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Night School

**Basic Steel Design**  
Louis F. Geschwindner




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Stronger.  
Steel.



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## Session Description

### 22.3 Compression Members February 11, 2020

The design of columns – compression members, is the focus of this session. The session will review the strength of compression members as defined by the Specification. The session will review steel shapes and their behavior in compression. The session will discuss the limit states of flexural buckling, local buckling, torsional buckling, and flexural-torsional buckling. Members with and without slender elements are reviewed. Design examples will be presented.





### Learning Objectives:

- Describe the limit state of flexural buckling for the design of compression members.
- Describe the limit state of local buckling for the design of compression members.
- Describe the limit state of torsional buckling and flexural-torsional buckling for the design of compression members.
- List the design steps for members with and without slender elements.



Basic Steel Design: A review of the  
principles of steel design according to  
ANSI/AISC 360-16

Night School 22  
Lesson 3  
Compression Members



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## Lesson 3 – Compression

- Compression Members
  - Strength
  - Flexural buckling
  - Effective length
  - Local buckling
  - Torsional and flexural-torsional buckling
  - Built-up shapes



3.9

## Compression Members

B3.1. For LRFD, design shall be performed in accordance with:

Required Strength  $\leq$  Available Strength

$$R_u \leq \phi R_n \quad (\text{B3-1})$$

where

$R_u$  = required strength (LRFD) defined in Chapter C

$R_n$  = nominal strength specified in Chapter E

$\phi$  = resistance factor specified in Chapter E

$\phi R_n$  = design strength = resistance factor (nominal strength)



3.10

## Compression Members

B3.2. For ASD, design shall be performed in accordance with:

Required Strength  $\leq$  Available Strength

$$R_a \leq R_n / \Omega \quad (\text{B3-2})$$

where

$R_a$  = required strength (ASD) defined in Chapter C

$R_n$  = nominal strength specified in Chapter E

$\Omega$  = safety factor specified in Chapter E

$R_n / \Omega$  = allowable strength =  $\frac{\text{nominal strength}}{\text{safety factor}}$



3.11

## Compression Members

E1. “The design compressive strength,  $\phi_c P_n$ , and the allowable compressive strength,  $P_n / \Omega_c$ , are determined as follows:

The nominal compressive strength,  $P_n$ , shall be the lowest value obtained based on the applicable limit states of flexural buckling, torsional buckling, and flexural-torsional buckling.”

$$\phi_c = 0.90 \text{ (LRFD)}$$

$$\Omega_c = 1.67 \text{ (ASD)}$$



3.12

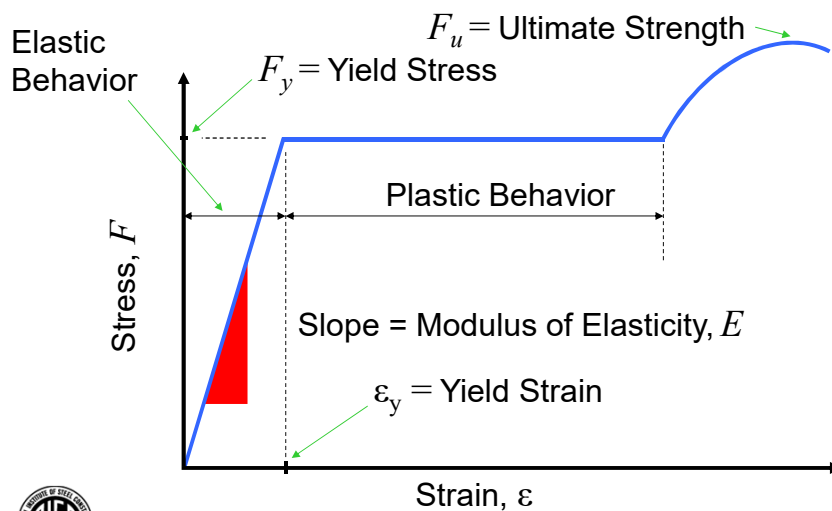
## Compression Members

- Limit States
  - **Yielding**: not mentioned in list of limit states to be checked. But, it is the upper limit for all shapes.
  - **Flexural buckling**: lateral buckling about a geometric axis, Euler Buckling, considered for all shapes.
  - **Torsional buckling**: Twist buckling of double symmetric shapes.
  - **Flexural-Torsional buckling**: Combined twist and lateral buckling for singly- and non-symmetric shapes.



3.13

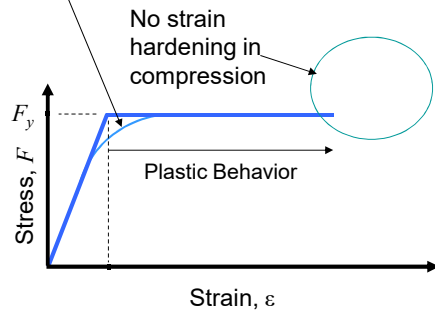
## Steel as a Material



3.14

# Compression Members

Inelastic behavior will be discussed next



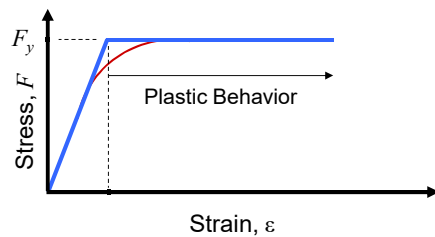
Yielding is the upper limit for all shapes. Only members with a length about twice the depth or less will reach yielding. A buckling limit state will occur first for longer members.



3.15

# Compression Members

Inelastic behavior results from the presence of residual stresses in the rolled shape. This will have an impact on column strength that will be shown later.



Stub column test where actual residual stresses impact stress-strain curve

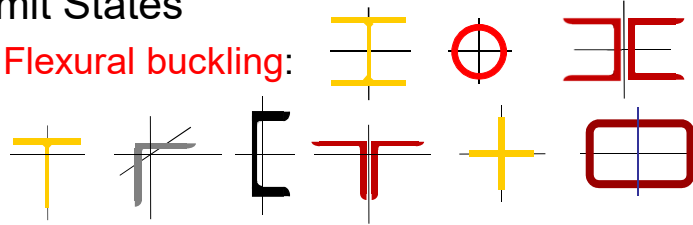


3.16

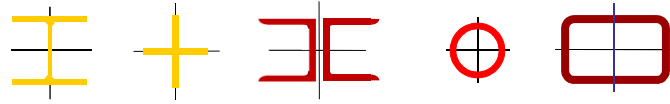
## Compression Members

- Limit States

- Flexural buckling:



- Torsional buckling:

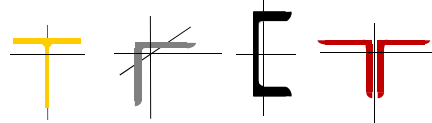


3.17

## Compression Members

- Limit States

- Flexural-Torsional buckling:



3.18

## Flexural Buckling

- Flexural buckling was first address by Leonhard Euler, a Swiss mathematician, about 1744. It is what we generally call Euler buckling.
- The theoretical derivation will not be addressed here but there are many references available.
- Remember Euler's Equation? It is given by

