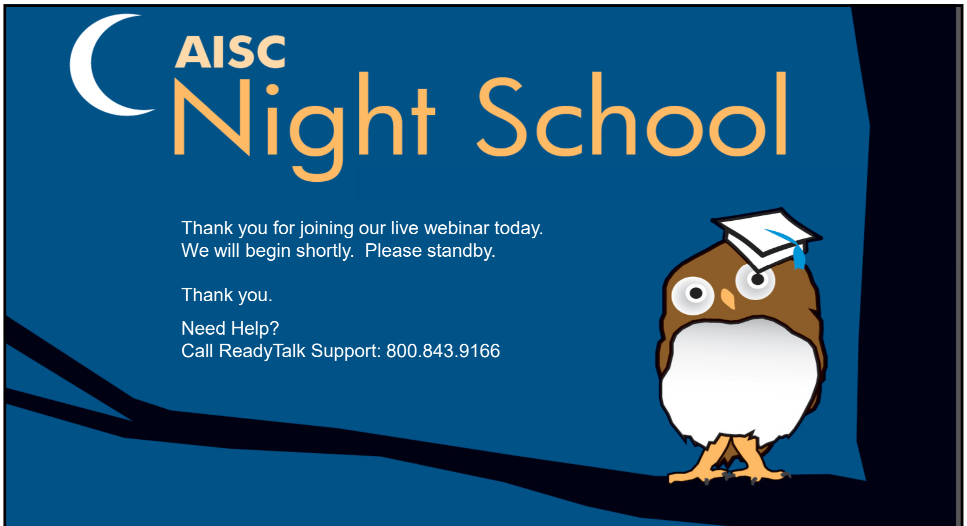

Classical Methods of Structural Analysis

Louis F. Geschwindner

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Session Description
20.8 Classical Approaches Applied to Second-Order Analysis
August 5, 2019

This lesson will discuss the "modern methods" of analysis and illustrate how they may be understood and developed from the classical methods. It will also include a discussion of second-order analysis. The classical methods of analysis will be used in an iterative way to carry out a second-order analysis and to help explain the concepts of second-order analysis. The link between moment distribution and the approximate second-order analysis approach of AISC 360-16 Appendix 8 will be discussed.

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Learning Objectives:

- Compare modern methods of analysis with classical methods.
- List the second-order analysis requirements.
- Investigate second-order effects by iterative approaches.
- Describe the link between moment distribution and approximate second-order analysis method in the AISC Specification.

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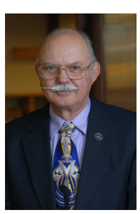
Classical Methods of Structural Analysis

Session 8: Classical Approaches Applied to Second-Order Analysis
August 5, 2019



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Classical Methods of Structural Analysis:

How we did it before computers

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Lesson 8

Classical Approaches Applied to Second-Order Analysis

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Lesson 8

Classical Approaches Applied to Second-Order Analysis

– Investigate second-order effects by iterative approaches.
– Consider the use of slope-deflection for second-order analysis.
– Relate moment distribution to the B_1-B_2 method of AISC 360-16 Appendix 8
– Compare classical methods with modern methods

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Second-Order Analysis

- For many, the need to consider second-order effects is believed to be something new.
- Although it may be more clearly stated as a requirement in the AISC Specification since 2005 than it has been in the AISC Specification since the 1960's.
- The need to consider second-order effects is not new.

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AISC 360-16

C1. General Stability Requirements

The effects of all of the following on the stability of the structure and its elements shall be considered:

a) flexural, shear and axial member deformations and all other component and connection deformations that contribute to displacements of the structure;
b) second-order effects (including P-Δ and P-δ effects);
c) geometric imperfections;
d) Stiffness reductions due to inelasticity, including the effect of partial yielding of the cross section which may be accentuated by the presence of residual stresses; and
e) uncertainty in system, member, and connection strength and stiffness.

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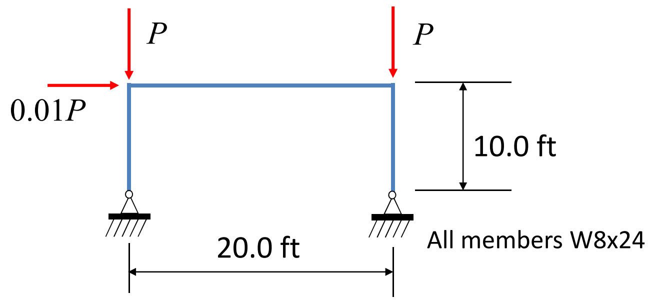
Second-Order Analysis

- Consider a simple frame and look at 3 types of analyses that may be carried out, first-order, second-order, and buckling.



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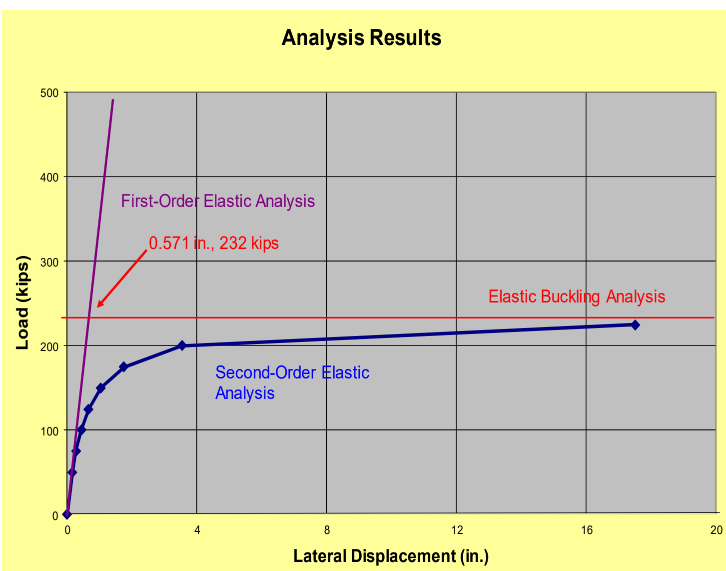

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Second-Order Analysis

- Second-order effects change
  - Moments in beams and columns
  - Shear and axial forces in beams and columns
  - Forces on connections and foundations
- Second-order moments may have a different distribution than first-order moments.
- Superposition does not apply.

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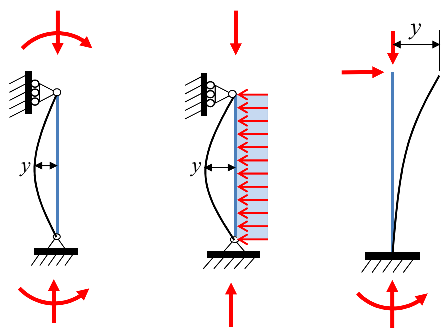
Second-Order Analysis

- Look at 3 beam-columns with a step-by-step application of classical methods.



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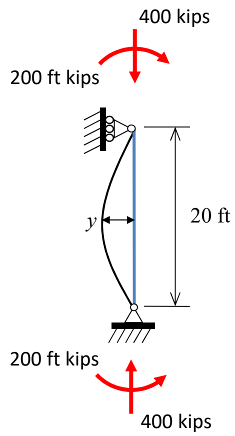
Second-Order Analysis



- A second-order analysis means that we recognize that once the member displaces, as shown, if we take moments about a point on the member in its deformed position, there will be some additional moment in the member due to the axial force times the displacement.

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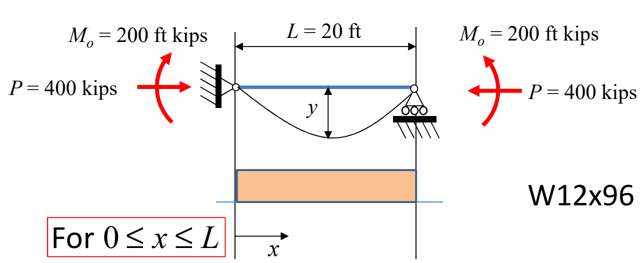
Second-Order Analysis

- Determine the equation for the deflection curve due to the end moments.



$$M_x = M_o$$

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Second-Order Analysis

- Using double integration

$$\frac{d^2y}{dx^2} = \frac{M_o}{EI}$$

$$\left(\frac{dy}{dx}\right)_1 = \int \frac{M_o}{EI} dx = \frac{M_o x}{EI} + C_1$$

at $x = \frac{L}{2}$, $\left(\frac{dy}{dx}\right)_1 = 0$, therefore $C_1 = -\frac{M_o L}{2EI}$ and

$$\left(\frac{dy}{dx}\right)_1 = \frac{M_o}{EI}\left(x - \frac{L}{2}\right)$$

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Second-Order Analysis

- Using double integration

$$\left(\frac{dy}{dx}\right)_1 = \frac{M_o}{EI}\left(x - \frac{L}{2}\right)$$

$$y_1 = \int \frac{M_o}{EI}\left(x - \frac{L}{2}\right) dx = \frac{M_o}{EI}\left(\frac{x^2}{2} - \frac{Lx}{2} + C_2\right)$$

at $x = 0$, $y_1 = 0$, therefore $C_2 = 0$ and

$$y_1 = \frac{M_o}{2EI}\left(x^2 - Lx\right)$$

This is the equation for deflection due to the applied end moments.

