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Classical Methods of Structural Analysis

Louis F. Geschwindner

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Session Description
20.5 Indeterminate Structures and the General Method
July 15, 2019

This lesson will continue the analysis of indeterminate structures by conjugate beam. Calculated deflections will be used with the method of consistent deflections to analyze indeterminate structures. The AISC 360-10 Specification for Structural Steel Buildings requirement that all deformations be considered in an analysis will be discussed and examples will be presented to illustrate the relative influence of these deformations on redundant forces and moments.

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Learning Objectives:

- Describe the analysis of indeterminate structures by the conjugate beam method.
- Describe the analysis of indeterminate structures by the general method.
- Apply the general method to several types of structures.
- Describe how structural behavior is altered as member stiffnesses are altered.

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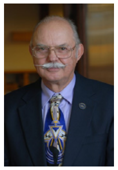
Night School 20
Classical Methods of Structural Analysis

Session 5: Indeterminate Structures and the General Method
July 15, 2019



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Classical Methods of Structural Analysis:
How we did it before computers

Night School 20
Lesson 5
Indeterminate Structures and the General Method

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Lesson 5

Indeterminate Structures and the General Method

- Indeterminate structures directly by conjugate beam.
- Indeterminate structures by the general method.
  - Truss with external redundants
  - Truss with internal redundants
  - Truss with support settlement
  - Beam and frame with flexure
  - Combined axial and flexure

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Conjugate Beam

- In Lesson 4 we saw that the fixed end moments of an indeterminate beam could be determined directly by the conjugate beam method.
- Other, more complicated, indeterminate beams may also be solved directly using this method.

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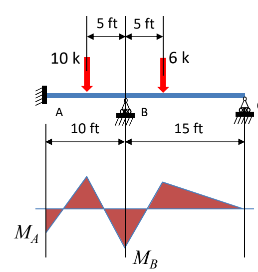
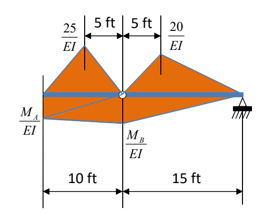
Conjugate Beam

- Consider the following beam, two degrees indeterminate, with a constant EI.



Real Beam



Conjugate Beam

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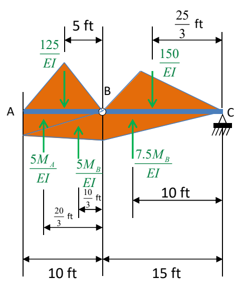
Conjugate Beam



1. The conjugate beam must be in equilibrium.
2. There are two unknown moments, one at A and one at B. Thus, two equations of equilibrium will be required.
3. For the first equation, take the sum of the moments about B for segment AB,

$$\frac{5M_A}{EI}\left(\frac{20}{3}\right)+\frac{5M_B}{EI}\left(\frac{10}{3}\right)-\frac{125}{EI}\left(\frac{10}{2}\right)=0$$

from which

$$2M_A+M_B=37.5$$

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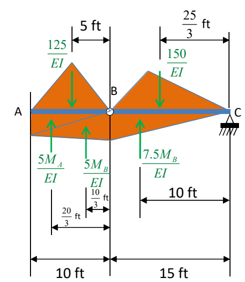
Conjugate Beam



4. There are several possibilities for the second equation. One could be sum of the moments about C must be zero. Another could be that the pin at B, as a free body, must be in equilibrium.
5. For this example, take moments about C for the entire conjugate beam,

$$\frac{5M_A}{EI}(21.67)+\frac{5M_B}{EI}(18.33)-\frac{125}{EI}(20)+\frac{7.5M_B}{EI}(10)-\frac{150}{EI}\left(\frac{25}{3}\right)=0$$

from which

$$M_A+1.54M_B=34.6$$

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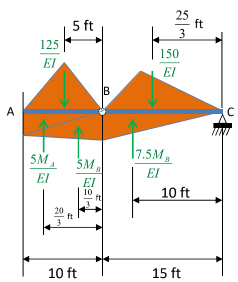
14

Conjugate Beam



6. Now, solve these two equations simultaneously.

$$2M_A+M_B=37.5$$
$$M_A+1.54M_B=34.6$$

Thus,

$$M_A=11.1\text{ ft-k}\qquad M_B=15.3\text{ ft-k}$$

The + sign on the results means that we assumed the moments in the correct direction.

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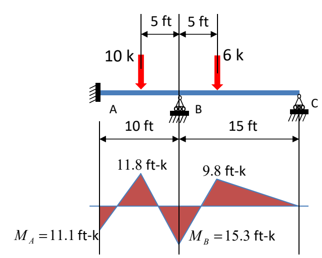
15

Conjugate Beam



Real Beam

7. Determine the moment at the load on segment AB.

$$25-\left(\frac{11.1+15.3}{2}\right)=11.8\text{ ft-k}$$

8. Determine the moment at the load on segment BC.

$$20-\left(\frac{2(15.3)}{3}\right)=9.8\text{ ft-k}$$

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16

Conjugate Beam

- Some further considerations
  - The conjugate beam method is only capable of incorporating flexural deformations.
  - Although it can be extended to frame structures and beams of unusual geometries, the complexity of the solution is not normally worth the effort required.
  - Other, more useful techniques to analyze indeterminate structures without first calculating deflections and rotations will be covered in Lessons 6 and 7.

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17

General Method

- The general method for solution of indeterminate structures is also known as the method of consistent deflections, method of consistent deformations, and method of consistent displacements.
- It was first introduced by Clerk Maxwell and called the general method in 1864. His presentation did not include examples and was so abstract that it attracted little attention.

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18

General Method

- In 1874, Otto Mohr developed the same method, independent of Maxwell's work. He used somewhat different principles to arrive at his method.
- In 1886, Heinrich Müller-Breslau published his variation of these two previous efforts. His is the method most commonly used and that which we will consider.

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19

General Method

- Indeterminate Structures
  - In Lesson 1 we introduced the idea that a structure could have more unknown reactions than could be determined through application of the available equations of equilibrium.
  - We also illustrated an example where the contact force between two beams could be determined by making the deflection on each beam equal.
  - The general method is a systematic approach to applying the required deformation relationships.

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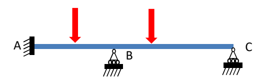
20

Indeterminate Structures

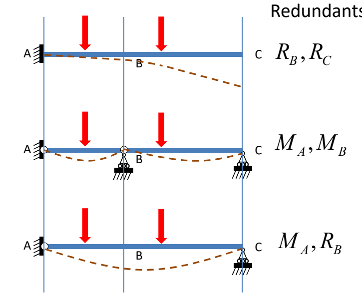
- When a structure is indeterminate, there are more unknowns than can be determined through application of the equilibrium and condition equations.
- If these "extra" unknowns are removed from the structure, the resulting "cut back" structure will be both stable and determinate.
  - These removed unknowns are referred to as "redundants" because they are extra.

