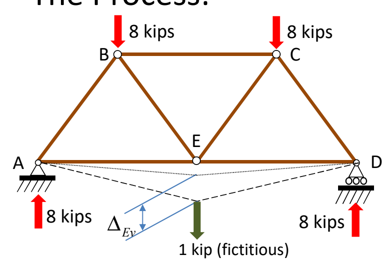
15

Virtual Work

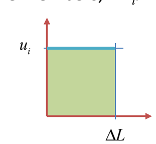
The Process:



8. The change in length of the members due to those forces can be calculated.

$$\Delta L_i = \frac{S_i L_i}{A_i E}$$

9. The internal virtual work, virtual strain energy, can be determined since the internal virtual forces, u_i, move through the change in length of the members, ΔL_i.



$$w_i = u_i \Delta L_i = \frac{u_i S_i L_i}{A_i E}$$

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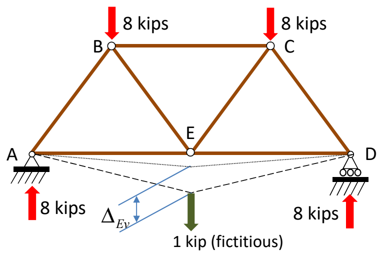
16

Virtual Work

The Process:



10. The total internal virtual work is

$$W_i = \sum \frac{uSL}{AE}$$

11. From the principle of virtual work

$$1\Delta_{Ev} = \sum \frac{uSL}{AE}$$

Member	Length, L, (ft)	Force , S, (kips)	Force, u, (kips)
AB	25.0	-10.0	-0.625
BC	30.0	-6.0	-0.75
CD	25.0	-10.0	-0.625
AE	30.0	+6.0	+0.375
ED	30.0	+6.0	+0.375
BE	25.0	0	+0.625
CE	25.0	0	+0.625

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17

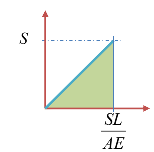
Virtual Work

- Compare strain energy with internal virtual work.
  - Axial

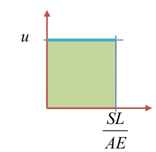
$$W_i = \sum \frac{S^2 L}{2AE}$$

$$W_i = \sum \frac{uSL}{AE}$$

Strain Energy



Virtual Work



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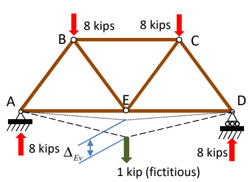
18

Virtual Work-Axial

Determine Δ_{Ev}



Member	Length, L, (ft)	Force , S, (kips)	Force, u, (kips)	Area, A, (in.²)	$\frac{uSL}{A}$
AB	25.0	-10.0	-0.625	0.5	312.5
BC	30.0	-6.0	-0.75	0.5	270
CD	25.0	-10.0	-0.625	0.5	312.5
AE	30.0	+6.0	+0.375	0.5	135
ED	30.0	+6.0	+0.375	0.5	135
BE	25.0	0	+0.625	0.5	0
CE	25.0	0	+0.625	0.5	0
				Σ	1165

The member areas used in these and subsequent examples are for illustration purposes only. An actual steel truss would have much larger members with varying areas for tension and compression.

$$1(\text{kip})\Delta_{Ev} = \sum \frac{uSL}{AE} = \frac{1165}{29,000} = 0.0402 \ (\text{kip-ft})$$

$$\Delta_{Ev} = 0.0402 \text{ ft}(12 \text{ in./ft}) = 0.482 \text{ in.}$$

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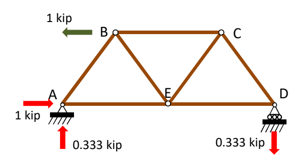
19

Virtual Work-Axial

Determine, Δ_{Bh}



Since Δ_{Bh} is negative, the horizontal deflection at node B is opposite to what we had assumed, thus, to the right.

Member	Length, L, (ft)	Force , S, (kips)	Force, u, (kips)	Area, A, (in.²)	$\frac{uSL}{A}$
AB	25.0	-10.0	-0.417	0.5	208.5
BC	30.0	-6.0	+0.50	0.5	-180
CD	25.0	-10.0	+0.417	0.5	-208.5
AE	30.0	+6.0	-0.75	0.5	-270
ED	30.0	+6.0	-0.25	0.5	-90
BE	25.0	0	+0.417	0.5	0
CE	25.0	0	-0.417	0.5	0
				Σ	-540

$$1(\text{kip})\Delta_{Bh} = \sum \frac{uSL}{AE} = \frac{-540}{29,000} = -0.0186 \ (\text{kip-ft})$$

$$\Delta_{Bh} = -0.0186 \text{ ft}(12 \text{ in./ft}) = -0.223 \text{ in.}$$

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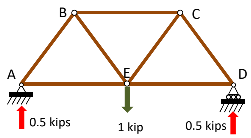
20

Virtual Work-Axial



Member	Length, L, (ft)	Force , S, (kips)	Force, u, (kips)	Area, A, (in.^{2})	$\frac{uSL}{A}$
AB	25.0	-10.0	-0.625	0.5	312.5
BC	30.0	-6.0	-0.75	0.5	270
CD	25.0	-10.0	-0.625	0.5	312.5
AE	30.0	+6.0	+0.375	0.5	135
ED	30.0	+6.0	+0.375	0.5	135
BE	25.0	0	+0.625	0.5	0
CE	25.0	0	+0.625	0.5	0
				Σ	1165

Questions to consider.

1. If we double the area of all members, what happens to the vertical deflection at E?
2. What happens to the vertical deflection at E if we just increase the area of members BE and CE?
3. How much does member BC contribute to the total vertical deflection at E?

$$\Delta_{Ev}$$

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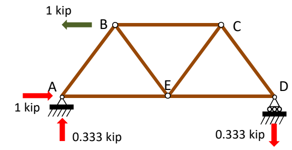
Virtual Work-Axial



Member	Length, L, (ft)	Force , S, (kips)	Force, u, (kips)	Area, A, (in.^{2})	$\frac{uSL}{A}$
AB	25.0	-10.0	-0.417	0.5	208.5
BC	30.0	-6.0	+0.50	0.5	-180
CD	25.0	-10.0	+0.417	0.5	-208.5
AE	30.0	+6.0	-0.75	0.5	-270
ED	30.0	+6.0	-0.25	0.5	-90
BE	25.0	0	+0.417	0.5	0
CE	25.0	0	-0.417	0.5	0
				Σ	-540

Questions to consider.

