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Classical Methods of Structural Analysis

Louis F. Geschwindner

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Session Description
20.7 Approximate Methods and Moment Distribution
July 29, 2019

The portal and cantilever methods of approximate frame analysis will be presented. The method of moment distribution will be introduced and the special approach for treatment of frames that exhibit sidesway will be presented. In addition, an approach that permits a somewhat independent checking of the moment distribution process will be developed.

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Learning Objectives:

- Describe the moment distribution method for continuous beams and frame analysis.
- Identify an independent method (slope deflection) for checking of the moment distribution process.
- Compare the portal and cantilever methods.
- Identify ways to simplify multi-story frames for gravity load analysis.

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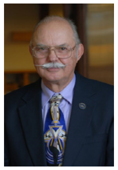
Night School 20
Classical Methods of Structural Analysis

Session 7: Approximate Methods and Moment Distribution
July 29, 2019



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Classical Methods of Structural Analysis:
How we did it before computers

Night School 20
Lesson 7
Approximate Methods and Moment Distribution

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9

Lesson 7
Approximate Methods and Moment Distribution

- Introduce the Moment Distribution Method for continuous beams and frames.
- Apply Moment Distribution to frames when sidesway is permitted.
- Use slope deflection to develop a technique to check the math in the process of moment distribution.
- Look at ways to simplify multi-story frames for gravity load analysis.
- Develop and compare the Portal Method and the Cantilever Method for multi-story frames.

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Moment Distribution

- You can get 80% of the truth with a simple approach to structural analysis.
- The most complicated approach will get you at best an additional 10% toward the truth.

Attributed to Hardy Cross

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11

Moment Distribution

- Hardy Cross taught the method of moment distribution to his students at the University of Illinois in 1924.
- He first published the method in the *Proceedings* of ASCE in May, 1930.
- During the years after its publication, it received much attention in professional journals.
- Up to the time of the introduction of modern computing capability, it was perhaps the most widely used method for analysis of multi-story structures.

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12

Moment Distribution

- Two properties that we developed for use with Slope Deflection (Lesson 6) will be useful for Moment Distribution.
  - Absolute stiffness: The moment required to produce a unit rotation at the simply supported end of a beam when the other end is fixed.
  - Carry-over factor: The relationship between the moment applied to the simply supported end of the beam and the resisting moment developed at the other end.

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Moment Distribution

- For prismatic members, the absolute flexural stiffness, K_{AB}, is

$$K_{AB} = \frac{4EI}{L}$$

and the carry-over factor, C_{AB}, is

$$C_{AB} = \frac{1}{2}$$

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Moment Distribution

- Relative Stiffness: In one of the examples from Lesson 6 we introduced the concept of relative stiffness, K'. It will be helpful to formalize that for moment distribution. Since $4E$ is a constant we can ignore it in our relative stiffness.

Absolute Stiffness $K_{AB} = \frac{4E_{AB}I_{AB}}{L_{AB}}$

Relative Stiffness $K'_{AB} = \frac{I_{AB}}{L_{AB}}$

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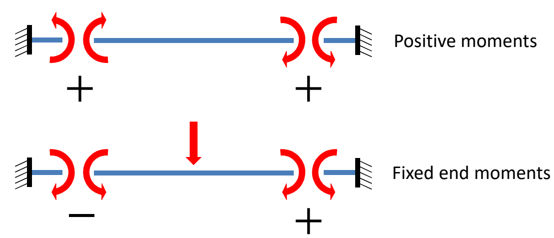
Moment Distribution

- Sign convention: clockwise on the end of the member is positive. Thus, counterclockwise on the joint is positive. (The same as for slope deflection)



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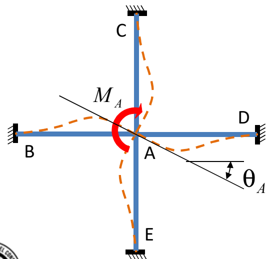
Moment Distribution

- Distribution Factor: The portion of the moment applied at a joint that is induced into a specific member at that joint.



$$M_{AB} = \frac{4EI_{AB}}{L_{AB}}\theta_A \quad M_{AC} = \frac{4EI_{AC}}{L_{AC}}\theta_A$$

$$M_{AD} = \frac{4EI_{AD}}{L_{AD}}\theta_A \quad M_{AE} = \frac{4EI_{AE}}{L_{AE}}\theta_A$$

$$M_A = M_{AB} + M_{AC} + M_{AD} + M_{AE}$$

If each of 4 members have the same stiffness, each member will take ¼ of the moment M_A.

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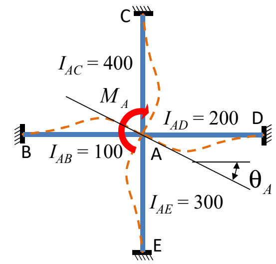
17

Moment Distribution

- If the members have differing stiffnesses,



$$M_{AB} = \frac{4E(100)}{L}\theta_A \quad M_{AC} = \frac{4E(400)}{L}\theta_A$$

$$M_{AD} = \frac{4E(200)}{L}\theta_A \quad M_{AE} = \frac{4E(300)}{L}\theta_A$$

$$M_A = M_{AB} + M_{AC} + M_{AD} + M_{AE}$$

$$M_A = \frac{4E(1000)}{L}\theta_A$$

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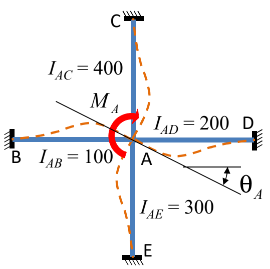
18

Moment Distribution

- If the members have differing stiffnesses,



$$\frac{4E\theta_A}{L} = \frac{M_A}{(1000)}$$

$$M_{AB} = \frac{100}{1000}M_A \quad M_{AC} = \frac{400}{1000}M_A$$

$$M_{AD} = \frac{200}{1000}M_A \quad M_{AE} = \frac{300}{1000}M_A$$

The moment, M_A, will be distributed according to member stiffness with respect to the total stiffness at the joint.

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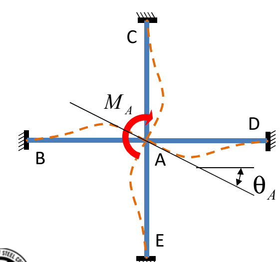
Moment Distribution

- Distribution Factor: The stiffness of the member framing into a joint divided by the sum of the stiffnesses of all members framing into the joint



$$DF_{NF} = \frac{K_{NF}}{\sum K_N}$$

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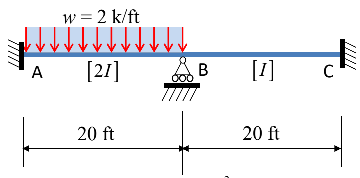
20

Moment Distribution

- Introduce Moment Distribution with a simple example.



$$FEM_{AB} = -\frac{wL^2}{12} = -\frac{2(20)^2}{12} = -66.7 \text{ ft-kip}$$

$$FEM_{BA} = \frac{wL^2}{12} = \frac{2(20)^2}{12} = 66.7 \text{ ft-kip}$$

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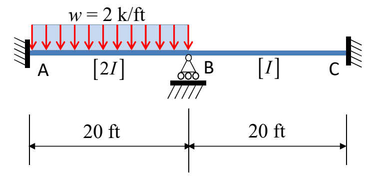
Moment Distribution



$$K'_{AB} = \frac{2I}{20} = 2 \qquad K'_{BC} \frac{I}{20} = 1$$

$$DF_{BA} = \frac{2}{(2+1)} = 0.67 \quad DF_{BC} = \frac{1}{(2+1)} = 0.33$$

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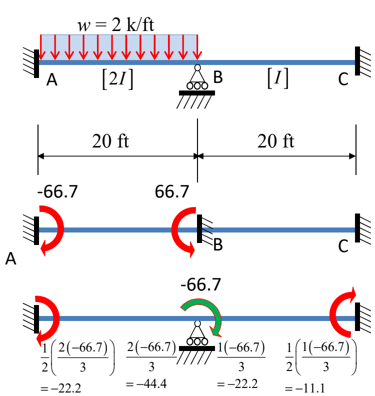
Moment Distribution



Fixed end moments on the joint

Release joint B and apply -66.7 ft-kips to put joint in equilibrium.