Indeterminate Structures

- Redundants may be either external or internal.



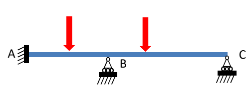
Indeterminate Beam



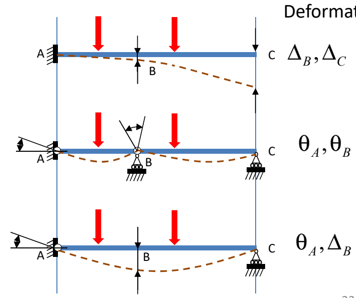
3 ways to cut back the beam so that it becomes determinate.

Indeterminate Structures

- As redundants are removed, displacements or rotations are permitted that were not permitted in the real structure.

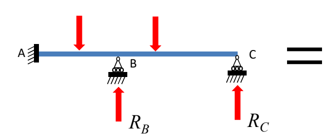


Indeterminate Beam

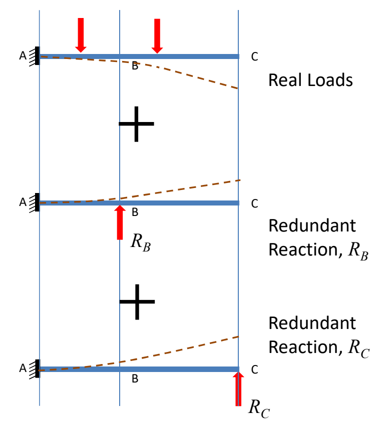


Indeterminate Structures

- For the first cut back beam, superposition shows that the real beam is the sum of the three determinate beams.



Indeterminate Beam



Indeterminate Structures

- For this beam, compatibility means that superposition of the three loadings just shown must result in zero deflection at B and C.
- Therefore, label the required deflections and assemble the appropriate compatibility equations.

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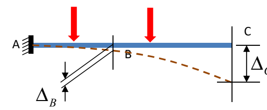
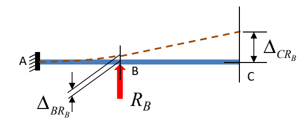
Indeterminate Structures

- Deflections



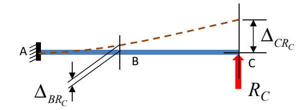
Real Loads



Redundant Reaction, R_B

Remember the notation scheme we introduced in Lesson 2.

Δ_{BR_B} deflection at / due to



Redundant Reaction, R_C

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Indeterminate Structures

- Compatibility equations

$$\Delta_B + \Delta_{BR_B} + \Delta_{BR_C} = 0$$
$$\Delta_C + \Delta_{CR_B} + \Delta_{CR_C} = 0$$

- To solve for the unknown reactions seen on the previous slide, we must find a way to incorporate them into the above compatibility equations.
- We can do that by using unit loads and determining the associated deflections.

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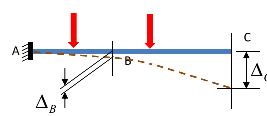
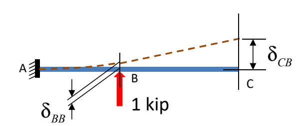
27

Indeterminate Structures

- Unit loads and Deflections



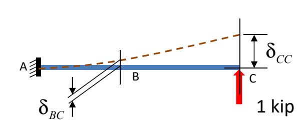
Real Loads



Unit Redundant at B

Remember, using unit loads we can determine deflections, as introduced in Lesson 2.

$$\Delta_{BR_B} = R_B \delta_{BB}$$



Unit Redundant at C

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Indeterminate Structures

- From superposition, the compatibility equations become

$$\Delta_B + R_B\delta_{BB} + R_C\delta_{BC} = 0$$
$$\Delta_C + R_B\delta_{CB} + R_C\delta_{CC} = 0$$

- Each of these deflections can be determined.
- The two equations may then be solved simultaneously for the unknown reactions.

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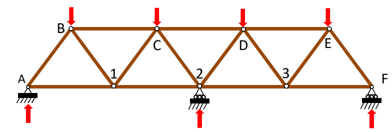
29

General Method

- Apply the general method to the single degree indeterminate truss shown.



- Assume that all members have the same area for our initial calculations.

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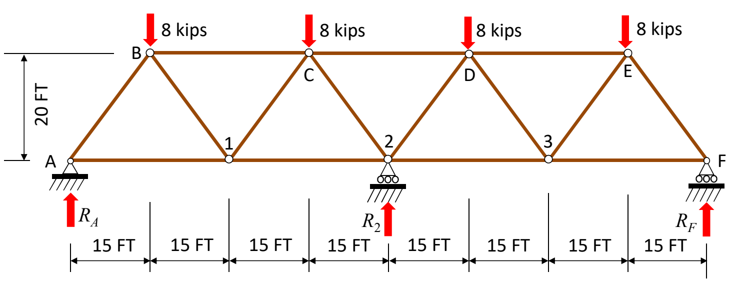
30

General Method

- Truss geometry and loading.



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31

General Method

- If we remove the redundant reaction at 2, the truss will be both stable and determinate but it will deflect, Δ_2, at that node.
- The reaction, R_2, must push the truss back to its original position, zero deflection.
- Thus, the consistent deflection equation for this one condition is

$$\Delta_2 + R_2\delta_{22} = 0$$

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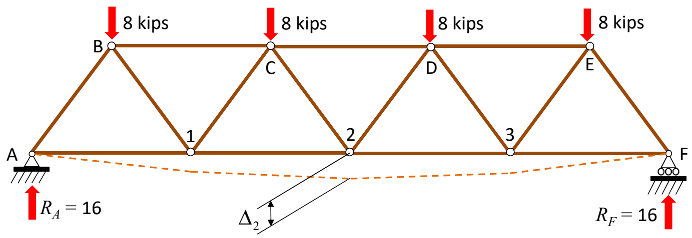
32

General Method

- Analyze the cut back structure and determine the deflection at node 2 by virtual work.



Real loads on the cutback structure yield S forces.