1. What happens to the horizontal deflection at B if we increase the area of member AB?
2. To maintain symmetry, any change in area of members AB and CD will be the same. What impact will an increase in area have on the horizontal deflection at B.

$$\Delta_{Bh}$$

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Polling Question

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Polling Question

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Virtual Work-Axial

- Temperature change
  - Use the method of virtual work to determine the impact of a temperature change on the deflection of the truss.
  - Assume that both bottom chord members see an increase in temperature of 70°F.
  - Determine the vertical deflection at point E.
  - The coefficient of thermal expansion for steel is:

$$\alpha = 0.0000065 \text{ in./in./}^{\circ}\text{F}$$

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25

Virtual Work-Axial

- Temperature change
  - The virtual load is applied just as it was to calculate deflection due to a real loading system.
  - The temperature change results in an elongation or shortening of the member length, ΔL, analogous to SL/AE.

For a member 30 ft in length with a 70 °F temperature change.

$$\Delta L = 0.0000065(70)(30)(12) = 0.164 \text{ in.}$$

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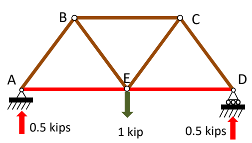
26

Virtual Work-Axial



Member	Length, L, (ft)	Force, u, (kips)	ΔL, (in.)	$u\Delta L$
AB	25.0	-0.625	0.0	0
BC	30.0	-0.75	0.0	0
CD	25.0	-0.625	0.0	0
AE	30.0	+0.375	0.164	0.0615
ED	30.0	+0.375	0.164	0.0615
BE	25.0	+0.625	0.0	0
CE	25.0	+0.625	0.0	0
			Σ	0.123

Temperature change

A 70 °F increase in the temperature of the bottom chord results in a downward deflection of

$$1(\text{kip})\Delta_{Ev} = 0.123 \text{ kip-in.}$$

$$\Delta_{Ev} = 0.123 \text{ in.}$$

Note that member area does not influence deflection due to temperature change.

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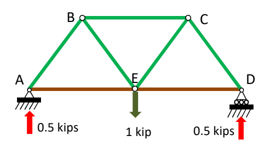
27

Virtual Work-Axial



Member	Length, L, (ft)	Force, u, (kips)	ΔL, (in.)	$u\Delta L$
AB	25.0	-0.625	-0.1365	0.085
BC	30.0	-0.75	-0.164	0.123
CD	25.0	-0.625	-0.1365	0.085
AE	30.0	+0.375	0	0
ED	30.0	+0.375	0	0
BE	25.0	+0.625	-0.1365	-0.085
CE	25.0	+0.625	-0.1365	-0.085
			Σ	0.123

Temperature change

A 70°F decrease in the temperature of all but the bottom chord results in a downward deflection of

$$1(\text{kip})\Delta_{Ev} = 0.123 \text{ kip-in.}$$

$$\Delta_{Ev} = 0.123 \text{ in.}$$

Note that these two approaches yield the same deflection.

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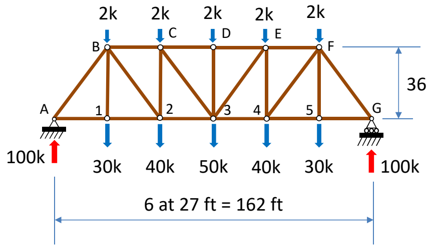
28

Virtual Work-Axial

Consider the long span steel truss under the given load.



Again, the member areas used in this and subsequent examples are for illustration purposes only. They may not be sufficient to carry the imposed load.

Member	Length, L, (ft)	Area, A, (in.²)	Force, S, (kips)
AB	45	6.0	-125
BC	27	4.0	-126
CD	27	5.0	-145.5
DE	27	5.0	-145.5
EF	27	4.0	-126
FG	45	6.0	-125
A1	27	3.0	+75
12	27	3.0	+75
23	27	5.0	+126
34	27	5.0	+126
45	27	3.0	+75
5G	27	3.0	+75
B1	36	2.0	+30
C2	36	2.0	-28
D3	36	2.0	-2
E4	36	2.0	-28
F5	36	2.0	+30
B2	45	5.0	+85
C3	45	5.0	+32.5
E3	45	5.0	+32.5
F4	45	5.0	+85

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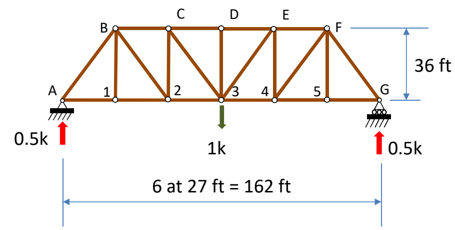
29

Virtual Work-Axial

Determine the vertical deflection at node 3 and the rotation of member 2-3.



For the deflection of node 3.

Member	Length, L, (ft)	Area, A, (in.²)	Force, S, (kips)	Force, u, (kips)
AB	45	6.0	-125	-0.625
BC	27	4.0	-126	-0.750
CD	27	5.0	-145.5	-1.125
DE	27	5.0	-145.5	-1.125
EF	27	4.0	-126	-0.750
FG	45	6.0	-125	-0.625
A1	27	3.0	+75	+0.375
12	27	3.0	+75	+0.375
23	27	5.0	+126	+0.750
34	27	5.0	+126	+0.750
45	27	3.0	+75	+0.375
5G	27	3.0	+75	+0.375
B1	36	2.0	+30	0
C2	36	2.0	-28	-0.500
D3	36	2.0	-2	0
E4	36	2.0	-28	-0.500
F5	36	2.0	+30	0
B2	45	5.0	+85	+0.625
C3	45	5.0	+32.5	+0.625
E3	45	5.0	+32.5	+0.625
F4	45	5.0	+85	+0.625