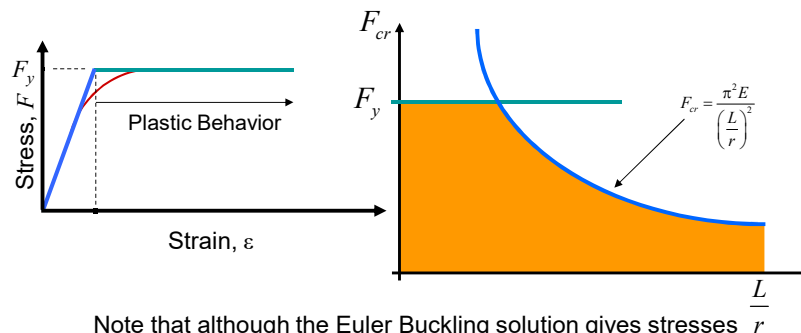
$$P_{cr} = \frac{\pi^2 EI}{L^2} \quad \text{or} \quad F_{cr} = \frac{\pi^2 E}{\left(\frac{L}{r}\right)^2}$$



3.19

## Flexural Buckling

- Yielding and elastic flexural buckling



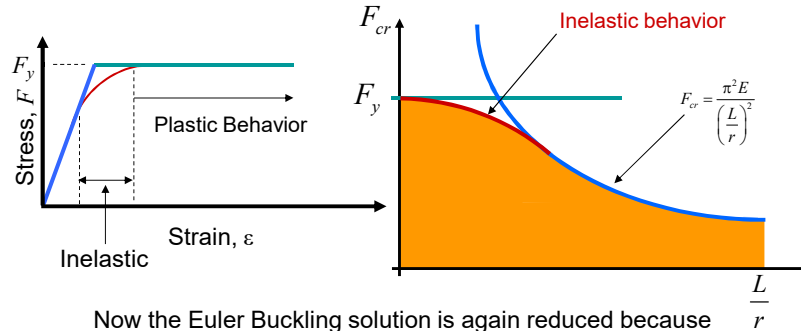
Note that although the Euler Buckling solution gives stresses greater than  $F_y$ , the column can not physically carry stresses that high, thus the curve is "cut off" at  $F_y$ .



3.20

## Flexural Buckling

- Inelastic flexural buckling



Now the Euler Buckling solution is again reduced because of the presence of residual stresses in the real compression member.



3.21

## Flexural Buckling

- Have already considered:
  - Yielding
  - Elastic flexural buckling
  - Inelastic flexural buckling
- Additional factors influencing column behavior that must be addressed to produce the *Specification* provisions:
  - End conditions: **the  $K$ -factor and effective length are introduced**
  - Out-of-straightness: **the 0.877 multiplier is used**



3.22

## Flexural Buckling

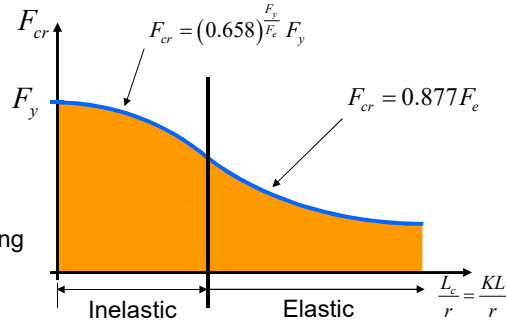
- Specification equations

Elastic buckling stress

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2}$$

Dividing line between inelastic and elastic buckling

$$\frac{KL}{r} = 4.71 \sqrt{\frac{E}{F_y}}$$



Both curves are reduced from theoretical to account for out-of-straightness



3.23

## Compression Members

### E2. Effective Length

“The effective length,  $L_c$ , for calculation of member slenderness,  $L_c/r$ , shall be determined in accordance with Chapter C or Appendix 7,”

where

$L_c = KL =$  effective length of member, in. (mm)

$K =$  effective length factor

$L =$  laterally unbraced length of the member, in. (mm)

$r =$  radius of gyration, in. (mm)

**User Note:** For members designed on the basis of compression, the effective slenderness ratio,  $L_c/r$ , preferably should not exceed 200.



3.24

# Effective Length Factor

**TABLE C-A-7.1**  
**Approximate Values of Effective Length Factor,  $K$**

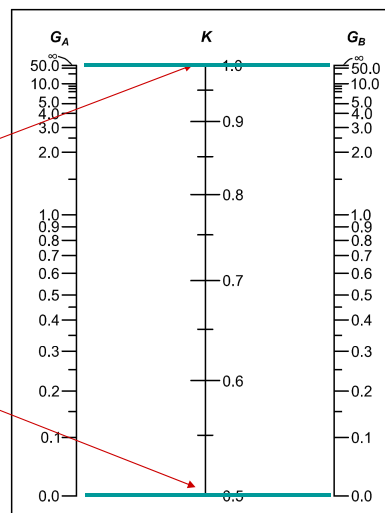
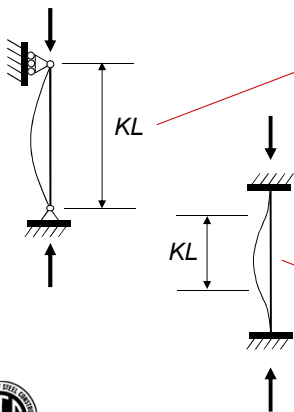
	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical $K$ value	0.5	0.7	1.0	1.0	2.0	2.0
Recommended design value when ideal conditions are approximated	0.65	0.80	1.2	1.0	2.1	2.0
End condition code						



3.25

# Effective Length Factor

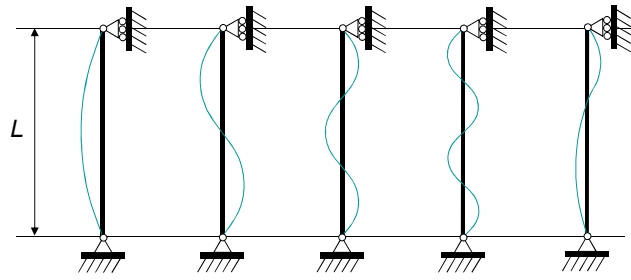
Braced frame members: ends do not sway relative to each other



3.26

## Effective Length Factor

- Compression member bracing



$K = 1.0$     $K = 0.5$     $K = 0.33$     $K = 0.25$     $K = 0.67$

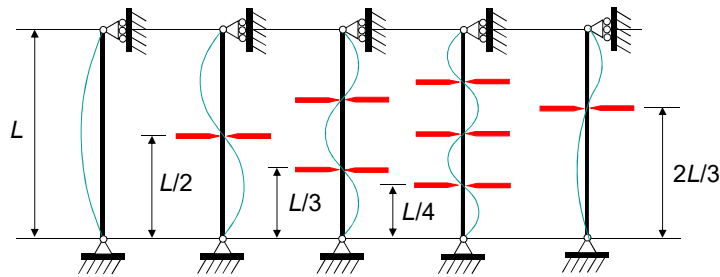
The definition of  $K$  is dependent on the definition of  $L$



3.27

## Effective Length Factor

- Compression member bracing



$K = 1.0$     $K = 0.5$     $K = 0.33$     $K = 0.25$     $K = 0.67$

The definition of  $K$  is dependent on the definition of  $L$



3.28

## Flexural Buckling

### E3. Flexural Buckling of Members (**Without Slender Elements**)

Nominal Compressive Strength

$$P_n = F_{cr} A_g \quad (\text{E3-1})$$



3.29

## Flexural Buckling

- Elastic buckling stress is based on

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} \quad (\text{E3-4})$$

or Appendix 7.2.3(b) where  $F_e$  shall be determined from a sidesway buckling analysis for moment frames.