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Second-Order Analysis

- For the given member, the maximum displacement, at mid-span, is

$$y_{1\max} = \frac{M_o}{2EI}\left(x^2 - Lx\right)$$
$$= \frac{200}{2(29000)(833)}\left(10^2 - 20(10)\right)(1728) = -0.715 \text{ in.}$$

- Therefore, the additional moment at mid-span due to the axial force is

$$M_2 = Py_{1\max} = \frac{400(0.715)}{12} = 23.8 \text{ ft-kips}$$

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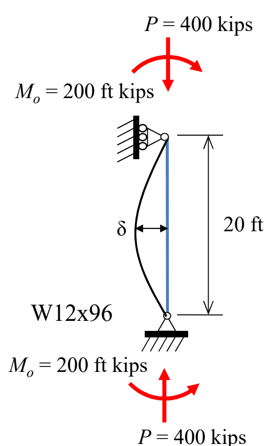
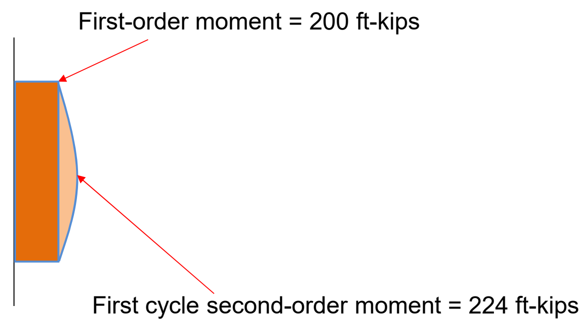
Second-Order Analysis



First Iteration on member effect



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Second-Order Analysis

- This additional moment, which varies along the member length, will cause additional deflection. We carry out the double integration again using that varying moment diagram, thus

$$M_2 = Py_1 = P\left[\frac{M_o}{2EI}\left(x^2 - Lx\right)\right]$$

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Second-Order Analysis

- Using double integration

$$\left(\frac{d^2y}{dx^2}\right)_2 = \frac{M_2}{EI} = \frac{PM_o}{2(EI)^2}\left(x^2 - Lx\right)$$

$$\left(\frac{dy}{dx}\right)_2 = \int \frac{PM_o}{2(EI)^2}\left(x^2 - Lx\right)dx = \frac{PM_o}{2(EI)^2}\left(\frac{x^3}{3} - \frac{Lx^2}{2} + C_3\right)$$

at $x = \frac{L}{2}$, $\left(\frac{dy}{dx}\right)_2 = 0$, therefore $C_3 = \frac{L^3}{12}$ and

$$\left(\frac{dy}{dx}\right)_2 = \frac{PM_o}{2(EI)^2}\left(\frac{x^3}{3} - \frac{Lx^2}{2} + \frac{L^3}{12}\right)$$

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Second-Order Analysis

- Using double integration

$$\left(\frac{dy}{dx}\right)_2 = \frac{PM_o}{2(EI)^2}\left(\frac{x^3}{3} - \frac{Lx^2}{2} + \frac{L^3}{12}\right)$$

$$y_2 = \int \frac{PM_o}{2(EI)^2}\left(\frac{x^3}{3} - \frac{Lx^2}{2} + \frac{L^3}{12}\right)dx = \frac{PM_o}{2(EI)^2}\left(\frac{x^4}{12} - \frac{Lx^3}{6} + \frac{L^3x}{12} + C_4\right)$$

at $x = 0$, $y_2 = 0$, therefore $C_4 = 0$ and

$$y_2 = \frac{PM_o}{24(EI)^2}\left(x^4 - 2Lx^3 + L^3x\right)$$

This is the deflection due to the moment caused by axial load times our first calculation of displacement.

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Second-Order Analysis

- The additional displacement at mid-span due to our first estimate of second-order moment is

$$y_{2\max} = \frac{PM_o}{24(EI)^2}\left(x^4 - 2Lx^3 + L^3x\right)$$

$$= \frac{-400(200)}{24(29000)^2(833)^2}\left(10^4 - 2\left(20(10)^3\right) + 20^3(10)\right)(12)^5 = -0.0711 \text{ in.}$$

- Therefore, the additional moment at mid-span due the axial force is

$$M_3 = Py_{2\max} = \frac{400(0.0711)}{12} = 2.37 \text{ ft-kips}$$

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Second-Order Analysis

- Using double integration a third time using this new variation in moment and jumping to the solutions

$$M_3 = Py_2$$

$$\left(\frac{d^2y}{dx^2}\right)_3 = \frac{P^2M_o}{24(EI)^3}\left(x^4 - 2Lx^3 + L^3x\right)$$

$$\left(\frac{dy}{dx}\right)_3 = \frac{P^2M_o}{24(EI)^3}\left(\frac{x^5}{5} - \frac{2Lx^4}{4} + \frac{L^3x^2}{2} - \frac{L^5}{10}\right)$$

$$y_3 = \frac{P^2M_o}{2880(EI)^3}\left(4x^6 - 12Lx^5 + 20L^3x^3 - 12L^5x\right)$$

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Second-Order Analysis

- The deflection at mid-span for this cycle is

$$y_{3\max} = -0.00689 \text{ in.}$$

- And the additional moment is

$$M_4 = Py_{3\max} = \frac{400(0.00689)}{12} = 0.230 \text{ ft-kips}$$

- Although our solution is approximate, it is clear that any additional effort is not warranted. There really was no need to determine M_4 since it is so small.

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Second-Order Analysis

- The amplification of moment is

$$M_{2nd-order} = M_o + M_2 + M_3 + M_4$$
$$= 200 + 23.8 + 2.37 + 0.230 = 226 \text{ ft-kips}$$
$$A.F. = \frac{226}{200} = 1.13$$

- The amplification of deflection is

$$y_{2nd-order} = y_1 + y_2 + y_3$$
$$= 0.715 + 0.0711 + 0.00689 = 0.793$$
$$A.F. = \frac{0.793}{0.715} = 1.11$$

Note that the amplification for moment and deflection are not the same

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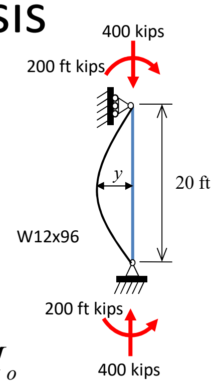
Second-Order Analysis

- An exact theoretical solution is available in the literature.



$$\mu = \sqrt{\frac{PL^2}{4EI}} = \sqrt{\frac{400(20)^2(144)}{4(29000)(833)}} = 0.488$$

$$M_{MAX} = M_o \sec\mu = M_o \sec(0.488) = 1.13 M_o$$

$$y = \frac{M_o L^2}{8EI}\left(\frac{2(1-\cos\mu)}{\mu^2 \cos\mu}\right) = \frac{M_o L^2}{8EI}\left(\frac{2(1-\cos(0.488))}{(0.488)^2 \cos(0.488)}\right) = 1.11\frac{M_o L^2}{8EI}$$

Note that these amplification factors are the same as our solution.

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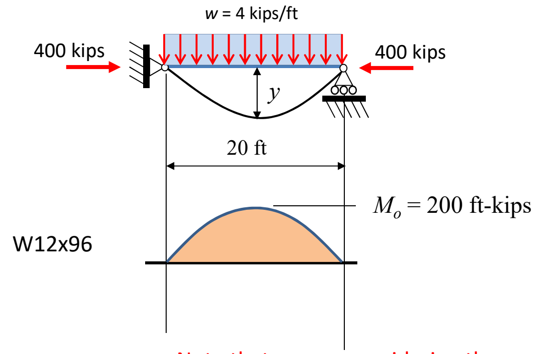
Second-Order Analysis

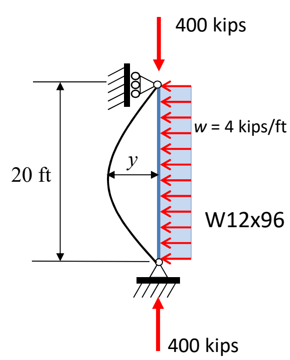
Next consider a beam with uniform load and axial force.