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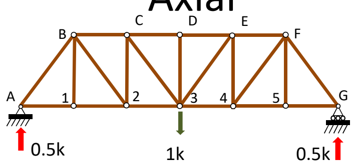
Virtual Work-Axial



$$\sum \frac{uSL}{A} = 8{,}074 \text{ kip}^2\text{-ft/in.}^2$$

$$(1 \text{ kip})\Delta_{3v} = \sum \frac{uSL}{AE}$$

$$= \frac{8{,}074 \text{ kip}^2\text{-ft/in.}^2}{29{,}000 \text{ ksi}}$$

$$= 0.278 \text{ kip-ft.}$$

$$\Delta_{3v} = 0.278(12) = 3.34 \text{ in.}$$

Member	Length, L, (ft)	Area, A, (in.²)	Force, S, (kips)	Force, u, (kips)	$\frac{uSL}{A}$
AB	45	6.0	-125	-0.625	586
BC	27	4.0	-126	-0.750	638
CD	27	5.0	-145.5	-1.125	884
DE	27	5.0	-145.5	-1.125	884
EF	27	4.0	-126	-0.750	638
FG	45	6.0	-125	-0.625	586
A1	27	3.0	+75	+0.375	253
12	27	3.0	+75	+0.375	253
23	27	5.0	+126	+0.750	510
34	27	5.0	+126	+0.750	510
45	27	3.0	+75	+0.375	253
5G	27	3.0	+75	+0.375	253
B1	36	2.0	+30	0	0
C2	36	2.0	-28	-0.500	252
D3	36	2.0	-2	0	0
E4	36	2.0	-28	-0.500	252
F5	36	2.0	+30	0	0
B2	45	5.0	+85	+0.625	478
C3	45	5.0	+32.5	+0.625	183
E3	45	5.0	+32.5	+0.625	183
F4	45	5.0	+85	+0.625	478

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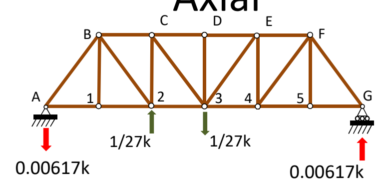
Virtual Work-Axial



For rotation of member 2-3.

$$\sum \frac{uSL}{A} = 54.0 \text{ kip}^2\text{-ft/in.}^2$$

$$(1 \text{ ft-kip})\alpha_{23} = \sum \frac{uSL}{AE}$$

$$= \frac{54.0 \text{ kip}^2\text{-ft/in.}^2}{29{,}000 \text{ ksi}}$$

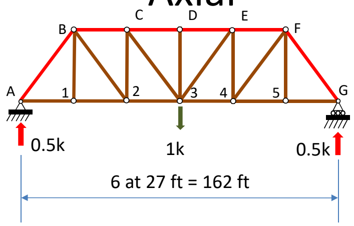
$$= 1.86 \times 10^{-3} \text{ kip-ft.}$$

$$\alpha_{23} = 1.86 \times 10^{-3} \text{ rad}$$

Member	Length, L, (ft)	Area, A, (in.²)	Force, S, (kips)	Force, u, (kips)	$\frac{uSL}{A}$
AB	45	6.0	-125	+0.00771	-7.23
BC	27	4.0	-126	+0.00926	-7.88
CD	27	5.0	-145.5	-0.0139	10.92
DE	27	5.0	-145.5	-0.0139	10.92
EF	27	4.0	-126	-0.00926	7.88
FG	45	6.0	-125	-0.00771	7.23
A1	27	3.0	+75	-0.00463	-3.13
12	27	3.0	+75	-0.00463	-3.13
23	27	5.0	+126	-0.00926	-6.30
34	27	5.0	+126	+0.00926	6.30
45	27	3.0	+75	+0.00463	3.13
5G	27	3.0	+75	+0.00463	3.13
B1	36	2.0	+30	0	0.00
C2	36	2.0	-28	-0.0308	15.52
D3	36	2.0	-2	0	0.00
E4	36	2.0	-28	-0.00617	3.11
F5	36	2.0	+30	0	0.00
B2	45	5.0	+85	-0.00771	-5.90
C3	45	5.0	+32.5	+0.0385	11.26
E3	45	5.0	+32.5	+0.00771	2.26
F4	45	5.0	+85	+0.00771	5.90

Smarter. Stronger. Steel.

Virtual Work- Axial



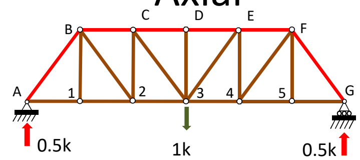
It is desired to impose an upward camber of 3.0 in. at node 3 of the given truss. The top chord and end post members will be made longer than their given dimension by the same amount.

Assume a change in length of 0.1 in.

Member	Length, L, (ft)	Area, A, (in.^{2})	ΔL (in.)	Force, u, (kips)	$u\Delta L$
AB	45	6.0	0.1	-0.625	-0.0625
BC	27	4.0	0.1	-0.750	-0.0750
CD	27	5.0	0.1	-1.125	-0.1125
DE	27	5.0	0.1	-1.125	-0.1125
EF	27	4.0	0.1	-0.750	-0.0750
FG	45	6.0	0.1	-0.625	-0.0625
A1	27	3.0	0	+0.375	0
12	27	3.0	0	+0.375	0
23	27	5.0	0	+0.750	0
34	27	5.0	0	+0.750	0
45	27	3.0	0	+0.375	0
5G	27	3.0	0	+0.375	0
B1	36	2.0	0	0	0
C2	36	2.0	0	-0.500	0
D3	36	2.0	0	0	0
E4	36	2.0	0	-0.500	0
F5	36	2.0	0	0	0
B2	45	5.0	0	+0.625	0
C3	45	5.0	0	+0.625	0
E3	45	5.0	0	+0.625	0
F4	45	5.0	0	+0.625	0

Smarter. Stronger. Steel.

Virtual Work- Axial



$$\sum u\Delta L = -0.5 \text{ kip-in.}$$

$$(1 \text{ kip})\Delta_{3v} = \sum u\Delta L = -0.5 \text{ kip-in.}$$

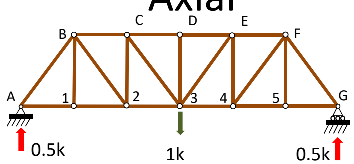
$$\Delta_{3v} = -0.5$$

Thus, to attain an upward camber of 3.0 in., the top chord and end post members will each have to be fabricated (3/0.5)(0.1) = 0.6 in. longer than dimensioned.