3.30

## Flexural Buckling

- Inelastic Response  $\frac{L_c}{r} \leq 4.71 \sqrt{\frac{E}{F_y}}$  or  $\frac{F_y}{F_e} \leq 2.25$

$$F_{cr} = (0.658)^{\frac{F_y}{F_e}} F_y \quad (\text{E3-2})$$

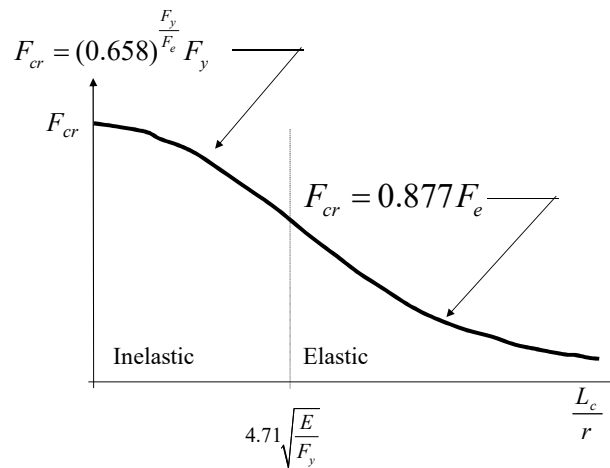
- Elastic Response  $\frac{L_c}{r} > 4.71 \sqrt{\frac{E}{F_y}}$  or  $\frac{F_y}{F_e} > 2.25$

$$F_{cr} = 0.877 F_e \quad (\text{E3-3})$$



3.31

## Flexural Buckling



3.32

# Flexural Buckling

- ASD

$$\frac{P_n}{\Omega_c} = \frac{F_{cr} A_g}{1.67} = 0.6 F_{cr} A_g$$

Allowable Stress

- LRFD

$$\phi_c P_n = 0.90 F_{cr} A_g$$

These are the available stress values tabulated in Table 4-14

Design Stress



3.33

# Flexural Buckling

Table 4-14  
Available Critical Stress for  
Compression Members


$\frac{L_c}{r}$	$F_y = 35 \text{ ksi}$		$F_y = 36 \text{ ksi}$		$F_y = 46 \text{ ksi}$		$F_y = 50 \text{ ksi}$		$F_y = 65 \text{ ksi}$		$F_y = 70 \text{ ksi}$	
	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$
	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
1	21.0	31.5	21.6	32.4	27.5	41.4	29.9	45.0	38.9	58.5	41.9	63.0
2	21.0	31.5	21.6	32.4	27.5	41.4	29.9	45.0	38.9	58.5	41.9	63.0
3	20.9	31.5	21.5	32.4	27.5	41.4	29.9	45.0	38.9	58.4	41.9	62.9
4	20.9	31.5	21.5	32.4	27.5	41.4	29.9	44.9	38.9	58.4	41.8	62.9
5	20.9	31.5	21.5	32.4	27.5	41.3	29.9	44.9	38.8	58.4	41.8	62.8
6	20.9	31.4	21.5	32.3	27.5	41.3	29.9	44.9	38.8	58.3	41.8	62.8
7	20.9	31.4	21.5	32.3	27.5	41.3	29.8	44.8	38.7	58.2	41.7	62.7
35	19.7	29.6	20.2	30.4	25.4	38.1	27.4	41.2	34.6	52.1	37.0	55.6
36	19.6	29.5	20.1	30.3	25.2	37.9	27.2	40.9	34.4	51.7	36.7	55.2
37	19.5	29.4	20.1	30.1	25.1	37.8	27.1	40.7	34.2	51.4	36.4	54.8
38	19.5	29.3	20.0	30.0	25.0	37.6	26.9	40.5	33.9	51.0	36.2	54.3
39	19.4	29.1	19.9	29.9	24.9	37.4	26.8	40.3	33.7	50.6	35.9	53.9
40	19.3	29.0	19.8	29.8	24.7	37.2	26.6	40.0	33.4	50.2	35.6	53.5
ASD	LRFD											
	$\Omega_c = 1.67$	$\phi_c = 0.90$										



3.34

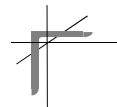
## Flexural Buckling

- A992 Wide flange members,  $F_y = 50$  ksi



$$\frac{L_c}{r} = 4.71 \sqrt{\frac{E}{F_y}} = 113$$

- A36 Angles,  $F_y = 36$  ksi



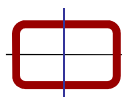
$$\frac{L_c}{r} = 4.71 \sqrt{\frac{E}{F_y}} = 134$$