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General Method

Member	Length, ft	Area	Force, S, kips
AB	25.0	1	-20.0
BC	30.0	1	-18.0
CD	30.0	1	-24.0
DE	30.0	1	-18.0
EF	25.0	1	-20.0
A1	30.0	1	+12.0
12	30.0	1	+24.0
23	30.0	1	+24.0
3F	30.0	1	+12.0
B1	25.0	1	+10.0
1C	25.0	1	-10.0
C2	25.0	1	0
2D	25.0	1	0
D3	25.0	1	-10.0
3E	25.0	1	+10.0

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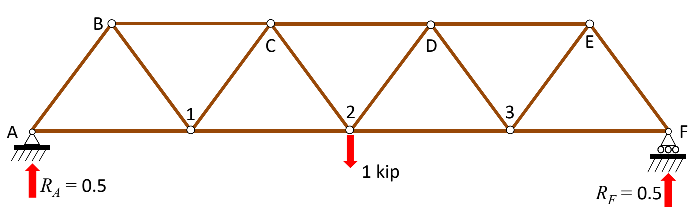
General Method

- Apply a unit virtual load at 2 to determine Δ_2



Unit load on the cutback structure yields u forces.

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General Method

Member	Length, ft	Area	Force, S, kips	Force, u, kips
AB	25.0	1	-20.0	-0.625
BC	30.0	1	-18.0	-0.750
CD	30.0	1	-24.0	-1.500
DE	30.0	1	-18.0	-0.750
EF	25.0	1	-20.0	-0.625
A1	30.0	1	+12.0	+0.375
12	30.0	1	+24.0	+1.125
23	30.0	1	+24.0	+1.125
3F	30.0	1	+12.0	+0.375
B1	25.0	1	+10.0	+0.625
1C	25.0	1	-10.0	-0.625
C2	25.0	1	0	+0.625
2D	25.0	1	0	+0.625
D3	25.0	1	-10.0	-0.625
3E	25.0	1	+10.0	+0.625

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General Method

Member	Length, ft	Area	Force, S, kips	Force, u, kips	uSL
AB	25.0	1	-20.0	-0.625	313
BC	30.0	1	-18.0	-0.750	405
CD	30.0	1	-24.0	-1.500	1080
DE	30.0	1	-18.0	-0.750	405
EF	25.0	1	-20.0	-0.625	313
A1	30.0	1	+12.0	+0.375	135
12	30.0	1	+24.0	+1.125	810
23	30.0	1	+24.0	+1.125	810
3F	30.0	1	+12.0	+0.375	135
B1	25.0	1	+10.0	+0.625	156
1C	25.0	1	-10.0	-0.625	156
C2	25.0	1	0	+0.625	0
2D	25.0	1	0	+0.625	0
D3	25.0	1	-10.0	-0.625	156
3E	25.0	1	+10.0	+0.625	156
				Σ	5030

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37

General Method

- Determine the displacement at node 2 on the cut back structure.

$$(1\text{ k})\Delta_2 = \sum \frac{uSL}{AE} = \frac{5030}{AE}\text{k}^2\text{-ft}$$

$$\Delta_2 = \frac{5030}{AE}\text{k-ft}$$

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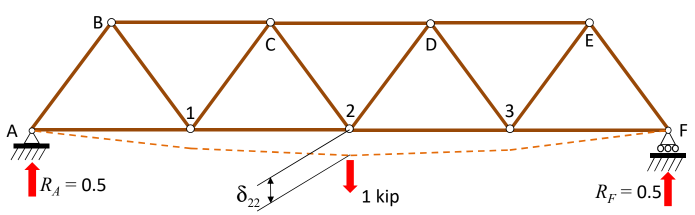
38

General Method

- Determine the deflection at node 2 due to the 1 kip load. Since we are finding deflection due to the 1 kip load, we can replace S with u.



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39

General Method

Member	Length, ft	Area	Force, S, kips	Force, u, kips	uSL	uuL
AB	25.0	1	-20.0	-0.625	313	9.8
BC	30.0	1	-18.0	-0.750	405	16.9
CD	30.0	1	-24.0	-1.500	1080	67.5
DE	30.0	1	-18.0	-0.750	405	16.9
EF	25.0	1	-20.0	-0.625	313	9.8
A1	30.0	1	+12.0	+0.375	135	4.2
12	30.0	1	+24.0	+1.125	810	38.0
23	30.0	1	+24.0	+1.125	810	38.0
3F	30.0	1	+12.0	+0.375	135	4.2
B1	25.0	1	+10.0	+0.625	156	9.8
1C	25.0	1	-10.0	-0.625	156	9.8
C2	25.0	1	0	+0.625	0	9.8
2D	25.0	1	0	+0.625	0	9.8
D3	25.0	1	-10.0	-0.625	156	9.8
3E	25.0	1	+10.0	+0.625	156	9.8
				Σ	5030	263.8

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40

General Method

- Determine the deflection at node 2 due to the unit load at node 2.

$$(1\text{ k})\delta_{22} = \sum \frac{u^2 L}{AE} = \frac{263.8}{AE}\text{k}^2\text{-ft}$$

$$\delta_{22} = \frac{263.8}{AE}\text{k-ft}$$

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41

General Method

- From the consistent deflection equation

$$\Delta_2 + R_2\delta_{22} = 0$$

$$\frac{5030}{AE} + R_2\left(\frac{263.8}{AE}\right) = 0$$

$$R_2 = \frac{-5030}{263.8} = -19.1\text{ kips}$$

The (-) sign means that our assumption of a downward reaction is not correct.

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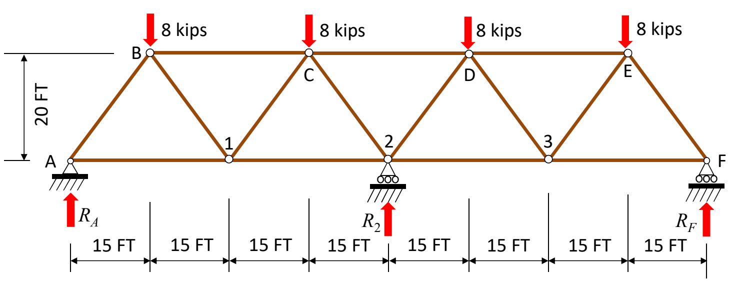
42

General Method

- Reconsider the one degree indeterminate truss.



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43

General Method

- In this case we will cut the redundant member between nodes C and D. We must not “remove” the member because it will still contribute to our deflection calculations. The truss is again both stable and determinate.
- Because of the cut in the member, the cut faces will move with respect to each other.
- Thus, the consistent deformation equation for this one condition is

$$\Delta_{CD} + T_{CD}\delta_{CDCD} = 0$$

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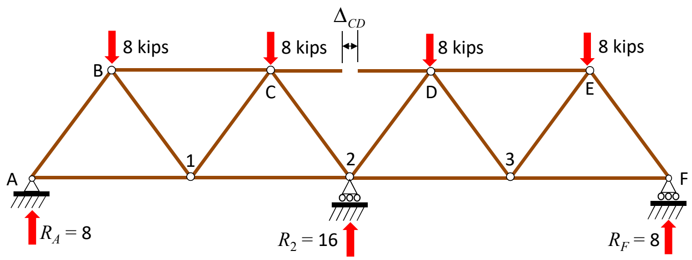
44

General Method

- Analyze the cut back structure and determine the deformation at the cut in member CD.