Member	Length, L, (ft)	Area, A, (in.^{2})	ΔL (in.)	Force, u, (kips)	$u\Delta L$
AB	45	6.0	0.1	-0.625	-0.0625
BC	27	4.0	0.1	-0.750	-0.0750
CD	27	5.0	0.1	-1.125	-0.1125
DE	27	5.0	0.1	-1.125	-0.1125
EF	27	4.0	0.1	-0.750	-0.0750
FG	45	6.0	0.1	-0.625	-0.0625
A1	27	3.0	0	+0.375	0
12	27	3.0	0	+0.375	0
23	27	5.0	0	+0.750	0
34	27	5.0	0	+0.750	0
45	27	3.0	0	+0.375	0
5G	27	3.0	0	+0.375	0
B1	36	2.0	0	0	0
C2	36	2.0	0	-0.500	0
D3	36	2.0	0	0	0
E4	36	2.0	0	-0.500	0
F5	36	2.0	0	0	0
B2	45	5.0	0	+0.625	0
C3	45	5.0	0	+0.625	0
E3	45	5.0	0	+0.625	0
F4	45	5.0	0	+0.625	0

Smarter. Stronger. Steel.

Virtual Work- Axial



Question to consider

1. Is it more effective to change the length of the top chord and end post as done here, or the bottom chord.

Member	Length, L, (ft)	Area, A, (in.^{2})	ΔL (in.)	Force, u, (kips)	$u\Delta L$
AB	45	6.0	0.1	-0.625	-0.0625
BC	27	4.0	0.1	-0.750	-0.0750
CD	27	5.0	0.1	-1.125	-0.1125
DE	27	5.0	0.1	-1.125	-0.1125
EF	27	4.0	0.1	-0.750	-0.0750
FG	45	6.0	0.1	-0.625	-0.0625
A1	27	3.0	0	+0.375	0
12	27	3.0	0	+0.375	0
23	27	5.0	0	+0.750	0
34	27	5.0	0	+0.750	0
45	27	3.0	0	+0.375	0
5G	27	3.0	0	+0.375	0
B1	36	2.0	0	0	0
C2	36	2.0	0	-0.500	0
D3	36	2.0	0	0	0
E4	36	2.0	0	-0.500	0
F5	36	2.0	0	0	0
B2	45	5.0	0	+0.625	0
C3	45	5.0	0	+0.625	0
E3	45	5.0	0	+0.625	0
F4	45	5.0	0	+0.625	0

Smarter. Stronger. Steel.

Polling Question

Smarter. Stronger. Steel.

36

Virtual Work-Flexural

- Flexural virtual work
  - The formulation of the external virtual work will not be affected by which internal strains are considered to contribute to the deflection.
  - The internal virtual work will be done by the internal virtual moments , m_x, caused by the virtual load moving through the rotation caused by the real loads. Thus,

$$W_i = \int_0^L \frac{m_x M_x}{EI} dx$$

Smarter. Stronger. Steel.

37

Virtual Work-Flexural

- Compare strain energy with internal virtual work.
  - Axial
  - Flexural

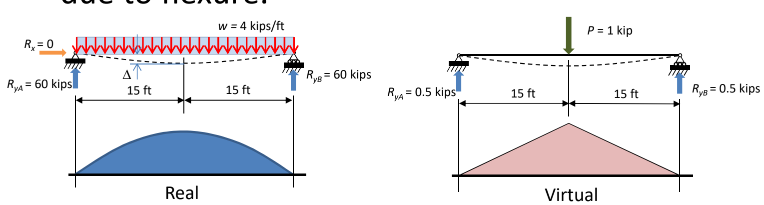
	Strain Energy	Virtual Work
Axial	$W_i = \sum \frac{S^2 L}{2AE}$	$W_i = \sum \frac{uSL}{AE}$
Flexural	$W_i = \int_0^L \frac{M_x^2}{2EI} dx$	$W_i = \int_0^L \frac{m_x M_x}{EI} dx$

Smarter. Stronger. Steel.

38

Virtual Work-Flexural

- Determine the vertical deflection at mid-span due to flexure.



Considerations when writing moment equations.

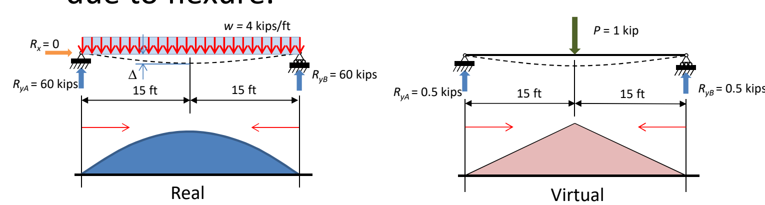
- Try to write so that x starts at 0.
- Try to write so that moment value starts at 0.
- Be sure to look to both real and virtual moment diagrams for guidance.

Smarter. Stronger. Steel.

39

Virtual Work-Flexural

- Determine the vertical deflection at mid-span due to flexure.



The simplest solution will be obtained if the moment equations are written from 0 to L/2 starting at each end of the beam.

$$M_x = 60x - \frac{4.0x^2}{2} \qquad m_x = 0.5x$$

Smarter. Stronger. Steel.

40

Virtual Work-Flexural

If the beam is a W14x90, I = 999 in.^{4}

$$1(\text{kip})\Delta = 2\int_0^{15} \frac{0.5x\left(60x - \frac{4.0x^2}{2}\right)}{EI}dx = \frac{2}{EI}\int_0^{15}\left(30x^2 - x^3\right)dx$$

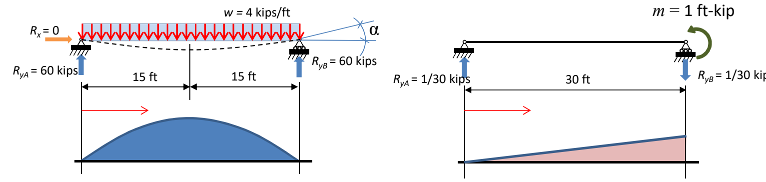
$$= \frac{1}{EI}\left(20x^3 - \frac{x^4}{2}\right)\Bigg|_0^{15} = \frac{42{,}187\ (\text{ft}^3\text{-kips}^2)}{EI}$$

$$1(\text{kip})\Delta = \frac{42{,}187\ (\text{ft}^3\text{-kips}^2)}{EI} = \frac{(42{,}187\ \text{ft}^3\text{-kips}^2)(1728\ \text{in.}^3/\text{ft}^3)}{(29{,}000\ \text{ksi})(999\ \text{in.}^4)} = 2.52\ \text{kip-in.}$$

$$\Delta = 2.52\ \text{in.}$$

Virtual Work-Flexural

- Determine the rotation at the right support due to flexure..



