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Classical Methods of Structural Analysis
Louis F. Geschwindner




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Session Description

20.6 Indeterminate Structures by Slope Deflection

July 22, 2019

This lesson will continue the analysis of indeterminate structures. Analysis of indeterminate structures by the slope-deflection method will be developed using the principles previously developed for deflection calculations. Sidesway and non-sidesway will be presented and discussed. Approaches for modeling structures will be illustrated, including continuous beams and multi-story sway- and nonsway-frames.



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Learning Objectives:

- Explain the analysis of indeterminate structures.
- Develop the slope-deflection method for analysis of indeterminate structures.
- Analyze continuous beams using the slope deflection method.
- Describe how to apply the method to frames exhibiting sway.
- Compare the slope deflection method to the modern stiffness method.



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Night School 20 Classical Methods of Structural Analysis

Session 6: Indeterminate Structures by Slope Deflection
July 22, 2019



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Classical Methods of Structural Analysis: How we did it before computers

Night School 20

Lesson 6

Indeterminate Structures by Slope
Deflection



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Lesson 6

Indeterminate Structures by Slope Deflection

- Develop the absolute stiffness and carry-over factor for flexural members.
- Derive the method of slope deflection.
- Consider additional equations for fixed end moments.
- Analyze continuous beams using the slope deflection method.
- Consider support settlements.
- Apply the method to frames exhibiting sway.
- Determine the effect of temperature change on a beam.
- Compare the slope deflection method to the modern stiffness method.



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Slope Deflection

- A method presented in 1880 by Heinrich Manderla for the solution of secondary stresses in articulated structures was somewhat similar to slope deflection and may have lead to its development.
- In 1892, Otto Mohr presented an improved method which is what is now called slope deflection.



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Slope Deflection

- Axel Bendixen published a book in 1914 that presented the method in greater detail.
- Then, a year later, in 1915 the person generally given credit for the method, G. A. Maney, published his development of the method.



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Slope Deflection

- Advantages of slope deflection:
 - “Most **computers** will find the formulation of the equations by this process (*slope deflection*) much simpler than that required for elastic equations (*general method*)” Parcel and Moorman, *Analysis of Statically Indeterminate Structures*, 1955.
 - 1. Reduced number of equations.
 - 2. Easy equations to set up.
 - 3. Good for simple frames with lots of fixed joints.



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The computers referenced here were people, not machines. Remember the movie, *Hidden Figures*.

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Slope Deflection

- Before the actual slope deflection method can be developed, two properties of flexural members will need to be developed.
 - **Absolute stiffness**: The moment required to produce a unit rotation at the simply supported end of a beam when the other end is fixed.
 - **Carry-over factor**: The relationship between the moment applied to the simply supported end of the beam and the resisting moment developed at the other end.

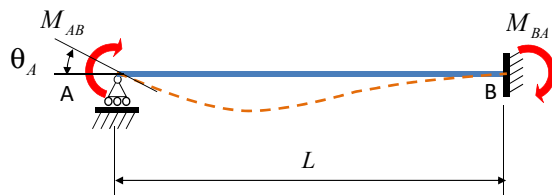


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Slope Deflection

- Sign convention



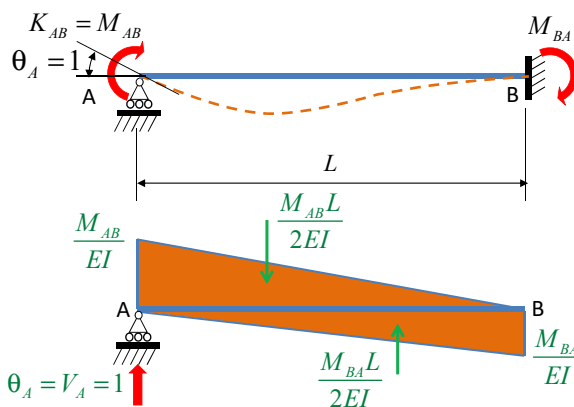
Clockwise moment on the end of the member is positive.
 Clockwise rotation of a joint is positive.



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Slope Deflection

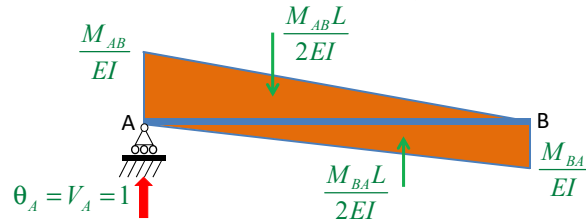
- Absolute stiffness and Carry-over factor



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Slope Deflection

- Absolute stiffness and Carry-over factor



Sum of the moments about A $\frac{M_{AB}L}{2EI} \left(\frac{L}{3} \right) - \frac{M_{BA}L}{2EI} \left(\frac{2L}{3} \right) = 0$

Sum of the vertical forces $\frac{M_{AB}L}{2EI} - \frac{M_{BA}L}{2EI} = 1$



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Slope Deflection

- Solve for the moments

From the first equation $\frac{M_{AB}L}{2EI} \left(\frac{L}{3} \right) - \frac{M_{BA}L}{2EI} \left(\frac{2L}{3} \right) = 0$
 $\rightarrow M_{BA} = \frac{M_{AB}}{2}$

From the second equation $\frac{M_{AB}L}{2EI} - \left(\frac{M_{AB}}{2} \right) \frac{L}{2EI} = 1$
 $\rightarrow M_{AB} = \frac{4EI}{L}$



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Slope Deflection

- By definition, the absolute stiffness of the beam, K_{AB} , is

$$K_{AB} = M_{AB} = \frac{4EI}{L}$$

- The carry-over factor, C_{AB} , is

$$M_{BA} = C_{AB}M_{AB} = \frac{M_{AB}}{2}$$

$$C_{AB} = \frac{1}{2}$$

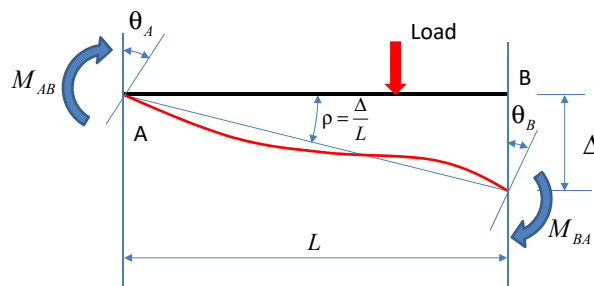


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Slope Deflection

- Develop the method by looking at the impact of load, deflection, and rotation on a bending member. In its final configuration, we have



All variables shown positive.

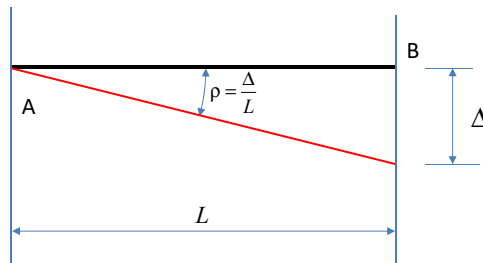


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Slope Deflection

1. Consider the unloaded and unrestrained member rotated through ρ .

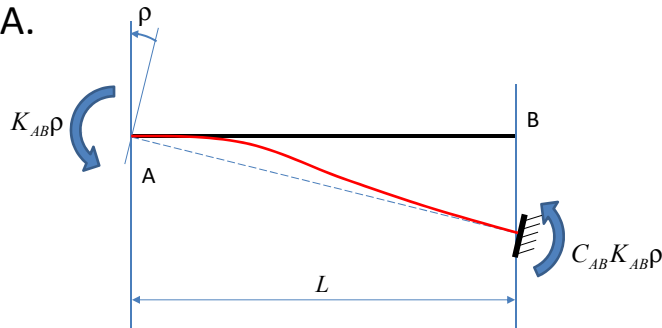


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Slope Deflection

2. Fix end B in this position and rotate end A back, through ρ , so the member is horizontal at A.

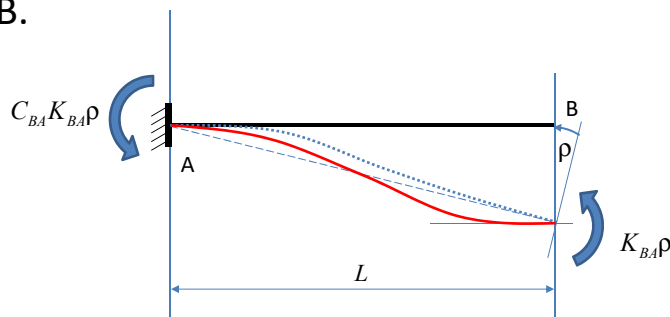


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Slope Deflection

- Fix the member at A and rotate end B through ρ so that the member is horizontal at B.