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45

General Method

Member	Length, ft	Area	Force, S, kips
AB	25.0	1	-10.0
BC	30.0	1	-6.0
CD	30.0	1	0
DE	30.0	1	-6.0
EF	25.0	1	-10.0
A1	30.0	1	+6.0
12	30.0	1	+6.0
23	30.0	1	+6.0
3F	30.0	1	+6.0
B1	25.0	1	0
1C	25.0	1	0
C2	25.0	1	-10.0
2D	25.0	1	-10.0
D3	25.0	1	0
3E	25.0	1	0

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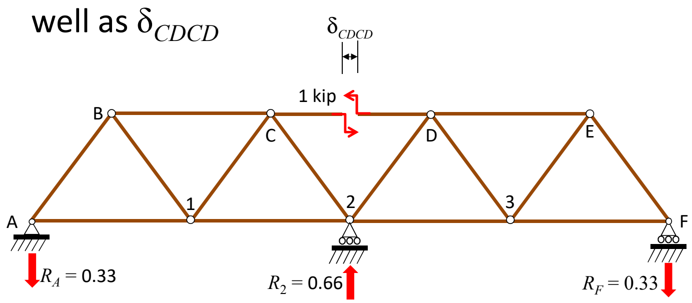
46

General Method

- Apply a unit virtual load to determine Δ_{CD} as well as δ_{CDCD}



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47

General Method

Cut member →

Member	Length, ft	Area	Force, S, kips	Force, u, kips
AB	25.0	1	-10.0	+0.417
BC	30.0	1	-6.0	+0.500
CD	30.0	1	0	+1.00
DE	30.0	1	-6.0	+0.500
EF	25.0	1	-10.0	+0.417
A1	30.0	1	+6.0	-0.250
12	30.0	1	+6.0	-0.750
23	30.0	1	+6.0	-0.750
3F	30.0	1	+6.0	-0.250
B1	25.0	1	0	-0.417
1C	25.0	1	0	+0.417
C2	25.0	1	-10.0	-0.417
2D	25.0	1	-10.0	-0.417
D3	25.0	1	0	+0.417
3E	25.0	1	0	-0.417

Smarter. Stronger. Steel.

48

General Method

Cut member → CD

Member	Length, ft	Area	Force, S, kips	Force, u, kips	uSL	uuL
AB	25.0	1	-10.0	+0.417	-104.3	4.3
BC	30.0	1	-6.0	+0.500	-90.0	7.5
CD	30.0	1	0	+1.00	0.0	30.0
DE	30.0	1	-6.0	+0.500	-90.0	7.5
EF	25.0	1	-10.0	+0.417	-104.3	4.3
A1	30.0	1	+6.0	-0.250	-45.0	1.9
12	30.0	1	+6.0	-0.750	-135.0	16.9
23	30.0	1	+6.0	-0.750	-135.0	16.9
3F	30.0	1	+6.0	-0.250	-45.0	1.9
B1	25.0	1	0	-0.417	0.0	4.3
1C	25.0	1	0	+0.417	0.0	4.3
C2	25.0	1	-10.0	-0.417	104.3	4.3
2D	25.0	1	-10.0	-0.417	104.3	4.3
D3	25.0	1	0	+0.417	0.0	4.3
3E	25.0	1	0	-0.417	0.0	4.3
				Σ	-540.0	117.3

Smarter. Stronger. Steel.

General Method

- Determine the deformation at the cut in member CD.

$$(1\text{ k})\Delta_{CD} = \sum \frac{uSL}{AE} = \frac{-540}{AE}\text{k}^2\text{-ft} \quad \rightarrow \Delta_{CD} = \frac{-540}{AE}\text{k-ft}$$

- Determine the deformation at the cut due to the unit load

$$(1\text{ k})\delta_{CDCD} = \sum \frac{u^2L}{AE} = \frac{117.3}{AE}\text{k}^2\text{-ft} \quad \rightarrow \delta_{CDCD} = \frac{117.3}{AE}\text{k-ft}$$

Smarter. Stronger. Steel.

General Method

- From the consistent deformation equation

$$\Delta_{CD} + T_{CD}\delta_{CDCD} = 0$$

$$\frac{-540}{AE} + T_{CD}\left(\frac{117.3}{AE}\right) = 0$$

$$T_{CD} = \frac{540}{117.3} = 4.6\text{ kips}$$

The + sign means that our assumption of a tension force is correct.

Smarter. Stronger. Steel.

General Method

- Using the first solution, where the reaction at node 2 was the redundant, confirm that the force in member CD that we just determined is correct. Using superposition

From the table on slide 36.

$S_{CD} = -24$ kips

$u_{CDR_2} = -1.5$ kips

$R_2 = -19.1$ kips

$$T_{CD} = S_{CD} + R_2 u_{CDR_2}$$
$$= -24 - 19.1(-1.5) = 4.6\text{ kips}$$

Smarter. Stronger. Steel.

General Method

- Questions to consider
  - For the force in member CD,
    - What would be the impact on the force if all the member areas were increased from 1.0 in. to 2.0 in.?
    - What would be the impact if only members C2 and 2D, the center diagonals, had increased area?
  - For the reaction at 2,
    - What would be the impact if only members C2 and 2D had increased area?

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53

Polling Question

Smarter. Stronger. Steel.

54

Polling Question

Smarter. Stronger. Steel.

55

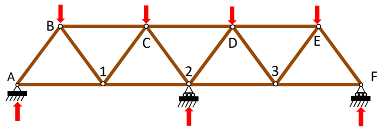
Polling Question

Smarter. Stronger. Steel.

56

General Method

- Again consider the truss of our previous problems.



- What will be the impact on the loaded structure if node 2 settles 1.0 in?

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57

General Method

- The compatibility equation we had written was

$$\Delta_2 + R_2\delta_{22} = 0$$

- If node 2 actually is permitted to displace 1.0 in. down in its final configuration, then the compatibility equation becomes

$$\Delta_2 + R_2\delta_{22} = 1.0$$

The 1.0 in. is + because the unit redundant was down.

Smarter. Stronger. Steel.

58

General Method

- Substituting for the deflection calculations we have already carried out yields

$$\frac{5030}{AE} + R_2\left(\frac{263.8}{AE}\right) = 1.0$$

- Clearly we can not solve for the unknown reaction without knowing the member areas. Even knowing that they are all the same is not sufficient.
- The same can be said regarding E.

Smarter. Stronger. Steel.