$$M_x = 60x - \frac{4.0x^2}{2} \qquad m_x = \frac{x}{30}$$

$$1(\text{ft-kip})\alpha = \int_0^L \frac{m_x M_x}{EI}dx$$

Virtual Work-Flexural

If the beam is a W14x90, I = 999 in.^{4}

$$1(\text{ft-kip})\alpha = \int_0^{30} \frac{\frac{x}{30}\left(60x - \frac{4.0x^2}{2}\right)}{EI}dx = \frac{1}{EI}\int_0^{30}\left(2x^2 - \frac{x^3}{15}\right)dx$$

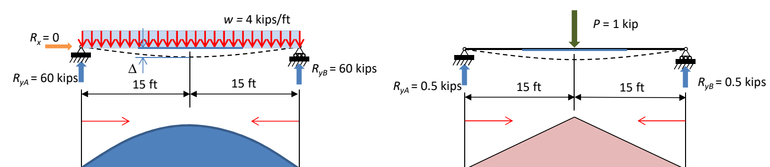
$$= \frac{1}{EI}\left(\frac{2x^3}{3} - \frac{x^4}{60}\right)\Bigg|_0^{30} = \frac{4{,}500\ (\text{ft}^3\text{-kips}^2)}{EI}$$

$$1(\text{ft-kip})\alpha = \frac{4{,}500\ (\text{ft}^3\text{-kips}^2)}{EI} = \frac{(4{,}500\ \text{ft}^3\text{-kips}^2)(144\ \text{in.}^2/\text{ft}^2)}{(29{,}000\ \text{ksi})(999\ \text{in.}^4)} = 0.0224\ \text{ft-kip}$$

$$\alpha = 0.0224\ \text{rad}$$

Virtual Work-Flexural

- Determine the vertical deflection at mid-span due to flexure if the moment of inertia in the middle half is twice that at the ends.



$$M_x = 60x - \frac{4.0x^2}{2} \qquad m_x = 0.5x$$

Virtual Work-Flexural

If the beam is a coverplated W14x90, I = 999 in.^{4}

$$1(\text{kip})\Delta = 2\int_0^{7.5} \frac{0.5x\left(60x - \frac{4.0x^2}{2}\right)}{EI}dx + 2\int_{7.5}^{15} \frac{0.5x\left(60x - \frac{4.0x^2}{2}\right)}{E(2I)}dx$$

$$= \frac{2}{EI}\int_0^{7.5}\left(30x^2 - x^3\right)dx + \frac{2}{E(2I)}\int_{7.5}^{15}\left(30x^2 - x^3\right)dx$$

$$= \frac{1}{EI}\left(20x^3 - \frac{x^4}{2}\right)\Bigg|_0^{7.5} + \frac{1}{E(2I)}\left(20x^3 - \frac{x^4}{2}\right)\Bigg|_{7.5}^{15}$$

Smarter. Stronger. Steel.

45

Virtual Work-Flexural

If the beam is a coverplated W14x90, I = 999 in.^{4}

$$1(\text{kip})\Delta = \frac{6{,}855\ (\text{ft}^3\text{-kips}^2)}{EI} + \frac{42{,}187\ (\text{ft}^3\text{-kips}^2)}{E(2I)} - \frac{6{,}855\ (\text{ft}^3\text{-kips}^2)}{E(2I)}$$

$$= \frac{24{,}521\ (\text{ft}^3\text{-kips}^2)}{EI}$$

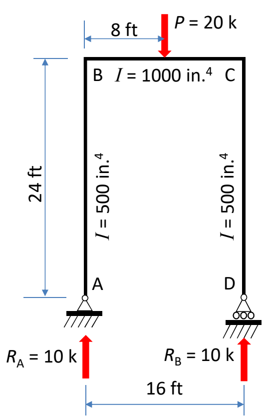
$$1(\text{kip})\Delta = \frac{(24{,}521\ \text{ft}^3\text{-kips}^2)(1728\ \text{in.}^3/\text{ft}^3)}{(29{,}000\ \text{ksi})(999\ \text{in.}^4)} = 1.46\ \text{kip-in.}$$

$$\Delta = 1.46\ \text{in.}$$

Smarter. Stronger. Steel.

46

Virtual Work-Flexural



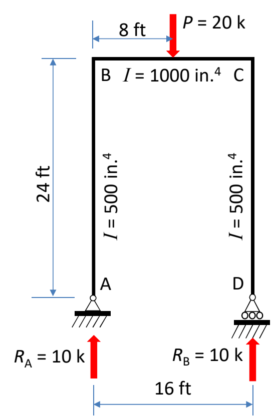
For the steel frame shown, with the properties given, determine the horizontal deflection of support D due to flexure.

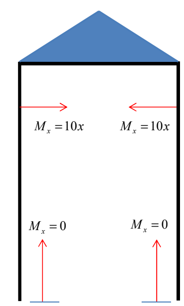
Which direction do you expect the support to move?

Smarter. Stronger. Steel.

47

Virtual Work-Flexural



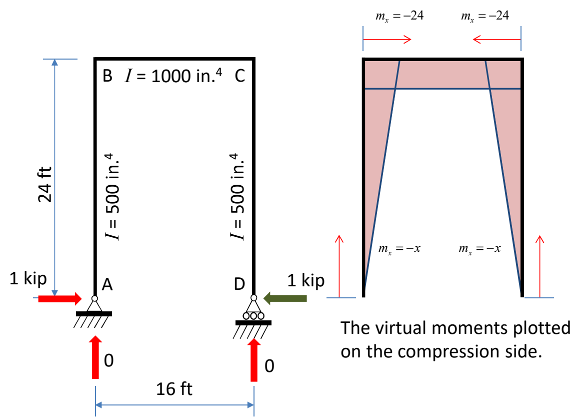


The real moments plotted on the compression side.