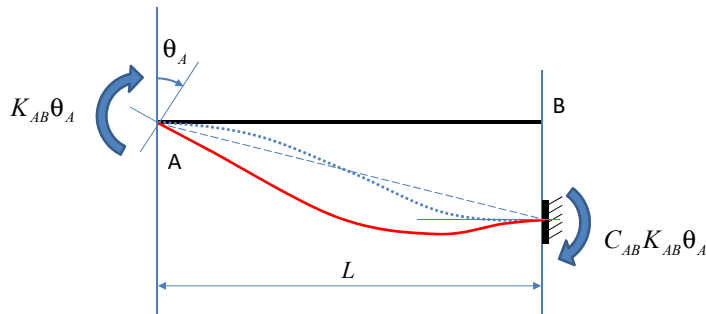


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Slope Deflection

- Fix end B and rotate end A through θ_A

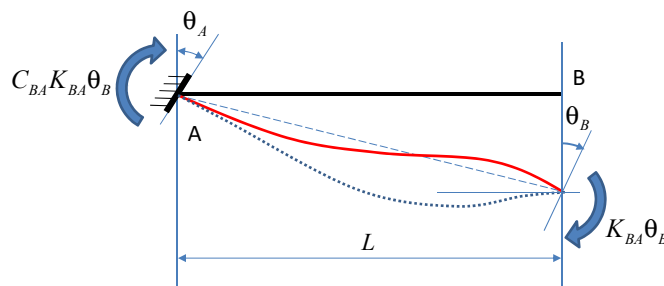


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Slope Deflection

5. Fix end A and rotate end B through θ_B

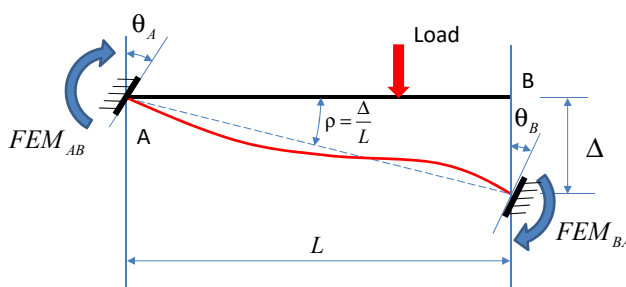


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Slope Deflection

6. Fix both ends and apply the load.



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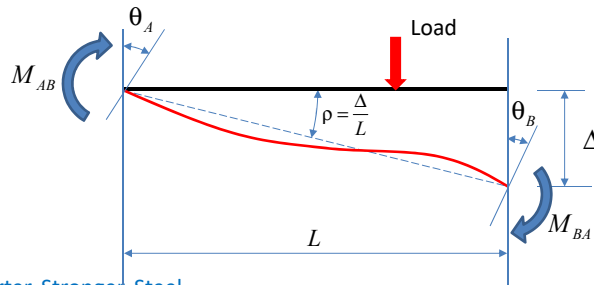
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Slope Deflection

7. Add the moments for each step at A and at B.

$$M_{AB} = -K_{AB}\rho - C_{BA}K_{BA}\rho + K_{AB}\theta_A + C_{BA}K_{BA}\theta_B + FEM_{AB}$$

$$M_{BA} = -C_{AB}K_{AB}\rho - K_{BA}\rho + C_{AB}K_{AB}\theta_A + K_{BA}\theta_B + FEM_{BA}$$



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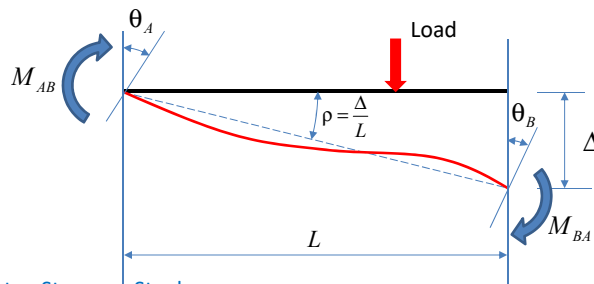
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Slope Deflection

8. To generalize, replace the subscripts with N for the end where the moment is being determined (near) and F for the other end (far).

$$M_N = -K_N\rho - C_F K_F \rho + K_N\theta_N + C_F K_F \theta_F + FEM_N$$

$$M_N = -C_F K_F \rho - K_N\rho + C_F K_F \theta_F + K_N\theta_N + FEM_N$$



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Slope Deflection

- As can be seen, the equations are the same, reorganize and write as a single slope deflection equation.

$$M_N = K_N \theta_N + C_F K_F \theta_F - (K_N + C_F K_F) \rho + FEM_N$$

- For a prismatic member, considering symmetry, as we saw earlier when we derived absolute stiffness and carry-over factor. (slide 19)

$$K_N = K_F = \frac{4EI}{L} \quad C_N = C_F = \frac{1}{2}$$



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Slope Deflection

- Substitute for stiffness and carry-over factor and simplify

$$M_N = \frac{4EI}{L} \theta_N + \frac{2EI}{L} \theta_F - \frac{6EI}{L} \rho + FEM_N$$

This is the slope deflection equation for the moment on the end of a prismatic flexural member.

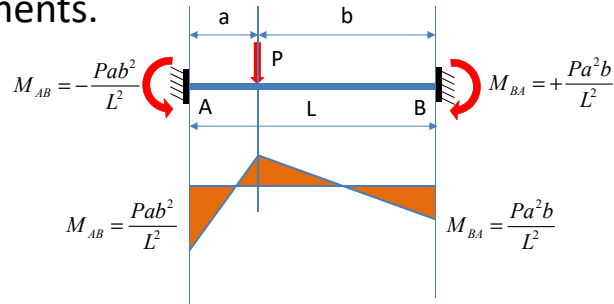


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Slope Deflection

- Lesson 4 illustrated the use of the conjugate beam method for determining fixed end moments.

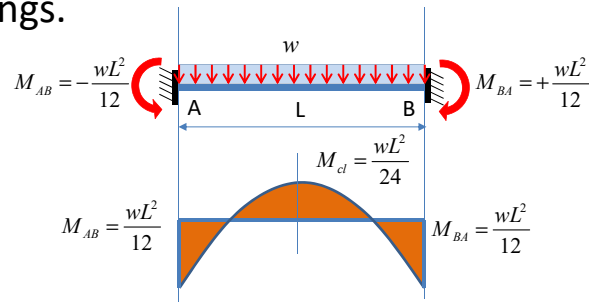


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Slope Deflection

- The same approach is quite useful for determining fixed end moments for other loadings.

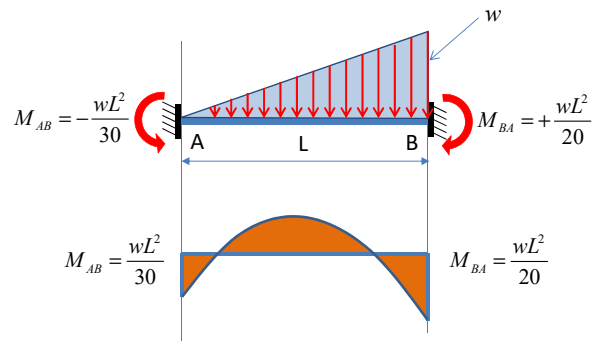


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Slope Deflection

- Triangular loading

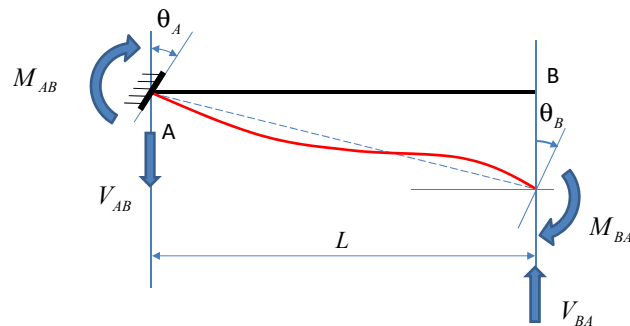


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Slope Deflection

Consider the positive shears that correspond to the final positive end moments.