59

General Method

- If each member area is actually 1.0 in.^{2}

$$\Delta_2 = \frac{5030}{AE}\text{k-ft} = \frac{5030(12)}{1.0(29{,}000)} = 2.08 \text{ in.}$$

$$\delta_{22} = \frac{263.8}{AE}\text{k-ft} = \frac{263.8(12)}{(1.0)(29{,}000)} = 0.109 \text{ in.}$$

$$\frac{5030}{AE} + R_2\left(\frac{263.8}{AE}\right) = 1.0 = 2.08 + R_2(0.109)$$

$$R_2 = -9.91 \text{ kips}$$

Thus, the reaction is up, opposite of the assumed direction. This compares to 19.1 kips with no settlement.

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60

General Method

- Some thoughts on the general method for trusses
  - It would have been a simple task to include differing member areas in our solution.
  - The impact of a temperature change on the redundant reaction can be determined by applying what we used to determine temperature impact on deflection.
  - The selection of internal redundants often leads to a simpler solution than selection of external reactions.
  - Inclusion of flexural members within a truss will be addressed after we look at flexure alone.

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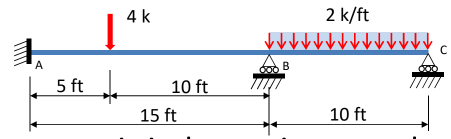
61

General Method

- Consider the beam with loading shown.



- The structure is indeterminate to the second degree. Solve for the redundants by the general method.
- Select the moments at A and B as the redundants.

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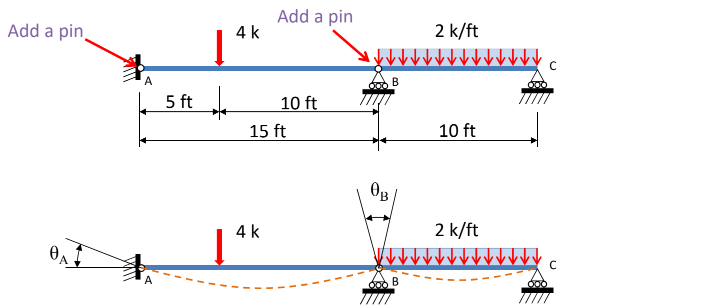
62

General Method

- The cut back structure



You end up with 2 simple beams and 2 rotations to determine.

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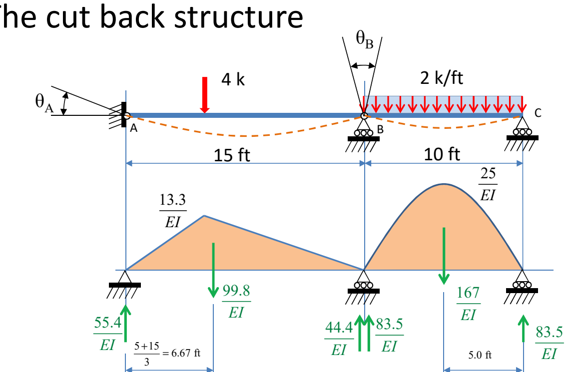
63

General Method

- The cut back structure



Use Conjugate Beam

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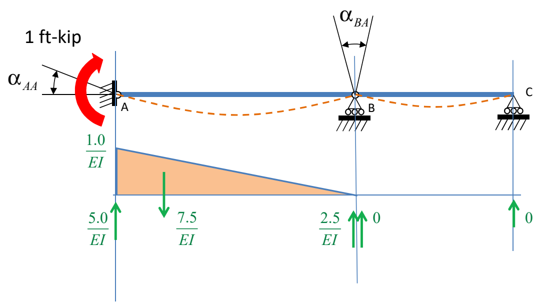
64

General Method

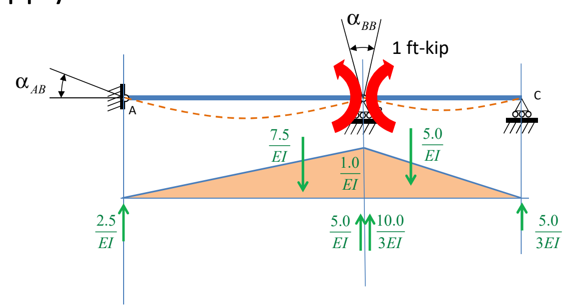
- Apply the unit redundant moment at A



Smarter. Stronger. Steel.

General Method

- Apply the unit redundant moment at B



Smarter. Stronger. Steel.

General Method

- Rotations on the real beam are shears on the conjugate beam. Thus, from previous 3 slides

$$\theta_A = \frac{55.4}{EI} \qquad \theta_B = \frac{44.4}{EI} + \frac{83.5}{EI} = \frac{127.9}{EI}$$

$$\alpha_{AA} = \frac{5.0}{EI} \qquad \alpha_{BA} = \frac{2.5}{EI} + 0 = \frac{2.5}{EI}$$

$$\alpha_{AB} = \frac{2.5}{EI} \qquad \alpha_{BB} = \frac{5.0}{EI} + \frac{10.0}{3EI} = \frac{8.33}{EI}$$

Smarter. Stronger. Steel.

General Method

- Rotations on the real beam are shears on the conjugate beam. Thus, from previous 3 slides

Note:
Maxwell's Law of Reciprocal Deflections from Lesson 2

$$\theta_A = \frac{55.4}{EI} \qquad \theta_B = \frac{44.4}{EI} + \frac{83.5}{EI} = \frac{127.9}{EI}$$

$$\alpha_{AA} = \frac{5.0}{EI} \qquad \alpha_{BA} = \frac{2.5}{EI} + 0 = \frac{2.5}{EI}$$

$$\alpha_{AB} = \frac{2.5}{EI} \qquad \alpha_{BB} = \frac{5.0}{EI} + \frac{10.0}{3EI} = \frac{8.33}{EI}$$

Smarter. Stronger. Steel.

General Method

- Consistent Deformation equations

$$\theta_A + M_A\alpha_{AA} + M_B\alpha_{AB} = 0$$
$$\theta_B + M_A\alpha_{BA} + M_B\alpha_{BB} = 0$$

- Substitute results for rotations

$$\frac{55.4}{EI} + M_A\left(\frac{5.0}{EI}\right) + M_B\left(\frac{2.5}{EI}\right) = 0$$
$$\frac{127.9}{EI} + M_A\left(\frac{2.5}{EI}\right) + M_B\left(\frac{8.33}{EI}\right) = 0$$

Smarter. Stronger. Steel.

69

General Method

- Rearrange and solve for the moments

$$5.0M_A + 2.5M_B = -55.4$$
$$2.5M_A + 8.33M_B = -127.9$$

$M_A = -3.91$ ft-kips $\qquad M_B = -14.2$ ft-kips

The negative sign indicates that our assumption that the redundant moments put compression on the top was incorrect. They each put compression on the bottom.