Note that we are considering the same axial force and maximum moment as our previous solution, but with different distribution of the moment.

Second-Order Analysis

Deflection due to uniform load



$$y_{1\max} = \frac{5wL^4}{384EI} = \frac{5(4.0)(20)^4(1728)}{384(29000)(833)} = 0.596 \text{ in.}$$

$$M_2 = Py_{1\max} = \frac{(400(0.596))}{12} = 19.9 \text{ ft-kips}$$

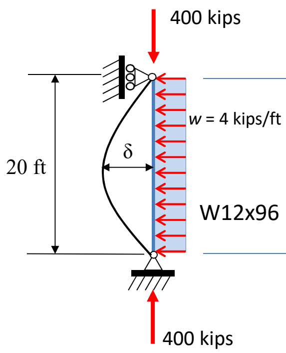
$$M_{2nd-order} = 200 + 19.9 = 220 \text{ ft-kips}$$

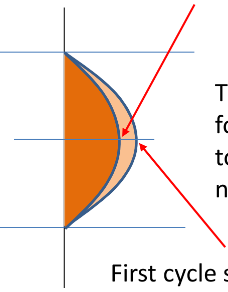
$$\text{Amplification Factor} = \frac{220}{200} = 1.10$$

We are not using double integration since we know the deflection equation and shape.

Second-Order Analysis

Consider the deflection due to this additional moment.





The shape of the moment diagram for the second cycle is very similar to that for the first cycle so we will not use double integration.

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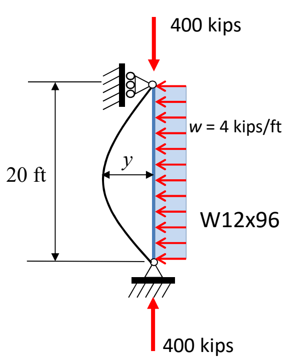
Second-Order Analysis

Second Iteration on member effect using a parabolic moment diagram (Approximation)



$$y_{2\max} = \frac{5M_2 l^2}{48EI} = \frac{5(19.9)(20)^2(1728)}{48(29000)(833)} = 0.0593 \text{ in.}$$

$$M_3 = Py_{2\max} = \frac{(400(0.0593))}{12} = 1.98 \text{ ft-kips}$$

$$M_{2nd-order} = 200 + 19.9 + 1.98 = 222 \text{ ft-kips}$$

$$\text{Amplification Factor } = \frac{222}{200} = 1.11$$

This can be expected to accurately estimate the amplification since we used a reasonable moment diagram.

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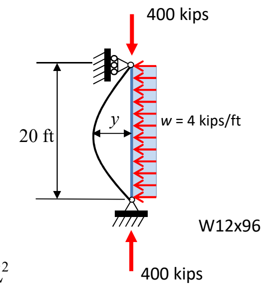
Second-Order Analysis

- The exact Theoretical Solution is available as AISC 360-05 Commentary Benchmark Problem Case 1.



$$\mu = \sqrt{\frac{PL^2}{4EI}} = \sqrt{\frac{400(20)^2 144}{4(29000)(833)}} = 0.488$$

$$M_{MAX} = \frac{wL^2}{8}\left[\frac{2(\sec\mu - 1)}{\mu^2}\right] = \frac{wL^2}{8}\left[\frac{2(\sec(0.488) - 1)}{(0.488)^2}\right] = 1.11\frac{wL^2}{8}$$

$$y_{MAX} = \frac{5wL^4}{384EI}\left[\frac{12\left(2\sec\mu - \mu^2 - 2\right)}{5\mu^4}\right] = \frac{5wL^4}{384EI}\left[\frac{12\left(2\sec(0.488) - (0.488)^2 - 2\right)}{5(0.488)^4}\right] = 1.11\frac{5wL^4}{384EI}$$

Our solution and the exact solution are the same.

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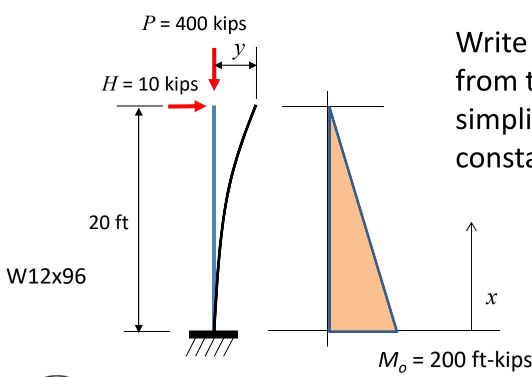
Second-Order Analysis

Consider the flagpole column shown. The axial force and the maximum moment are the same as for the previous two examples.



Write the moment equation from the fixed end. This will simplify determination of constants of integration.

For $0 \le x \le L$

$$M_x = HL - Hx$$

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Second-Order Analysis

- Using double integration

$$\frac{d^2y}{dx^2} = \frac{M_x}{EI} = \frac{H}{EI}(L-x)$$

$$\left(\frac{dy}{dx}\right)_1 = \int \frac{H}{EI}(L-x)dx = \frac{H}{EI}\left(Lx - \frac{x^2}{2} + C_1\right)$$

at $x = 0,\ \left(\frac{dy}{dx}\right)_1 = 0,$ therefore $C_1 = 0$ and

$$\left(\frac{dy}{dx}\right)_1 = \frac{H}{EI}\left(Lx - \frac{x^2}{2}\right)$$

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Second-Order Analysis

- Using double integration

$$\left(\frac{dy}{dx}\right)_1 = \frac{H}{EI}\left(Lx - \frac{x^2}{2}\right)$$

$$y_1 = \int \frac{H}{EI}\left(Lx - \frac{x^2}{2}\right)dx = \frac{H}{EI}\left(\frac{Lx^2}{2} - \frac{x^3}{6} + C_2\right)$$

at $x = 0,\ y_1 = 0,$ therefore $C_2 = 0$ and

$$y_1 = \frac{H}{EI}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right)$$

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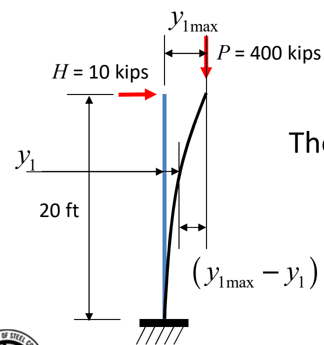
Second-Order Analysis

- Unlike the previous two examples, the additional moment is not Py.



$$M_2 = P\left(y_{1max} - y_1\right)$$

The maximum displacement is at $x = L$, thus

$$y_{1max} = \frac{H}{EI}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) = \frac{HL^3}{3EI}$$

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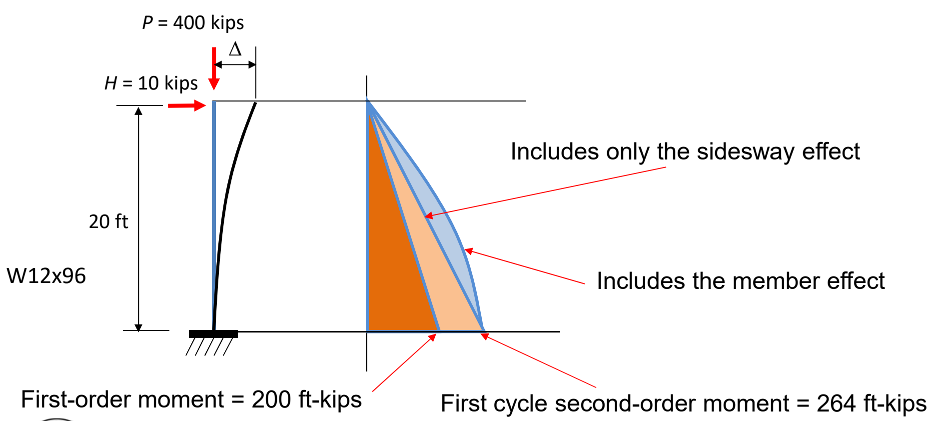
Second-Order Analysis