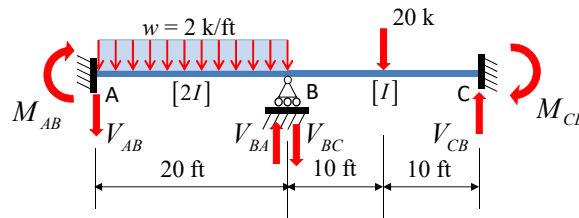


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Slope Deflection

- Using the slope deflection method, determine the member end moments and plot the final moment diagram.

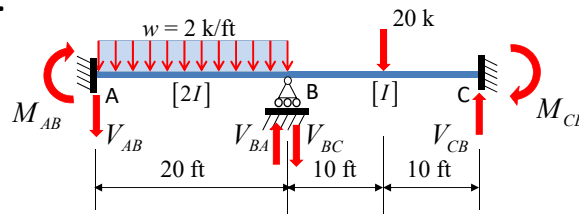


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Slope Deflection

- If we were solving this beam by consistent deformations, there would be 4 redundants to remove. Thus, 4 simultaneous equations to solve.

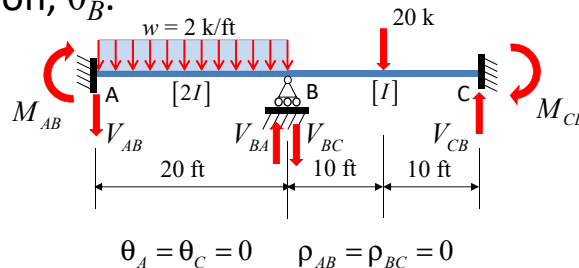


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Slope Deflection

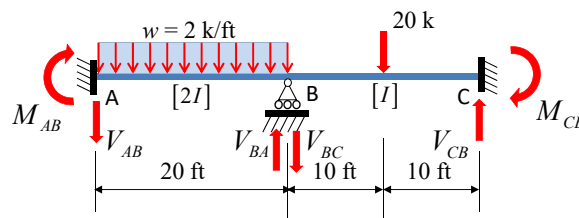
- With slope deflection, we are concerned with rotation of the members and rotation of the joints. Here, we only have one unknown rotation, θ_B .



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Slope Deflection



Fixed end moments

$$FEM_{AB} = -\frac{wL^2}{12} = -\frac{2(20)^2}{12} = -66.7 \text{ ft-kips}$$

$$FEM_{BC} = -\frac{PL}{8} = -\frac{20(20)}{8} = -50.0 \text{ ft-kips}$$

$$FEM_{BA} = \frac{wL^2}{12} = \frac{2(20)^2}{12} = 66.7 \text{ ft-kips}$$

$$FEM_{CB} = \frac{PL}{8} = \frac{20(20)}{8} = 50.0 \text{ ft-kips}$$



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Slope Deflection

$$M_N = \frac{4EI}{L}\theta_N + \frac{2EI}{L}\theta_F - \frac{6EI}{L}\rho + FEM_N$$

$$M_{AB} = \frac{2E(2I)}{20}\theta_B - 66.7$$

$$M_{BA} = \frac{4E(2I)}{20}\theta_B + 66.7$$

$$M_{BC} = \frac{4E(I)}{20}\theta_B - 50.0$$

$$M_{CB} = \frac{2E(I)}{20}\theta_B + 50.0$$



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Slope Deflection

- The only unknown rotation is θ_B . In order to determine this rotation, an equilibrium equation “parallel to” or “consistent with” this unknown must be found.
- In this instance, the equilibrium equation is sum of the moments at B must be equal to zero.

$$\sum M_B = 0 = M_{BA} + M_{BC}$$



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Slope Deflection

Equilibrium at joint B

$$\sum M_B = 0 = M_{BA} + M_{BC}$$

Note that positive moment on the joint is counterclockwise



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Slope Deflection


- Substituting the member end moment equations yields (slide 39)

$$M_{BA} + M_{BC} = \left[\frac{4E(2I)}{20} \theta_B + 66.7 \right] + \left[\frac{4E(I)}{20} \theta_B - 50.0 \right] = 0$$

- This equilibrium equation can be solved for

$$\frac{12E(I)}{20} \theta_B = -16.7$$

$$EI\theta_B = -27.8$$



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Slope Deflection

- Substitute for $EI\theta_B$ and determine the member end moments. (slide 39)

$$M_{AB} = \frac{2E(2I)}{20}\theta_B - 66.7 = \frac{4(-27.8)}{20} - 66.7 = -72.3$$

$$M_{BA} = \frac{4E(2I)}{20}\theta_B + 66.7 = \frac{8(-27.8)}{20} + 66.7 = +55.6$$

$$M_{BC} = \frac{4E(I)}{20}\theta_B - 50.0 = \frac{4(-27.8)}{20} - 50.0 = -55.6$$

$$M_{CB} = \frac{2E(I)}{20}\theta_B + 50.0 = \frac{2(-27.8)}{20} + 50.0 = +47.2$$

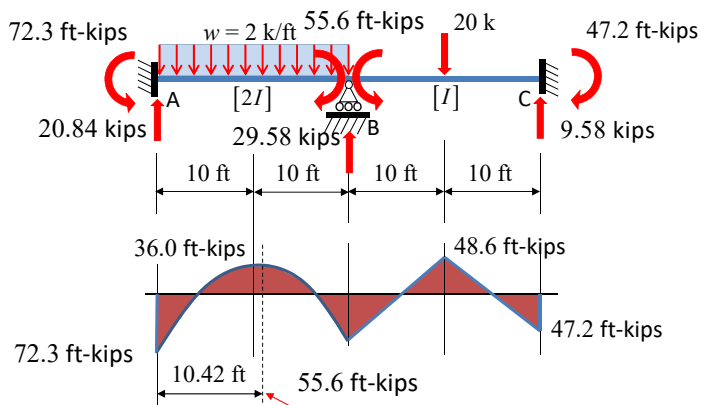


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Slope Deflection

- Solution



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Slope Deflection

- Some thoughts on slope deflection.
 - We do not need the actual stiffness of the members to determine member end moments, only the relative stiffness.
 - With relative stiffness, we are able to determine the direction of the unknown rotations but not the magnitude.
 - The sign convention used is clockwise on the end of the member is positive.



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Slope Deflection

- Determine the rotation in radians at joint B.
 - From solution of the equilibrium equation we have

$$EI\theta_B = -27.8$$

- But what are the units? Note that from our member end moment equation we used fixed end moments with units of ft-kips.

$$M_{BC} = \frac{4E(I)}{20}\theta_B - 50.0$$



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Slope Deflection

- For units to be consistent the other terms must also be in ft-kips. Thus $EI\theta_B$ must be in units of ft²-kips so that when we divide by span in feet, as we did, the term has units of ft-kips.
- If member BC is a W16x31, $I = 375 \text{ in.}^4$ Thus,

$$EI\theta_B = -27.8 \text{ ft}^2\text{-kips}$$

$$\theta_B = \frac{-27.8(144)}{29,000(375)} = -0.000368 \text{ rad}$$



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Slope Deflection

- Some questions to consider.
 - What would happen to the **magnitude** of the rotation at B if the moment of inertia of member AB were made the same as member BC?
 - How would you expect the moment at B to respond?
 - How would you expect the moment at C to respond?



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Polling Question



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Polling Question



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Polling Question

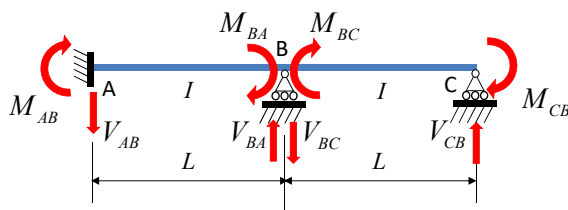


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Slope Deflection

- Consider a prismatic beam where the support at the end is a pin.