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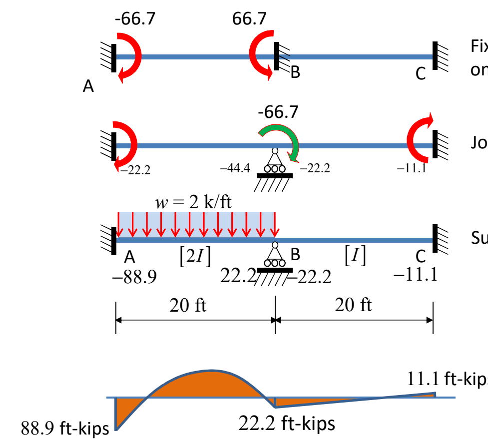
Moment Distribution



Fixed end moments on the joint

Joint balanced

Sum of moments

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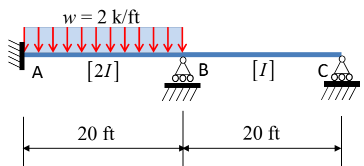
Moment Distribution

- Consider the same beam with end C a pin.



$$FEM_{AB} = -\frac{wL^2}{12} = -\frac{2(20)^2}{12} = -66.7 \text{ ft-kip}$$

$$FEM_{BA} = \frac{wL^2}{12} = \frac{2(20)^2}{12} = 66.7 \text{ ft-kip}$$

No change in fixed end moments

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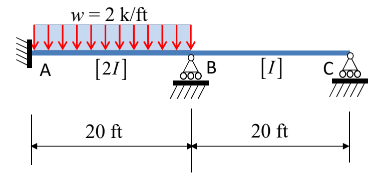
Moment Distribution



$$K'_{AB} = \frac{2I}{20} = 2 \qquad K'_{BC} = \frac{I}{20} = 1$$

$$DF_{BA} = \frac{2}{3} = 0.67 \qquad DF_{BC} = \frac{1}{3} = 0.33$$

No change in relative stiffness or distribution factors

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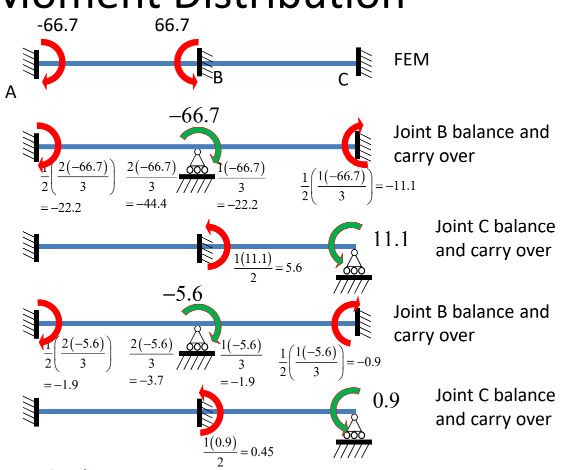
26

Moment Distribution



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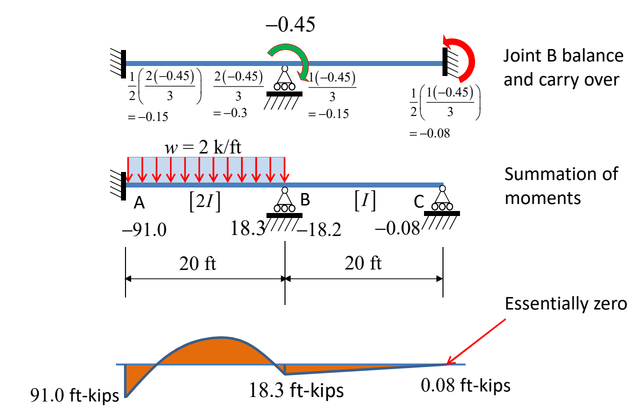
27

Moment Distribution



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28

Moment Distribution

- The first approach to this tabulation was to write the numbers under the beam in a way similar to what we just saw in the preceding two examples.
- However, when the structure under consideration is more complex than just a beam, such as a frame with columns and beams, this tabulation approach can become messy.

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Moment Distribution

- Thus, a useful approach is to use a table, completely disconnected from the graphic of the structure. For this example this will be:

Joint	A	B		C
Member	AB	BA	BC	CB
K	2	2	1	1
DF	0	0.67	0.33	1
FEM	-66.7	66.7		
	-22.2 ←	-44.4	-22.2 →	-11.1
			5.6 ←	11.1
	-1.9 ←	-3.7	-1.9 →	-0.9
			0.45 ←	0.9
	-0.15 ←	-0.3	-0.15 →	?
Total	-90.95	18.3	-18.2	0

Note that the fixed end just keeps soaking up the induced moment.

How do we know when to stop?

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Moment Distribution

- Note that when the far end is a pin, we do the same thing every time.

Joint	A	B		C
Member	AB	BA	BC	CB
K	2	2	1	1
DF	0	0.67	0.33	1
FEM	-66.7	66.7		
	-22.2 ←	-44.4	-22.2 →	-11.1
			5.6 ←	11.1
	-1.9 ←	-3.7	-1.9 →	-0.9
			0.45 ←	0.9
	-0.15 ←	-0.3	-0.15	
Total	-90.95	18.3	-18.2	0

When a moment is induced into the near end (-22.2), ½ is carried over to the pin (-11.1) and then -½ of that is carried back to the near end (5.6) and the pin remains at zero.

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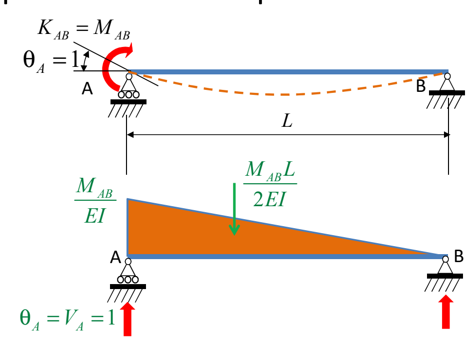
Moment Distribution

- The absolute stiffness of a beam with the far end pinned can be helpful in this situation.