3.35

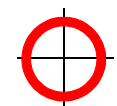
## Flexural Buckling

- A500 Gr C Rectangular HSS,  $F_y = 50$  ksi



$$\frac{L_c}{r} = 4.71 \sqrt{\frac{E}{F_y}} = 113$$

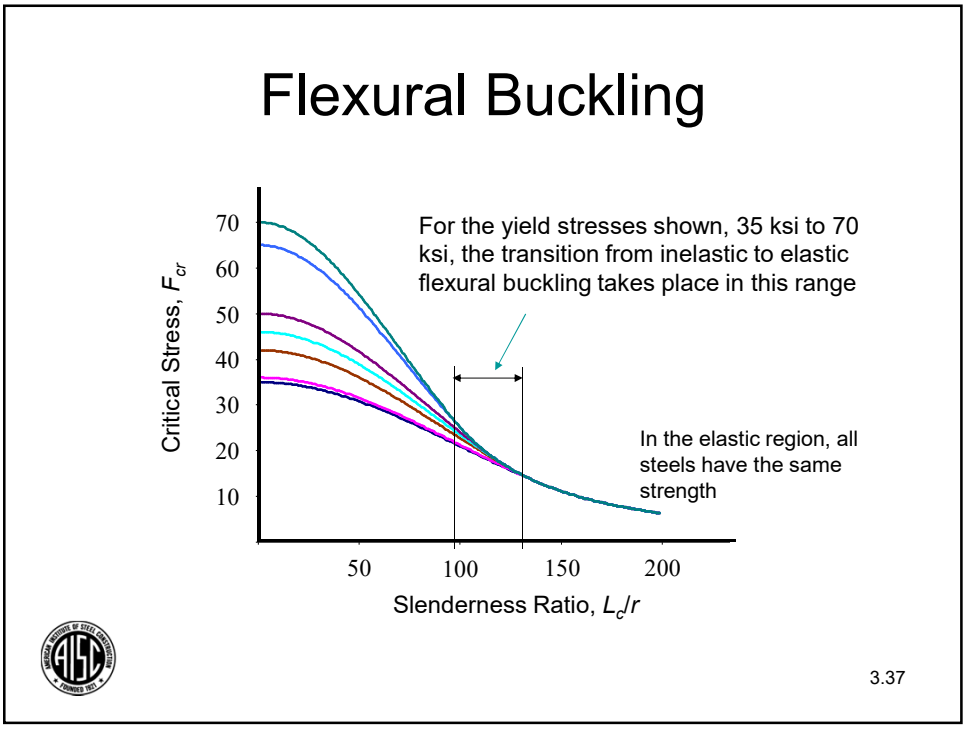
- A500 Gr C Round HSS,  $F_y = 46$  ksi



$$\frac{L_c}{r} = 4.71 \sqrt{\frac{E}{F_y}} = 118$$



3.36



# Flexural Buckling

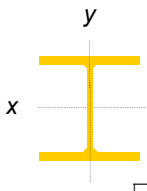
Table 4-14 (continued)  
Available Critical Stress for  
Compression Members

$\lambda_c$	$F_y = 35 \text{ ksi}$		$F_y = 46 \text{ ksi}$		$F_y = 58 \text{ ksi}$		$F_y = 70 \text{ ksi}$		$F_y = 85 \text{ ksi}$		$F_y = 100 \text{ ksi}$		$F_y = 120 \text{ ksi}$		$F_y = 150 \text{ ksi}$		
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	
134	8.37	12.6	134	8.37	12.6	134	8.37	12.6	134	8.37	12.6	134	8.37	12.6	134	8.37	12.6
135	8.25	12.4	135	8.25	12.4	135	8.25	12.4	135	8.25	12.4	135	8.25	12.4	135	8.25	12.4
136	8.13	12.2	136	8.13	12.2	136	8.13	12.2	136	8.13	12.2	136	8.13	12.2	136	8.13	12.2
137	8.01	12.0	137	8.01	12.0	137	8.01	12.0	137	8.01	12.0	137	8.01	12.0	137	8.01	12.0
138	7.89	11.9	138	7.89	11.9	138	7.89	11.9	138	7.89	11.9	138	7.89	11.9	138	7.89	11.9
139	7.78	11.7	139	7.78	11.7	139	7.78	11.7	139	7.78	11.7	139	7.78	11.7	139	7.78	11.7
140	7.67	11.5	140	7.67	11.5	140	7.67	11.5	140	7.67	11.5	140	7.67	11.5	140	7.67	11.5
141	7.56	11.4	141	7.56	11.4	141	7.56	11.4	141	7.56	11.4	141	7.56	11.4	141	7.56	11.4
142	7.45	11.2	142	7.45	11.2	142	7.45	11.2	142	7.45	11.2	142	7.45	11.2	142	7.45	11.2
143	7.35	11.0	143	7.35	11.0	143	7.35	11.0	143	7.35	11.0	143	7.35	11.0	143	7.35	11.0
144	7.25	10.9	144	7.25	10.9	144	7.25	10.9	144	7.25	10.9	144	7.25	10.9	144	7.25	10.9
145	7.15	10.7	145	7.15	10.7	145	7.15	10.7	145	7.15	10.7	145	7.15	10.7	145	7.15	10.7
146	7.05	10.6	146	7.05	10.6	146	7.05	10.6	146	7.05	10.6	146	7.05	10.6	146	7.05	10.6
151	6.42	9.85	151	6.42	9.85	151	6.42	9.85	151	6.42	9.85	151	6.42	9.85	151	6.42	9.85
152	6.34	9.53	152	6.34	9.53	152	6.34	9.53	152	6.34	9.53	152	6.34	9.53	152	6.34	9.53
153	6.26	9.40	153	6.26	9.40	153	6.26	9.40	153	6.26	9.40	153	6.26	9.40	153	6.26	9.40
154	6.18	9.28	154	6.18	9.28	154	6.18	9.28	154	6.18	9.28	154	6.18	9.28	154	6.18	9.28
155	6.10	9.17	155	6.10	9.17	155	6.10	9.17	155	6.10	9.17	155	6.10	9.17	155	6.10	9.17
156	6.02	9.05	156	6.02	9.05	156	6.02	9.05	156	6.02	9.05	156	6.02	9.05	156	6.02	9.05
157	5.95	8.94	157	5.95	8.94	157	5.95	8.94	157	5.95	8.94	157	5.95	8.94	157	5.95	8.94
158	5.87	8.82	158	5.87	8.82	158	5.87	8.82	158	5.87	8.82	158	5.87	8.82	158	5.87	8.82
159	5.87	8.82	159	5.87	8.82	159	5.87	8.82	159	5.87	8.82	159	5.87	8.82	159	5.87	8.82
160	5.87	8.82	160	5.87	8.82	160	5.87	8.82	160	5.87	8.82	160	5.87	8.82	160	5.87	8.82

$F_y$  has no impact on available strength

3.38

## Flexural Buckling



**Table 4-1a (continued)**  
**Available Strength in Axial Compression, kips**  $F_y = 50$  ksi

**W-Shapes**

Shape lb/ft	W14*											
	145		132		120		109		99		90	
Design	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793	1190
6	1250	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160
7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150
8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140
9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120

**Table 4-1b**  
 $F_y = 65$  ksi

**Table 4-1c**  
 $F_y = 70$  ksi

Effective length,  $L_c$  (ft), with respect to least radius of gyration,  $r_y$

Effective length, $L_c$ (ft)	W14*											
	145		132		120		109		99		90	
Design	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000
16	1080	1620	960	1440	872	1310	789	1190	716	1080	652	979
17	1060	1590	937	1410	850	1280	770	1160	698	1050	635	955
18	1030	1550	913	1370	828	1240	750	1130	680	1020	618	929
19	1010	1510	888	1330	805	1210	729	1100	661	994	601	903
20	990	1470	869	1290	789	1180	708	1060	642	964	582	877

For W-shapes,  $r_y$  is the least radius of gyration

Effective length, $L_c$ (ft)	W14*											
	145		132		120		109		99		90	
Design	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$	$P_u/\phi_c P_n$	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
26	810	1230	702	1000	635	955	574	863	519	761	472	709
28	759	1140	648	974	586	880	529	796	478	719	434	653
30	703	1060	594	883	537	807	485	729	436	658	397	587
32	647	973	542	814	489	735	441	663	398	598	361	543

3.39



## Example 1

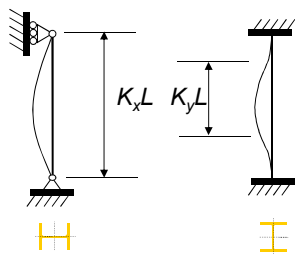
- Consider a W14 x 120 column (A992)
  - Shape without slender elements
- Determine the available compressive strength by ASD and LRFD

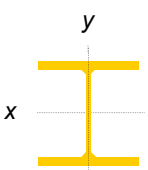
$L = 30$  ft

$F_y = 50$  ksi

$K_x = 1.0$

$K_y = 0.5$





$r_x = 6.24$  in.