First iteration on sidesway effect



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Second-Order Analysis

- Using double integration

$$M_2 = P\left(y_{1\max} - y_1\right) = \frac{PH}{EI}\left(\frac{L^3}{3} - \frac{Lx^2}{2} + \frac{x^3}{6}\right)$$

$$\left(\frac{d^2y}{dx^2}\right)_2 = \frac{M_2}{EI} = \frac{PH}{(EI)^2}\left(\frac{L^3}{3} - \frac{Lx^2}{2} + \frac{x^3}{6}\right)$$

$$\left(\frac{dy}{dx}\right)_2 = \frac{PH}{(EI)^2}\left(\frac{L^3x}{3} - \frac{Lx^3}{6} + \frac{x^4}{24} + C_3\right)$$

at $x = 0,\ \left(\frac{dy}{dx}\right)_2 = 0,$ therefore $C_3 = 0$ and

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Second-Order Analysis

- Using double integration

$$y_2 = \int \frac{PH}{(EI)^2}\left(\frac{L^3x}{3} - \frac{Lx^3}{6} + \frac{x^4}{24}\right)dx$$

$$= \frac{PH}{(EI)^2}\left(\frac{L^3x^2}{6} - \frac{Lx^4}{24} + \frac{x^5}{120} + C_4\right)$$

at $x = 0,\ y_2 = 0,$ therefore $C_4 = 0$ and

$$y_2 = \frac{PH}{(EI)^2}\left(\frac{L^3x^2}{6} - \frac{Lx^4}{24} + \frac{x^5}{120}\right) \text{ and } y_{2\max} = \frac{2PHL^5}{15(EI)^2}$$

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Second-Order Analysis

- The results for the next step are:

$$M_3 = P\left(y_{2\max} - y_2\right) = \frac{P^2 H}{\left(EI\right)^2}\left(\frac{2L^5}{15} - \frac{L^3 x^2}{6} + \frac{Lx^4}{24} - \frac{x^5}{120}\right)$$

$$\left(\frac{dy}{dx}\right)_3 = \frac{P^2 H}{\left(EI\right)^3}\left(\frac{2L^5 x}{15} - \frac{L^3 x^3}{18} + \frac{Lx^5}{120} - \frac{x^6}{720}\right)$$

$$y_3 = \frac{P^2 H}{\left(EI\right)^3}\left(\frac{2L^5 x^2}{30} - \frac{L^3 x^4}{72} + \frac{Lx^6}{720} - \frac{x^7}{5040}\right) \text{ and } y_{3\max} = \frac{17P^2 HL^7}{315\left(EI\right)^3}$$

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Second-Order Analysis

- And again:

$$M_4 = P\left(y_{3\max} - y_3\right) = \frac{P^3 H}{\left(EI\right)^3}\left(\frac{17L^7}{315} - \frac{2L^5 x^2}{30} + \frac{L^3 x^4}{72} - \frac{Lx^6}{720} + \frac{x^7}{5040}\right)$$

$$\left(\frac{dy}{dx}\right)_4 = \frac{P^3 H}{\left(EI\right)^4}\left(\frac{17L^7 x}{315} - \frac{L^5 x^3}{45} + \frac{L^3 x^5}{360} - \frac{Lx^7}{5040} + \frac{x^8}{40320}\right)$$

$$y_4 = \frac{P^3 H}{\left(EI\right)^4}\left(\frac{17L^7 x^2}{630} - \frac{L^5 x^4}{180} + \frac{L^3 x^6}{2160} - \frac{Lx^8}{8\left(5040\right)} + \frac{x^9}{9\left(40320\right)}\right)$$

$$y_{4\max} = 0.0219\frac{P^3 HL^9}{\left(EI\right)^4}$$

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Second-Order Analysis

- Determine the maximum deflections

$$y_{1\max} = \frac{HL^3}{3EI} = \frac{10(20)^3(12)^3}{3(29000)(833)} = 1.908 \text{ in.}$$

$$y_{2\max} = \frac{2PHL^5}{15(EI)^2} = \frac{2(400)(10)(20)^5(12)^5}{15((29000)(833))^2} = 0.728 \text{ in.}$$

$$y_{3\max} = \frac{17P^2HL^7}{315(EI)^3} = \frac{17(400)^2(10)(20)^7(12)^7}{315((29000)(833))^3} = 0.281 \text{ in.}$$

$$y_{4\max} = 0.0219\frac{P^3HL^9}{(EI)^4} = \frac{0.0219(400)^3(10)(20)^9(12)^9}{((29000)(833))^4} = 0.109 \text{ in}$$

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Second-Order Analysis

- Determine the total second-order deflection

$$\begin{aligned} y_{\text{second-order}} &= y_{1\max} + y_{2\max} + y_{3\max} + y_{4\max} \\ &= 1.908 \text{ in.} + 0.728 \text{ in.} + 0.281 \text{ in.} + 0.109 \text{ in.} \\ &= 3.026 \text{ in.} \end{aligned}$$

$$A.F. = \frac{3.026}{1.908} = 1.59$$

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Second-Order Analysis

- And the additional second-order moment.

$$M_{\text{second-order}} = H(L) + Py_{1\max} + Py_{2\max} + Py_{3\max} + Py_{4\max}$$
$$= 10(20) + \frac{400(1.908 \text{ in.})}{12} + \frac{400(0.728 \text{ in.})}{12}$$
$$+ \frac{400(0.281 \text{ in.})}{12} + \frac{400(0.109 \text{ in.})}{12}$$
$$= 200 + 63.6 + 24.3 + 9.4 + 3.6$$
$$= 301 \text{ ft-kips}$$

$$A.F. = \frac{301}{200} = 1.50$$

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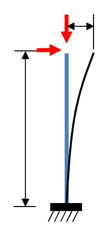
Second-Order Analysis

- The exact Theoretical Solution is available as AISC 360-05 Commentary Benchmark Problem Case 2.



$$\alpha = \sqrt{\frac{PL^2}{EI}} = \sqrt{\frac{400(20)^2(144)}{29000(833)}} = 0.977$$

$$M_{MAX} = HL\left(\frac{\tan\alpha}{\alpha}\right) = HL\left(\frac{\tan(0.977)}{0.977}\right) = 1.52HL$$

$$y_{MAX} = \frac{HL^3}{3EI}\left(\frac{3(\tan\alpha - \alpha)}{\alpha^3}\right) = \frac{HL^3}{3EI}\left(\frac{3(\tan(0.977) - 0.977)}{(0.977)^3}\right) = 1.62\left(\frac{HL^3}{3EI}\right)$$

Compared to 1.50 and 1.59 from our calculations.

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Second-Order Analysis

- Question to consider.
  - Which of the following would you consider to generally be true. (Select all that you believe are true.)
    - Second-order effects can be significant.
    - Based on our 3 examples, sway effect looks to be more significant than member effect.
    - Consideration of second-order effects is not a new requirement in the AISC Specifications.
    - Consideration of second-order effects is too complicated for every day practice.
    - It is acceptable for simple structures to ignore second-order effects.

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Polling Question

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Second-Order Analysis

- Second-order analysis by iteration
  - Hopefully, this has helped us understand what a second-order analysis is really trying to assess.
  - Clearly it can become a very complicated process, especially when carried out this way.
  - This is not likely to be a method that any one of us would propose using on a real structure.
  - We can get closer to an approach for real structures by looking further at slope deflection.

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Slope Deflection

- The slope deflection equation developed in Lesson 6 for prismatic members was

$$M_N = K_N \theta_N + C_F K_F \theta_F - (K_N + C_F K_F)\rho + FEM_N$$

where

$$K_N = K_F = \frac{4EI}{L} \qquad C_N = C_F = \frac{1}{2}$$

- This can be modified to include the effects of axial forces on the stiffness by use of stability functions first introduced by Freidrich Bleich in 1919.