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Slope Deflection

- For member BC, write the slope deflection equations assuming that the member may rotate.

$$M_{BC} = \frac{4EI}{L}\theta_B + \frac{2EI}{L}\theta_C - \frac{6EI}{L}\rho + FEM_{BC}$$

$$M_{CB} = \frac{4EI}{L}\theta_C + \frac{2EI}{L}\theta_B - \frac{6EI}{L}\rho + FEM_{CB}$$



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Slope Deflection

- Since we already know that $M_{CB} = 0$, we can use that equation to eliminate an unknown.

$$M_{CB} = \frac{4EI}{L}\theta_C + \frac{2EI}{L}\theta_B - \frac{6EI}{L}\rho + FEM_{CB} = 0$$

$$\frac{EI}{L}\theta_C = \left[-FEM_{CB} - \frac{2EI}{L}\theta_B + \frac{6EI}{L}\rho \right] \frac{1}{4}$$



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Slope Deflection

- If we substitute this into the equation for the moment M_{BC} we will be able to eliminate θ_C from our unknowns.

$$M_{BC} = \frac{4EI}{L}\theta_B + 2\left[-FEM_{CB} - \frac{2EI}{L}\theta_B + \frac{6EI}{L}\rho\right]\left(\frac{1}{4}\right) - \frac{6EI}{L}\rho + FEM_{BC}$$

- Which simplifies to

$$M_{BC} = \frac{3EI}{L}\theta_B - \frac{3EI}{L}\rho + \left[FEM_{BC} - \frac{FEM_{CB}}{2}\right]$$



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Slope Deflection

- In the N (near) and F (far) format this becomes

$$M_N = \frac{3EI}{L}\theta_N - \frac{3EI}{L}\rho + \left[FEM_N - \frac{FEM_F}{2}\right]$$

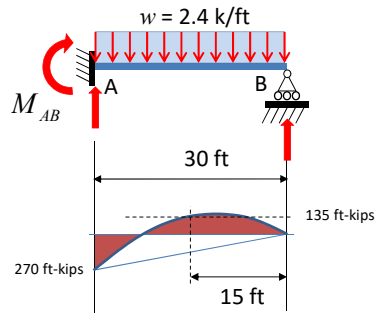


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Slope Deflection

- The simplest case that this may be applied to is the propped cantilever.



$$FEM_{AB} = -\frac{wL^2}{12} = -\frac{2.4(30)^2}{12} = -180 \text{ ft-kips}$$

$$FEM_{BA} = \frac{wL^2}{12} = \frac{2.4(30)^2}{12} = 180 \text{ ft-kips}$$

Since we know that $\theta_A = \rho = 0$,

$$M_{AB} = \frac{3EI}{L}\theta_A - \frac{3EI}{L}\rho + \left[FEM_{AB} - \frac{FEM_{BA}}{2} \right]$$

$$= \left[-180 - \left(\frac{180}{2} \right) \right] = -270 \text{ ft-kips}$$

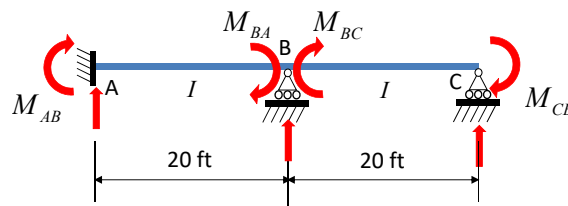


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Slope Deflection

- For the W21x83 two span beam shown, what would the moment be if the support at B were to settle 1.0 in.?

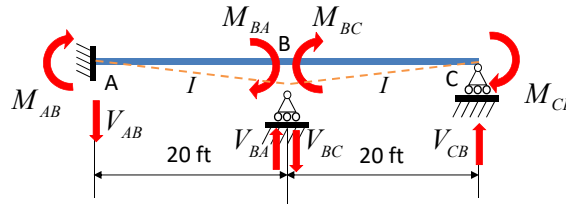


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Slope Deflection

- Settlement of joint B



$$\rho_{AB} = \frac{\Delta}{L} = \frac{1.0}{20(12)} = 0.00417 \text{ rad}$$

$$\rho_{BC} = \frac{\Delta}{L} = -\frac{1.0}{20(12)} = -0.00417 \text{ rad}$$



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Slope Deflection

- Member end moment equations

$$M_{AB} = \frac{2EI}{L} \theta_B - \frac{6EI}{L} (0.00417)$$

$$M_{BA} = \frac{4EI}{L} \theta_B - \frac{6EI}{L} (0.00417)$$

$$M_{BC} = \frac{3EI}{L} \theta_B - \frac{3EI}{L} (-0.00417)$$



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Slope Deflection

- Substituting for EI *EI has units of k-in.² so need to divide by 144 to convert to ft²*

$$\frac{6EI}{L}(0.00417) = \frac{6(29,000)(1830)}{20(144)}(0.00417) = 461 \text{ ft-kips}$$

This is essentially a fixed end moment for settlement.

$$M_{AB} = \frac{2EI}{L}\theta_B - \frac{6EI}{L}(0.00417) = \frac{2EI}{L}\theta_B - 461$$

$$M_{BA} = \frac{4EI}{L}\theta_B - \frac{6EI}{L}(0.00417) = \frac{4EI}{L}\theta_B - 461$$

$$M_{BC} = \frac{3EI}{L}\theta_B - \frac{3EI}{L}(-0.00417) = \frac{3EI}{L}\theta_B + 231$$



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Slope Deflection

- Sum moments about joint B

$$\sum M_B = M_{BA} + M_{BC} = 0$$

$$\left[\frac{4EI}{L}\theta_B - 461 \right] + \left[\frac{3EI}{L}\theta_B + 231 \right] = 0$$

$$\frac{7EI}{L}\theta_B = 230$$

$$\frac{EI}{L}\theta_B = 32.9$$



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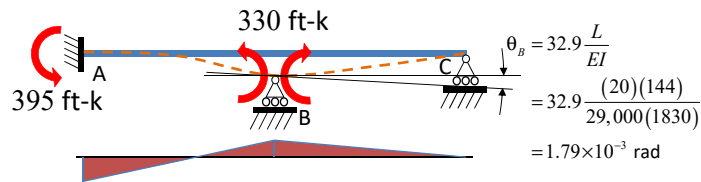
Slope Deflection

- Substituting, determine the member end moments.

$$M_{AB} = \frac{2EI}{L} \theta_B - 461 = 2(32.9) - 461 = -395 \text{ ft-kips}$$

$$M_{BA} = \frac{4EI}{L} \theta_B - 461 = 4(32.9) - 461 = -329 \text{ ft-kips}$$

$$M_{BC} = \frac{3EI}{L} \theta_B + 231 = 3(32.9) + 231 = 330 \text{ ft-kips}$$

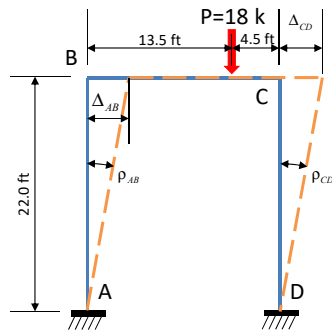


Smarter. Stronger. Steel.

63

Slope Deflection

- Next consider a frame with an unknown amount of side sway. (Which way does it sway?)



Sidesway is shown to the right. This produces a positive member rotation for both members AB and CD.

Considering flexural deformations only and small angle theory,

$$\Delta_{AB} = \Delta_{CD} = \Delta$$

$$\rho_{AB} = \rho_{CD} = \rho = \frac{\Delta}{22}$$

Also,

$$\theta_A = \theta_D = 0$$

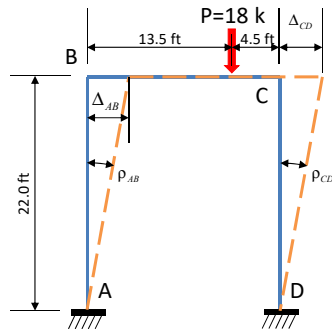


Smarter. Stronger. Steel.

64

Slope Deflection

- Next consider a frame with an unknown amount of side sway.