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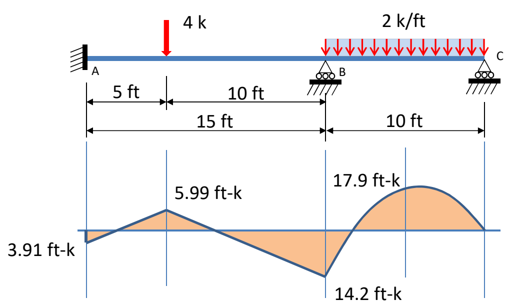
70

General Method

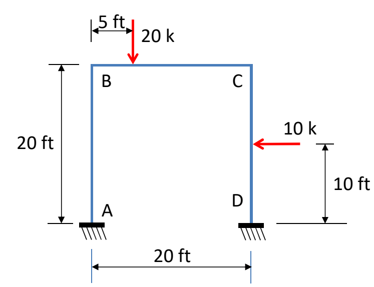


Smarter. Stronger. Steel.

71

General Method

- Apply the general method to a portal frame considering only flexural deformations.



Assume that EI is constant for the entire frame.

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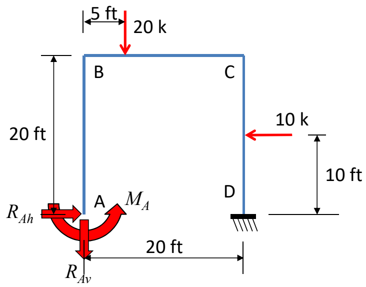
72

General Method

- Select the three reaction components at A as the redundants. Thus, the cut back structure becomes

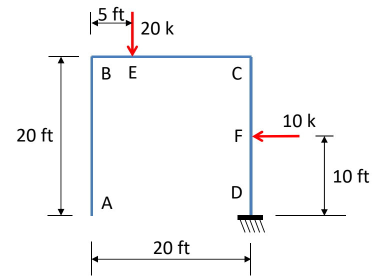
and the unknown redundants are R_{Av}, R_{Ah}, and M_A shown in the positive direction.



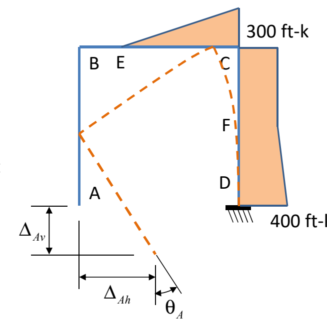
Smarter. Stronger. Steel.

73

General Method



Cut Back Structure

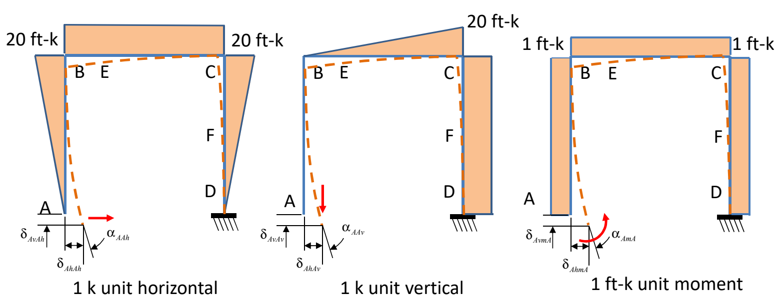


Real Load Moments and deflected shape

Smarter. Stronger. Steel.

74

General Method



Virtual moments and deflected shapes.
All moments are plotted on the tension side.

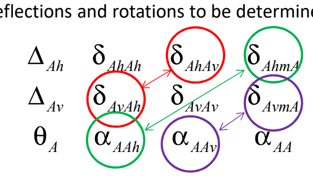
Smarter. Stronger. Steel.

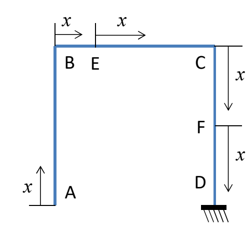
75

General Method

Segment	Range of x	M	m_h	m_v	m_α
AB	0-20	0	x	0	1
BE	0-5	0	20	x	1
EC	0-15	$20x$	20	$5+x$	1
CF	0-10	300	$20-x$	20	1
FD	0-10	$300+10x$	$10-x$	20	1

Deflections and rotations to be determined





Smarter. Stronger. Steel.

76

General Method

Calculate displacements and rotations.

$$\Delta_{Ah} = \frac{106,667}{EI} \qquad \Delta_{Av} = \frac{163,750}{EI} \qquad \theta_{A} = \frac{8,750}{EI}$$

$$\delta_{AhAh} = \frac{13,333}{EI} \qquad \delta_{AvAh} = \frac{8,000}{EI} \qquad \alpha_{AAh} = \frac{800}{EI}$$

$$\delta_{AhAv} = \frac{8,000}{EI} \qquad \delta_{AvAv} = \frac{10,667}{EI} \qquad \alpha_{AAv} = \frac{600}{EI}$$

$$\delta_{AhmA} = \frac{800}{EI} \qquad \delta_{AvmA} = \frac{600}{EI} \qquad \alpha_{AmA} = \frac{60}{EI}$$

Smarter. Stronger. Steel.

77

General Method

- Consistent deformation equations

$$\Delta_{Ah} + R_{Ah}\delta_{AhAh} + R_{Av}\delta_{AhAv} + M_{A}\delta_{AhmA} = 0$$
$$\Delta_{Av} + R_{Ah}\delta_{AvAh} + R_{Av}\delta_{AvAv} + M_{A}\delta_{AvmA} = 0$$
$$\theta_{A} + R_{Ah}\alpha_{AAh} + R_{Av}\alpha_{AAv} + M_{A}\alpha_{AmA} = 0$$

Smarter. Stronger. Steel.

78

General Method

- Substitute for the known displacements

$$\frac{106,667}{EI} + R_{Ah}\left(\frac{13,333}{EI}\right) + R_{Av}\left(\frac{8,000}{EI}\right) + M_{A}\left(\frac{800}{EI}\right) = 0$$
$$\frac{163,750}{EI} + R_{Ah}\left(\frac{8,000}{EI}\right) + R_{Av}\left(\frac{10,667}{EI}\right) + M_{A}\left(\frac{600}{EI}\right) = 0$$
$$\frac{8,750}{EI} + R_{Ah}\left(\frac{800}{EI}\right) + R_{Av}\left(\frac{600}{EI}\right) + M_{A}\left(\frac{60}{EI}\right) = 0$$

Your assignment, solve the 3 simultaneous equations.