Smarter. Stronger. Steel.

48

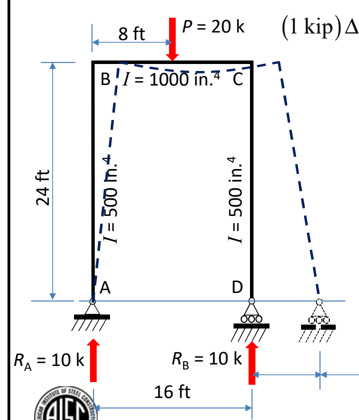
Virtual Work-Flexural



The virtual moments plotted on the compression side.

Smarter. Stronger. Steel. 49

Virtual Work-Flexural



$$(1\text{ kip})\Delta_{Dh} = \int_0^{24}\frac{-x(0)}{500E}dx + 2\int_0^{8}\frac{-24(10x)}{1000E}dx + \int_0^{24}\frac{-x(0)}{500E}dx$$

$$= \int_0^{8}\frac{-0.48x}{E}dx = \frac{-0.48x^2}{2E}\Bigg|_0^8$$

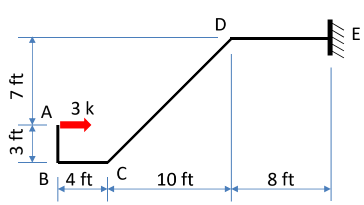
$$= \frac{-15.36(1728)}{29{,}000} = -0.915\text{ kip-in.}$$

$$\Delta_{Dh} = -0.915\text{ in.}$$

Was your guess on direction correct?

Smarter. Stronger. Steel. 50

Virtual Work-Flexural

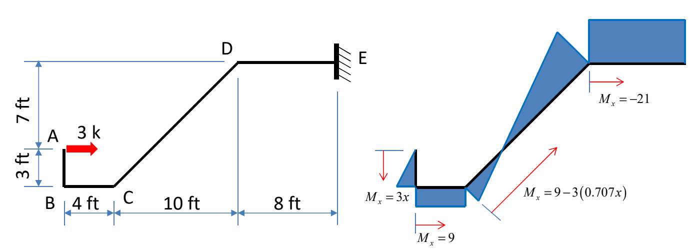


For the steel member with the geometry and loading given, determine the horizontal and vertical deflections at A and the rotation at A.

The member has a moment of inertia of 200 in.^{4}

Smarter. Stronger. Steel. 51

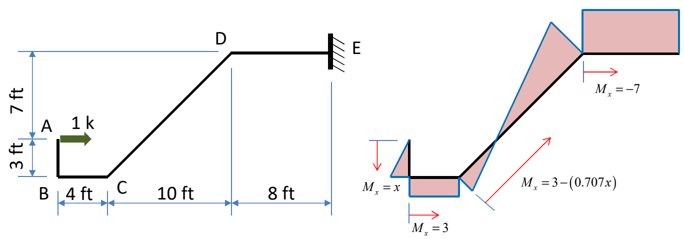
Virtual Work-Flexural



Real moment plotted on tension side

Smarter. Stronger. Steel. 52

Virtual Work-Flexural



Virtual moment for Δ_{Ah}, plotted on tension side.

Smarter. Stronger. Steel.

Virtual Work-Flexural

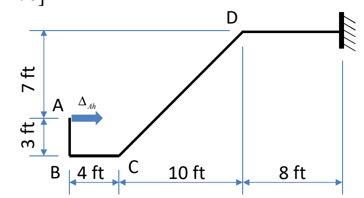
$$(1\text{ kip})\Delta_{Ah} = \frac{1}{EI}\left[\int_0^3 x(3x)dx + \int_0^4 3(9)dx + \int_0^{14.1}(3-0.707x)(9-3(0.707x))dx + \int_0^8 -7(-21)dx\right]$$

$$= \frac{1}{EI}\left[x^3\Big|_0^3 + 27x\Big|_0^4 + 27x - 6.363x^2 + 0.5x^3\Big|_0^{14.1} + 147x\Big|_0^8\right]$$

$$= \frac{1}{EI}\left[27+108+381-1265+1402+1176\right]$$

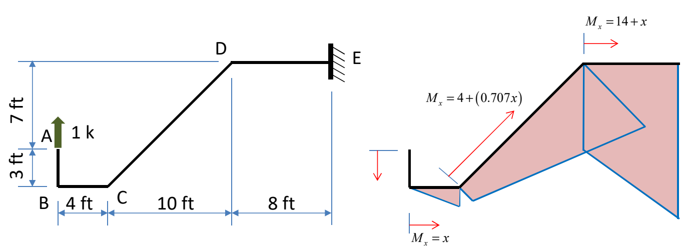
$$= \frac{1829(1728)}{29{,}000(200)} = 0.545\text{ kip-in.}$$

$$\Delta_{Ah} = 0.545\text{ in.}$$



Smarter. Stronger. Steel.

Virtual Work-Flexural



Virtual moment for Δ_{Av}, plotted on tension side.

Smarter. Stronger. Steel.

Virtual Work-Flexural

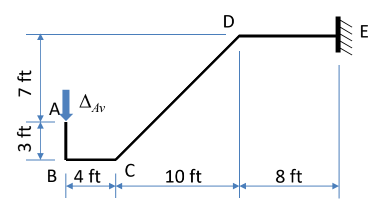
$$(1\text{ kip})\Delta_{Av} = \frac{1}{EI}\left[\int_0^3 0(3x)dx + \int_0^4 x(9)dx + \int_0^{14.1}(4+0.707x)(9-3(0.707x))dx + \int_0^8 (14+x)(-21)dx\right]$$

$$= \frac{1}{EI}\left[0\Big|_0^3 + 4.5x^2\Big|_0^4 + 36x - 1.06x^2 - 0.5x^3\Big|_0^{14.1} - 294x - 10.5x^2\Big|_0^8\right]$$

$$= \frac{1}{EI}\left[0+72+508-211-1402-2352-672\right]$$

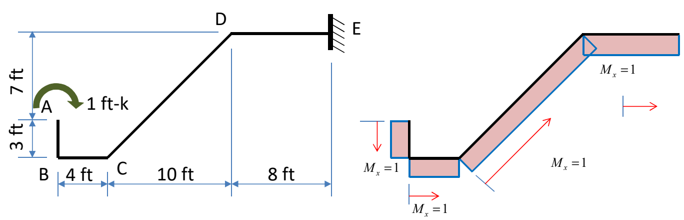
$$= \frac{-4057(1728)}{29{,}000(200)} = -1.21\text{ kip-in.}$$

$$\Delta_{Av} = 1.21\text{ in.}$$



Smarter. Stronger. Steel.