Only member BC will have fixed end moments.

$$FEM_{BC} = -\frac{18(13.5)(4.5)^2}{(18.0)^2} = -15.2 \text{ ft-k}$$

$$FEM_{CB} = +\frac{18(4.5)(13.5)^2}{(18.0)^2} = +45.6 \text{ ft-k}$$

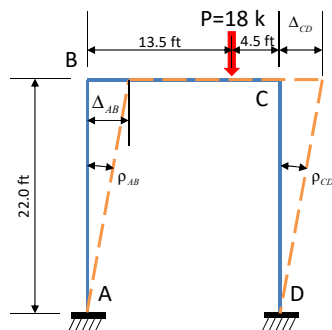


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65

Slope Deflection

- Member end moments



$$M_{AB} = \frac{2EI}{22} \theta_B - \frac{6EI}{22} \rho$$

$$M_{BA} = \frac{4EI}{22} \theta_B - \frac{6EI}{22} \rho$$

$$M_{BC} = \frac{4EI}{18} \theta_B + \frac{2EI}{18} \theta_C - 15.2$$

$$M_{CB} = \frac{4EI}{18} \theta_C + \frac{2EI}{18} \theta_B + 45.6$$

$$M_{CD} = \frac{4EI}{22} \theta_C - \frac{6EI}{22} \rho$$

$$M_{DC} = \frac{2EI}{22} \theta_C - \frac{6EI}{22} \rho$$



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66

Slope Deflection

- Equilibrium Equations at B and C

$$\begin{aligned}\sum M_B &= M_{BA} + M_{BC} = 0 \\ &= \frac{4EI}{22}\theta_B - \frac{6EI}{22}\rho + \frac{4EI}{18}\theta_B + \frac{2EI}{18}\theta_C - 15.2 \\ &= 0.404EI\theta_B + 0.111EI\theta_C - 0.273EI\rho - 15.2 = 0\end{aligned}$$

$$\begin{aligned}\sum M_C &= M_{CB} + M_{CD} = 0 \\ &= \frac{4EI}{18}\theta_C + \frac{2EI}{18}\theta_B + 45.6 + \frac{4EI}{22}\theta_C - \frac{6EI}{22}\rho \\ &= 0.111EI\theta_B + 0.404EI\theta_C - 0.273EI\rho + 45.6 = 0\end{aligned}$$

2 equations, 3 unknowns



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67

Slope Deflection

- A third equilibrium equation is required. The first two were associated with the unknown rotations at B and C. The third must be associated with the unknown sway of the frame.
- To develop this equation, we will look at horizontal equilibrium of the frame.



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Slope Deflection

- Horizontal equilibrium.

$$H_A = \frac{M_{AB} + M_{BA}}{22}$$

$$H_D = \frac{M_{CD} + M_{DC}}{22}$$

$$H_A + H_D = 0$$

$$\frac{M_{AB} + M_{BA}}{22} + \frac{M_{CD} + M_{DC}}{22} = 0$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} = 0$$

Take moments about B and C

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69

Slope Deflection

- Horizontal equilibrium.

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} = 0$$

$$\left[\frac{2EI}{22} \theta_B - \frac{6EI}{22} \rho \right] + \left[\frac{4EI}{22} \theta_B - \frac{6EI}{22} \rho \right] +$$

$$\left[\frac{4EI}{22} \theta_C - \frac{6EI}{22} \rho \right] + \left[\frac{2EI}{22} \theta_C - \frac{6EI}{22} \rho \right] = 0$$

$$0.273EI\theta_B + 0.273EI\theta_C - 1.09EI\rho = 0$$

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70

Slope Deflection

- The resulting three simultaneous equilibrium equations are:

$$0.404EI\theta_B + 0.111EI\theta_C - 0.273EI\rho = 15.2$$

$$0.111EI\theta_B + 0.404EI\theta_C - 0.273EI\rho = -45.6$$

$$0.273EI\theta_B + 0.273EI\theta_C - 1.09EI\rho = 0$$

- The solution:

$$EI\theta_B = 63.6 \quad EI\theta_C = -144. \quad EI\rho = -20.1$$



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Slope Deflection

- The resulting three simultaneous equilibrium equations are:

$$0.404EI\theta_B + 0.111EI\theta_C - 0.273EI\rho = 15.2$$

$$0.111EI\theta_B + 0.404EI\theta_C - 0.273EI\rho = -45.6$$

$$0.273EI\theta_B + 0.273EI\theta_C - 1.09EI\rho = 0$$

- The solution:

$$EI\theta_B = 63.6 \quad EI\theta_C = -144. \quad EI\rho = -20.1$$



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Note the symmetry, if you multiply the last equation by -1. We have seen this before.

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Slope Deflection

- Substituting back into the member end moment equations yields

$$M_{AB} = \frac{2EI}{22} \theta_B - \frac{6EI}{22} \rho = 11.3$$

$$M_{BA} = \frac{4EI}{22} \theta_B - \frac{6EI}{22} \rho = 17.0$$

$$M_{BC} = \frac{4EI}{18} \theta_B + \frac{2EI}{18} \theta_C - 15.2 = -17.1$$

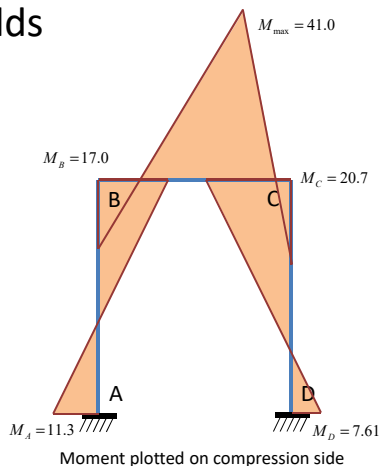
$$M_{CB} = \frac{4EI}{18} \theta_C + \frac{2EI}{18} \theta_B + 45.6 = 20.7$$

$$M_{CD} = \frac{4EI}{22} \theta_C - \frac{6EI}{22} \rho = -20.7$$

$$M_{DC} = \frac{2EI}{22} \theta_C - \frac{6EI}{22} \rho = -7.61$$



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Slope Deflection

- Deflected shape for a W14x22

$$\theta_B = \frac{63.6}{EI} = \frac{63.6(144)}{29,000(199)} = 0.00159 \text{ rad}$$

$$\theta_C = -\frac{144.}{EI} = -\frac{144.(144)}{29,000(199)} = -0.00359 \text{ rad}$$

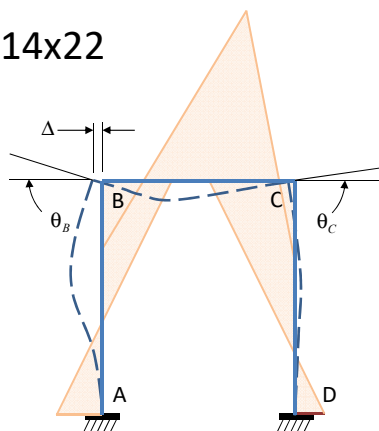
$$\rho = \frac{-20.1}{EI} = \frac{-20.1(144)}{29,000(199)} = -0.000501$$

$$\rho = \frac{\Delta}{L} = \frac{\Delta}{22} \therefore \Delta = \rho(22)$$

$$\Delta = -0.000501(22) = -0.0110 \text{ ft} \rightarrow -0.132 \text{ in.}$$



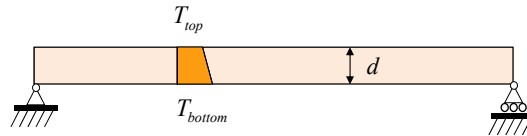
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74

Slope Deflection

- Consider a beam experiencing a temperature gradient from top to bottom.



