AISC Night School

Thank you for joining our live webinar today.
We will begin shortly. Please standby.

Thank you.

Need Help?
Call ReadyTalk Support: 800.843.9166

Classical Methods of Structural Analysis

Louis F. Geschwindner

Smarter. Stronger. Steel.

AISC Night School

Today's audio will be broadcast through the internet.

Alternatively, to hear the audio through the phone, dial 866-519-2796. Passcode: 158650

Smarter. Stronger. Steel.

AISC Night School

Today's live webinar will begin shortly.
Please standby.

As a reminder, all lines have been muted. Please type any questions or comments through the Chat feature on the left portion of your screen.

Today's audio will be broadcast through the internet.

Alternatively, to hear the audio through the phone, dial 866-519-2796. Passcode: 158650

Smarter. Stronger. Steel.

AISC Night School

AIA Credit

AISC is a Registered Provider with The American Institute of Architects Continuing Education Systems (AIA/CES). Credit(s) earned on completion of this program will be reported to AIA/CES for AIA members. Certificates of Completion for both AIA members and non-AIA members are available upon request.

This program is registered with AIA/CES for continuing professional education. As such, it does not include content that may be deemed or construed to be an approval or endorsement by the AIA of any material of construction or any method or manner of handling, using, distributing, or dealing in any material or product.

Questions related to specific materials, methods, and services will be addressed at the conclusion of this presentation.

Smarter. Stronger. Steel.

AISC Night School

Copyright Materials

Smarter. Stronger. Steel.

AISC Night School

Session Description
20.6 Indeterminate Structures by Slope Deflection
July 22, 2019

This lesson will continue the analysis of indeterminate structures. Analysis of indeterminate structures by the slope-deflection method will be developed using the principles previously developed for deflection calculations. Sidesway and non-sidesway will be presented and discussed. Approaches for modeling structures will be illustrated, including continuous beams and multi-story sway- and nonsway-frames.

Smarter. Stronger. Steel.

AISC Night School

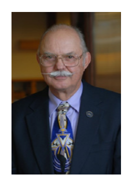
Learning Objectives:

- Explain the analysis of indeterminate structures.
- Develop the slope-deflection method for analysis of indeterminate structures.
- Analyze continuous beams using the slope deflection method.
- Describe how to apply the method to frames exhibiting sway.
- Compare the slope deflection method to the modern stiffness method.

Smarter. Stronger. Steel.

Night School 20
Classical Methods of Structural Analysis

Session 6: Indeterminate Structures by Slope Deflection
July 22, 2019



Louis F. Geschwindner, PE, PhD
Professor Emeritus, Penn State University,
Former Vice President, AISC, and
Senior Consultant, Providence Engineering
State College, Pennsylvania

Smarter. Stronger. Steel.

Classical Methods of Structural Analysis:
How we did it before computers

Night School 20

Lesson 6

Indeterminate Structures by Slope Deflection

Smarter. Stronger. Steel. 9

Lesson 6
Indeterminate Structures by Slope Deflection

- Develop the absolute stiffness and carry-over factor for flexural members.
- Derive the method of slope deflection.
- Consider additional equations for fixed end moments.
- Analyze continuous beams using the slope deflection method.
- Consider support settlements.
- Apply the method to frames exhibiting sway.
- Determine the effect of temperature change on a beam.
- Compare the slope deflection method to the modern stiffness method.

Smarter. Stronger. Steel. 10

Slope Deflection

- A method presented in 1880 by Heinrich Manderla for the solution of secondary stresses in articulated structures was somewhat similar to slope deflection and may have lead to its development.
- In 1892, Otto Mohr presented an improved method which is what is now called slope deflection.

Smarter. Stronger. Steel. 11

Slope Deflection

- Axel Bendixen published a book in 1914 that presented the method in greater detail.
- Then, a year later, in 1915 the person generally given credit for the method, G. A. Maney, published his development of the method.

Smarter. Stronger. Steel. 12

Slope Deflection

- Advantages of slope deflection:

  "Most computers will find the formulation of the equations by this process (*slope deflection*) much simpler than that required for elastic equations (*general method*)" Parcel and Moorman, *Analysis of Statically Indeterminate Structures*, 1955.

  1. Reduced number of equations.
  2. Easy equations to set up.
  3. Good for simple frames with lots of fixed joints.

The computers referenced here were people, not machines. Remember the movie, *Hidden Figures*.

Smarter. Stronger. Steel. 13

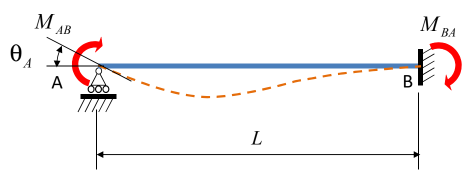
Slope Deflection

- Before the actual slope deflection method can be developed, two properties of flexural members will need to be developed.
  - Absolute stiffness: The moment required to produce a unit rotation at the simply supported end of a beam when the other end is fixed.
  - Carry-over factor: The relationship between the moment applied to the simply supported end of the beam and the resisting moment developed at the other end.

Smarter. Stronger. Steel. 14

Slope Deflection

- Sign convention

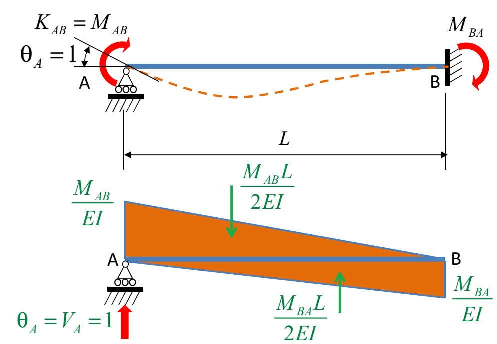


Clockwise moment on the end of the member is positive.
Clockwise rotation of a joint is positive.

Smarter. Stronger. Steel. 15

Slope Deflection

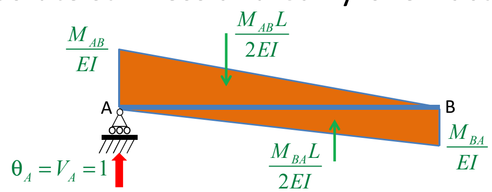
- Absolute stiffness and Carry-over factor



Smarter. Stronger. Steel. 16

Slope Deflection

- Absolute stiffness and Carry-over factor



Sum of the moments about A $\quad \frac{M_{AB}L}{2EI}\left(\frac{L}{3}\right) - \frac{M_{BA}L}{2EI}\left(\frac{2L}{3}\right) = 0$

Sum of the vertical forces $\quad \frac{M_{AB}L}{2EI} - \frac{M_{BA}L}{2EI} = 1$

Smarter. Stronger. Steel.

17

Slope Deflection

- Solve for the moments

From the first equation
$$\frac{M_{AB}L}{2EI}\left(\frac{L}{3}\right) - \frac{M_{BA}L}{2EI}\left(\frac{2L}{3}\right) = 0$$
$$\rightarrow M_{BA} = \frac{M_{AB}}{2}$$

From the second equation
$$\frac{M_{AB}L}{2EI} - \left(\frac{M_{AB}}{2}\right)\frac{L}{2EI} = 1$$
$$\rightarrow M_{AB} = \frac{4EI}{L}$$

Smarter. Stronger. Steel.

18

Slope Deflection

- By definition, the absolute stiffness of the beam, K_{AB}, is

$$K_{AB} = M_{AB} = \frac{4EI}{L}$$

- The carry-over factor, C_{AB}, is

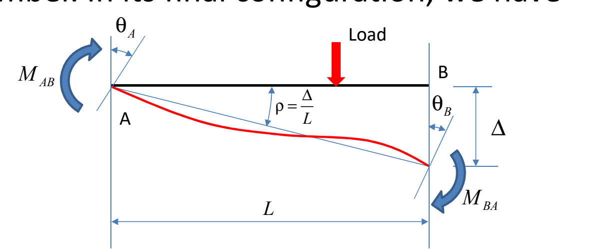
$$M_{BA} = C_{AB}M_{AB} = \frac{M_{AB}}{2}$$
$$C_{AB} = \frac{1}{2}$$

Smarter. Stronger. Steel.