Virtual Work-Flexural



Virtual moment for α_A , plotted on tension side.

Smarter. Stronger. Steel.

Virtual Work-Flexural

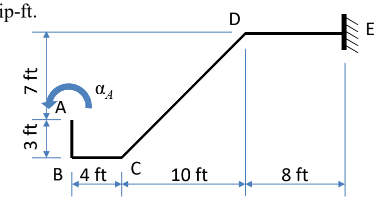
$$(1\text{ ft-kip})\,\alpha_A = \frac{1}{EI}\left[\int_0^3 1(3x)dx + \int_0^4 1(9)dx + \int_0^{14.1}(1)\big(9-3(0.707x)\big)dx + \int_0^8 (1)(-21)dx\right]$$

$$= \frac{1}{EI}\left[1.5x^2\Big|_0^3 + 9x\Big|_0^4 + 9x - 1.06x^2\Big|_0^{14.1} - 21x\Big|_0^8\right]$$

$$= \frac{1}{EI}\left[13.5+36+127-211-168\right]$$

$$= \frac{-203(144)}{29{,}000(200)} = -0.00504 \text{ kip-ft.}$$

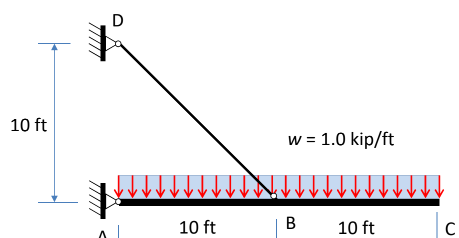
$$\alpha_A = 0.00504 \text{ rad}$$



Smarter. Stronger. Steel.

Virtual Work-Flexural

Using axial and flexural virtual work, determine the vertical deflection at C. Beam ABC is a W10x22 and member DB is 1.5 in. diameter steel rod.

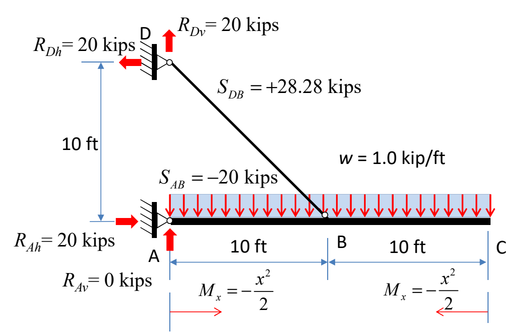


W10x22
A = 6.49 in.^{2}
I = 118 in.^{4}

1.5 in. Rod
A = 1.77 in.^{2}

Smarter. Stronger. Steel.

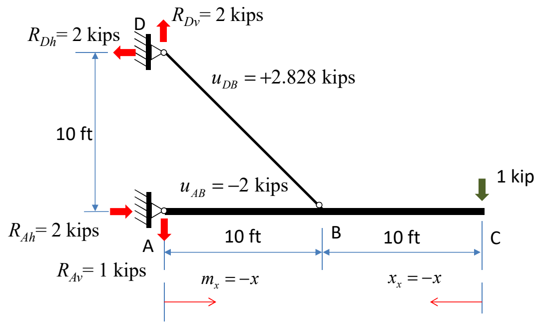
Virtual Work-Flexural



Real forces and moments

Smarter. Stronger. Steel.

Virtual Work-Flexural



Virtual forces and moments

Virtual Work-Flexural

$$(1\text{ kip})\Delta_{Cv} = \frac{2}{EI}\int_0^{10} -x\left(\frac{-x^2}{2}\right)dx + \frac{2.828(28.28)(14.14)}{A_{rod}E} + \frac{2(20)(10)}{A_W E}$$

$$= \frac{1}{EI}\left(\frac{x^4}{4}\right)\Bigg|_0^{10} + \frac{1131}{A_{rod}E} + \frac{400}{A_W E}$$

$$= \frac{1}{E}\left[\frac{2500(1728)}{118} + \frac{1131(12)}{1.77} + \frac{400(12)}{6.49}\right]$$

$$= \frac{1}{E}\left[36,610 + 7,668 + 740\right]$$

$$= \frac{45,081}{29,000} = 1.55\text{ kip-in.}$$

$$\Delta_{Cv} = 1.55\text{ in.}$$

Questions to consider

1. If we had ignored axial deformations, would the answer have been sufficiently accurate?
2. If the rod had an infinite area, is it reasonable to ignore axial?

Polling Question

Polling Question

Virtual Work-Shear

- Shear virtual work
  - As before, the formulation of the external virtual work will not be affected by which internal strains are considered to contribute to the deflection.
  - The internal virtual work will be done by the internal virtual shear forces caused by the virtual load, v_x, moving through the displacement caused by the real loads. Thus, for a W-shape.

For a W-shape, *K*=1.0

$$W_i = \int_0^L \frac{v_x V_x}{A_w G} dx$$

Smarter. Stronger. Steel.

65

Virtual Work-Shear

- Compare strain energy with internal virtual work.
  - Axial
  - Flexural
  - Shear

	Strain Energy	Virtual Work
Axial	$W_i = \sum \frac{S^2 L}{2AE}$	$W_i = \sum \frac{uSL}{AE}$
Flexural	$W_i = \int_0^L \frac{M_x^2}{2EI} dx$	$W_i = \int_0^L \frac{m_x M_x}{EI} dx$
Shear	$W_i = \int_0^L \frac{V_x^2}{2A_w G} dx$	$W_i = \int_0^L \frac{v_x V_x}{A_w G} dx$

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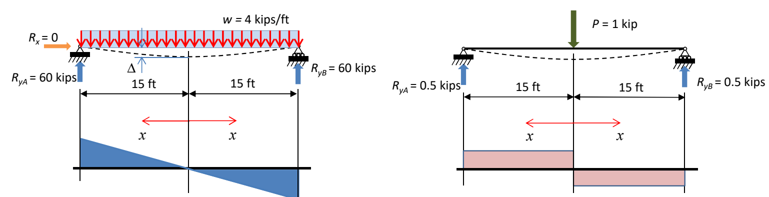
66

Virtual Work-Shear

- Determine the vertical deflection at mid-span due to shear.