For a temperature gradient, defined by the temperature difference, $\Delta T = T_{bottom} - T_{top}$, the fixed end moments can be shown to be

$$FEM_{NF} = -\frac{EI\alpha\Delta T}{d}$$

$$FEM_{FN} = \frac{EI\alpha\Delta T}{d}$$

and for steel,

$$\alpha = 6.5 \times 10^{-6} \text{ per degree F}$$

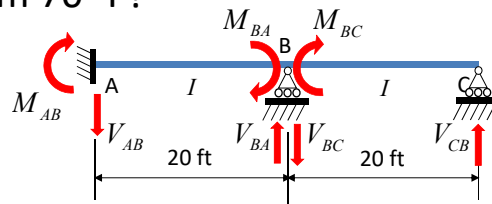


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Slope Deflection

- For the W21x83 two span beam considered earlier, what would the moment be if the temperature on the top were 20 °F and on the bottom 70 °F?



$$\Delta T = T_{bottom} - T_{top} = 50^\circ \text{F}$$



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Slope Deflection

- Fixed end moments Note that beam length has no influence on FEM.

$$FEM_{AB} = FEM_{BC} = -\frac{EI\alpha\Delta T}{d} = -\frac{29,000(1830)(0.0000065)(50)}{21.4(12)} = -67.2 \text{ ft-kips}$$

$$FEM_{BA} = FEM_{CB} = \frac{EI\alpha\Delta T}{d} = \frac{29,000(1830)(0.0000065)(50)}{21.4(12)} = 67.2 \text{ ft-kips}$$

- End moment equations

$$M_{AB} = \frac{2EI}{L}\theta_B - 67.2$$

$$M_{BA} = \frac{4EI}{L}\theta_B + 67.2$$

$$M_{BC} = \frac{3EI}{L}\theta_B + \left(-67.2 - \frac{+67.2}{2}\right)$$



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Slope Deflection

- Summation of moments at B

$$\begin{aligned} M_{BA} + M_{BC} &= \frac{4EI}{L}\theta_B + 67.2 + \frac{3EI}{L}\theta_B + \left(-67.2 - \frac{+67.2}{2}\right) = 0 \\ &= \frac{7EI}{L}\theta_B - 33.6 = 0 \end{aligned}$$

- Solve for the unknown

$$\frac{EI}{L}\theta_B = \frac{33.6}{7} = 4.8$$



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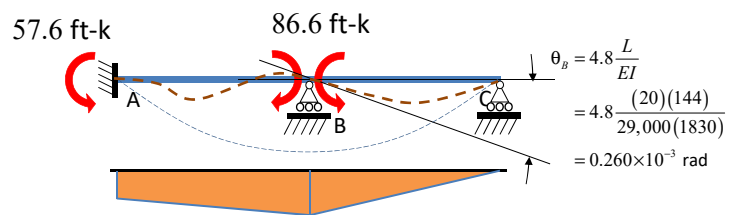
Slope Deflection

- Determine the end moments

$$M_{AB} = \frac{2EI}{L}\theta_B - 67.2 = 2(4.8) - 67.2 = -57.6 \text{ ft-kips}$$

$$M_{BA} = \frac{4EI}{L}\theta_B + 67.2 = 4(4.8) + 67.2 = 86.4 \text{ ft-kips}$$

$$M_{BC} = \frac{3EI}{L}\theta_B + \left(-67.2 - \frac{+67.2}{2}\right) = 3(4.8) - 101 = -86.6 \text{ ft-kips}$$

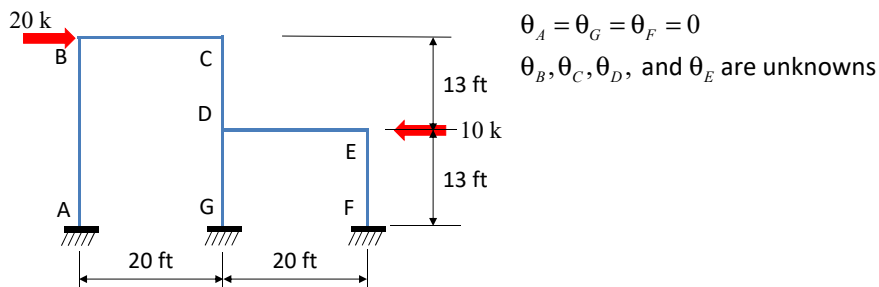


Smarter. Stronger. Steel. Moment diagram just due to the end moments, not considering internal temperature change effect.

79

Slope Deflection

- Determine the unknown joint rotations, θ , and member rotations, ρ , for the frame shown.

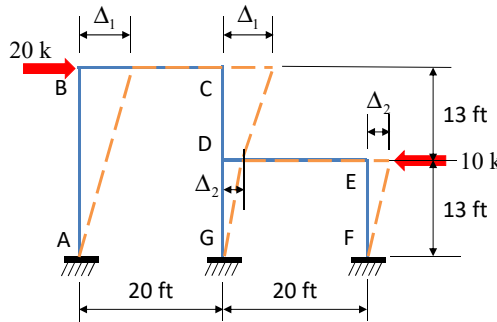


Smarter. Stronger. Steel.

80

Slope Deflection

- Determine the unknown joint rotations, θ , and member rotations, ρ , for the frame shown.



$$\theta_A = \theta_G = \theta_F = 0$$

$\theta_B, \theta_C, \theta_D,$ and θ_E are unknowns

Δ to the right is positive, to the left is negative.

Define

$$\rho_1 = \frac{\Delta_1}{26} \quad \rho_2 = \frac{\Delta_2}{13}$$

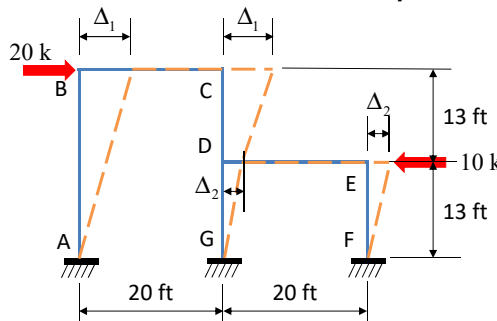


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81

Slope Deflection

- Determine the unknown joint rotations, θ , and member rotations, ρ , for the frame shown.



$$\rho_{AB} = \frac{\Delta_1}{26} = \rho_1$$

$$\rho_{DC} = \frac{\Delta_1 - \Delta_2}{13} = 2\rho_1 - \rho_2$$

$$\rho_{GD} = \frac{\Delta_2}{13} = \rho_2$$

$$\rho_{EF} = \frac{\Delta_2}{13} = \rho_2$$



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82

Slope Deflection

- Joint equilibrium equations

$$\sum M_B = 0 = M_{BA} + M_{BC}$$

$$\sum M_C = 0 = M_{CB} + M_{CD}$$

$$\sum M_D = 0 = M_{DC} + M_{DE} + M_{DG}$$

$$\sum M_E = 0 = M_{ED} + M_{EF}$$

- This would permit determination of 4 unknowns (the 4 θ 's) but we have 2 more unknowns, Δ_1 and Δ_2 .

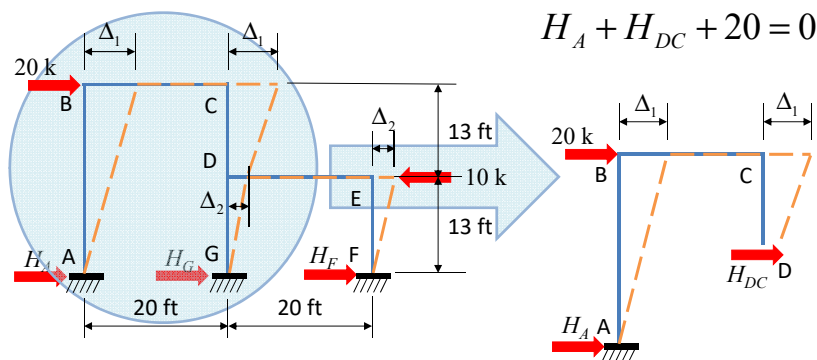


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83

Slope Deflection

- Horizontal equilibrium equations

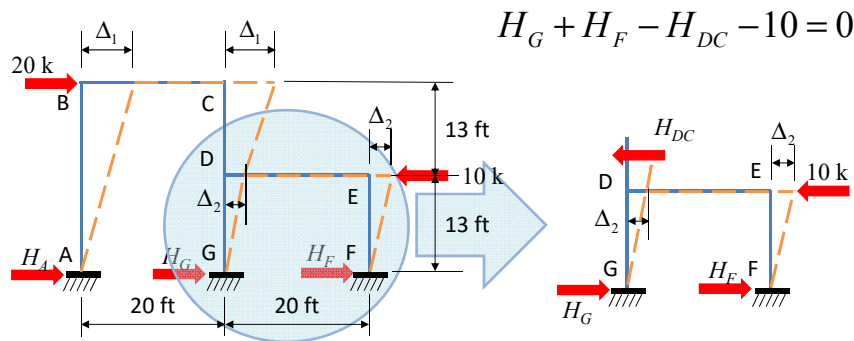


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84

Slope Deflection

- Horizontal equilibrium equations



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85

Slope Deflection

- Relative stiffness: Since E and I are constants and we do not actually need to know the values of E and I to determine moments, we can introduce the relative stiffness.