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Slope Deflection

- The first step is to substitute for the stiffnesses and carry-over factors so that

$$M_N = K_N\theta_N + C_F K_F \theta_F - (K_N + C_F K_F)\rho + FEM_N$$

becomes

$$M_N = \frac{EI}{L}\left[C\theta_N + S\theta_F - (C+S)\rho\right] + FEM_N$$

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Slope Deflection

- The first step is to substitute for the stiffnesses and carry-over factors so that

$$M_N = K_N\theta_N + C_F K_F \theta_F - (K_N + C_F K_F)\rho + FEM_N$$

becomes

$$M_N = \frac{EI}{L}\left[C\theta_N + S\theta_F - (C+S)\rho\right] + FEM_N$$

For a member without axial load

$$C = 4 \quad \text{and} \quad S = 2$$

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Slope Deflection

- When considering axial load C and S become

$$C = \frac{c}{c^2 - s^2} \quad \text{and} \quad S = \frac{s}{c^2 - s^2}$$

where

$$c = \frac{1 - L\sqrt{P/EI} \Big/ \tan\left(L\sqrt{P/EI}\right)}{PL^2/EI} \qquad s = \frac{L\sqrt{P/EI} \Big/ \sin\left(L\sqrt{P/EI}\right) - 1}{PL^2/EI}$$

We see now that the slope deflection coefficients are dependent on P, which we don't actually know.

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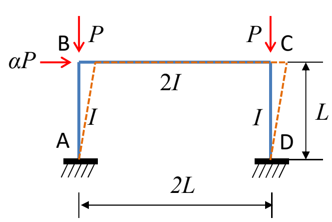
Slope Deflection

- Look at what this means for a simple frame.



These member end moment equations are similar to the equations written for the sway frame in Lesson 6 although here there are no fixed end moments and the C and S variables are used.

$$M_{AB} = \frac{EI}{L}[S_{AB}\theta_B - (C_{AB} + S_{AB})\rho]$$

$$M_{BA} = \frac{EI}{L}[C_{AB}\theta_B - (C_{AB} + S_{AB})\rho]$$

$$M_{BC} = \frac{2EI}{2L}[C_{BC}\theta_B + S_{BC}\theta_C]$$

$$M_{CB} = \frac{2EI}{2L}[S_{BC}\theta_B + C_{BC}\theta_C]$$

$$M_{CD} = \frac{EI}{L}[C_{CD}\theta_C - (C_{CD} + S_{CD})\rho]$$

$$M_{DC} = \frac{EI}{L}[S_{CD}\theta_C - (C_{CD} + S_{CD})\rho]$$

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Slope Deflection

- Equilibrium Equations at B and C

$$\sum M_B = M_{BA} + M_{BC} = 0$$
$$\sum M_C = M_{CB} + M_{CD} = 0$$

- The horizontal equilibrium equation must be written in the deformed (swayed) position and the horizontal load, αP, included.

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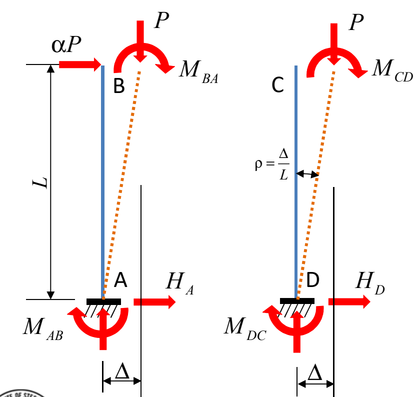
Slope Deflection

- Horizontal equilibrium.

Take moments about B and C in the displaced position, thus, $P\Delta$ is included.



$$H_A = \frac{M_{AB} + M_{BA} + P\Delta}{L}$$
$$H_D = \frac{M_{CD} + M_{DC} + P\Delta}{L}$$
$$\sum F_x = H_A + H_D + \alpha P = 0$$
$$M_{AB} + M_{BA} + M_{CD} + M_{DC} + 2PL\rho = -\alpha PL$$

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Slope Deflection

- If we assume there is no axial force in BC and that P is the same in AB and CD, we can substitute the member end moment equations into the three equilibrium equations and rearranging we get

$$(C+4)\theta_B + 2\theta_C - (C+S)\rho = 0$$
$$2\theta_B + (C+4)\theta_C - (C+S)\rho = 0$$
$$(C+S)\theta_B + (C+S)\theta_C + \left[\frac{2PL^2}{EI} - 4(C+S)\right]\rho = -\frac{\alpha PL^2}{EI}$$

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Slope Deflection

- The effect of the axial load on the sway, the P-Δ effect, is included here

$$(C+4)\theta_B + 2\theta_C - (C+S)\rho = 0$$
$$2\theta_B + (C+4)\theta_C - (C+S)\rho = 0$$
$$(C+S)\theta_B + (C+S)\theta_C + \left[\frac{2PL^2}{EI} - 4(C+S)\right]\rho = -\frac{\alpha PL^2}{EI}$$

The effect of axial force on the stiffness of the members is included in C and S.

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Slope Deflection

- To help grasp the impact of axial force on the stiffness of a member, consider the values of C and S. Using the equations from slide 56;

$L\sqrt{P/EI}$	C	S
0	4.00	2.00
1.0	3.8649	2.0344
2.0	3.4361	2.1519
3.0	2.6242	2.4115
3.14	2.4692	2.4667

This is equivalent to the Euler buckling load on a non-sway pin-pin column.

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Slope Deflection

- The solution to our three simultaneous equations is

$$\theta_B = \theta_C = \frac{\alpha\left(PL^2/EI\right)(C+S)}{2(C+S)(C-S+12)-2\left(PL^2/EI\right)(C+6)}$$

$$\rho = \frac{\alpha\left(PL^2/EI\right)(C+6)}{2(C+S)(C-S+12)-2\left(PL^2/EI\right)(C+6)}$$

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Slope Deflection

- And the final moments are

$$M_{AB} = M_{DC} = \alpha PL\left[\frac{S(C+S)-(C+S)(C+6)}{2(C+S)(C-S+12)-2\left(PL^2/EI\right)(C+6)}\right]$$

$$M_{BA} = M_{CD} = -M_{BC} = -M_{CB} = \alpha PL\left[\frac{C(C+S)-(C+S)(C+6)}{2(C+S)(C-S+12)-2\left(PL^2/EI\right)(C+6)}\right]$$

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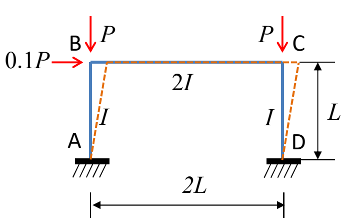
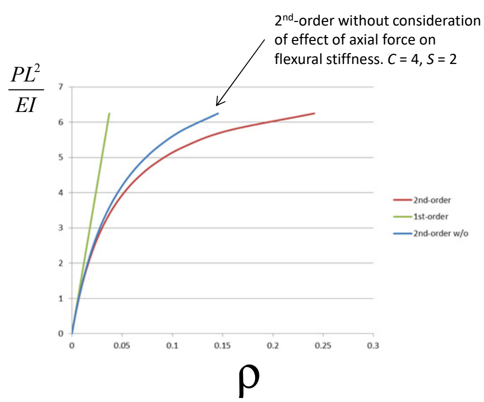
Slope Deflection

- Results





Reference:
Galambos, Theodore V., *Structural Members and Frames*, Prentice-Hall, 1968.

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Approximate Second-Order Analysis

- AISC 360-16 Appendix 8

  This appendix provides an approximate procedure to account for second-order effects in structures by amplifying the required strengths indicated by two first-order analyses.

$$M_r = B_1 M_{nt} + B_2 M_{lt} \quad \text{(A-8-1)}$$

$$P_r = P_{nt} + B_2 P_{lt} \quad \text{(A-8-2)}$$

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Approximate Second-Order Analysis

- AISC 360-16 Appendix 8

  The terms B_1 and B_2 are amplification factors, similar to the amplification factors that we determined for the three specific cases we addressed by double integration. They are defined as

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} \geq 1 \quad \text{(A-8-3)}$$

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\,story}}} \geq 1 \quad \text{(A-8-6)}$$

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Approximate Second-Order Analysis

- AISC 360-16 Appendix 8

  We will not look at the B_1-B_2 terms but will look at the other terms in the amplified equations, the terms with subscripts nt and lt, because these have to do with the analysis we will perform.

  The nt means with no translation while the lt means with translation.

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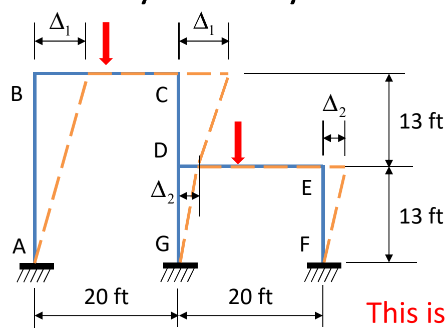
Approximate Second-Order Analysis

- Remember this structure from Lesson 6. It is permitted to sway sideways as shown.