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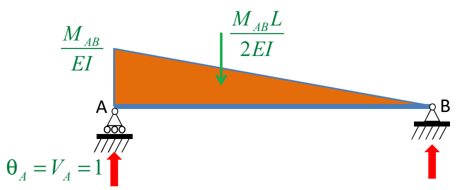
32

Moment Distribution

- Absolute stiffness



Sum of the moments about B $\quad \frac{M_{AB}L}{2EI}\left(\frac{2L}{3}\right) - \theta_A L = 0 \rightarrow M_{AB} = \frac{3EI}{L}\theta_A$

$$K_{AB} = \frac{3EI}{L}$$

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Moment Distribution

- Relative Stiffness: Now look at relative stiffness when the far end is a pin.

Absolute Stiffness $\quad K_{AB} = \frac{4E_{AB}I_{AB}}{L_{AB}}$

Relative Stiffness $\quad K'_{AB} = \frac{I_{AB}}{L_{AB}}$

Relative Stiffness, far end pinned

$$K_{AB} = \frac{3E_{AB}I_{AB}}{L_{AB}} = \frac{3}{4}\left[\frac{4E_{AB}I_{AB}}{L_{AB}}\right] \rightarrow K'_{AB} = \frac{3}{4}\frac{I_{AB}}{L_{AB}}$$

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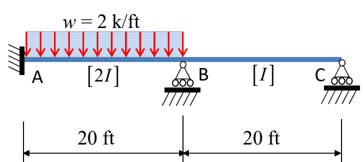
Moment Distribution

With the end of the member a pin, take advantage of the relative stiffness for that condition.



$K'_{AB} = \frac{2I}{20} = 2 \qquad K'_{BC} = \frac{3}{4}\left[\frac{I}{20}\right] = 0.75$

$DF_{BA} = \frac{2}{2.75} = 0.73 \qquad DF_{BC} = \frac{0.75}{2.75} = 0.27$

The change in relative stiffness means a change in the distribution factors

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Moment Distribution

- Using the same table as the previous example illustrates quickly how much less space (and thus calculation effort) is required.

Joint	A	B		C
Member	AB	BA	BC	CB
K	2	2	0.75(1)	1
DF	0	0.73	0.27	1
FEM	-66.7	66.7		
	-24.3 ←	-48.7	-18.0 →	
Total	-91.0	18.0	-18.0	0

When we use the reduced stiffness for the far end pinned, we never carry over to that end.

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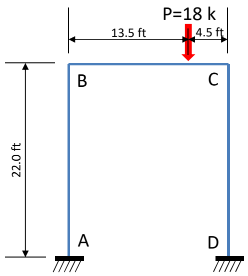
36

Moment Distribution

- Now consider a frame. In this case with constant moment of inertia.



$$K'_{AB} = K'_{CD} = \frac{I}{22} = 1.0$$

$$K'_{BC} = \frac{I}{18} = 1.22$$

$$FEM_{BC} = \frac{Pab^2}{L^2} = \frac{18(13.5)(4.5)^2}{18^2} = -15.2 \text{ ft-kips}$$

$$FEM_{CB} = \frac{Pa^2b}{L^2} = \frac{18(13.5)^2(4.5)}{18^2} = 45.6 \text{ ft-kips}$$

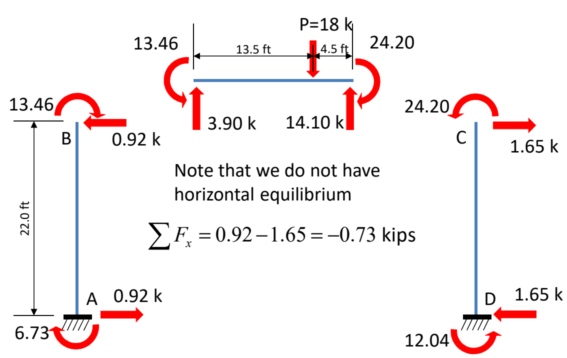
Moment Distribution

- Distribution table

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
K	1	1	1.22	1.22	1	1
DF	0	0.45	0.55	0.55	0.45	0
FEM			-15.19	45.56		
	3.42 ←	6.84	8.35 →	4.18		
			-13.68 ←	-27.36	-22.38 →	-11.19
	3.08 ←	6.15	7.52 →	3.76		
			-1.03 ←	-2.07	-1.69 →	-0.85
	0.23 ←	0.47	0.57 →	0.28		
				-0.16	-0.13	
Total	6.73	13.46	-13.46	24.20	-24.20	-12.04

Moment Distribution

- Member shears

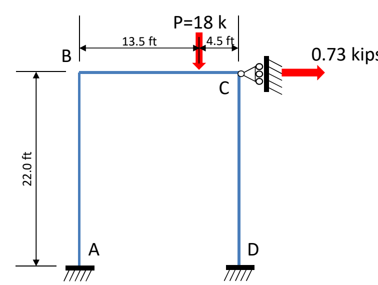


Note that we do not have horizontal equilibrium

$$\sum F_x = 0.92 - 1.65 = -0.73 \text{ kips}$$

Moment Distribution

- The structure we have actually analyzed is

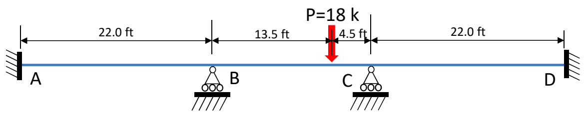


The horizontal reaction is not supposed to be there. It is sometimes called the Artificial Joint Restraint (AJR).

Note that the horizontal reaction is to the right, this means that the frame is trying to sway to the left.

Moment Distribution

- The solution is the same as what we would obtain for a 3-span continuous beam as shown.



- This beam has the same stiffness, distribution factors and fixed end moments as the frame.

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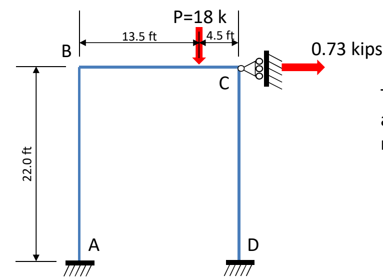
41

Moment Distribution

- The structure we have actually analyzed is



To get back to the structure that we are really looking for, we must remove this AJR.

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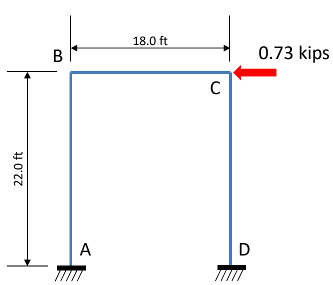
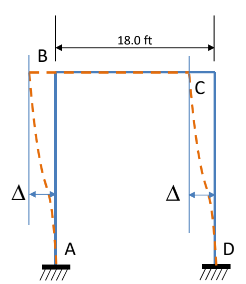
Moment Distribution

- We must analyze the structure for this loading.





Fix the ends against rotation and displace the structure as shown.

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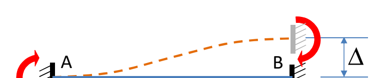
Moment Distribution

- Fixed end moment for sway or support settlement.



$$FEM_{AB} = \frac{6EI\Delta}{L^2} \qquad FEM_{BA} = \frac{6EI\Delta}{L^2}$$

These fixed end moments can be derived through the conjugate beam or slope deflection methods.

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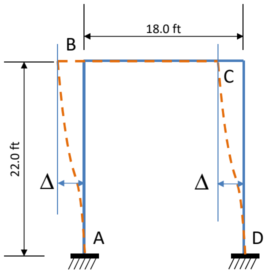
Moment Distribution

- In our case, we do not know the magnitude of the sway, Δ.