19

Slope Deflection

- Develop the method by looking at the impact of load, deflection, and rotation on a bending member. In its final configuration, we have



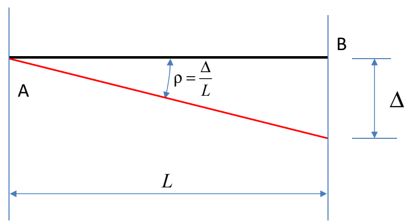
All variables shown positive.

Smarter. Stronger. Steel.

20

Slope Deflection

1. Consider the unloaded and unrestrained member rotated through ρ.

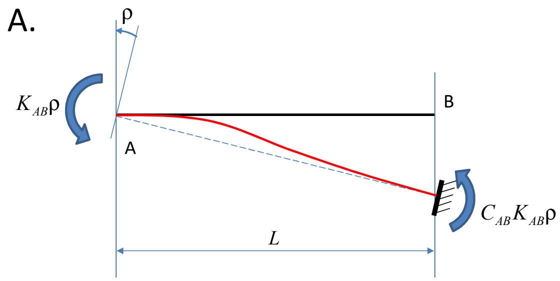


Smarter. Stronger. Steel.

21

Slope Deflection

2. Fix end B in this position and rotate end A back, through ρ, so the member is horizontal at A.

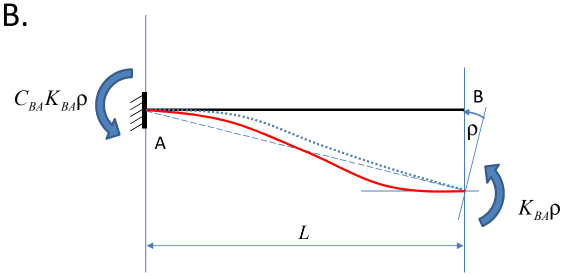


Smarter. Stronger. Steel.

22

Slope Deflection

3. Fix the member at A and rotate end B through ρ so that the member is horizontal at B.

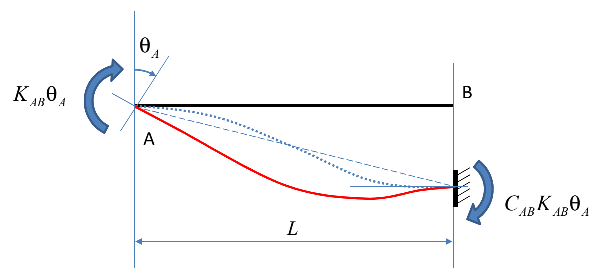


Smarter. Stronger. Steel.

23

Slope Deflection

4. Fix end B and rotate end A though θ_A

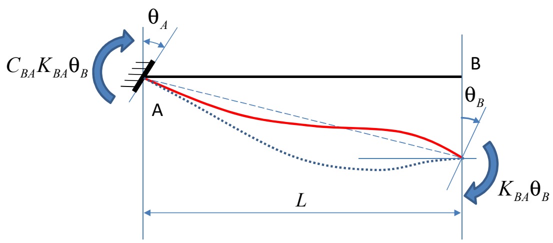


Smarter. Stronger. Steel.

24

Slope Deflection

5. Fix end A and rotate end B through θ_B

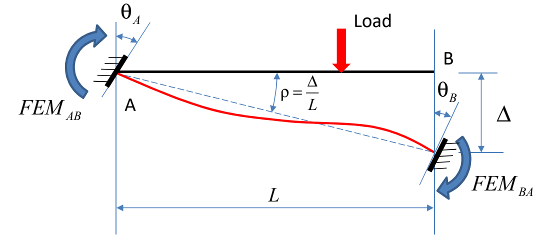


Smarter. Stronger. Steel.

25

Slope Deflection

6. Fix both ends and apply the load.



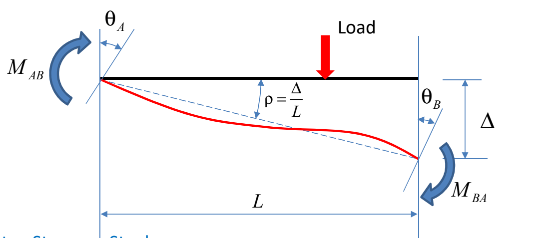
Smarter. Stronger. Steel.

26

Slope Deflection

7. Add the moments for each step at A and at B.

$$M_{AB} = -K_{AB}\rho - C_{BA}K_{BA}\rho + K_{AB}\theta_A + C_{BA}K_{BA}\theta_B + FEM_{AB}$$
$$M_{BA} = -C_{AB}K_{AB}\rho - K_{BA}\rho + C_{AB}K_{AB}\theta_A + K_{BA}\theta_B + FEM_{BA}$$



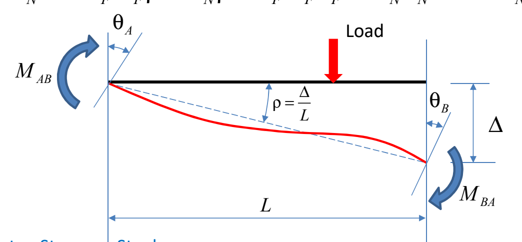
Smarter. Stronger. Steel.

27

Slope Deflection

8. To generalize, replace the subscripts with N for the end where the moment is being determined (near) and F for the other end (far).

$$M_N = -K_N\rho - C_F K_F\rho + K_N\theta_N + C_F K_F\theta_F + FEM_N$$
$$M_N = -C_F K_F\rho - K_N\rho + C_F K_F\theta_F + K_N\theta_N + FEM_N$$



Smarter. Stronger. Steel.

28

Slope Deflection

- As can be seen, the equations are the same, reorganize and write as a single slope deflection equation.

$$M_N = K_N\theta_N + C_F K_F \theta_F - (K_N + C_F K_F)\rho + FEM_N$$

- For a prismatic member, considering symmetry, as we saw earlier when we derived absolute stiffness and carry-over factor. (slide 19)

$$K_N = K_F = \frac{4EI}{L} \qquad C_N = C_F = \frac{1}{2}$$

Smarter. Stronger. Steel.

29

Slope Deflection

- Substitute for stiffness and carry-over factor and simplify

$$M_N = \frac{4EI}{L}\theta_N + \frac{2EI}{L}\theta_F - \frac{6EI}{L}\rho + FEM_N$$

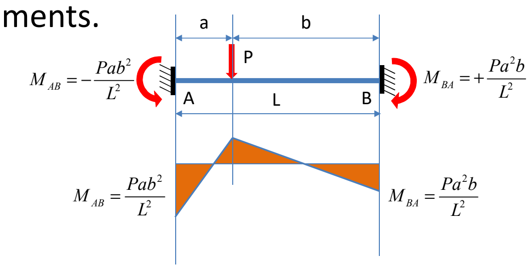
This is the slope deflection equation for the moment on the end of a prismatic flexural member.

Smarter. Stronger. Steel.

30

Slope Deflection

- Lesson 4 illustrated the use of the conjugate beam method for determining fixed end moments.

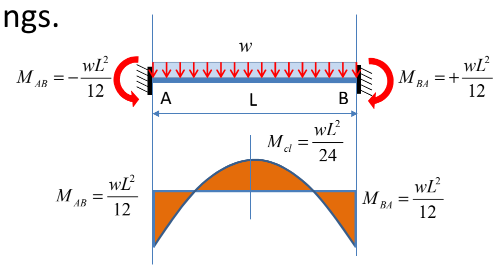


Smarter. Stronger. Steel.

31

Slope Deflection

- The same approach is quite useful for determining fixed end moments for other loadings.

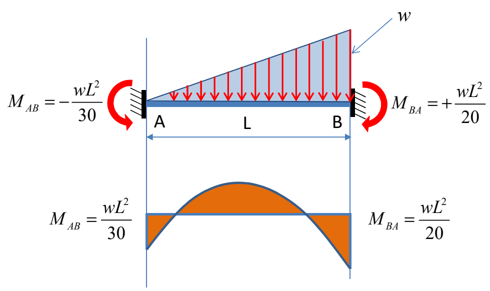


Smarter. Stronger. Steel.