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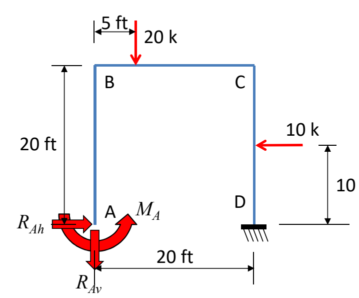
79

General Method

- Redundants

$R_{Ah} = 3.75$ kips
$R_{Av} = -16.3$ kips
$M_{A} = -32.5$ ft-kips

The assumed direction for the horizontal reaction was correct, the other two were incorrect.



Assumed redundant directions

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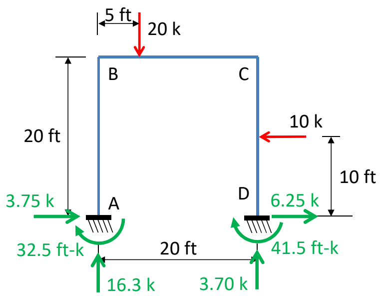
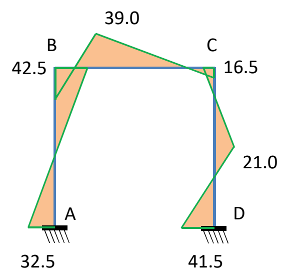
80

General Method



Final Reactions



Final Moment Diagram (ft-k) on the compression side

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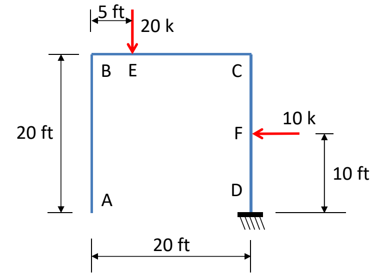
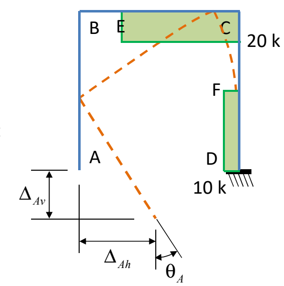
81

General Method

- Now consider the influence of shear deformation



Cut Back Structure

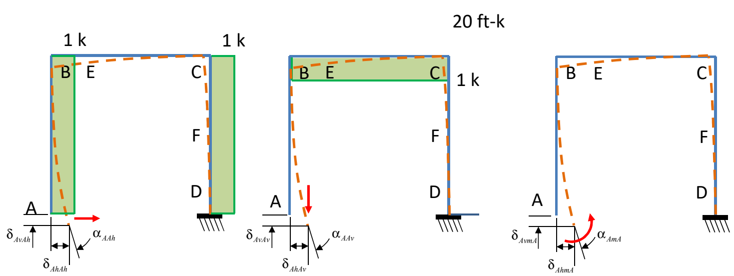


Real Load Shears and deflected shape

Smarter. Stronger. Steel.

82

General Method



1 k unit horizontal

1 k unit vertical

1 ft-k unit moment

Virtual shears and deflected shapes.

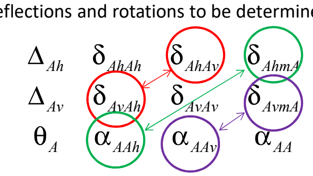
Smarter. Stronger. Steel.

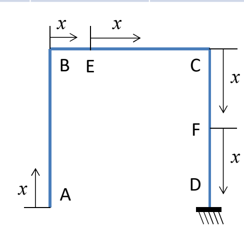
83

General Method

Segment	*Range of x*	*V*	v_h	v_v	v_α
AB	0-20	0	1	0	0
BE	0-5	0	0	1	0
EC	0-15	20	0	1	0
CF	0-10	0	1	0	0
FD	0-10	-10	1	0	0

Deflections and rotations to be determined





Smarter. Stronger. Steel.

84

General Method

Calculate displacements and rotations.

$$\Delta_{Ah} = \frac{-100}{AG} \qquad \Delta_{Av} = \frac{300}{AG} \qquad \theta_A = 0$$

$$\delta_{AhAh} = \frac{40}{AG} \qquad \delta_{AvAh} = 0 \qquad \alpha_{AAh} = 0$$

$$\delta_{AhAv} = 0 \qquad \delta_{AvAv} = \frac{20}{AG} \qquad \alpha_{AAv} = 0$$

$$\delta_{AhmA} = 0 \qquad \delta_{AvmA} = 0 \qquad \alpha_{AmA} = 0$$

Smarter. Stronger. Steel.

General Method

- Consistent deformation equations

$$\Delta_{Ah} + R_{Ah}\delta_{AhAh} + R_{Av}\delta_{AhAv} + M_A\delta_{AhmA} = 0$$
$$\Delta_{Av} + R_{Ah}\delta_{AvAh} + R_{Av}\delta_{AvAv} + M_A\delta_{AvmA} = 0$$
$$\theta_A + R_{Ah}\alpha_{AAh} + R_{Av}\alpha_{AAv} + M_A\alpha_{AmA} = 0$$

Smarter. Stronger. Steel.

General Method

- Add the shear contributions to the flexural already considered.

$$\frac{106,667}{EI} - \frac{100}{AG} + R_{Ah}\left(\frac{13,333}{EI} + \frac{40}{AG}\right) + R_{Av}\left(\frac{8,000}{EI}\right) + M_A\left(\frac{800}{EI}\right) = 0$$
$$\frac{163,750}{EI} + \frac{300}{AG} + R_{Ah}\left(\frac{8,000}{EI}\right) + R_{Av}\left(\frac{10,667}{EI} + \frac{20}{AG}\right) + M_A\left(\frac{600}{EI}\right) = 0$$
$$\frac{8,750}{EI} + R_{Ah}\left(\frac{800}{EI}\right) + R_{Av}\left(\frac{600}{EI}\right) + M_A\left(\frac{60}{EI}\right) = 0$$

To solve these simultaneous equations, we must know the values of I and A for each member. Remember, we used the shape factor, 1.0, for shear in a wide flange.