This is the real structure

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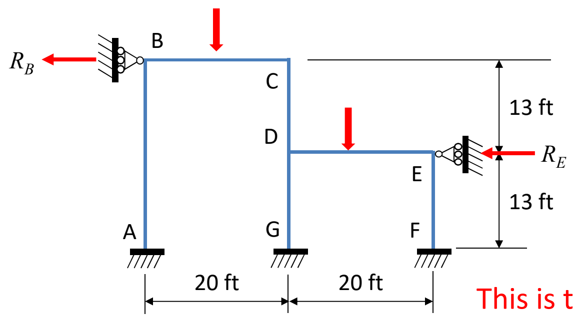
Approximate Second-Order Analysis

- If we add reactions at B and E, we can keep the frame from swaying sideways.



This is the *nt* structure

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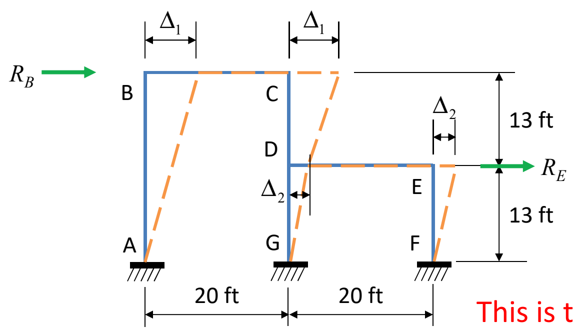
Approximate Second-Order Analysis

- But the real structure requires that we remove these reactions.



This is the *lt* structure

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Approximate Second-Order Analysis

- These unwanted reactions, R_B and R_E, should remind us of the artificial joint restraint that we needed to use in Lesson 7 when we treated sway frames by moment distribution.
- They are exactly the same thing. They give us a way to obtain the influence of sway on the moments and forces in the frame.

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Approximate Second-Order Analysis

- The nt structure permits us to obtain the influence of the member second-order effects (*P*-δ), similar to the first two examples we looked at by iteration where there was no relative displacement of the column ends.
- The lt structure permits us to address the influence of the sway or structure second-order effect (*P*-Δ) as in the flagpole column.

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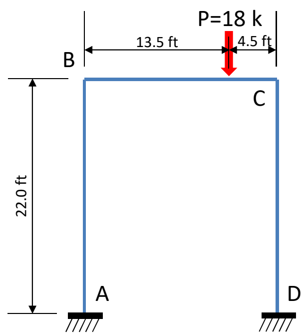
Moment Distribution

- Return to this sway frame from Lesson 7.



$$K'_{AB} = K_{CD} = \frac{I}{22} = 1.0$$

$$K'_{BC} = \frac{I}{18} = 1.22$$

$$FEM_{BC} = \frac{Pab^2}{L^2} = \frac{18(13.5)(4.5)^2}{18^2} = -15.2 \text{ ft-kips}$$

$$FEM_{BC} = \frac{Pa^2b}{L^2} = \frac{18(13.5)^2(4.5)}{18^2} = 45.6 \text{ ft-kips}$$

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Moment Distribution

- The original distribution table

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
K	1	1	1.22	1.22	1	1
DF	0	0.45	0.55	0.55	0.45	0
FEM			-15.19	45.56		
	3.42	6.84	8.35	4.18		
			-13.68	-27.36	-22.38	-11.19
	3.08	6.15	7.52	3.76		
			-1.03	-2.07	-1.69	-0.85
	0.23	0.47	0.57	0.28		
				-0.16	-0.13	
Total	6.73	13.46	-13.46	24.20	-24.20	-12.04

These are the 1st cycle moments from moment distribution and also the *nt* moments for an approximate second-order analysis

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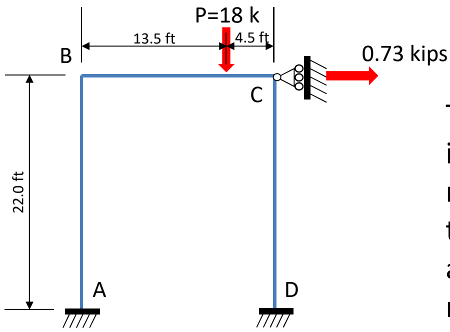
Moment Distribution

- The structure we actually analyzed was



The moments determined in the first cycle of moment distribution are the *nt* moments because, as analyzed, the frame was not permitted to sway.

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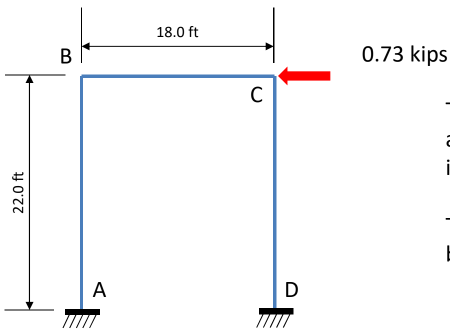
Moment Distribution

- The structures response to sway is given by



To get back to the structure that we are really looking for, we must include the influence of sway.

Thus, the results of this analysis will be the *lt* moments.

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Moment Distribution

- Original distribution table for sway

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
K	1	1	1.22	1.22	1	1
DF	0	0.45	0.55	0.55	0.45	0
FEM	10.0	10.0			10.0	10.0
	-2.25	-4.50	-5.50	-2.75		
			-1.99	-3.99	-3.26	-1.63
	0.45	0.9	1.10	0.55		
			-0.15	-0.30	-0.25	-0.13
		0.07	0.08			
Total	8.20	6.47	-6.46	-6.49	6.49	8.24

These are the sway moments but for an arbitrary magnitude of sway since we started with an arbitrary 10 ft-kip moment.

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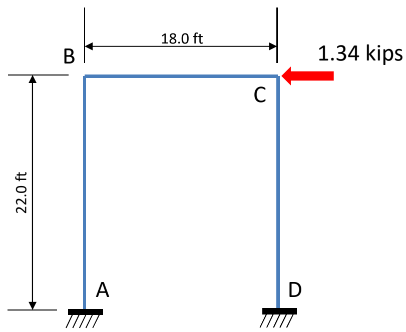
Moment Distribution

- Now the structure we actually analyzed is



The horizontal force was determined from the moments on the previous slide.

But it is not the magnitude force that we need, we need z times that.

$$0.73 = z(1.34)$$

$$z = \frac{0.73}{1.34} = 0.54$$

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Moment Distribution

- Modify the sway results by z.

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
K	1	1	1.22	1.22	1	1
DF	0	0.45	0.55	0.55	0.45	0
FEM	10.0	10.0			10.0	10.0
	-2.25	-4.50	-5.50	-2.75		
			-1.99	-3.99	-3.26	-1.63
	0.45	0.9	1.10	0.55		
			-0.15	-0.30	-0.25	-0.13
		0.07	0.08			
Total	8.20	6.47	-6.46	-6.49	6.49	8.24
z(Total)	4.43	3.49	-3.49	-3.49	3.49	4.45

These are the sway moments for the correct magnitude of sway. They are the lt moments.

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Moment Distribution

- Final results table for a first-order analysis.

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
1st cycle	6.73	13.46	-13.46	24.20	-24.20	-12.04
2nd cycle	8.20	6.47	-6.46	-6.49	6.49	8.24
z(2nd)	4.43	3.49	-3.49	-3.49	3.49	4.45
Final	11.16	16.95	-16.95	20.71	-20.71	-7.59

The nt moments

The lt moments

The final first-order moments

- Note that at joint B, the second-order moment in the beam and the column will be

$$M_{BA} = B_1(13.46) + B_2(3.49)$$

$$M_{BC} = B_1(-13.46) + B_2(-3.49)$$

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Second-Order Analysis

- An assumption often made in practice is that gravity loads do not produce translation and that lateral loads do.
- We now can see clearly that there can be sway due to gravity loads.
- The extent of that sway will be a function of the lack of symmetry of the structure and loads.
- It may not be a good idea to use this commonly accepted assumption.

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Modern Methods

- Since most structural analysis of large systems is not actually carried out using these classical methods, it will be useful to see how some of the classical methods are related to the common modern methods.
- We will specifically look at the general method and the slope deflection method.

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Modern Methods

- Modern approaches to structural analysis might generally be categorized as matrix methods and finite element methods.
  - When I think of matrix methods I generally think of beam and column type structures.
  - When I think of finite element methods I think of plates, shells, and other continuous structures.
  - However, matrix methods can really be thought of as a subset of finite element methods.