$$K_{AB} = \frac{EI}{26} = K'$$

$$K_{BC} = K_{DE} = \frac{EI}{20} = 1.3K'$$

$$K_{CD} = K_{DG} = K_{EF} = \frac{EI}{13} = 2K'$$



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86

Slope Deflection

- Member end moments

$$M_{AB} = 2K'\theta_B - 6K'\rho_1$$

$$M_{BA} = 4K'\theta_B - 6K'\rho_1$$

$$M_{BC} = 4(1.3K')\theta_B + 2(1.3K')\theta_C$$

$$M_{CB} = 4(1.3K')\theta_C + 2(1.3K')\theta_B$$

$$M_{CD} = 4(2K')\theta_C + 2(2K')\theta_D - 6(2K')(2\rho_1 - \rho_2)$$

$$M_{DC} = 4(2K')\theta_D + 2(2K')\theta_C - 6(2K')(2\rho_1 - \rho_2)$$



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87

Slope Deflection

- Member end moments

$$M_{DG} = 4(2K')\theta_D - 6(2K')\rho_2$$

$$M_{GD} = 2(2K')\theta_D - 6(2K')\rho_2$$

$$M_{DE} = 4(1.3K')\theta_D + 2(1.3K')\theta_E$$

$$M_{ED} = 4(1.3K')\theta_E + 2(1.3K')\theta_D$$

$$M_{EF} = 4(2K')\theta_E - 6(2K')\rho_2$$

$$M_{FE} = 2(2K')\theta_E - 6(2K')\rho_2$$



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88

Slope Deflection

- Joint equilibrium equations

$$\begin{aligned} \sum M_B = 0 &= M_{BA} + M_{BC} = 4K'\theta_B - 6K'\rho_1 + 4(1.3K')\theta_B + 2(1.3K')\theta_C \\ \sum M_C = 0 &= M_{CB} + M_{CD} = 4(1.3K')\theta_C + 2(1.3K')\theta_B + 4(2K')\theta_C + 2(2K')\theta_D \\ &\quad - 6(2K')(2\rho_1 - \rho_2) \\ \sum M_D = 0 &= M_{DC} + M_{DE} + M_{DG} = 4(2K')\theta_D + 2(2K')\theta_C \\ &\quad - 6(2K')(2\rho_1 - \rho_2) + 4(1.3K')\theta_D + 2(1.3K')\theta_E + 4(2K')\theta_D - 6(2K')\rho_2 \\ \sum M_E = 0 &= M_{ED} + M_{EF} = 4(1.3K')\theta_E + 2(1.3K')\theta_D + 4(2K')\theta_E - 6(2K')\rho_2 \end{aligned}$$



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Slope Deflection

- Horizontal equilibrium

$$\begin{aligned} H_G + H_F - H_{DC} - 10 &= 0 \\ \frac{M_{GD} + M_{DG}}{13} + \frac{M_{FE} + M_{EF}}{13} - \frac{M_{DC} + M_{CD}}{13} - 10 &= 0 \\ M_{GD} + M_{DG} + M_{FE} + M_{EF} - M_{DC} - M_{CD} &= 130 \end{aligned}$$



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90

Slope Deflection

- Substitute for the moments

$$M_{GD} + M_{DG} + M_{FE} + M_{EF} - M_{DC} - M_{CD} = 130$$

$$\begin{aligned} &4(2K')\theta_D - 6(2K')\rho_2 + 2(2K')\theta_D - 6(2K')\rho_2 \\ &+ 4(2K')\theta_E - 6(2K')\rho_2 + 2(2K')\theta_E - 6(2K')\rho_2 \\ &- 4(2K')\theta_C - 2(2K')\theta_D + 6(2K')(2\rho_1 - \rho_2) \\ &- 4(2K')\theta_D - 2(2K')\theta_C + 6(2K')(2\rho_1 - \rho_2) = 130 \end{aligned}$$

$$-12K'\theta_C + 12K'\theta_E + 48K'\rho_1 - 72K'\rho_2 = 130$$



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91

Slope Deflection

- Horizontal equilibrium

$$H_A + H_{DC} + 20 = 0$$

$$\frac{M_{AB} + M_{BA}}{26} + \frac{M_{DC} + M_{CD}}{13} + 20 = 0$$

$$M_{AB} + M_{BA} + 2M_{DC} + 2M_{CD} = -520$$



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Slope Deflection

- Substitute for the moments

$$M_{AB} + M_{BA} + 2M_{DC} + 2M_{CD} = -520$$

$$2K'\theta_B - 6K'\rho_1 + 4K'\theta_B - 6K'\rho_1$$

$$+ 2[4(2K')\theta_C + 2(2K')\theta_D - 6(2K')(2\rho_1 - \rho_2)]$$

$$+ 2[4(2K')\theta_D + 2(2K')\theta_C - 6(2K')(2\rho_1 - \rho_2)] = -520$$

$$6K'\theta_B + 24K'\theta_C + 24K'\theta_D - 108K'\rho_1 + 48K'\rho_2 = -520$$



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Slope Deflection

- Resulting 6 simultaneous equations

$9.2K'\theta_B$	$+2.6K'\theta_C$			$-6K'\rho_1$		$= 0$
$2.6K'\theta_B$	$+13.2K'\theta_C$	$+4K'\theta_D$		$-24K'\rho_1$	$+12K'\rho_2$	$= 0$
	$+4K'\theta_C$	$+21.2K'\theta_D$	$+2.6K'\theta_E$	$-24K'\rho_1$		$= 0$
		$+2.6K'\theta_D$	$+13.2K'\theta_E$		$-12K'\rho_2$	$= 0$
$6K'\theta_B$	$+24K'\theta_C$	$+24K'\theta_D$		$-108K'\rho_1$	$+48K'\rho_2$	$= -520$
	$-12K'\theta_C$		$+12K'\theta_E$	$+48K'\rho_1$	$-72K'\rho_2$	$= 130$



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Slope Deflection

- Question to consider
 - How does symmetry play a role in our solutions?
 - Only for a symmetric structure will the equilibrium equations be symmetric.
 - The equilibrium equations will always automatically come out symmetric.
 - The equilibrium equations can always be made symmetric.
 - Symmetry depends on how we define our variables.



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Polling Questions



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96

Slope Deflection

- Resulting 6 simultaneous equations

$$\begin{array}{rcccccccl}
 9.2K'\theta_B & +2.6K'\theta_C & & & & -6K'\rho_1 & & = & 0 \\
 2.6K'\theta_B & +13.2K'\theta_C & +4K'\theta_D & & & -24K'\rho_1 & +12K'\rho_2 & = & 0 \\
 & +4K'\theta_C & +21.2K'\theta_D & +2.6K'\theta_E & & -24K'\rho_1 & & = & 0 \\
 & & +2.6K'\theta_D & +13.2K'\theta_E & & & -12K'\rho_2 & = & 0 \\
 6K'\theta_B & +24K'\theta_C & +24K'\theta_D & & & -108K'\rho_1 & +48K'\rho_2 & = & -520 \\
 & -12K'\theta_C & & +12K'\theta_E & & +48K'\rho_1 & -72K'\rho_2 & = & 130
 \end{array}$$

See how the 5th row and 5th column illustrate symmetry if we multiply the last 2 equations by -1.