32

Slope Deflection

- Triangular loading

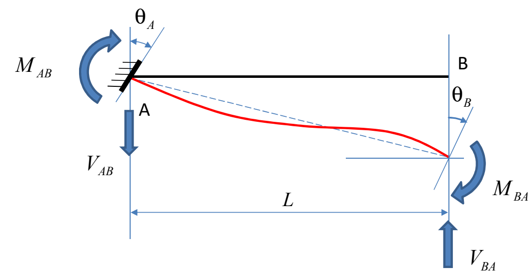


Smarter. Stronger. Steel.

33

Slope Deflection

Consider the positive shears that correspond to the final positive end moments.

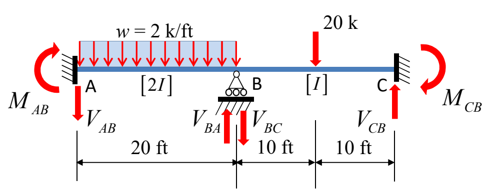


Smarter. Stronger. Steel.

34

Slope Deflection

- Using the slope deflection method, determine the member end moments and plot the final moment diagram.

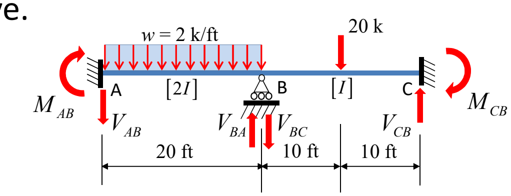


Smarter. Stronger. Steel.

35

Slope Deflection

- If we were solving this beam by consistent deformations, there would be 4 redundants to remove. Thus, 4 simultaneous equations to solve.

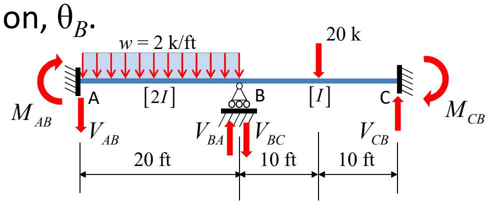


Smarter. Stronger. Steel.

36

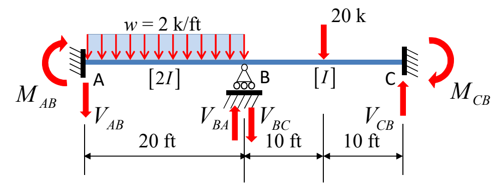
Slope Deflection

- With slope deflection, we are concerned with rotation of the members and rotation of the joints. Here, we only have one unknown rotation, θ_B.



$$\theta_A = \theta_C = 0 \qquad \rho_{AB} = \rho_{BC} = 0$$

Slope Deflection



Fixed end moments

$$FEM_{AB} = -\frac{wL^2}{12} = -\frac{2(20)^2}{12} = -66.7 \text{ ft-kips} \qquad FEM_{BC} = -\frac{PL}{8} = -\frac{20(20)}{8} = -50.0 \text{ ft-kips}$$

$$FEM_{BA} = \frac{wL^2}{12} = \frac{2(20)^2}{12} = 66.7 \text{ ft-kips} \qquad FEM_{CB} = \frac{PL}{8} = \frac{20(20)}{8} = 50.0 \text{ ft-kips}$$

Slope Deflection

$$M_N = \frac{4EI}{L}\theta_N + \frac{2EI}{L}\theta_F - \frac{6EI}{L}\rho + FEM_N$$

$$M_{AB} = \frac{2E(2I)}{20}\theta_B - 66.7$$

$$M_{BA} = \frac{4E(2I)}{20}\theta_B + 66.7$$

$$M_{BC} = \frac{4E(I)}{20}\theta_B - 50.0$$

$$M_{CB} = \frac{2E(I)}{20}\theta_B + 50.0$$

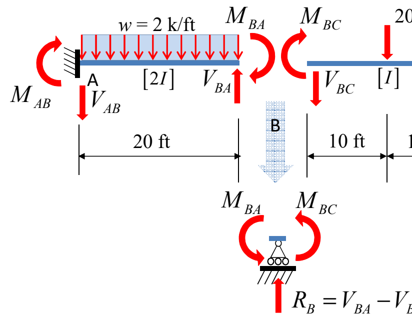
Slope Deflection

- The only unknown rotation is θ_B. In order to determine this rotation, an equilibrium equation "parallel to" or "consistent with" this unknown must be found.
- In this instance, the equilibrium equation is sum of the moments at B must be equal to zero.

$$\sum M_B = 0 = M_{BA} + M_{BC}$$

Slope Deflection

Equilibrium at joint B



Note that positive moment on the joint is counterclockwise

$$\sum M_B = 0 = M_{BA} + M_{BC}$$

Slope Deflection

- Substituting the member end moment equations yields (slide 39)

$$M_{BA} + M_{BC} = \left[\frac{4E(2I)}{20}\theta_B + 66.7\right] + \left[\frac{4E(I)}{20}\theta_B - 50.0\right] = 0$$

- This equilibrium equation can be solved for

$$\frac{12E(I)}{20}\theta_B = -16.7$$

$$EI\theta_B = -27.8$$

Slope Deflection

- Substitute for $EI\theta_B$ and determine the member end moments. (slide 39)

$$M_{AB} = \frac{2E(2I)}{20}\theta_B - 66.7 = \frac{4(-27.8)}{20} - 66.7 = -72.3$$

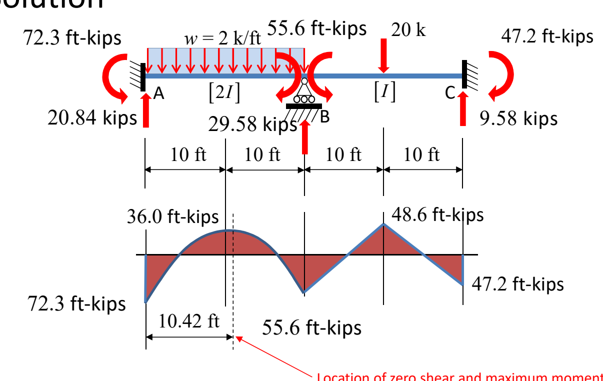
$$M_{BA} = \frac{4E(2I)}{20}\theta_B + 66.7 = \frac{8(-27.8)}{20} + 66.7 = +55.6$$

$$M_{BC} = \frac{4E(I)}{20}\theta_B - 50.0 = \frac{4(-27.8)}{20} - 50.0 = -55.6$$

$$M_{CB} = \frac{2E(I)}{20}\theta_B + 50.0 = \frac{2(-27.8)}{20} + 50.0 = +47.2$$

Slope Deflection

- Solution



Slope Deflection

- Some thoughts on slope deflection.
  - We do not need the actual stiffness of the members to determine member end moments, only the relative stiffness.
  - With relative stiffness, we are able to determine the direction of the unknown rotations but not the magnitude.
  - The sign convention used is clockwise on the end of the member is positive.

Smarter. Stronger. Steel.

45

Slope Deflection

- Determine the rotation in radians at joint B.
  - From solution of the equilibrium equation we have

$$EI\theta_B = -27.8$$

  - But what are the units? Note that from our member end moment equation we used fixed end moments with units of ft-kips.

$$M_{BC} = \frac{4E(I)}{20}\theta_B - 50.0$$

Smarter. Stronger. Steel.

46

Slope Deflection

- For units to be consistent the other terms must also be in ft-kips. Thus $EI\theta_B$ must be in units of ft^2-kips so that when we divide by span in feet, as we did, the term has units of ft-kips.
- If member BC is a W16x31, I = 375 in.^{4} Thus,

$$EI\theta_B = -27.8 \text{ ft}^2\text{-kips}$$

$$\theta_B = \frac{-27.8(144)}{29,000(375)} = -0.000368 \text{ rad}$$

Smarter. Stronger. Steel.

47

Slope Deflection

- Some questions to consider.
  - What would happen to the magnitude of the rotation at B if the moment of inertia of member AB were made the same as member BC?
  - How would you expect the moment at B to respond?
  - How would you expect the moment at C to respond?

Smarter. Stronger. Steel.

48

Polling Question

Smarter. Stronger. Steel.

49

Polling Question

Smarter. Stronger. Steel.

50

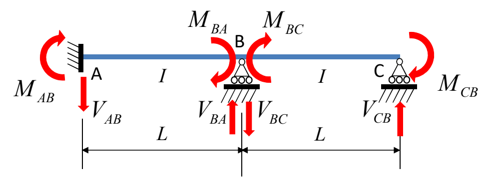
Polling Question

Smarter. Stronger. Steel.