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Modern Methods

- The use of matrices to solve systems of simultaneous equations goes back over 2000 years.
- Matrix methods of structural analysis began appearing in the technical literature in the early 1950's.
- Their use was aided by the development of greatly improved digital computers.

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Modern Methods

- Several of the classical methods we discussed resulted in a series of simultaneous equations.
- We did not discuss much how we might solve those equations but they certainly could have been cast into the form of a matrix equation and then be solved using matrix algebra.
- These simultaneous equations could be considered the precursor to matrix analysis of structures.

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Modern Methods

- Matrix structural analysis may be considered through two basic methods, the *flexibility method* and the *stiffness method*.
  - Flexibility method uses compatibility equations and results in the determination of a set of redundant forces. This is also called the compatibility method.
  - Stiffness method uses equilibrium equations and results in the determination of a set of displacements. This is also called the equilibrium method.

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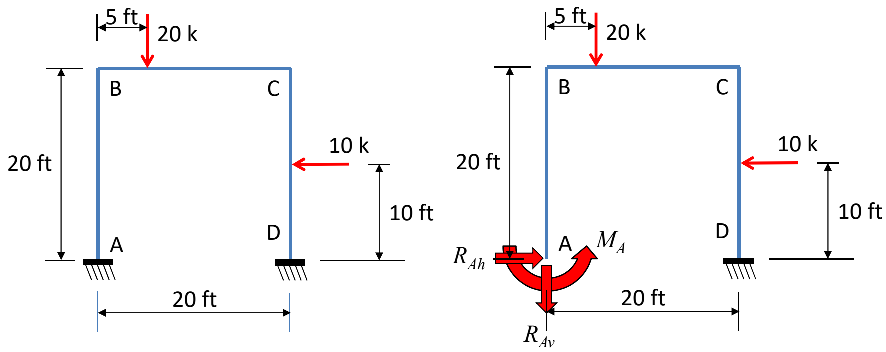
General Method

- Remember the general method from Lesson 5.



Real Structure

Cut Back Structure

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General Method

- The consistent deformation equations are compatibility equations. They represent the requirement that certain displacement conditions must be satisfied.

$$\Delta_{Ah} + R_{Ah}\delta_{AhAh} + R_{Av}\delta_{AhAv} + M_A\delta_{AhmA} = 0$$
$$\Delta_{Av} + R_{Ah}\delta_{AvAh} + R_{Av}\delta_{AvAv} + M_A\delta_{AvmA} = 0$$
$$\theta_A + R_{Ah}\alpha_{AAh} + R_{Av}\alpha_{AAv} + M_A\alpha_{AmA} = 0$$

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General Method

- Upon substituting for the displacements, including the shear contributions, we had

$$\frac{106,667}{EI}-\frac{100}{AG}+R_{Ah}\left(\frac{13,333}{EI}+\frac{40}{AG}\right)+R_{Av}\left(\frac{8,000}{EI}\right)+M_{A}\left(\frac{800}{EI}\right)=0$$

$$\frac{163,750}{EI}+\frac{300}{AG}+R_{Ah}\left(\frac{8,000}{EI}\right)+R_{Av}\left(\frac{10,667}{EI}+\frac{20}{AG}\right)+M_{A}\left(\frac{600}{EI}\right)=0$$

$$\frac{8,750}{EI}+R_{Ah}\left(\frac{800}{EI}\right)+R_{Av}\left(\frac{600}{EI}\right)+M_{A}\left(\frac{60}{EI}\right)=0$$

Lesson 5 Slide 87

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General Method

- Casting the compatibility equations into matrix form yields:

$$\begin{bmatrix}\delta_{AhAh} & \delta_{AhAv} & \delta_{AhmA}\\ \delta_{AvAh} & \delta_{AvAv} & \delta_{AvmA}\\ \alpha_{AAh} & \alpha_{AAv} & \alpha_{AmA}\end{bmatrix}\begin{Bmatrix}R_{Ah}\\ R_{Av}\\ M_{A}\end{Bmatrix}=\begin{Bmatrix}-\Delta_{Ah}\\ -\Delta_{Av}\\ -\theta_{A}\end{Bmatrix}$$

Again we note symmetry

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General Method

- Substituting for the displacements, including the shear contributions we have

$$\begin{bmatrix} \left(\frac{13,333}{EI}+\frac{40}{AG}\right) & \frac{8,000}{EI} & \frac{800}{EI} \\ \frac{8,000}{EI} & \left(\frac{10,667}{EI}+\frac{20}{AG}\right) & \frac{600}{EI} \\ \frac{800}{EI} & \frac{600}{EI} & \frac{60}{EI} \end{bmatrix} \begin{Bmatrix} R_{Ah} \\ R_{Av} \\ M_A \end{Bmatrix} = \begin{Bmatrix} -\frac{106,667}{EI}+\frac{100}{AG} \\ -\frac{163,750}{EI}-\frac{300}{AG} \\ -\frac{8,750}{EI} \end{Bmatrix}$$

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General Method

- The general method is a compatibility method, also referred to as the flexibility method.
- The flexibility matrix, in this case a 3x3 matrix, is made up of displacements.
- The unknown vector is a vector of unknown forces, in this case a 3x1 vector, $\{R_{Ah}\ R_{Av}\ M_A\}$.

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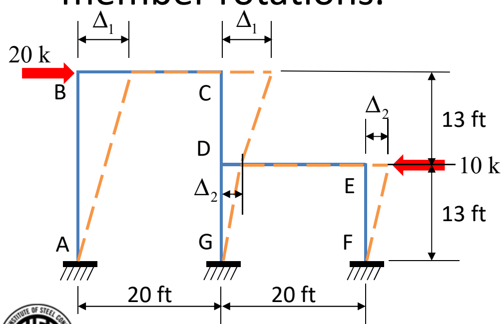
Slope Deflection

- Now look at the slope deflection method from Lesson 6. The unknowns are joint and member rotations.



$\theta_A = \theta_G = \theta_F = 0$

$\theta_B, \theta_C, \theta_D,$ and θ_E are unknowns

Δ to the right is positive, to the left is negative.

Define

$$\rho_1 = \frac{\Delta_1}{26} \qquad \rho_2 = \frac{\Delta_2}{13}$$

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Slope Deflection

- These 6 simultaneous equations are from that example in Lesson 6. They are equilibrium equations in terms of stiffness.

$$\begin{array}{llllllll}
9.2K'\theta_B & +2.6K'\theta_C & & & -6K'\rho_1 & & = & 0 \\
2.6K'\theta_B & +13.2K'\theta_C & +4K'\theta_D & & -24K'\rho_1 & +12K'\rho_2 & = & 0 \\
 & +4K'\theta_C & +21.2K'\theta_D & +2.6K'\theta_E & -24K'\rho_1 & & = & 0 \\
 & & +2.6K'\theta_D & +13.2K'\theta_E & & -12K'\rho_2 & = & 0 \\
6K'\theta_B & +24K'\theta_C & +24K'\theta_D & & -108K'\rho_1 & +48K'\rho_2 & = & -520 \\
 & -12K'\theta_C & & +12K'\theta_E & +48K'\rho_1 & -72K'\rho_2 & = & 130
\end{array}$$

This is Lesson 6 Slide 94. Again, remember that we pointed out symmetry in Lesson 6

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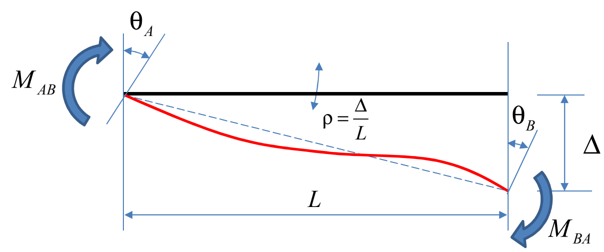
Slope Deflection

- If we return to the single member and the end moment equations, without load, we had

$$M_{AB} = \frac{4EI}{L}\theta_A + \frac{2EI}{L}\theta_B - \frac{6EI}{L}\rho$$

$$M_{BA} = \frac{2EI}{L}\theta_A + \frac{4EI}{L}\theta_B - \frac{6EI}{L}\rho$$



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Slope Deflection

- In matrix format, the member end moment equations can be written, based on the slope deflection equations, as

$$\begin{Bmatrix} M_{AB} \\ M_{BA} \end{Bmatrix} = \begin{bmatrix} \frac{4EI}{L} & \frac{2EI}{L} & -\frac{6EI}{L} \\ \frac{2EI}{L} & \frac{4EI}{L} & -\frac{6EI}{L} \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \\ \rho_{AB} \end{Bmatrix}$$

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Slope Deflection

- Since this formulation was based on slope deflection, it only includes flexural deformations.
- The stiffness method would include axial forces, shear forces, and torsional moments in its complete form.
- We can look at that formulation with just a few more definitions.