$r_y = 3.74$  in.

3.40

## Example 1

### Critical Slenderness

$$\frac{L_c}{r_y} = \frac{0.5(30)(12)}{3.74} = 48.1 \quad \frac{L_c}{r_x} = \frac{1.0(30)(12)}{6.24} = 57.7$$



$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} = \frac{\pi^2 (29,000)}{(57.7)^2} = 86.0 \text{ ksi}$$



3.41

## Example 1

$$\frac{L_c}{r} = 57.7 < 113 \quad \text{therefore use Eq. E3-2}$$

$$F_{cr} = (0.658)^{\frac{50}{86.0}} (50) = 39.2 \text{ ksi}$$

$$P_n = (39.2)(35.3) = 1380 \text{ kips}$$



3.42

# Example 1

• **ASD**  $\frac{P_n}{\Omega_c} = \frac{1380}{1.67} = 826 \text{ kips}$

To use Table 4-1a, when x-axis is critical we must determine an equivalent  $L_{cy}$

$$\frac{(L_{cy})_{\text{equivalent}}}{r_y} = \frac{L_{cx}}{r_x}$$

$$(L_{cy})_{\text{equivalent}} = \frac{L_{cx}}{r_x/r_y}$$

$$= \frac{30}{1.67} = 18.0 \text{ ft}$$



**Table 4-1a (continued)**  
**Available Strength in Axial Compression, kips**  $F_y = 50 \text{ ksi}$   
**W-Shapes**

W14

Shape	W14*											
	145		132		120		109		99		90	
lb/ft	$P_n/\Omega_c$	$\phi P_n$	$P_n/\Omega_c$	$\phi P_n$	$P_n/\Omega_c$	$\phi P_n$	$P_n/\Omega_c$	$\phi P_n$	$P_n/\Omega_c$	$\phi P_n$	$P_n/\Omega_c$	$\phi P_n$
Design	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793	1190
6	1260	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160
7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150
8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140
9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120
10	1200	1800	1080	1620	980	1470	889	1340	807	1210	735	1100
11	1180	1770	1060	1600	965	1450	874	1310	794	1190	723	1090
12	1160	1750	1040	1570	948	1430	859	1290	780	1170	710	1070
13	1140	1720	1020	1540	931	1400	843	1270	766	1150	697	1050
14	1120	1690	1000	1510	912	1370	826	1240	750	1130	682	1030
15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000
16	1080	1620	960	1440	872	1310	789	1190	716	1080	652	979
17	1060	1590	937	1410	850	1280	770	1160	698	1050	635	955
18	1030	1550	913	1370	828	1240	750	1130	680	1020	618	929
19	1010	1510	888	1330	805	1210	729	1100	661	994	601	903
20	990	1470	862	1300	782	1180	708	1060	642	964	583	877

*r<sub>y</sub>*, in. 3.98 3.76 3.74 3.73 3.71 3.70  
*r<sub>x</sub>*, in. 1.59 1.67 1.67 1.67 1.66 1.66  
 $P_n/A_g/10^3$ , k-in.<sup>2</sup> 48900 43800 39500 35500 31800 28800  
 $P_n/A_g/10^3$ , k-in.<sup>2</sup> 19400 15700 14200 12800 11500 10400

ASD LRFD  
 $\Omega_c = 1.67$   $\phi_c = 0.90$

# Example 1

• **LRFD**  $\phi_c P_n = 0.9(1380) = 1240 \text{ kips}$

To use Table 4-1a, when x-axis is critical we must determine an equivalent  $L_{cy}$

$$\frac{(L_{cy})_{\text{equivalent}}}{r_y} = \frac{L_{cx}}{r_x}$$

$$(L_{cy})_{\text{equivalent}} = \frac{L_{cx}}{r_x/r_y}$$

$$= \frac{30}{1.67} = 18.0 \text{ ft}$$



**Table 4-1a (continued)**  
**Available Strength in Axial Compression, kips**  $F_y = 50 \text{ ksi}$   
**W-Shapes**

W14

Shape	W14*											
	145		132		120		109		99		90	
lb/ft	$P_n/\Omega_c$	$\phi P_n$	$P_n/\Omega_c$	$\phi P_n$	$P_n/\Omega_c$	$\phi P_n$	$P_n/\Omega_c$	$\phi P_n$	$P_n/\Omega_c$	$\phi P_n$	$P_n/\Omega_c$	$\phi P_n$
Design	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793	1190
6	1260	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160
7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150
8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140
9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120
10	1200	1800	1080	1620	980	1470	889	1340	807	1210	735	1100
11	1180	1770	1060	1600	965	1450	874	1310	794	1190	723	1090
12	1160	1750	1040	1570	948	1430	859	1290	780	1170	710	1070
13	1140	1720	1020	1540	931	1400	843	1270	766	1150	697	1050
14	1120	1690	1000	1510	912	1370	826	1240	750	1130	682	1030
15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000
16	1080	1620	960	1440	872	1310	789	1190	716	1080	652	979
17	1060	1590	937	1410	850	1280	770	1160	698	1050	635	955
18	1030	1550	913	1370	828	1240	750	1130	680	1020	618	929
19	1010	1510	888	1330	805	1210	729	1100	661	994	601	903
20	990	1470	862	1300	782	1180	708	1060	642	964	583	877

*r<sub>y</sub>*, in. 3.98 3.76 3.74 3.73 3.71 3.70  
*r<sub>x</sub>*, in. 1.59 1.67 1.67 1.67 1.66 1.66  
 $P_n/A_g/10^3$ , k-in.<sup>2</sup> 48900 43800 39500 35500 31800 28800  
 $P_n/A_g/10^3$ , k-in.<sup>2</sup> 19400 15700 14200 12800 11500 10400

ASD LRFD  
 $\Omega_c = 1.67$   $\phi_c = 0.90$

## Compression Member Design

- For a compression member design
  - We likely know
    - Required strength
    - Member length
    - Some idea of effective length factor
  - Unlike for tension members we don't know
    - The critical stress
  - However, we could estimate
    - Radius of gyration which leads to a slenderness ratio which leads to critical stress
  - Or rely on the design tables from the Manual