51

Slope Deflection

- Consider a prismatic beam where the support at the end is a pin.



Smarter. Stronger. Steel.

52

Slope Deflection

- For member BC, write the slope deflection equations assuming that the member may rotate.

$$M_{BC} = \frac{4EI}{L}\theta_B + \frac{2EI}{L}\theta_C - \frac{6EI}{L}\rho + FEM_{BC}$$
$$M_{CB} = \frac{4EI}{L}\theta_C + \frac{2EI}{L}\theta_B - \frac{6EI}{L}\rho + FEM_{CB}$$

Smarter. Stronger. Steel.

53

Slope Deflection

- Since we already know that $M_{CB} = 0$, we can use that equation to eliminate an unknown.

$$M_{CB} = \frac{4EI}{L}\theta_C + \frac{2EI}{L}\theta_B - \frac{6EI}{L}\rho + FEM_{CB} = 0$$
$$\frac{EI}{L}\theta_C = \left[-FEM_{CB} - \frac{2EI}{L}\theta_B + \frac{6EI}{L}\rho\right]\frac{1}{4}$$

Smarter. Stronger. Steel.

54

Slope Deflection

- If we substitute this into the equation for the moment M_{BC} we will be able to eliminate θ_C from our unknowns.

$$M_{BC} = \frac{4EI}{L}\theta_B + 2\left[-FEM_{CB} - \frac{2EI}{L}\theta_B + \frac{6EI}{L}\rho\right]\left(\frac{1}{4}\right) - \frac{6EI}{L}\rho + FEM_{BC}$$

- Which simplifies to

$$M_{BC} = \frac{3EI}{L}\theta_B - \frac{3EI}{L}\rho + \left[FEM_{BC} - \frac{FEM_{CB}}{2}\right]$$

Smarter. Stronger. Steel.

55

Slope Deflection

- In the N (near) and F (far) format this becomes

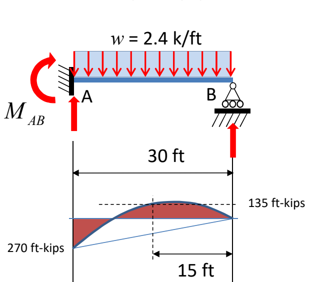
$$M_N = \frac{3EI}{L}\theta_N - \frac{3EI}{L}\rho + \left[FEM_N - \frac{FEM_F}{2}\right]$$

Smarter. Stronger. Steel.

56

Slope Deflection

- The simplest case that this may be applied to is the propped cantilever.



$$FEM_{AB}=-\frac{wL^2}{12}=-\frac{2.4(30)^2}{12}=-180\text{ ft-kips}$$

$$FEM_{BA}=\frac{wL^2}{12}=\frac{2.4(30)^2}{12}=180\text{ ft-kips}$$

Since we know that $\theta_A=\rho=0$,

$$M_{AB}=\frac{3EI}{L}\theta_A-\frac{3EI}{L}\rho+\left[FEM_{AB}-\frac{FEM_{BA}}{2}\right]$$

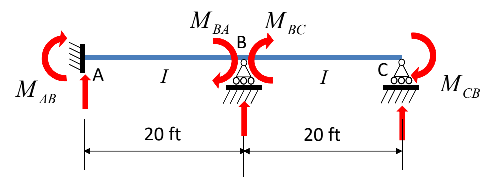
$$=\left[-180-\left(\frac{180}{2}\right)\right]=-270\text{ ft-kips}$$

Smarter. Stronger. Steel.

57

Slope Deflection

- For the W21x83 two span beam shown, what would the moment be if the support at B were to settle 1.0 in.?

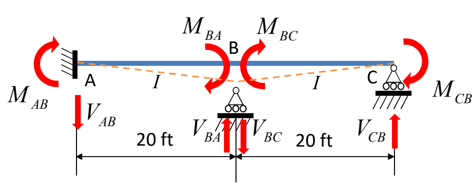


Smarter. Stronger. Steel.

58

Slope Deflection

- Settlement of joint B



$$\rho_{AB}=\frac{\Delta}{L}=\frac{1.0}{20(12)}=0.00417\text{ rad}$$

$$\rho_{BC}=\frac{\Delta}{L}=-\frac{1.0}{20(12)}=-0.00417\text{ rad}$$

Smarter. Stronger. Steel.

59

Slope Deflection

- Member end moment equations

$$M_{AB}=\frac{2EI}{L}\theta_B-\frac{6EI}{L}(0.00417)$$

$$M_{BA}=\frac{4EI}{L}\theta_B-\frac{6EI}{L}(0.00417)$$

$$M_{BC}=\frac{3EI}{L}\theta_B-\frac{3EI}{L}(-0.00417)$$

Smarter. Stronger. Steel.

60

Slope Deflection

- Substituting for EI — EI has units of k-in.[2] so need to divide by 144 to convert to ft[2]

$$\frac{6EI}{L}(0.00417)=\frac{6(29{,}000)(1830)}{20(144)}(0.00417)=461 \text{ ft-kips}$$

This is essentially a fixed end moment for settlement.

$$M_{AB}=\frac{2EI}{L}\theta_B-\frac{6EI}{L}(0.00417)=\frac{2EI}{L}\theta_B-461$$

$$M_{BA}=\frac{4EI}{L}\theta_B-\frac{6EI}{L}(0.00417)=\frac{4EI}{L}\theta_B-461$$

$$M_{BC}=\frac{3EI}{L}\theta_B-\frac{3EI}{L}(-0.00417)=\frac{3EI}{L}\theta_B+231$$

Smarter. Stronger. Steel.

61

Slope Deflection

- Sum moments about joint B

$$\sum M_B=M_{BA}+M_{BC}=0$$

$$\left[\frac{4EI}{L}\theta_B-461\right]+\left[\frac{3EI}{L}\theta_B+231\right]=0$$

$$\frac{7EI}{L}\theta_B=230$$

$$\frac{EI}{L}\theta_B=32.9$$

Smarter. Stronger. Steel.

62

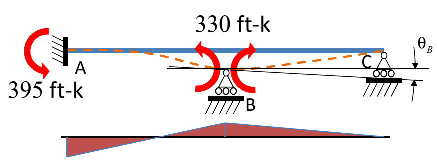
Slope Deflection

- Substituting, determine the member end moments.

$$M_{AB}=\frac{2EI}{L}\theta_B-461=2(32.9)-461=-395 \text{ ft-kips}$$

$$M_{BA}=\frac{4EI}{L}\theta_B-461=4(32.9)-461=-329 \text{ ft-kips}$$

$$M_{BC}=\frac{3EI}{L}\theta_B+231=3(32.9)+231=330 \text{ ft-kips}$$



$$\theta_B=32.9\frac{L}{EI}$$

$$=32.9\frac{(20)(144)}{29{,}000(1830)}$$

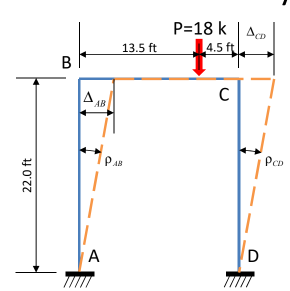
$$=1.79\times10^{-3} \text{ rad}$$

Smarter. Stronger. Steel.

63

Slope Deflection

- Next consider a frame with an unknown amount of side sway. (Which way does it sway?)



Sidesway is shown to the right. This produces a positive member rotation for both members AB and CD.

Considering flexural deformations only and small angle theory,

$$\Delta_{AB}=\Delta_{CD}=\Delta$$

$$\rho_{AB}=\rho_{CD}=\rho=\frac{\Delta}{22}$$

Also,

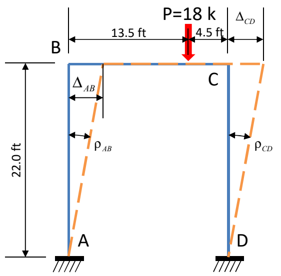
$$\theta_A=\theta_D=0$$

Smarter. Stronger. Steel.

64

Slope Deflection

- Next consider a frame with an unknown amount of side sway.