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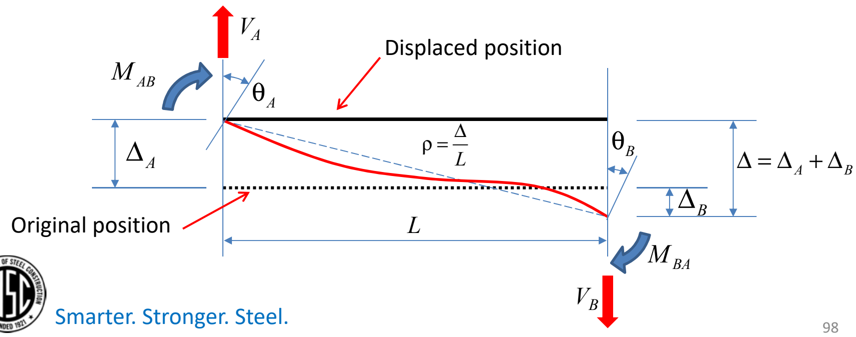
Modern Methods

- Looking at the same member from the element stiffness matrix perspective we approach the displacements a bit differently and we also look at the reactions. For flexure



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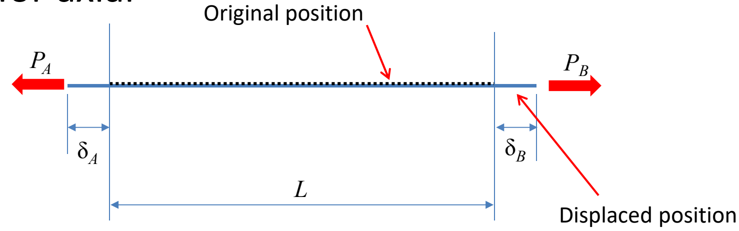
Modern Methods

- And for axial



Note that the axial deformation has no influence on member moments or shears.

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Modern Methods

- Considering axial and flexural deformations, and the new definitions for member end displacements, the member stiffness equations become

$$\begin{Bmatrix} P_A \\ V_A \\ M_{AB} \\ P_B \\ V_B \\ M_{BA} \end{Bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ 0 & \frac{-6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ \frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{Bmatrix} \delta_A \\ \Delta_A \\ \theta_A \\ \delta_B \\ \Delta_B \\ \theta_B \end{Bmatrix}$$

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Modern Methods

- Now considering axial, flexural and shearing deformations, the member stiffness equations become

$$\begin{Bmatrix} P_A \\ V_A \\ M_{AB} \\ P_B \\ V_B \\ M_{BA} \end{Bmatrix} = \frac{1}{1+2g} \left[\begin{array}{ccc|ccc} \frac{EA}{L}(1+2g) & 0 & 0 & \frac{EA}{L}(1+2g) & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ 0 & \frac{-6EI}{L^2} & \frac{4EI}{L}\left(1+\frac{g}{2}\right) & 0 & \frac{-6EI}{L^2} & \frac{2EI}{L}(1-g) \\ \hline \frac{EA}{L}(1+2g) & 0 & 0 & \frac{EA}{L}(1+2g) & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ 0 & \frac{-6EI}{L^2} & \frac{2EI}{L}(1-g) & 0 & \frac{-6EI}{L^2} & \frac{4EI}{L}\left(1+\frac{g}{2}\right) \end{array} \right] \begin{Bmatrix} \delta_A \\ \Delta_A \\ \theta_A \\ \delta_B \\ \Delta_B \\ \theta_B \end{Bmatrix}$$

For a W-shape $g = \frac{6EI}{GA_w L^2}$

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Modern Methods

- Most text books on matrix methods do not address shearing deformations in their development of the stiffness matrix. One reference that does is

    Weaver, W. Jr. and Gere, J. M., *Matrix Analysis of Framed Structures*, 2nd edition, D. Van Nostrand Co., 1980.

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Modern Methods

- So what really is the difference between the stiffness method and slope deflection?
  - As seen here, axial deformations are included.
  - Although not normally included in typical text books, shearing deformations can be included as shown.
  - Torsional moments and deformations can be included if we consider a three dimensional element.
  - But the real value is the systematic approach to formulating the structure equations from the member equations, which we have not illustrated.

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Modern Methods

- Question to consider
  - We have looked at classical and modern methods. Which of the following would you consider to be true? (Select all that are true.)
    - Modern methods do not require simplifications for the engineer to model the structure.
    - Classical methods require modeling simplifications that make them unsuitable for structural engineering in the 21st century.
    - The engineer must determine the level of simplification that is acceptable for their design.
    - There could be multiple correct answers to the solution of a real world structural analysis problem.

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Polling Question

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Summary

- For this lesson we have looked at second-order analysis and seen how the classical methods can help us understand the influence of these second-order effects.
- We have also looked at matrix methods and seen how the classical methods we have been studying can be help us to understand these matrix methods.

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Course Conclusion

- Over these 8 lessons we have studied what are generally known as the classical methods of structural analysis.
- We have addressed methods of calculating displacements and have used these displacements in the compatibility methods to determine force redundants.

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Course Conclusion

- We have also addressed equilibrium methods and found ways to determine displacements and then used those to directly determine forces and moments.
- We have investigated how these classical methods can help us carry-out second-order analysis.
- And we have concluded by comparing these classical methods to modern matrix methods.

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Course Conclusion

- Overall, the course intent was to emphasize an understanding of the behavior of structures so that the first thought, when faced with the analysis of a simple structure, is not to rush off to the computer but instead, to apply the appropriate classical method to the problem.

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Thank You

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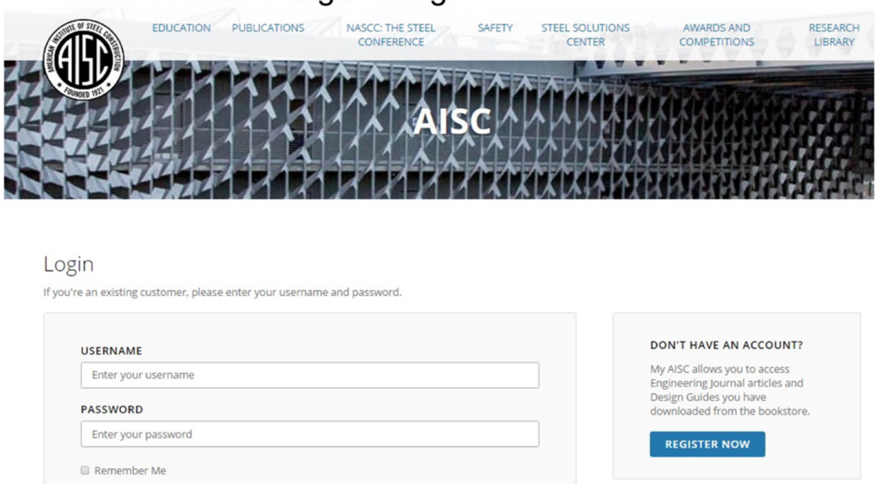

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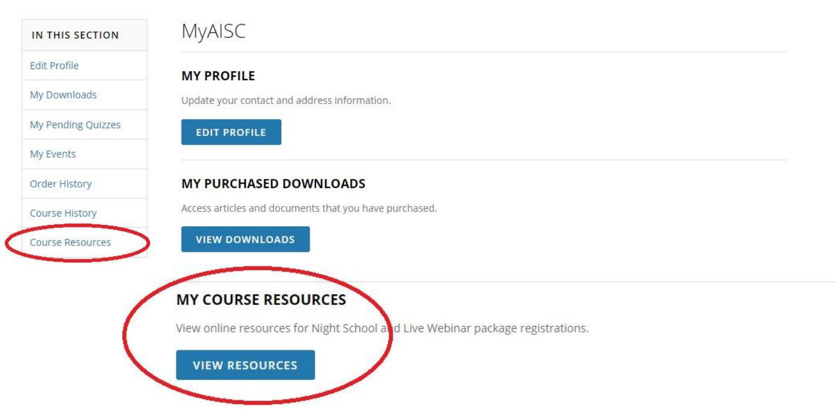

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