What we do know is that the fixed end moments on both columns will be the same.

Arbitrarily select a moment magnitude similar to what we already calculated when there was an AJR, say 10 ft-kips.

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Moment Distribution

- Distribution table for sway

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
K	1	1	1.22	1.22	1	1
DF	0	0.45	0.55	0.55	0.45	0
FEM	10.0	10.0			10.0	10.0
	-2.25 ←	-4.50	-5.50 →	-2.75		
			-1.99 ←	-3.99	-3.26 →	-1.63
	0.45 ←	0.9	1.10 →	0.55		
			-0.15 ←	-0.30	-0.25 →	-0.13
		0.07	0.08			
Total	8.20	6.47	-6.46	-6.49	6.49	8.24

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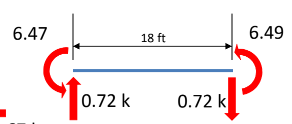
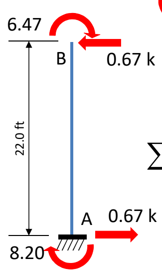
Moment Distribution

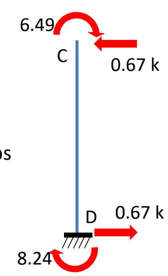
- Member shears due to sway





Note that we again do not have horizontal equilibrium

$$\sum F_x = 0.67 + 0.67 = 1.34 \text{ kips}$$



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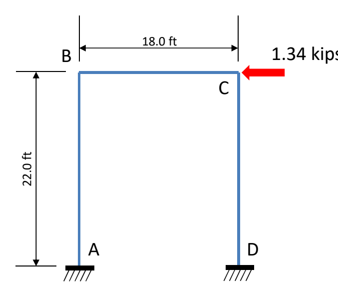
Moment Distribution

- Now the structure we actually analyzed is



The horizontal force is the result of sway. It is called the Consistent Joint Force (CJF).

But it is not the magnitude force that we need, we need z times that.

$$0.73 = z(1.34)$$

$$z = \frac{0.73}{1.34} = 0.54$$

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Moment Distribution

- Modify the sway results by $z = 0.54$.

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
K	1	1	1.22	1.22	1	1
DF	0	0.45	0.55	0.55	0.45	0
FEM	10.0	10.0			10.0	10.0
	-2.25	-4.50	-5.50	-2.75		
			-1.99	-3.99	-3.26	-1.63
	0.45	0.9	1.10	0.55		
			-0.15	-0.30	-0.25	-0.13
		0.07	0.08			
Total	8.20	6.47	-6.46	-6.49	6.49	8.24
z(Total)	4.43	3.49	-3.49	-3.49	3.49	4.45

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49

Moment Distribution

- Final Results Table.

Joint	A	B		C		D	
Member	AB	BA	BC	CB	CD	DC	
1st cycle	6.73	13.46	-13.46	24.20	-24.20	-12.04	Slide 38
2nd cycle	8.20	6.47	-6.46	-6.49	6.49	8.24	Slide 46
z(2nd)	4.43	3.49	-3.49	-3.49	3.49	4.45	Slide 49
Final	11.16	16.95	-16.95	20.71	-20.71	-7.59	

Final = 1st cycle + z(2nd cycle)

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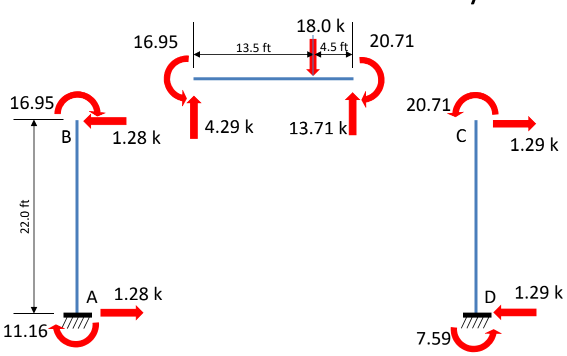
50

Moment Distribution

- Member shears due to load and sway



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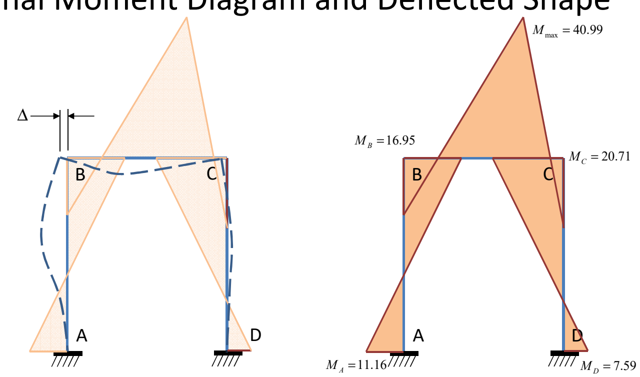
51

Moment Distribution

- Final Moment Diagram and Deflected Shape



Compare to Lesson 6 Slide 75

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52

Moment Distribution

- Check on moment distribution: Using the slope deflection equations, we can develop a simple approach for checking our math. It will only check the math, not the correctness of any of our starting values, such as FEM, K, or DF.

$$M_N = K_N\theta_N + C_F K_F \theta_F - (K_N + C_F K_F)\rho + FEM_N$$

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53

Moment Distribution

- Slope deflection equations without sway.

$$\begin{aligned} M_N &= K_N\theta_N + C_F K_F \theta_F + FEM_N \\ &= \Delta FEM_N + FEM_N \end{aligned}$$

- Therefore:

$$\Delta FEM_N = K_N\theta_N + C_F K_F \theta_F$$

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54

Moment Distribution

- Write the change in fixed end moment for member AB.

$$\Delta FEM_{AB} = K_{AB}\theta_A + C_{BA}K_{BA}\theta_B$$
$$\Delta FEM_{BA} = K_{BA}\theta_B + C_{AB}K_{AB}\theta_A$$

- Solve the first equation for θ_A then substitute into the second and solve for θ_B.

$$\theta_B = \frac{\Delta FEM_{BA} - C_{AB}\Delta FEM_{AB}}{K_{BA}(1 - C_{AB}C_{BA})}$$

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55

Moment Distribution

- Rewrite in terms of near and far and substitute for prismatic members.

$$\theta_B = \frac{\Delta FEM_{BA} - C_{AB}\Delta FEM_{AB}}{K_{BA}(1 - C_{AB}C_{BA})} = \frac{\Delta FEM_N - \frac{1}{2}\Delta FEM_F}{\left(\frac{4EI}{L}\right)\left(1 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\right)}$$

$$\theta_N = \frac{\Delta FEM_N - \frac{1}{2}\Delta FEM_F}{\left(\frac{3}{4}\right)\left(4EI/L\right)} = \frac{2\Delta FEM_N - \Delta FEM_F}{6EI/L}$$

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56

Moment Distribution

- Since we do not need the actual rotation, we can use relative stiffness.

$$(\theta_N)_{rel} = \frac{2\Delta FEM_N - \Delta FEM_F}{K'_N}$$

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57

Moment Distribution

- We now have an equation for the relative rotation at a joint based on the change in moment on the member framing into the joint and the change in moment at the other end of that member.
- For each member, the rotation at the joint should be the same.
- Thus, we can check the relative rotation at the end of each member framing into a joint and compare them.