Only member BC will have fixed end moments.

$$FEM_{BC} = -\frac{18(13.5)(4.5)^2}{(18.0)^2} = -15.2 \text{ ft-k}$$

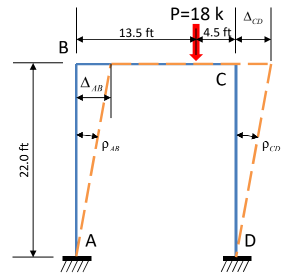
$$FEM_{CB} = +\frac{18(4.5)(13.5)^2}{(18.0)^2} = +45.6 \text{ ft-k}$$

Smarter. Stronger. Steel.

65

Slope Deflection

- Member end moments



$$M_{AB} = \frac{2EI}{22}\theta_B - \frac{6EI}{22}\rho$$

$$M_{BA} = \frac{4EI}{22}\theta_B - \frac{6EI}{22}\rho$$

$$M_{BC} = \frac{4EI}{18}\theta_B + \frac{2EI}{18}\theta_C - 15.2$$

$$M_{CB} = \frac{4EI}{18}\theta_C + \frac{2EI}{18}\theta_B + 45.6$$

$$M_{CD} = \frac{4EI}{22}\theta_C - \frac{6EI}{22}\rho$$

$$M_{DC} = \frac{2EI}{22}\theta_C - \frac{6EI}{22}\rho$$

Smarter. Stronger. Steel.

66

Slope Deflection

- Equilibrium Equations at B and C

$$\sum M_B = M_{BA} + M_{BC} = 0$$

$$= \frac{4EI}{22}\theta_B - \frac{6EI}{22}\rho + \frac{4EI}{18}\theta_B + \frac{2EI}{18}\theta_C - 15.2$$

$$= 0.404EI\theta_B + 0.111EI\theta_C - 0.273EI\rho - 15.2 = 0$$

$$\sum M_C = M_{CB} + M_{CD} = 0$$

$$= \frac{4EI}{18}\theta_C + \frac{2EI}{18}\theta_B + 45.6 + \frac{4EI}{22}\theta_C - \frac{6EI}{22}\rho$$

$$= 0.111EI\theta_B + 0.404EI\theta_C - 0.273EI\rho + 45.6 = 0$$

2 equations, 3 unknowns

Smarter. Stronger. Steel.

67

Slope Deflection

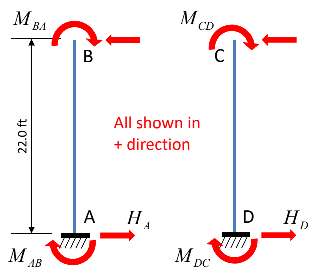
- A third equilibrium equation is required. The first two were associated with the unknown rotations at B and C. The third must be associated with the unknown sway of the frame.
- To develop this equation, we will look at horizontal equilibrium of the frame.

Smarter. Stronger. Steel.

68

Slope Deflection

- Horizontal equilibrium.



Take moments about B and C

$$H_A = \frac{M_{AB} + M_{BA}}{22}$$

$$H_D = \frac{M_{CD} + M_{DC}}{22}$$

$$H_A + H_D = 0$$

$$\frac{M_{AB} + M_{BA}}{22} + \frac{M_{CD} + M_{DC}}{22} = 0$$

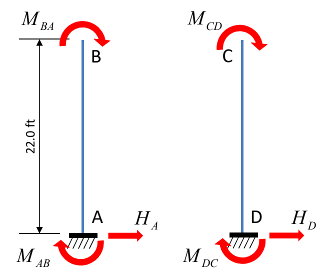
$$M_{AB} + M_{BA} + M_{CD} + M_{DC} = 0$$

Smarter. Stronger. Steel.

69

Slope Deflection

- Horizontal equilibrium.



$$M_{AB} + M_{BA} + M_{CD} + M_{DC} = 0$$

$$\left[\frac{2EI}{22}\theta_B - \frac{6EI}{22}\rho\right] + \left[\frac{4EI}{22}\theta_B - \frac{6EI}{22}\rho\right] +$$

$$\left[\frac{4EI}{22}\theta_C - \frac{6EI}{22}\rho\right] + \left[\frac{2EI}{22}\theta_C - \frac{6EI}{22}\rho\right] = 0$$

$$0.273EI\theta_B + 0.273EI\theta_C - 1.09EI\rho = 0$$

Smarter. Stronger. Steel.

70

Slope Deflection

- The resulting three simultaneous equilibrium equations are:

$$0.404EI\theta_B + 0.111EI\theta_C - 0.273EI\rho = 15.2$$

$$0.111EI\theta_B + 0.404EI\theta_C - 0.273EI\rho = -45.6$$

$$0.273EI\theta_B + 0.273EI\theta_C - 1.09EI\rho = 0$$

- The solution:

$$EI\theta_B = 63.6 \quad EI\theta_C = -144. \quad EI\rho = -20.1$$

Smarter. Stronger. Steel.

71

Slope Deflection

- The resulting three simultaneous equilibrium equations are:

$$0.404EI\theta_B + 0.111EI\theta_C - 0.273EI\rho = 15.2$$

$$0.111EI\theta_B + 0.404EI\theta_C - 0.273EI\rho = -45.6$$

$$0.273EI\theta_B + 0.273EI\theta_C - 1.09EI\rho = 0$$

- The solution:

$$EI\theta_B = 63.6 \quad EI\theta_C = -144. \quad EI\rho = -20.1$$

Note the symmetry, if you multiply the last equation by -1. We have seen this before.

Smarter. Stronger. Steel.

72

Slope Deflection

- Substituting back into the member end moment equations yields

$$M_{AB} = \frac{2EI}{22}\theta_B - \frac{6EI}{22}\rho = 11.3$$

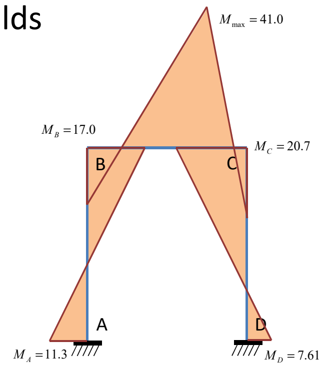
$$M_{BA} = \frac{4EI}{22}\theta_B - \frac{6EI}{22}\rho = 17.0$$

$$M_{BC} = \frac{4EI}{18}\theta_B + \frac{2EI}{18}\theta_C - 15.2 = -17.1$$

$$M_{CB} = \frac{4EI}{18}\theta_C + \frac{2EI}{18}\theta_B + 45.6 = 20.7$$

$$M_{CD} = \frac{4EI}{22}\theta_C - \frac{6EI}{22}\rho = -20.7$$

$$M_{DC} = \frac{2EI}{22}\theta_C - \frac{6EI}{22}\rho = -7.61$$



Moment plotted on compression side

Smarter. Stronger. Steel.

73

Slope Deflection

- Deflected shape for a W14x22

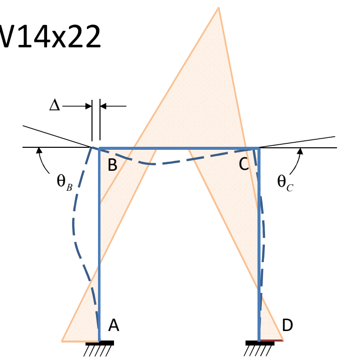
$$\theta_B = \frac{63.6}{EI} = \frac{63.6(144)}{29,000(199)} = 0.00159 \text{ rad}$$

$$\theta_C = -\frac{144.}{EI} = -\frac{144.(144)}{29,000(199)} = -0.00359 \text{ rad}$$

$$\rho = \frac{-20.1}{EI} = \frac{-20.1(144)}{29,000(199)} = -0.000501$$

$$\rho = \frac{\Delta}{L} = \frac{\Delta}{22} \quad \therefore \ \Delta = \rho(22)$$

$$\Delta = -0.000501(22) = -0.0110 \text{ ft} \rightarrow -0.132 \text{ in.}$$

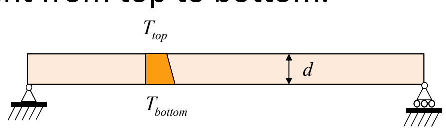


Smarter. Stronger. Steel.

74

Slope Deflection

- Consider a beam experiencing a temperature gradient from top to bottom.