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Slope Deflection

- Solution to simultaneous equations

$$K'\theta_B = 4.30$$

$$K'\theta_C = 15.19$$

$$K'\theta_D = 11.81$$

$$K'\theta_E = 2.03$$

$$K'\rho_1 = 13.18$$

$$K'\rho_2 = 4.79$$



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98

Slope Deflection

- Final Moments

$$M_{AB} = 2K'\theta_B - 6K'\rho_1 = 2(4.30) - 6(13.18) = -70.5$$

$$M_{BA} = 4K'\theta_B - 6K'\rho_1 = 4(4.30) - 6(13.18) = -61.9$$

$$M_{BC} = 4(1.3K')\theta_B + 2(1.3K')\theta_C = 5.2(4.30) + 2.6(15.19) = 61.9$$

$$M_{CB} = 4(1.3K')\theta_C + 2(1.3K')\theta_B = 5.2(15.19) + 2.6(4.30) = 90.2$$

$$M_{CD} = 4(2K')\theta_C + 2(2K')\theta_D - 6(2K')(2\rho_1 - \rho_2)$$

$$= 8(15.19) + 4(11.81) - 12(2(13.18) - 4.79) = -90.1$$

$$M_{DC} = 4(2K')\theta_D + 2(2K')\theta_C - 6(2K')(2\rho_1 - \rho_2)$$

$$= 8(11.81) + 4(15.19) - 12(2(13.18) - 4.79) = -103.6$$



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Slope Deflection

- Final Moments

$$M_{DG} = 4(2K')\theta_D - 6(2K')\rho_2 = 8(11.81) - 12(4.79) = 37.0$$

$$M_{GD} = 2(2K')\theta_D - 6(2K')\rho_2 = 4(11.81) - 12(4.79) = -10.24$$

$$M_{DE} = 4(1.3K')\theta_D + 2(1.3K')\theta_E = 5.2(11.81) + 2.6(2.03) = 66.7$$

$$M_{ED} = 4(1.3K')\theta_E + 2(1.3K')\theta_D = 5.2(2.03) + 2.6(11.81) = 41.2$$

$$M_{EF} = 4(2K')\theta_E - 6(2K')\rho_2 = 8(2.03) - 12(4.79) = -41.2$$

$$M_{FE} = 2(2K')\theta_E - 6(2K')\rho_2 = 4(2.03) - 12(4.79) = -49.4$$

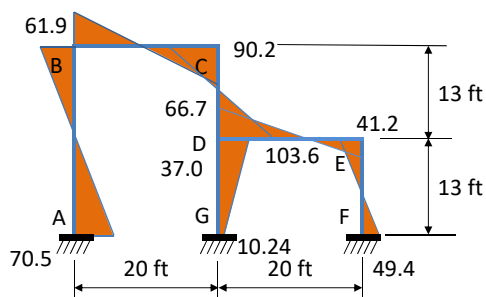


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100

Slope Deflection

- Final Moment diagram (ft-k)

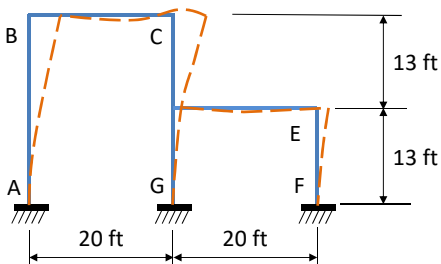


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Slope Deflection

- Deflected shape



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Modern Methods

- The 6 simultaneous equations from this example can be viewed in matrix form.

$$\begin{bmatrix} 9.2 & 2.6 & 0 & 0 & -6 & 0 \\ 2.6 & 13.2 & 4 & 0 & -24 & 12 \\ 0 & 4 & 21.2 & 2.6 & -24 & 0 \\ 0 & 0 & 2.6 & 13.2 & 0 & -12 \\ 6 & 24 & 24 & 0 & -108 & 48 \\ 0 & -12 & 0 & 12 & 48 & -72 \end{bmatrix} \begin{Bmatrix} K'\theta_B \\ K'\theta_C \\ K'\theta_D \\ K'\theta_E \\ K'p_1 \\ K'p_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -520 \\ 130 \end{Bmatrix}$$

- This solution includes only flexural deformations.
- If axial deformations and shearing deformations were included we would be approaching the stiffness method more closely.



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Summary

- Obtained equations for the absolute stiffness, carry-over factor and fixed-end moments for flexural members.
- Developed the method of slope deflection and applied it to continuous beams and sway permitted frames.
- Considered the special case of known support settlements.
- Determined the effect of temperature change on an indeterminate beam.
- Compared the slope deflection method to the modern stiffness method.



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Lesson 7

- **Approximate Methods and Moment Distribution**
 - Introduce the Portal Method and the Cantilever Method for multi-story frames.
 - Develop the Moment Distribution Method for continuous beams.
 - Introduce an approach to check the process of moment distribution.
 - Apply Moment Distribution to frames with side sway permitted.



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Thank You



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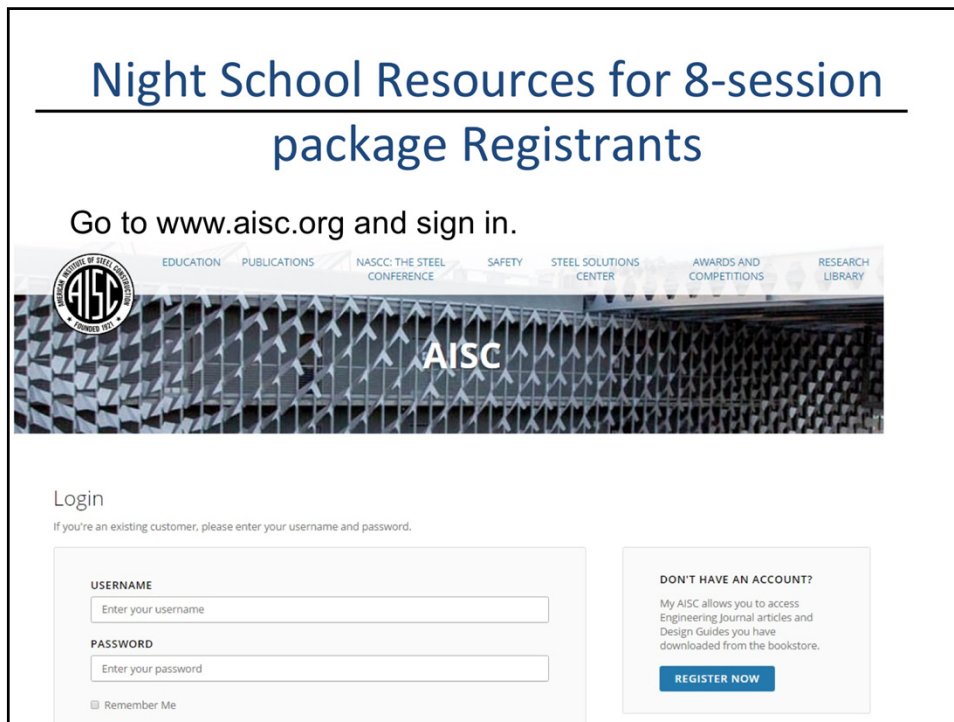
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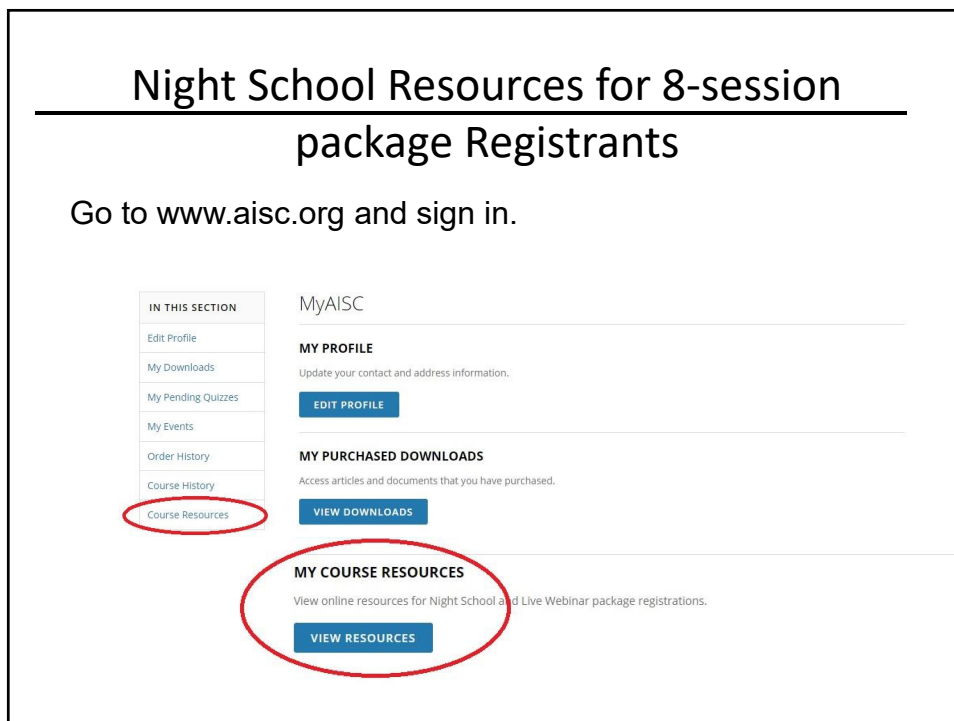
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