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58

Moment Distribution

- For joint B:

$$\theta_{BA} = \frac{2(13.46) - 6.73}{1.0} = 20.2$$

$$\theta_{BC} = \frac{2(1.73) - (-21.36)}{1.22} = 20.3$$

- For joint C:

$$\theta_{CB} = \frac{2(-21.36) - 1.73}{1.22} = -36.4$$

$$\theta_{CD} = \frac{2(-24.20) - (-12.04)}{1.0} = -36.4$$

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
K	1	1	1.22	1.22	1	1
DF	0	0.45	0.55	0.55	0.45	0
FEM			-15.19	45.56		
	3.42 ←	6.84	8.35 →	4.18		
			-13.68 ←	-27.36	-22.38 →	-11.19
	3.08 ←	6.15	7.52 →	3.76		
			-1.03 ←	-2.07	-1.69 →	-0.85
	0.23 ←	0.47	0.57 →	0.28		
				-0.16	-0.13	
Total	6.73	13.46	-13.46	24.20	-24.20	-12.04
ΔFEM	6.73	13.46	1.73	-21.36	-24.20	-12.04
θ		20.19	20.34	-36.43	-36.36	

This is the 1st cycle of our sway frame.

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59

Moment Distribution

- Questions to consider about Moment Distribution
  - When do you stop the balancing and carry over process?
    - After a fixed number of carry-overs
    - After you carry over something less than 1.0
    - Whenever you want
    - When it's "accurate enough?"

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60

Polling Question

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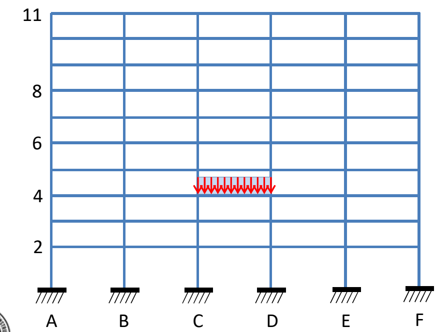
61

Approximate Methods

- Consider a multi-story frame.



What would it take to cut this structure back to make it determinate?

At each beam mid-span, cut the beam and remove the shear, moment, and axial force.

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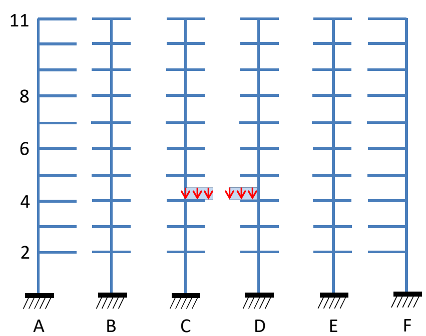
62

Approximate Methods

- Consider a multi-story frame.



What would it take to cut this structure back to make it determinate?

At each beam mid-span, cut the beam and remove the shear, moment, and axial force.

Thus, 3 x number of beams = degree of indeterminacy.

$$3 \times 10 \times 5 = 150$$

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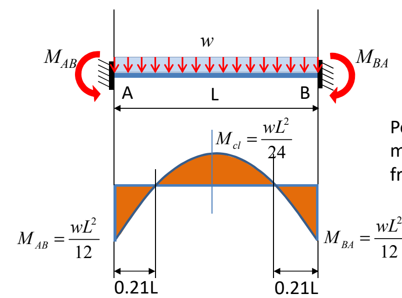
63

Approximate Methods

- How might we treat a beam within the frame?
- Fixed ends,



Point of zero moment at 0.21L from support

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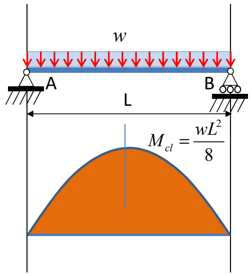
64

Approximate Methods

- How might we treat a beam within the frame?
- Pinned ends,



Point of zero moment at support

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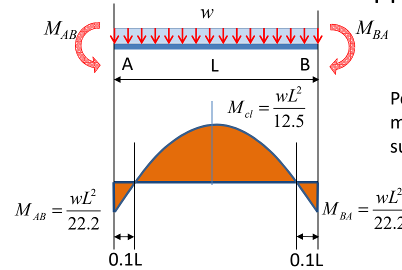
Approximate Methods

- How might we treat a beam within the frame?
- Since the ends will rotate, a good approximation might be zero moment at 0.1L from supports.



Point of zero moment at 0.1L from support

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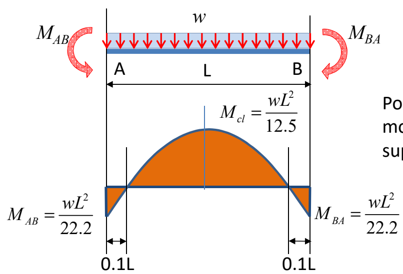
Approximate Methods

- Thus, we could obtain an approximate value for the moments without actually doing an analysis.



Point of zero moment at 0.1L from support

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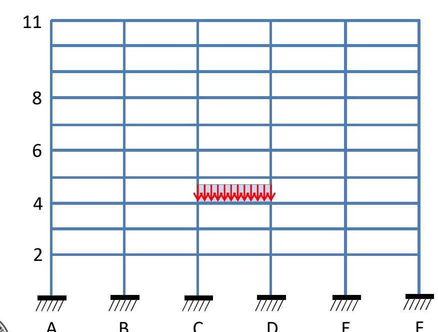
Approximate Methods

- Consider a multi-story frame again.



The farther away you get from the loaded span, the less impact the load will have on the members.

Remember from Lesson 1 the conceptual influence line discussion.

Picture how the deflections and rotations will become quite small as you move away from the loaded span.

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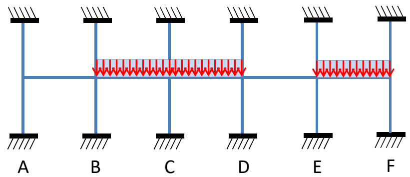
Approximate Methods

- For gravity load we can model the multi-story structure as one level of beams and the columns above and below.



This may be a better approach than assuming an inflection point location and the analysis could easily be carried out through moment distribution.

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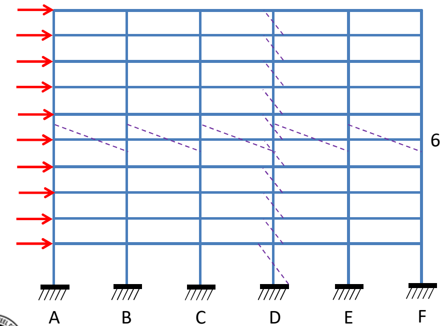
69

Approximate Methods

- For lateral load, what approximations might we make?



The expected moment diagram for each member in the frame loaded with concentrated lateral loads will be straight line segments as shown for column line D and the beam at level 6.

This suggests that inflection points be placed near mid-span and mid-height for all members

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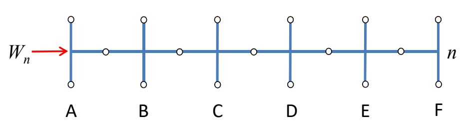
70

Approximate Methods

- The Portal Method (Albert Smith, 1915)
  1. Assume that all beams have an inflection point at mid-span.
  2. Assume that all columns have an inflection point at mid-height.