For a temperature gradient, defined by the temperature difference, $\Delta T = T_{bottom} - T_{top}$, the fixed end moments can be shown to be

$$FEM_{NF} = -\frac{EI\alpha\Delta T}{d}$$

$$FEM_{FN} = \frac{EI\alpha\Delta T}{d}$$

and for steel,

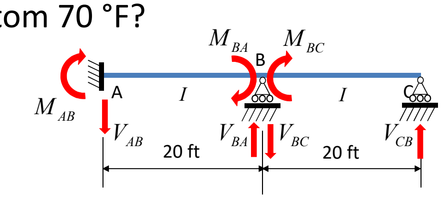
$\alpha = 6.5 \times 10^{-6}$ per degree F

Smarter. Stronger. Steel.

75

Slope Deflection

- For the W21x83 two span beam considered earlier, what would the moment be if the temperature on the top were 20 °F and on the bottom 70 °F?



$$\Delta T = T_{bottom} - T_{top} = 50^{\circ}\text{F}$$

Smarter. Stronger. Steel.

76

Slope Deflection

- Fixed end moments Note that beam length has no influence on FEM.

$$FEM_{AB} = FEM_{BC} = -\frac{EI\alpha\Delta T}{d} = -\frac{29,000(1830)(0.0000065)(50)}{21.4(12)} = -67.2 \text{ ft-kips}$$

$$FEM_{BA} = FEM_{CB} = \frac{EI\alpha\Delta T}{d} = \frac{29,000(1830)(0.0000065)(50)}{21.4(12)} = 67.2 \text{ ft-kips}$$

- End moment equations

$$M_{AB} = \frac{2EI}{L}\theta_B - 67.2$$

$$M_{BA} = \frac{4EI}{L}\theta_B + 67.2$$

$$M_{BC} = \frac{3EI}{L}\theta_B + \left(-67.2 - \frac{+67.2}{2}\right)$$

Smarter. Stronger. Steel.

77

Slope Deflection

- Summation of moments at B

$$M_{BA} + M_{BC} = \frac{4EI}{L}\theta_B + 67.2 + \frac{3EI}{L}\theta_B + \left(-67.2 - \frac{+67.2}{2}\right) = 0$$

$$= \frac{7EI}{L}\theta_B - 33.6 = 0$$

- Solve for the unknown

$$\frac{EI}{L}\theta_B = \frac{33.6}{7} = 4.8$$

Smarter. Stronger. Steel.

78

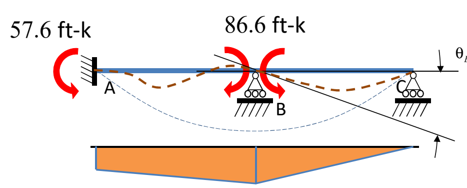
Slope Deflection

- Determine the end moments

$$M_{AB} = \frac{2EI}{L}\theta_B - 67.2 = 2(4.8) - 67.2 = -57.6 \text{ ft-kips}$$

$$M_{BA} = \frac{4EI}{L}\theta_B + 67.2 = 4(4.8) + 67.2 = 86.4 \text{ ft-kips}$$

$$M_{BC} = \frac{3EI}{L}\theta_B + \left(-67.2 - \frac{+67.2}{2}\right) = 3(4.8) - 101 = -86.6 \text{ ft-kips}$$



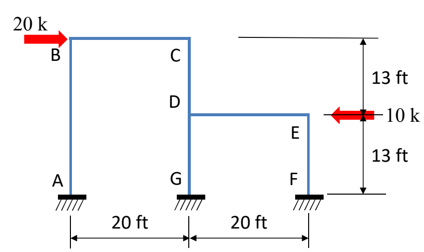
$$\theta_B = 4.8\frac{L}{EI} = 4.8\frac{(20)(144)}{29,000(1830)} = 0.260\times10^{-3} \text{ rad}$$

Smarter. Stronger. Steel. Moment diagram just due to the end moments, not considering internal temperature change effect.

79

Slope Deflection

- Determine the unknown joint rotations, θ, and member rotations, ρ, for the frame shown.



$\theta_A = \theta_G = \theta_F = 0$

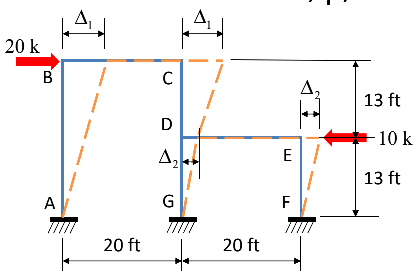
$\theta_B, \theta_C, \theta_D$, and θ_E are unknowns

Smarter. Stronger. Steel.

80

Slope Deflection

- Determine the unknown joint rotations, θ, and member rotations, ρ, for the frame shown.



$\theta_A = \theta_G = \theta_F = 0$

$\theta_B, \theta_C, \theta_D$, and θ_E are unknowns

Δ to the right is positive, to the left is negative.

Define

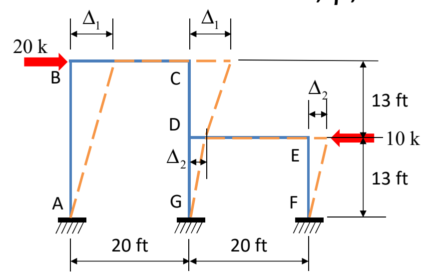
$\rho_1 = \frac{\Delta_1}{26} \qquad \rho_2 = \frac{\Delta_2}{13}$

Smarter. Stronger. Steel.

81

Slope Deflection

- Determine the unknown joint rotations, θ, and member rotations, ρ, for the frame shown.



$$\rho_{AB} = \frac{\Delta_1}{26} = \rho_1$$

$$\rho_{DC} = \frac{\Delta_1 - \Delta_2}{13} = 2\rho_1 - \rho_2$$

$$\rho_{GD} = \frac{\Delta_2}{13} = \rho_2$$

$$\rho_{EF} = \frac{\Delta_2}{13} = \rho_2$$

Smarter. Stronger. Steel.

82

Slope Deflection

- Joint equilibrium equations

$$\sum M_B = 0 = M_{BA} + M_{BC}$$
$$\sum M_C = 0 = M_{CB} + M_{CD}$$
$$\sum M_D = 0 = M_{DC} + M_{DE} + M_{DG}$$
$$\sum M_E = 0 = M_{ED} + M_{EF}$$

- This would permit determination of 4 unknowns (the 4 θ's) but we have 2 more unknowns, Δ_1 and Δ_2.

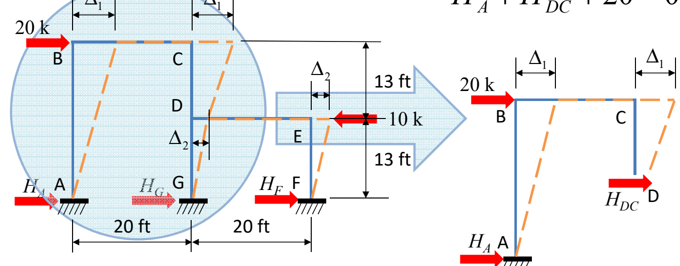
Smarter. Stronger. Steel.

83

Slope Deflection

- Horizontal equilibrium equations

$$H_A + H_{DC} + 20 = 0$$



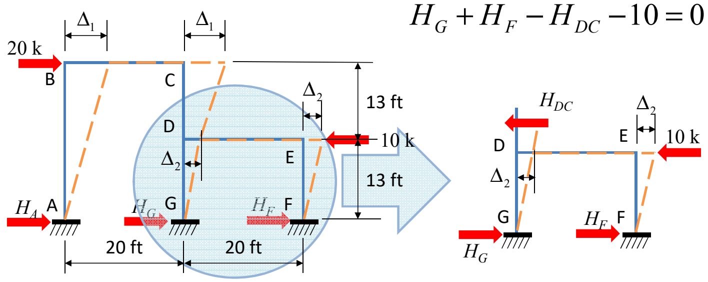
Smarter. Stronger. Steel.

84

Slope Deflection

- Horizontal equilibrium equations

$$H_G + H_F - H_{DC} - 10 = 0$$



Smarter. Stronger. Steel.