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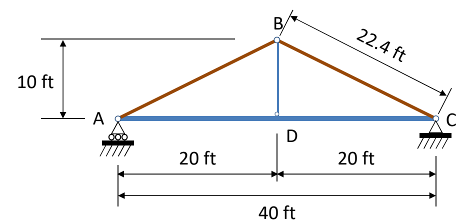
General Method

- Consider the King Post truss



Member ADC, W8x10
$I = 30.8$ in.^{4}
$A = 2.96$ in.^{2}
Members AB, BC
$A = 2.0$ in.^{2}
Member BD
$A = 0.5$ in.^{2}

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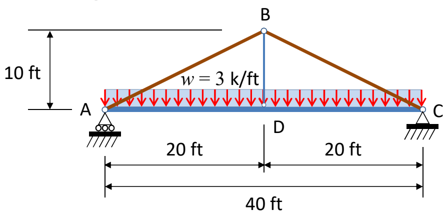
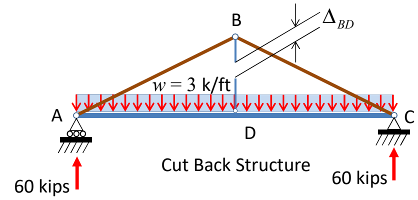
General Method

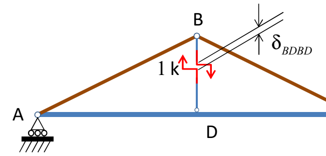
If the member ADC is continuous at node D, the structure is indeterminate internally to the first degree. There are various ways to cut back the structure. The member BD could be cut or a hinge could be inserted at D in member ADC.



For our solution, we will cut member BD.

General Method





Cut Back with unit redundant

Axial forces, all members

$S = 0$

Bending moments

$M_{A \to D} = 60x - \frac{3x^2}{2}$

Axial forces

$u_{BD} = 1.0$

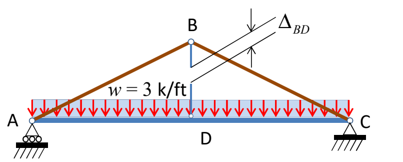
$u_{AB} = u_{BC} = -1.12$

$u_{AD} = u_{DC} = +1.0$

Bending moments

$m_{A \to D} = -0.5x$

General Method



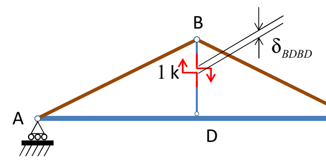
Cut Back Structure

$$\Delta_{BD} = \int \frac{m_x M_x}{EI_{ADC}} dx = 2\int_0^{20} \frac{(-0.5x)(60x - 1.5x^2)}{EI_{ADC}} dx = \frac{-100,000}{EI_{ADC}}$$

General Method

$$\delta_{BDBD} = 2\int_0^{20} \frac{(-0.5x)^2}{EI_{ADC}} dx + \frac{1^2(10)}{EA_{BD}} + \frac{2(-1.12)^2(22.4)}{EA_{AB}} + \frac{2(+1.0)^2(20)}{EA_{ADC}}$$

$$= \frac{1,333}{EI_{ADC}} + \frac{10}{EA_{BD}} + \frac{56.2}{EA_{AB}} + \frac{40}{EA_{ADC}}$$



Cut Back with unit redundant

General Method

- Consistent deformation equation

$$\Delta_{BD} + T_{BD}\delta_{BDBD} = 0$$

- Substitute for deformations and solve for the force, T_{BD}.

$$T_{BD} = \frac{-\Delta_{BD}}{\delta_{BDBD}} = \frac{100,000/EI_{ADC}}{1,333/EI_{ADC} + 10/EA_{BD} + 56.2/EA_{AB} + 40/EA_{ADC}}$$

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93

General Method

- Before any conclusions can be drawn, the member properties must be inserted and the equation corrected for units

$$T_{BD} = \frac{-\Delta_{BD}}{\delta_{BDBD}} = \frac{100,000(1728)/E(30.8)}{1,333(1728)/E(30.8) + 10(12)/E(0.5) + 56.2(12)/E(2.0) + 40(12)/E(2.96)}$$

$$T_{BD} = \frac{-\Delta_{BD}}{\delta_{BDBD}} = \frac{5,610,389/E}{74,786/E + 240/E + 337/E + 162/E}$$

$$T_{BD} = 74.3 \text{ k}$$

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94

General Method

- Questions to consider
  - If the area of all members was increased to infinity, the system would be axially very stiff.
    - How would the member force change?
  - If the bending stiffness of the beam was ignored, what type of behavior would this structure exhibit?

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Polling Question

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Polling Question

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- Section C1 requires that all deformations that contribute to the displacement of the structure be considered.
  - For our last example, it is clear that ignoring either axial or flexural deformations will have an impact on the resulting force distribution.
  - The impact of each type of deformation on the final results will vary as a function of the specific structure and its member stiffness.

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Summary

- Illustrated how to use the conjugate beam to directly determine redundant moments.
- Developed the general method, also know as consistent deformations.
- Applied the general method to several types of structures.
- Illustrated how structural behavior is altered as member stiffnesses are altered.

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Lesson 6

- Indeterminate Structures by Slope Deflection
  - Develop the method of slope deflection.
  - Develop equations for fixed end moments.
  - Analyze continuous beams using the slope deflection method.
  - Apply the method to frames exhibiting sway.
  - Compare the slope deflection method to the modern stiffness method.

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100

Thank You

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101

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8-Session Registrants

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Reasons for quiz:

- EEU – must take all quizzes and final to receive EEU
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- REINFORCEMENT – Reinforce what you learned tonight. Get more out of the course.

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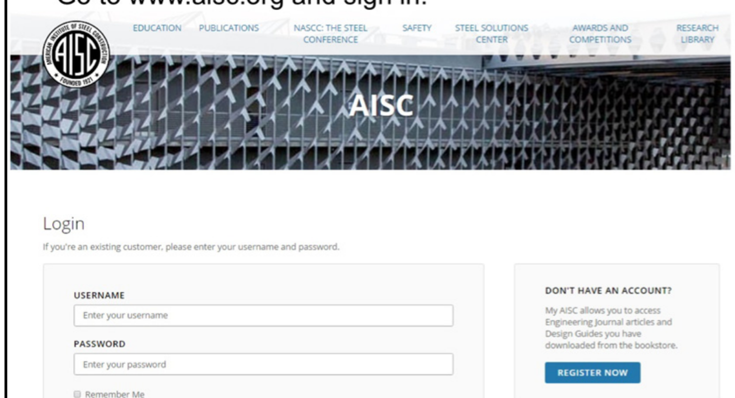

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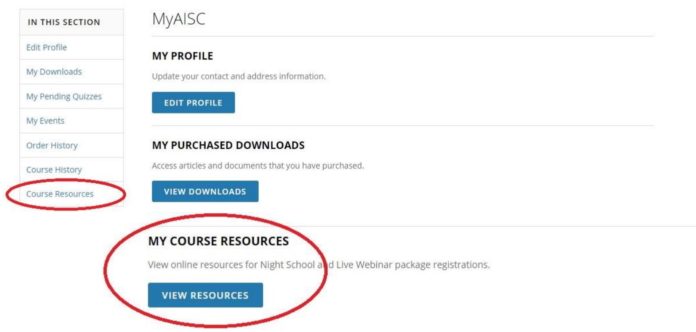

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