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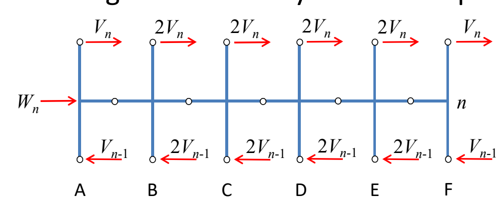
71

Approximate Methods

- The Portal Method
  3. The total horizontal shear on each story is divided between the columns of that story in a manner such that each column carries shear according to its tributary area. For equal spans,



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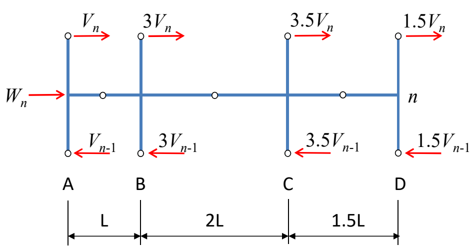
72

Approximate Methods

- The Portal Method
  3. For unequal spans,



Column A $\frac{0.5L}{4.5L} \rightarrow 1$

Column B $\frac{1.5L}{4.5L} \rightarrow 3.0$

Column C $\frac{1.75L}{4.5L} \rightarrow 3.5$

Column D $\frac{0.75L}{4.5L} \rightarrow 1.5$

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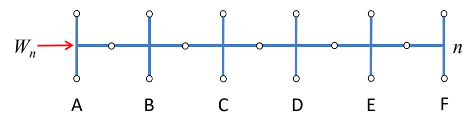
73

Approximate Methods

- The Cantilever Method (A. C. Wilson, 1908)
  1. Assume that all beams have an inflection point at mid-span.
  2. Assume that all columns have an inflection point at mid-height.



The same as for the Portal Method

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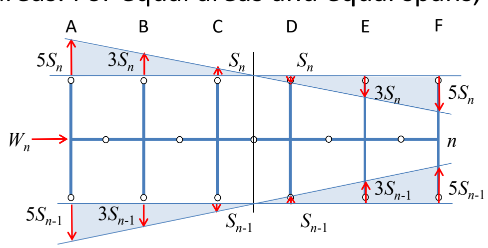
74

Approximate Methods

- The Cantilever Method
  3. The column axial force is proportional to its distance from the center of gravity of the column areas. For equal areas and equal spans,



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75

Approximate Methods

- The portal method, when introduced, was said to be satisfactory for most buildings up to 25 stories.
- The cantilever method is said to be a little more desirable for high narrow buildings than the portal method and is said to be satisfactory for buildings not in excess of 25 to 35 stories.

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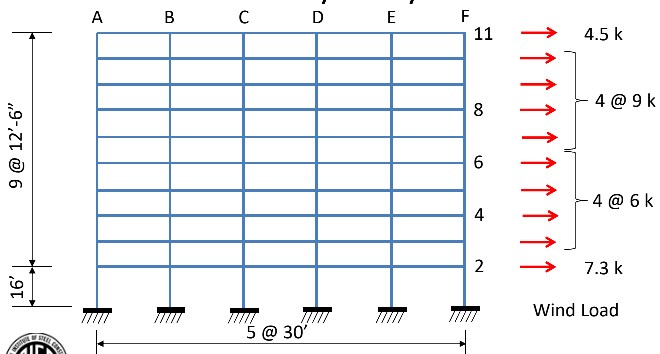
76

Approximate Methods

- Consider a 10-story 5-bay structure.



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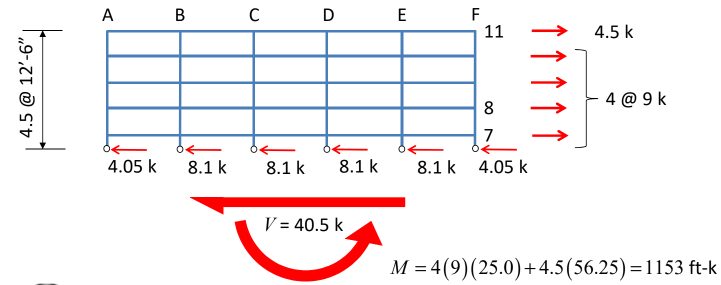
77

Approximate Methods

- Take a free body diagram above Level 6 and distribute the lateral load. (Portal Method)



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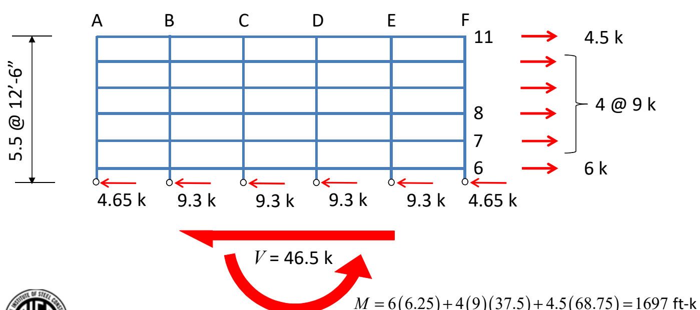
78

Approximate Methods

- Take a free body diagram below Level 6 and distribute the lateral load. (Portal Method)



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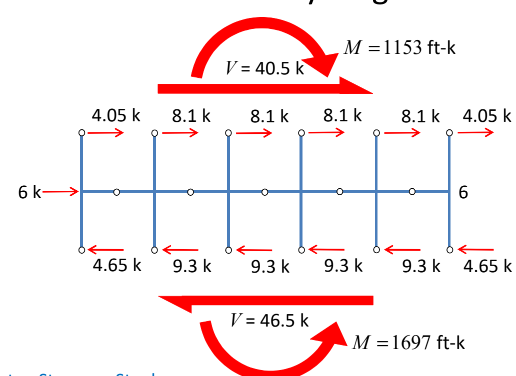
79

Approximate Methods

- Now look at the free body diagram of level 6.



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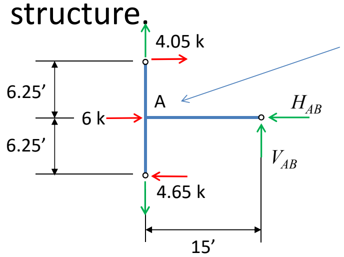
80

Approximate Methods

- To determine the moments in the girders, we will start at the left end and work across the structure.



Sum the moments about the intersection of the beam and column

$$4.05(6.25)+4.65(6.25)-V_{AB}(15.0)=0$$
$$V_{AB}=3.625\text{ k}$$

Sum the forces in the horizontal direction

$$4.05+6.0-4.65-H_{AB}=0$$
$$H_{AB}=5.4$$

Note that at this point, there is no way to determine the column axial forces.

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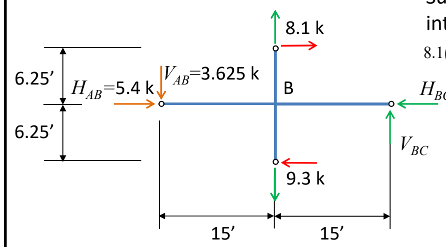
81

Approximate Methods

- Next, apply these now known forces on the next free body and continue the solution.