85

Slope Deflection

- Relative stiffness: Since E and I are constants and we do not actually need to know the values of E and I to determine moments, we can introduce the relative stiffness.

$$K_{AB} = \frac{EI}{26} = K'$$

$$K_{BC} = K_{DE} = \frac{EI}{20} = 1.3K'$$

$$K_{CD} = K_{DG} = K_{EF} = \frac{EI}{13} = 2K'$$

Smarter. Stronger. Steel.

86

Slope Deflection

- Member end moments

$$M_{AB} = 2K'\theta_B - 6K'\rho_1$$
$$M_{BA} = 4K'\theta_B - 6K'\rho_1$$
$$M_{BC} = 4(1.3K')\theta_B + 2(1.3K')\theta_C$$
$$M_{CB} = 4(1.3K')\theta_C + 2(1.3K')\theta_B$$
$$M_{CD} = 4(2K')\theta_C + 2(2K')\theta_D - 6(2K')(2\rho_1 - \rho_2)$$
$$M_{DC} = 4(2K')\theta_D + 2(2K')\theta_C - 6(2K')(2\rho_1 - \rho_2)$$

Smarter. Stronger. Steel.

87

Slope Deflection

- Member end moments

$$M_{DG} = 4(2K')\theta_D - 6(2K')\rho_2$$
$$M_{GD} = 2(2K')\theta_D - 6(2K')\rho_2$$
$$M_{DE} = 4(1.3K')\theta_D + 2(1.3K')\theta_E$$
$$M_{ED} = 4(1.3K')\theta_E + 2(1.3K')\theta_D$$
$$M_{EF} = 4(2K')\theta_E - 6(2K')\rho_2$$
$$M_{FE} = 2(2K')\theta_E - 6(2K')\rho_2$$

Smarter. Stronger. Steel.

88

Slope Deflection

- Joint equilibrium equations

$$\sum M_B = 0 = M_{BA} + M_{BC} = 4K'\theta_B - 6K'\rho_1 + 4(1.3K')\theta_B + 2(1.3K')\theta_C$$
$$\sum M_C = 0 = M_{CB} + M_{CD} = 4(1.3K')\theta_C + 2(1.3K')\theta_B + 4(2K')\theta_C + 2(2K')\theta_D - 6(2K')(2\rho_1 - \rho_2)$$
$$\sum M_D = 0 = M_{DC} + M_{DE} + M_{DG} = 4(2K')\theta_D + 2(2K')\theta_C - 6(2K')(2\rho_1 - \rho_2) + 4(1.3K')\theta_D + 2(1.3K')\theta_E + 4(2K')\theta_D - 6(2K')\rho_2$$
$$\sum M_E = 0 = M_{ED} + M_{EF} = 4(1.3K')\theta_E + 2(1.3K')\theta_D + 4(2K')\theta_E - 6(2K')\rho_2$$

Smarter. Stronger. Steel. 89

Slope Deflection

- Horizontal equilibrium

$$H_G + H_F - H_{DC} - 10 = 0$$
$$\frac{M_{GD} + M_{DG}}{13} + \frac{M_{FE} + M_{EF}}{13} - \frac{M_{DC} + M_{CD}}{13} - 10 = 0$$
$$M_{GD} + M_{DG} + M_{FE} + M_{EF} - M_{DC} - M_{CD} = 130$$

Smarter. Stronger. Steel. 90

Slope Deflection

- Substitute for the moments

$$M_{GD} + M_{DG} + M_{FE} + M_{EF} - M_{DC} - M_{CD} = 130$$
$$4(2K')\theta_D - 6(2K')\rho_2 + 2(2K')\theta_D - 6(2K')\rho_2$$
$$+4(2K')\theta_E - 6(2K')\rho_2 + 2(2K')\theta_E - 6(2K')\rho_2$$
$$-4(2K')\theta_C - 2(2K')\theta_D + 6(2K')(2\rho_1 - \rho_2)$$
$$-4(2K')\theta_D - 2(2K')\theta_C + 6(2K')(2\rho_1 - \rho_2) = 130$$

$$\boxed{-12K'\theta_C + 12K'\theta_E + 48K'\rho_1 - 72K'\rho_2 = 130}$$

Smarter. Stronger. Steel. 91

Slope Deflection

- Horizontal equilibrium

$$H_A + H_{DC} + 20 = 0$$
$$\frac{M_{AB} + M_{BA}}{26} + \frac{M_{DC} + M_{CD}}{13} + 20 = 0$$
$$M_{AB} + M_{BA} + 2M_{DC} + 2M_{CD} = -520$$

Smarter. Stronger. Steel. 92

Slope Deflection

- Substitute for the moments

$$M_{AB} + M_{BA} + 2M_{DC} + 2M_{CD} = -520$$

$$\begin{aligned} &2K'\theta_B - 6K'\rho_1 + 4K'\theta_B - 6K'\rho_1 \\ &+2\left[4(2K')\theta_C + 2(2K')\theta_D - 6(2K')(2\rho_1 - \rho_2)\right] \\ &+2\left[4(2K')\theta_D + 2(2K')\theta_C - 6(2K')(2\rho_1 - \rho_2)\right] = -520 \end{aligned}$$

$$6K'\theta_B + 24K'\theta_C + 24K'\theta_D - 108K'\rho_1 + 48K'\rho_2 = -520$$

Smarter. Stronger. Steel.

93

Slope Deflection

- Resulting 6 simultaneous equations

$9.2K'\theta_B$	$+2.6K'\theta_C$			$-6K'\rho_1$		=	0
$2.6K'\theta_B$	$+13.2K'\theta_C$	$+4K'\theta_D$		$-24K'\rho_1$	$+12K'\rho_2$	=	0
	$+4K'\theta_C$	$+21.2K'\theta_D$	$+2.6K'\theta_E$	$-24K'\rho_1$		=	0
		$+2.6K'\theta_D$	$+13.2K'\theta_E$		$-12K'\rho_2$	=	0
$6K'\theta_B$	$+24K'\theta_C$	$+24K'\theta_D$		$-108K'\rho_1$	$+48K'\rho_2$	=	−520
	$-12K'\theta_C$		$+12K'\theta_E$	$+48K'\rho_1$	$-72K'\rho_2$	=	130

Smarter. Stronger. Steel.

94

Slope Deflection

- Question to consider
  - How does symmetry play a role in our solutions?
    - Only for a symmetric structure will the equilibrium equations be symmetric.
    - The equilibrium equations will always automatically come out symmetric.
    - The equilibrium equations can always be made symmetric.
    - Symmetry depends on how we define our variables.

Smarter. Stronger. Steel.

95

Polling Questions

Smarter. Stronger. Steel.

96

Slope Deflection

- Resulting 6 simultaneous equations

$$
\begin{array}{ccccccccc}
9.2K'\theta_B & +2.6K'\theta_C & & & -6K'\rho_1 & & = & 0 \\
2.6K'\theta_B & +13.2K'\theta_C & +4K'\theta_D & & -24K'\rho_1 & +12K'\rho_2 & = & 0 \\
 & +4K'\theta_C & +21.2K'\theta_D & +2.6K'\theta_E & -24K'\rho_1 & & = & 0 \\
 & & +2.6K'\theta_D & +13.2K'\theta_E & & -12K'\rho_2 & = & 0 \\
6K'\theta_B & +24K'\theta_C & +24K'\theta_D & & -108K'\rho_1 & +48K'\rho_2 & = & -520 \\
 & -12K'\theta_C & & +12K'\theta_E & +48K'\rho_1 & -72K'\rho_2 & = & 130
\end{array}
$$

See how the 5th row and 5th column illustrate symmetry if we multiply the last 2 equations by -1.

Smarter. Stronger. Steel.

97

Slope Deflection

- Solution to simultaneous equations

$$K'\theta_B = 4.30$$
$$K'\theta_C = 15.19$$
$$K'\theta_D = 11.81$$
$$K'\theta_E = 2.03$$
$$K'\rho_1 = 13.18$$
$$K'\rho_2 = 4.79$$

Smarter. Stronger. Steel.