3.45

## Example 2 (ASD)

Select a column section by ASD

$$P_D = 275 \text{ kips}$$

$$P_L = 600 \text{ kips}$$

$$L_{cx} = L_{cy} = 18 \text{ ft}$$

ASD load combination (D+L)

$$P_a = 275 + 600 = 875 \text{ kips}$$



3.46

## Example 2 (ASD)

- Select W14x132
- Required Strength  
 $P_a = 875$  kips
- Available strength

$$\frac{P_n}{\Omega} = 913 \text{ kips}$$

- Therefore the column is adequate



Table 4-1a (continued)  
Available Strength in Axial Compression, kips  
W-Shapes  
 $F_y = 50$  ksi

Shape lb/ft	145				132				120				109				99				90				
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	
Design																									
0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793	1160	772	1160	772	1160	772	1160	772	1160	772	1160	772	1160	772
6	1250	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160	772	1160	772	1160	772	1160	772	1160	772	1160	772	1160	772
7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150	764	1150	764	1150	764	1150	764	1150	764	1150	764	1150	764
8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140	755	1140	755	1140	755	1140	755	1140	755	1140	755	1140	755
9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120	745	1120	745	1120	745	1120	745	1120	745	1120	745	1120	745
10	1200	1800	1080	1620	980	1470	888	1340	807	1210	735	1100	735	1100	735	1100	735	1100	735	1100	735	1100	735	1100	735
11	1180	1770	1060	1600	965	1450	874	1310	794	1190	723	1080	723	1080	723	1080	723	1080	723	1080	723	1080	723	1080	723
12	1160	1750	1040	1570	949	1430	859	1290	780	1170	710	1070	710	1070	710	1070	710	1070	710	1070	710	1070	710	1070	710
13	1140	1720	1020	1540	931	1400	843	1270	766	1150	697	1050	697	1050	697	1050	697	1050	697	1050	697	1050	697	1050	697
14	1120	1690	1000	1510	912	1370	826	1240	750	1130	682	1030	682	1030	682	1030	682	1030	682	1030	682	1030	682	1030	682
15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000	667	1000	667	1000	667	1000	667	1000	667	1000	667	1000	667
16	1080	1620	960	1440	872	1310	789	1190	716	1080	652	979	652	979	652	979	652	979	652	979	652	979	652	979	652
18	1030	1550	913	1320	826	1240	750	1120	689	1020	618	929	618	929	618	929	618	929	618	929	618	929	618	929	618
19	1010	1510	888	1300	805	1210	729	1100	661	994	601	905	601	905	601	905	601	905	601	905	601	905	601	905	601
20	990	1470	862	1300	782	1180	708	1060	642	964	583	877	583	877	583	877	583	877	583	877	583	877	583	877	583
22	972	1430	810	1220	734	1100	664	998	602	904	547	822	547	822	547	822	547	822	547	822	547	822	547	822	547
24	972	1310	756	1140	685	1030	620	931	561	843	509	766	509	766	509	766	509	766	509	766	509	766	509	766	509
26	816	1230	702	1060	635	955	574	863	519	781	472	709	472	709	472	709	472	709	472	709	472	709	472	709	472
28	759	1140	648	974	586	880	529	795	478	719	434	653	434	653	434	653	434	653	434	653	434	653	434	653	434
30	703	1060	594	893	537	807	483	728	438	658	397	597	397	597	397	597	397	597	397	597	397	597	397	597	397
32	647	973	542	814	489	735	441	663	398	598	361	543	361	543	361	543	361	543	361	543	361	543	361	543	361

3.47

## Example 2 (LRFD)

Select a column section by LRFD

$$P_D = 275 \text{ kips}$$

$$P_L = 600 \text{ kips}$$

$$L_{cx} = L_{cy} = 18 \text{ ft}$$

LRFD load combination (1.2D+1.6L)

$$P_u = 1.2(275) + 1.6(600) = 1290 \text{ kips}$$



3.48


## Example 2 (LRFD)

- Select W14x132
- Required Strength  
 $P_u = 1290$  kips
- Available strength

$$\phi P_n = 1370 \text{ kips}$$

- Therefore the column is adequate

Table 4-1a (continued)  
Available Strength in Axial Compression, kips  
 $F_y = 50$  ksi  
W-Shapes



Shape lb/ft	W14*											
	145		132		120		109		99		90	
Design	$P_n/\Omega_c$	$\phi_c P_n$	$P_n/\Omega_c$	$\phi_c P_n$	$P_n/\Omega_c$	$\phi_c P_n$	$P_n/\Omega_c$	$\phi_c P_n$	$P_n/\Omega_c$	$\phi_c P_n$	$P_n/\Omega_c$	$\phi_c P_n$
	0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793
6	1250	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160
7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150
8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140
9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120
10	1200	1800	1080	1620	980	1470	888	1340	807	1210	735	1100
11	1180	1770	1060	1600	965	1450	874	1310	794	1190	723	1080
12	1160	1750	1040	1570	949	1430	859	1290	780	1170	710	1070
13	1140	1720	1020	1540	931	1400	843	1270	766	1150	697	1050
14	1120	1690	1000	1510	912	1370	826	1240	750	1130	682	1030
15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000
16	1080	1620	960	1440	872	1310	789	1190	716	1080	652	979
17	1060	1590	937	1410	850	1280	770	1160	698	1050	635	955
18	1030	1550	913	1370	826	1240	750	1130	680	1020	618	929
19	1010	1510	888	1330	805	1210	729	1100	661	994	601	905
20	990	1470	862	1300	782	1180	708	1060	642	964	583	877
22	927	1390	810	1220	734	1100	664	998	602	904	547	822
24	872	1310	756	1140	685	1030	620	931	561	843	509	766
26	816	1230	702	1060	635	955	574	863	519	781	472	709
28	759	1140	648	974	586	880	529	795	478	719	434	653
30	703	1060	594	893	537	807	485	728	438	658	397	597
32	647	973	542	814	489	735	441	663	398	598	361	543

\*See length,  $L_c$ , (ft), with respect to least radius of gyration,  $r_y$ .



3.49

## Slender Elements

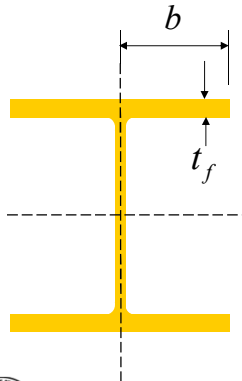
- An element is slender if it would buckle locally before it is able to reach yield.
- Two types of elements
  - *Unstiffened* elements: those supported along only one edge parallel to the direction of the compression force; such as flanges
  - *Stiffened* elements: those supported along two edges parallel to the direction of the compression force; such as webs



3.50

Unstiffened Elements

$$\frac{b}{t} = \frac{b_f}{2} \left( \frac{1}{t_f} \right) = \frac{b_f}{2t_f}$$



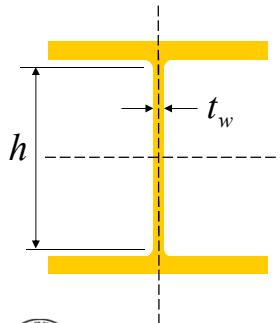
**TABLE B4.1a**  
**Width-to-Thickness Ratios: Compression Elements**  
**Members Subject to Axial Compression**

Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio $\lambda_c$ (nonslender/slender)	Examples
Unstiffened Elements	1 Flanges of rolled I-shaped sections, plates projecting from rolled I-shaped sections, outstanding legs of pairs of angles connected with continuous contact, flanges of channels, and flanges of tees	$b/t$	$0.56 \sqrt{\frac{E}{F_y}}$	
	2 Flanges of built-up I-shaped sections and plates or angle legs projecting from built-up I-shaped sections	$b/t$	$0.64 \sqrt{\frac{k_c E}{F_y}}$ <sup>(a)</sup>	
	3 Legs of single angles, legs of double angles with separators, and all other unstiffened elements	$b/t$	$0.45 \sqrt{\frac{E}{F_y}}$	
	4 Stems of tees	$d/t$	$0.75 \sqrt{\frac{E}{F_y}}$	

3.51

Stiffened Elements

$$\frac{h}{t_w}$$



**TABLE B4.1a**  
**Width-to-Thickness Ratios: Compression Elements**  
**Members Subject to Axial Compression**

Case	Description of Element	Width-to-Thickness Ratio	Limiting Width-to-Thickness Ratio $\lambda_c$ (nonslender/slender)	Examples
Stiffened Elements	5 Webs of doubly symmetric rolled and built-up I-shaped sections and channels	$h/t_w$	$1.49 \sqrt{\frac{E}{F_y}}$	
	6 Walls of rectangular HSS	$b/t$	$1.40 \sqrt{\frac{E}{F_y}}$	
	7 Flange cover plates and diaphragm plates between lines of fasteners or welds	$b/t$	$1.40 \sqrt{\frac{E}{F_y}}$	
	8 All other stiffened elements	$b/t$	$1.49 \sqrt{\frac{E}{F_y}}$	
	9 Round HSS	$D/t$	$0.11 \sqrt{\frac{E}{F_y}}$	

<sup>(a)</sup>  $k_c = 4\sqrt{h/t_w}$ , but shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes.

3.52

## Slender Elements

### Unstiffened Elements

#### W-shape Flange - Case 1

A992,  $F_y = 50$  ksi

$$\frac{b_f}{2t_f} = \frac{b}{t} \leq \lambda_{rf} = 0.56 \sqrt{\frac{E}{F_y}}$$

$$\lambda_{rf} = 13.5$$



3.53

## Slender Elements

### Stiffened Element

#### W-shape Web - Case 5

A992,  $F_y = 50$  ksi

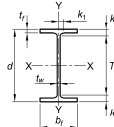
$$\frac{h}{t_w} \leq \lambda_{rw} = 1.49 \sqrt{\frac{E}{F_y}}$$

$$\lambda_{rw} = 35.9$$



3.54

# Slender Elements



**Table 1-1 (continued)  
W-Shapes  
Dimensions**

Shape	Area, A in. <sup>2</sup>	Depth, d in.	Web		Flange		Distance								
			Thickness, t <sub>w</sub> in.	$\frac{L_w}{2}$ in.	Width, b <sub>f</sub> in.	Thickness, t <sub>f</sub> in.	K <sub>des</sub> in.	K <sub>det</sub> in.	k <sub>1</sub> in.	T in.	Work-able Gage in.				
W27×129 <sup>c</sup>	37.8	27.6	27 <sup>5</sup> / <sub>16</sub>	0.610	<sup>5</sup> / <sub>16</sub>	<sup>5</sup> / <sub>16</sub>	10.0	10	1.10	1 <sup>1</sup> / <sub>16</sub>	1.70	2 <sup>1</sup> / <sub>16</sub>	1 <sup>1</sup> / <sub>2</sub>	23	5 <sup>1</sup> / <sub>2</sub>
×114 <sup>d</sup>	33.6	27.3	27 <sup>7</sup> / <sub>16</sub>	0.570	<sup>5</sup> / <sub>16</sub>	<sup>5</sup> / <sub>16</sub>	10.1	10 <sup>1</sup> / <sub>16</sub>	0.930	<sup>15</sup> / <sub>16</sub>	1.53	2 <sup>1</sup> / <sub>16</sub>	1 <sup>1</sup> / <sub>2</sub>		
×102 <sup>e</sup>	30.0	27.1	27 <sup>1</sup> / <sub>16</sub>	0.515	<sup>1</sup> / <sub>2</sub>	<sup>1</sup> / <sub>2</sub>	10.0	10	0.830	<sup>13</sup> / <sub>16</sub>	1.43	2 <sup>1</sup> / <sub>16</sub>	1 <sup>7</sup> / <sub>16</sub>		
×94 <sup>e</sup>	27.6	26.9	26 <sup>7</sup> / <sub>16</sub>	0.490	<sup>1</sup> / <sub>2</sub>	<sup>1</sup> / <sub>2</sub>	10.0	10	0.745	<sup>3</sup> / <sub>4</sub>	1.34	1 <sup>15</sup> / <sub>16</sub>	1 <sup>7</sup> / <sub>16</sub>		
×84 <sup>e</sup>	24.7	26.7	26 <sup>3</sup> / <sub>16</sub>	0.460	<sup>7</sup> / <sub>16</sub>	<sup>1</sup> / <sub>2</sub>	10.0	10	0.640	<sup>5</sup> / <sub>8</sub>	1.24	1 <sup>7</sup> / <sub>16</sub>	1 <sup>7</sup> / <sub>16</sub>		
W24×370 <sup>b</sup>	109	28.0	28	1.52	1 <sup>1</sup> / <sub>2</sub>	<sup>3</sup> / <sub>4</sub>	13.7	13 <sup>3</sup> / <sub>16</sub>	2.72	2 <sup>3</sup> / <sub>4</sub>	3.22	4	2	20	5 <sup>1</sup> / <sub>2</sub>
×335 <sup>b</sup>	98.3	27.5	27 <sup>1</sup> / <sub>2</sub>	1.38	1 <sup>3</sup> / <sub>16</sub>	<sup>1</sup> / <sub>2</sub>	13.5	13 <sup>1</sup> / <sub>16</sub>	2.48	2 <sup>1</sup> / <sub>2</sub>	2.98	3 <sup>3</sup> / <sub>4</sub>	1 <sup>7</sup> / <sub>16</sub>		
×306 <sup>b</sup>	89.7	27.1	27 <sup>1</sup> / <sub>16</sub>	1.26	1 <sup>1</sup> / <sub>4</sub>	<sup>5</sup> / <sub>8</sub>	13.4	13 <sup>3</sup> / <sub>16</sub>	2.28	2 <sup>1</sup> / <sub>2</sub>	2.78	3 <sup>3</sup> / <sub>16</sub>	1 <sup>13</sup> / <sub>16</sub>		
×279 <sup>b</sup>	81.9	26.7	26 <sup>3</sup> / <sub>16</sub>	1.16	1 <sup>3</sup> / <sub>16</sub>	<sup>5</sup> / <sub>8</sub>	13.3	13 <sup>1</sup> / <sub>16</sub>	2.09	2 <sup>1</sup> / <sub>16</sub>	2.59	3 <sup>3</sup> / <sub>16</sub>	1 <sup>13</sup> / <sub>16</sub>		
×250	73.5														
×229	67.2														
×207	60.7														
×192	56.5	25.5	25 <sup>1</sup> / <sub>2</sub>	0.810	<sup>13</sup> / <sub>16</sub>	<sup>5</sup> / <sub>16</sub>	13.0	13	1.46	1 <sup>7</sup> / <sub>16</sub>	1.96	2 <sup>1</sup> / <sub>16</sub>	1 <sup>9</sup> / <sub>16</sub>		
×176	51	25.2	25 <sup>3</sup> / <sub>16</sub>	0.750	<sup>3</sup> / <sub>4</sub>	<sup>5</sup> / <sub>16</sub>	12.9	12 <sup>7</sup> / <sub>16</sub>	1.34	1 <sup>3</sup> / <sub>16</sub>	1.84	2 <sup>1</sup> / <sub>16</sub>	1 <sup>9</sup> / <sub>16</sub>		
×162	46.8	25.0	25	0.705	<sup>13</sup> / <sub>16</sub>	<sup>5</sup> / <sub>16</sub>	13.0	13	1.22	1 <sup>3</sup> / <sub>4</sub>	1.72	2 <sup>1</sup> / <sub>16</sub>	1 <sup>9</sup> / <sub>16</sub>		
×148	43.0	24.7	24 <sup>1</sup> / <sub>16</sub>	0.650	<sup>5</sup> / <sub>8</sub>	<sup>5</sup> / <sub>16</sub>	12.9	12 <sup>1</sup> / <sub>16</sub>	1.09	1 <sup>1</sup> / <sub>16</sub>	1.59	2 <sup>1</sup> / <sub>16</sub>	1 <sup>9</sup> / <sub>16</sub>		
×131	38.6	24.5	24 <sup>1</sup> / <sub>2</sub>	0.605	<sup>5</sup> / <sub>8</sub>	<sup>5</sup> / <sub>16</sub>	12.9	12 <sup>1</sup> / <sub>16</sub>	0.960	<sup>15</sup> / <sub>16</sub>	1.46	2 <sup>1</sup> / <sub>16</sub>	1 <sup>9</sup> / <sub>16</sub>		
×117 <sup>c</sup>	34	24.3	24 <sup>1</sup> / <sub>16</sub>	0.550	<sup>9</sup> / <sub>16</sub>	<sup>5</sup> / <sub>16</sub>	12.8	12 <sup>3</sup> / <sub>16</sub>	0.850	<sup>7</sup> / <sub>8</sub>	1.35	2 <sup>1</sup> / <sub>16</sub>	1 <sup>9</sup> / <sub>16</sub>		
×104 <sup>c</sup>	30.7	24.1	24	0.500	<sup>1</sup> / <sub>2</sub>	<sup>1</sup> / <sub>2</sub>	12.8	12 <sup>3</sup> / <sub>16</sub>	0.750	<sup>3</sup> / <sub>4</sub>	1.25	2 <sup>1</sup> / <sub>16</sub>	1 <sup>7</sup> / <sub>16</sub>		
W24×103 <sup>c</sup>	30.3	24.5	24 <sup>1</sup> / <sub>2</sub>	0.550	<sup>9</sup> / <sub>16</sub>	<sup>5</sup> / <sub>16</sub>	9.00	9	0.980	1	1.48	2 <sup>1</sup> / <sub>16</sub>	1 <sup>1</sup> / <sub>2</sub>	20	5 <sup>1</sup> / <sub>2</sub>