Sum the moments about the intersection of the beam and column

$$8.1(6.25)+9.3(6.25)-3.625(15.0)-V_{BC}(15.0)=0$$
$$V_{BC}=3.625\text{ k}$$

Sum the forces in the horizontal direction

$$8.1+5.4-9.3-H_{BC}=0$$
$$H_{BC}=4.2$$

We are still unable to determine the column axial forces.

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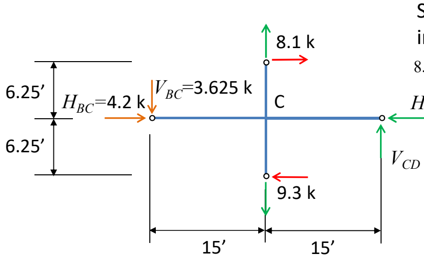
82

Approximate Methods

- Next, apply these now known forces on the next free body and continue the solution.



Sum the moments about the intersection of the beam and column

$$8.1(6.25)+9.3(6.25)-3.625(15.0)-V_{CD}(15.0)=0$$
$$V_{CD}=3.625\text{ k}$$

Sum the forces in the horizontal direction

$$8.1+4.2-9.3-H_{CD}=0$$
$$H_{CD}=3.0$$

We are still unable to determine the column axial forces.

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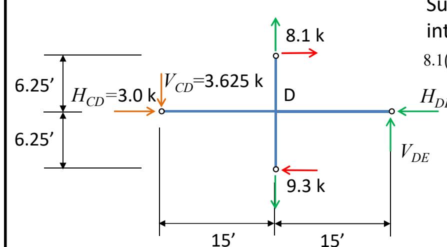
83

Approximate Methods

- Next, apply these now known forces on the next free body and continue the solution.



Sum the moments about the intersection of the beam and column

$$8.1(6.25)+9.3(6.25)-3.625(15.0)-V_{DE}(15.0)=0$$
$$V_{DE}=3.625\text{ k}$$

Sum the forces in the horizontal direction

$$8.1+3.0-9.3-H_{DE}=0$$
$$H_{DE}=1.8$$

We are still unable to determine the column axial forces.

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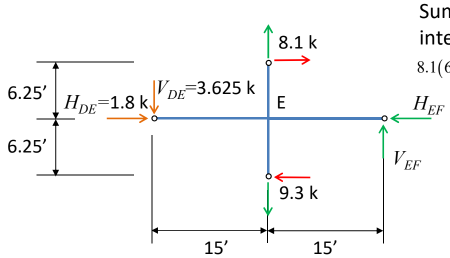
84

Approximate Methods

- Next, apply these now known forces on the next free body and continue the solution.



Sum the moments about the intersection of the beam and column

$$8.1(6.25)+9.3(6.25)-3.625(15.0)-V_{EF}(15.0)=0$$

$$V_{EF}=3.625\text{ k}$$

Sum the forces in the horizontal direction

$$8.1+1.8-9.3-H_{EF}=0$$

$$H_{EF}=0.6$$

We are still unable to determine the column axial forces.

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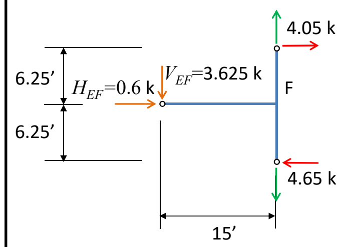
85

Approximate Methods

- Next, apply these now known forces on the next free body and continue the solution.



Sum the moments about the intersection of the beam and column.

$$4.05(6.25)+4.65(6.25)-3.625(15.0)=0$$

Sum the forces in the horizontal direction.

$$4.05+0.6-4.65=0$$

This confirms we have equilibrium.

But still no way to determine the column forces.

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86

Approximate Methods

- Thus, we see that the vertical shear at mid-span of each beam will be the same,
  - In this case, 3.625 k.
- If we were to start at the top level, we would determine that all the axial load is taken by the outer columns on lines A and F and that this carries all the way down to the support.

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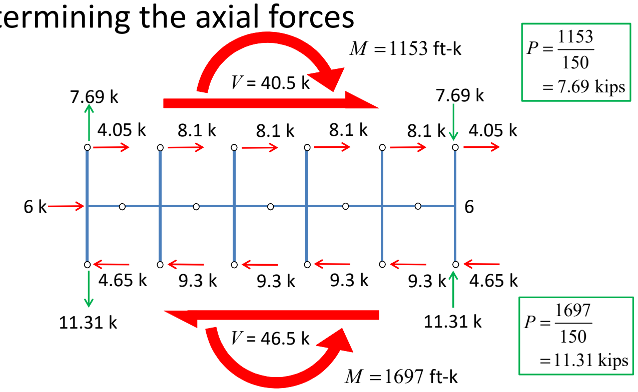
87

Approximate Methods

- Determining the axial forces



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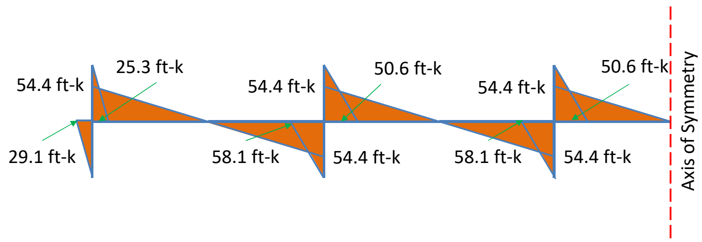
88

Approximate Methods

- A partial moment diagram can be developed



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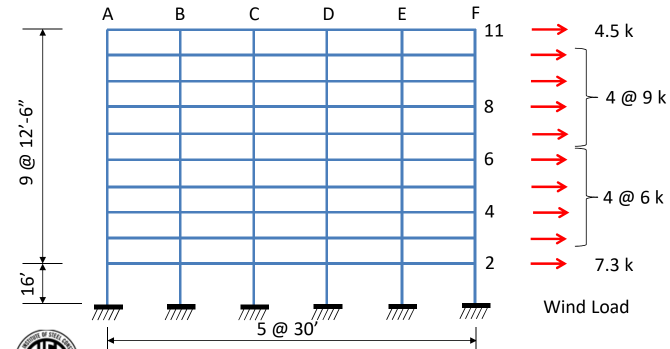
89

Approximate Methods

- Reconsider this structure by the cantilever method.