98

Slope Deflection

- Final Moments

$$M_{AB} = 2K'\theta_B - 6K'\rho_1 = 2(4.30) - 6(13.18) = -70.5$$
$$M_{BA} = 4K'\theta_B - 6K'\rho_1 = 4(4.30) - 6(13.18) = -61.9$$
$$M_{BC} = 4(1.3K')\theta_B + 2(1.3K')\theta_C = 5.2(4.30) + 2.6(15.19) = 61.9$$
$$M_{CB} = 4(1.3K')\theta_C + 2(1.3K')\theta_B = 5.2(15.19) + 2.6(4.30) = 90.2$$
$$M_{CD} = 4(2K')\theta_C + 2(2K')\theta_D - 6(2K')(2\rho_1 - \rho_2)$$
$$= 8(15.19) + 4(11.81) - 12(2(13.18) - 4.79) = -90.1$$
$$M_{DC} = 4(2K')\theta_D + 2(2K')\theta_C - 6(2K')(2\rho_1 - \rho_2)$$
$$= 8(11.81) + 4(15.19) - 12(2(13.18) - 4.79) = -103.6$$

Smarter. Stronger. Steel.

99

Slope Deflection

- Final Moments

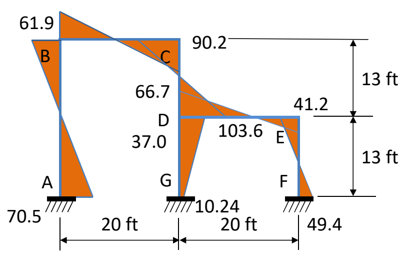
$$M_{DG} = 4(2K')\theta_D - 6(2K')\rho_2 = 8(11.81) - 12(4.79) = 37.0$$
$$M_{GD} = 2(2K')\theta_D - 6(2K')\rho_2 = 4(11.81) - 12(4.79) = -10.24$$
$$M_{DE} = 4(1.3K')\theta_D + 2(1.3K')\theta_E = 5.2(11.81) + 2.6(2.03) = 66.7$$
$$M_{ED} = 4(1.3K')\theta_E + 2(1.3K')\theta_D = 5.2(2.03) + 2.6(11.81) = 41.2$$
$$M_{EF} = 4(2K')\theta_E - 6(2K')\rho_2 = 8(2.03) - 12(4.79) = -41.2$$
$$M_{FE} = 2(2K')\theta_E - 6(2K')\rho_2 = 4(2.03) - 12(4.79) = -49.4$$

Smarter. Stronger. Steel.

100

Slope Deflection

- Final Moment diagram (ft-k)

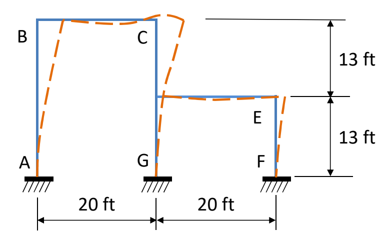


Smarter. Stronger. Steel.

101

Slope Deflection

- Deflected shape



Smarter. Stronger. Steel.

102

Modern Methods

- The 6 simultaneous equations from this example can be viewed in matrix form.

$$\begin{bmatrix} 9.2 & 2.6 & 0 & 0 & -6 & 0 \\ 2.6 & 13.2 & 4 & 0 & -24 & 12 \\ 0 & 4 & 21.2 & 2.6 & -24 & 0 \\ 0 & 0 & 2.6 & 13.2 & 0 & -12 \\ 6 & 24 & 24 & 0 & -108 & 48 \\ 0 & -12 & 0 & 12 & 48 & -72 \end{bmatrix} \begin{Bmatrix} K'\theta_B \\ K'\theta_C \\ K'\theta_D \\ K'\theta_E \\ K'\rho_1 \\ K'\rho_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -520 \\ 130 \end{Bmatrix}$$

- This solution includes only flexural deformations.
- If axial deformations and shearing deformations were included we would be approaching the stiffness method more closely.

Smarter. Stronger. Steel.

103

Summary

- Obtained equations for the absolute stiffness, carry-over factor and fixed-end moments for flexural members.
- Developed the method of slope deflection and applied it to continuous beams and sway permitted frames.
- Considered the special case of known support settlements.
- Determined the effect of temperature change on an indeterminate beam.
- Compared the slope deflection method to the modern stiffness method.

Smarter. Stronger. Steel.

104

Lesson 7

- Approximate Methods and Moment Distribution
  - Introduce the Portal Method and the Cantilever Method for multi-story frames.
  - Develop the Moment Distribution Method for continuous beams.
  - Introduce an approach to check the process of moment distribution.
  - Apply Moment Distribution to frames with side sway permitted.

Smarter. Stronger. Steel.

105

Thank You

American Institute of Steel Construction
130 East Randolph, Suite 2000
Chicago, IL 60601

Smarter. Stronger. Steel.

106

Individual Webinar Registrants

CEU/PDH Certificates

Within 2 business days…

- You will receive an email on how to report attendance from: registration@aisc.org.
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!

Smarter. Stronger. Steel.

Individual Webinar Registrants

CEU/PDH Certificates

Within 2 business days…

- New reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.

Smarter. Stronger. Steel.

8-Session Registrants

CEU/PDH Certificates

One certificate will be issued at the conclusion of all 8 sessions.

Smarter. Stronger. Steel.

8-Session Registrants

Access to the quiz: Information for accessing the quiz will be emailed to you by Wednesday. It will contain a link to access the quiz. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG

Quiz and Attendance records: Posted Tuesday mornings.
www.aisc.org/nightschool - click on Current Course Details.

Reasons for quiz:

- EEU – must take all quizzes and final to receive EEU
- CEUs/PDHS – If you watch a recorded session you must take quiz for CEUs/PDHs.
- REINFORCEMENT – Reinforce what you learned tonight. Get more out of the course.

NOTE: If you attend the live presentation, you do not have to take the quizzes to receive CEUs/PDHs.

Smarter. Stronger. Steel.

8-Session Registrants

Access to the recording: Information for accessing the recording will be emailed to you by this Wednesday. The recording will be available for three weeks. For 8-session registrants only. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG.

CEUs/PDHS – If you watch a recorded session you must take AND PASS the quiz for CEUs/PDHs.

Smarter. Stronger. Steel.

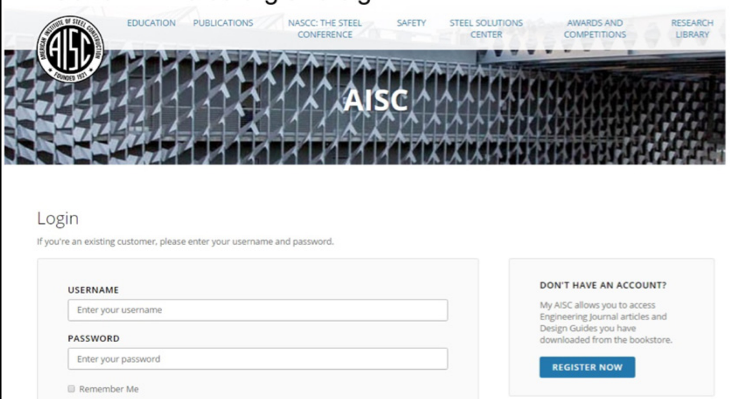
Night School Resources for 8-session package Registrants

Find all your handouts, quizzes and quiz scores, recording access, and attendance information all in one place!

Smarter. Stronger. Steel.

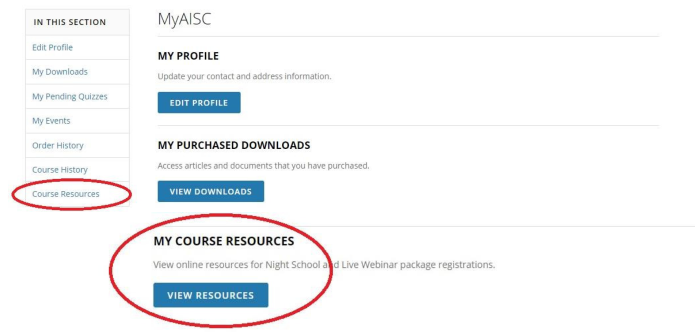
Night School Resources for 8-session package Registrants

Go to www.aisc.org and sign in.



Night School Resources for 8-session package Registrants

Go to www.aisc.org and sign in.



Please fill out our brief survey at the conclusion of the webinar. We greatly appreciate your feedback.

AISC | Thank you

Smarter.
Stronger.
Steel.

AISC | Thank you

Smarter.
Stronger.
Steel.