The simplest solution will be obtained if the shear equations are written from 0 to 15 ft starting at the mid-span of the beam.

$V_x = 4x$ and $V_x = -4x$ $\qquad$ $v_x = 0.5$ and $v_x = -0.5$

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Virtual Work-Shear

If the beam is a W14x90, $A_w = dt_w = 14.0(0.440) = 6.16\text{ in.}^2$

$$1(\text{kip})\Delta = 2\int_0^{15} \frac{4x(0.5)}{A_w G} dx = \frac{2}{A_w G}\int_0^{15}(2x)dx$$

$$= \frac{2}{A_w G}\left(\frac{2x^2}{2}\right)\Bigg|_0^{15} = \frac{450\text{ (kips}^2\text{-ft)}}{A_w G}$$

$$1(\text{kip})\Delta = \frac{450\text{ (kips}^2\text{-ft)}}{A_w G} = \frac{(450\text{ kips}^2\text{-ft})(12\text{ in./ft})}{(6.16\text{ in.}^2)(11{,}200\text{ ksi})} = 0.0783\text{ kip-in.}$$

$$\Delta = 0.0783\text{ in.}$$

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Virtual Work-Shear

- We are now in a position to determine the relationship between shearing deflection and flexural deflection for a uniformly loaded beam.

$$1\Delta = 2\int_0^{L/2} \frac{\frac{x}{2}\left(\frac{wL}{2}x - \frac{wx^2}{2}\right)dx}{EI} \rightarrow \Delta = \frac{5wL^4}{384EI}$$

$$1\Delta = 2\int_0^{L/2} \frac{0.5(wx)}{A_w G}dx \rightarrow \Delta = \frac{wL^2}{8A_w G}$$

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We previously addressed the beam with a concentrated load at mid-span.

69

Shear vs. Flexural Deflections

- The ratio of shear deflection to bending deflection is:

$$\frac{\Delta_V}{\Delta_M} = \frac{\frac{wL^2}{8A_w G}}{\frac{5wL^4}{384EI}} = \frac{48EI}{5A_w GL^2}$$

  - Making the following substitutions

$$A_w = dt_w \qquad \frac{E}{G} = 2.5$$

  - and multiplying by

$$\frac{d^2}{d^2}$$

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Shear vs. Flexural Deflections

- yields

$$\frac{\Delta_V}{\Delta_M} = \frac{24I}{t_w d^3}\left(\frac{d}{L}\right)^2 = C_1\left(\frac{d}{L}\right)^2$$

As was the case for a concentrated load, the relationship between shear and flexure contributions to deflection is a function of the depth to span ratio squared. Again confirming that the normal assumption that shear is an issue for short span deep beams.

The variable C_1 will be 0.8 times C_2 determined previously. It ranges from approximately 20.0 for a W14x90 to 6.2 for a W24x55.

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Shear vs. Flexural Deflections

- yields

$$\frac{\Delta_V}{\Delta_M} = \frac{24I}{t_w d^3}\left(\frac{d}{L}\right)^2 = C_1\left(\frac{d}{L}\right)^2$$

Again using the rule of thumb that the depth of a steel beam is 1/24 the span, for a W14x90

$$\frac{\Delta_V}{\Delta_M} = 20.0\left(\frac{1}{24}\right)^2 = 0.035$$

It should not be surprising that shear contribution for a uniformly loaded beam is less than for a beam with a concentrated load at mid-span. ($C_1 = 0.8C_2$)

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Summary

- Developed the procedure for applying the principle of virtual work.
- Applied the principle of virtual work to axial, flexural, and shear contributions to deflections and rotations.
- Investigated the influence of combined stresses on deflection.
- Considered the impact of temperature change and established an approach for inducing camber in trusses.
- Investigated the relationship between shear and flexural deflection for simple steel beams.

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Lesson 4

- Moment Area and Elastic Weights
  - Develop the direct integration approach for calculating flexural deflections.
  - Introduce the Moment Area theorems.
  - Develop the method of elastic weights, better known as the conjugate beam method.

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Thank You

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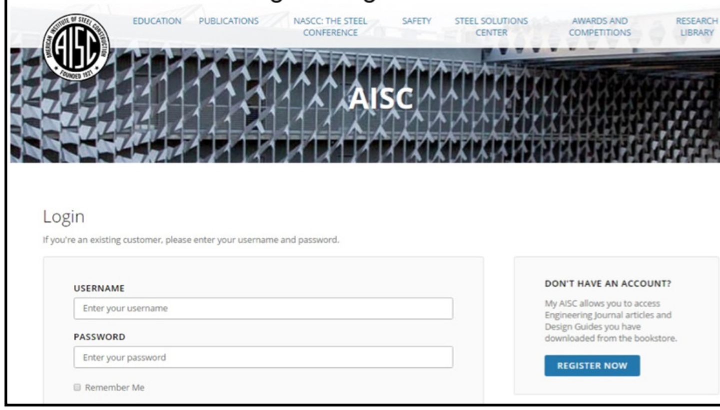

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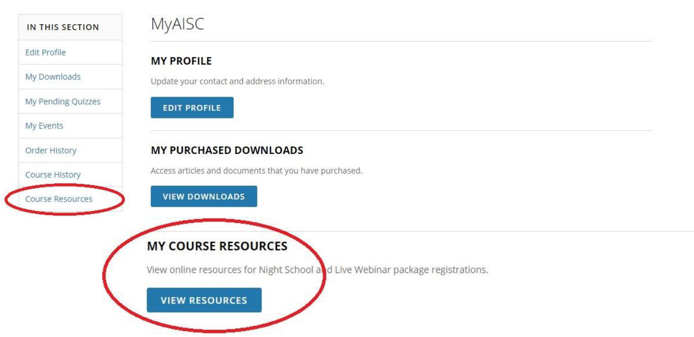

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