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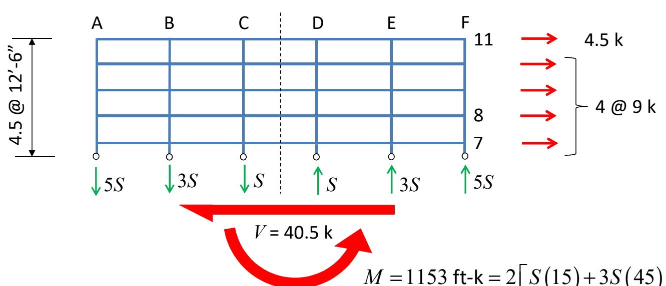
90

Approximate Methods

- Take a free body diagram above Level 6 and determine the axial forces.



$$M = 1153 \text{ ft-k} = 2\left[S(15)+3S(45)+5S(75)\right]$$
$$S = 1.10 \text{ k}$$

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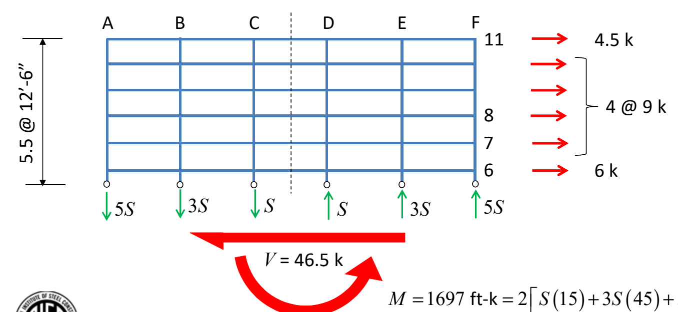
91

Approximate Methods

- Take a free body diagram below Level 6 and determine the axial forces.



$$M = 1697 \text{ ft-k} = 2\left[S(15)+3S(45)+5S(75)\right]$$
$$S = 1.62 \text{ k}$$

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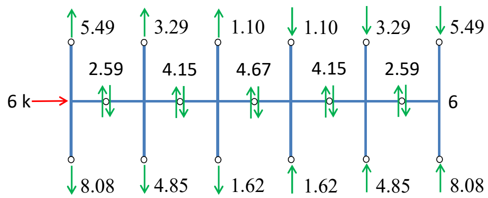
92

Approximate Methods

- Determine the beam shears from the axial forces.



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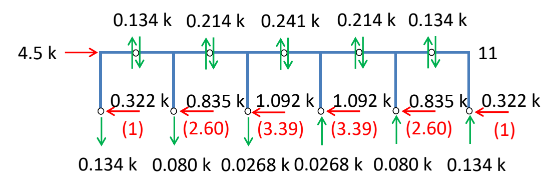
93

Approximate Methods

- To determine the column shears, we will need to start at the top level.
- The column shears will be in the same proportion (red) all the way down the structure.



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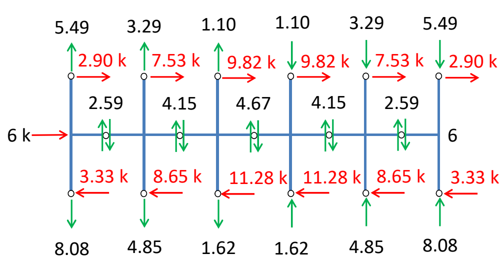
94

Approximate Methods

- So the column shears at level 6 can be determined



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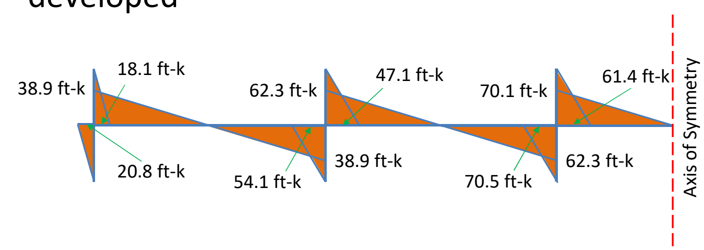
95

Approximate Methods

- And a partial moment diagram can be developed



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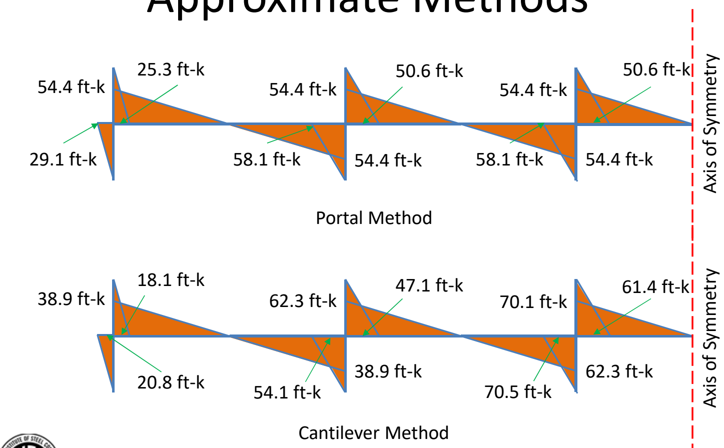
Approximate Methods



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97

Approximate Methods

- Question to consider about approximate methods.
  - Are the Portal Method and Cantilever Method accurate enough for final design?

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Polling Question

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Approximate Methods

- The moments from the lateral analysis and the gravity analysis can then be combined.
- Although these are only approximate answers, they are usually sufficient to start a design so that a more accurate estimate of the member sizes may be determined.
- It is also possible to analyze the structure at selected levels rather than at every level.

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Approximate Methods

- A useful reference that illustrates how to determine member properties for an entire multi-story building when properties at only 3 levels are known is:

Fleischer, Walter, "Simplified High-Rise Drift Analysis and Optimized Adjustment," AISC *Engineering Journal*, Vol. 9, No. 2, April, 1972, pp 50-69

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Summary

- Developed the Moment Distribution Method for continuous beams and frames.
- Applied Moment Distribution to frames when sidesway is permitted.
- Developed a technique to check the math in the process of moment distribution.
- Discussed approximations that could be applied for gravity load analysis of multi-story frames.
- Compared the Portal and Cantilever Methods for lateral load analysis of multi-story frames.

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Lesson 8

- Classical Approaches applied to Second-Order Analysis
  - Investigate second-order effects by iterative approaches.
  - Consider the use of slope-deflection for second-order analysis.
  - Relate moment distribution to the B_1-B_2 method of AISC 360-16 Appendix 8
  - Compare classical methods with modern methods

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104

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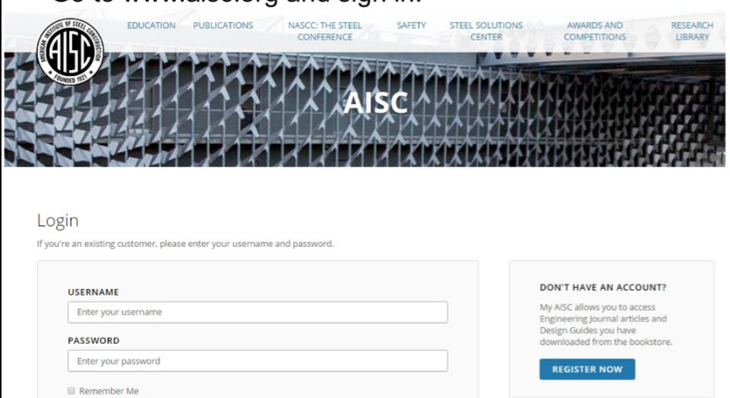

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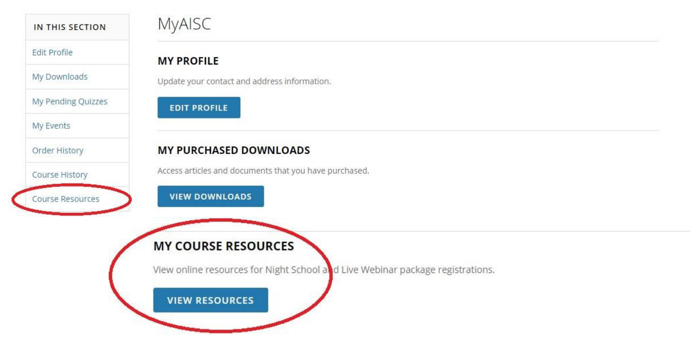

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