Note the footnote on the weight, 117<sup>c</sup>



3.55

# Slender Elements

**Table 1-1 (continued)  
W-Shapes  
Properties**



Nominal Wt.	Compact Section Criteria	Axis X-X					Axis Y-Y					r <sub>s</sub>	r <sub>o</sub>	Torsional Properties	
		I	S	r	Z	I	S	r	Z	J	C <sub>w</sub>				
		in. <sup>4</sup>	in. <sup>3</sup>	in.	in. <sup>3</sup>	in. <sup>4</sup>	in. <sup>3</sup>	in.	in. <sup>3</sup>	in.	in. <sup>4</sup>	in. <sup>6</sup>			
129	4.55	38.7	4760	345	11.2	395	184	36.8	2.21	57.6	2.66	26.5	11.1	32500	
114	5.41	42.5	4080	299	11.0	343	159	31.3	2.18	49.3	2.65	26.4	7.33	27800	
102	6.03	47.1	3620	267	11.0	305	139	27.8	2.15	43.4	2.62	26.3	5.28	24000	
94	6.70	49.5	3270	243	10.9	278	124	24.8	2.12	38.8	2.59	26.2	4.03	21300	
84	7.78	52.7	2850	213	10.7	244	106	21.2	2.07	33.2	2.54	26.1	2.81	17900	
370	2.51	14.2	13400	957	11.1	1130	1160	170	3.27	267	3.92	25.3	201	186000	
335	2.73	15.6	11900	864	11.0	1020	1030	152	3.23	238	3.86	25.0	152	161000	
306	2.94	17.1	10700	789	10.9	922	919	137	3.20	214	3.81	24.8	117	142000	
279	3.18	18.6	9600	718	10.8	835	823	124	3.17	193	3.76	24.6	93.5	125000	
250	3.49	20.7											66.6	108000	
229	3.79	22.5											51.3	96100	
207	4.14	24.8											38.3	84100	
192	4.43	26.6	6260	491	10.5	559	530	81.8	3.07	126	3.60	24.0	30.8	76300	
176	4.81	28.7	5580	450	10.5	511	479	74.3	3.04	115	3.57	23.9	23.9	68400	
162	5.31	30.6	5170	414	10.4	468	443	68.4	3.05	105	3.57	23.8	18.5	62600	
146	5.92	33.2	4580	371	10.3	418	391	60.5	3.01	93.2	3.53	23.6	13.4	54600	
131	6.43	35.6	420	329	10.2	370	340	53.0	2.97	81.5	3.49	23.5	9.50	47100	
117	7.13	39.2	3340	291	10.1	327	297	46.5	2.94	71.4	3.46	23.5	6.72	40800	
104	8.53	43.1	2100	258	10.1	289	259	40.7	2.91	62.4	3.42	23.4	4.72	35200	
103	4.59	39.2	3000	245	10.0	280	119	26.5	1.99	41.5	2.40	23.5	7.07	16600	

Note that h/t<sub>w</sub> exceeds 35.9



3.56

## Slender Elements

- All W-shapes have nonslender flanges for compression with  $F_y < 68$  ksi.
- Only one “column” section has a slender web for compression with A992 steel;  
W14x43
- Many W-shapes, meant to be used as beams, have slender webs for uniform compression. For example those just shown.



3.57

## Chapter E

- E7. Members with Slender Elements
  - Stiffened and unstiffened elements treated similarly (same effective width equation)
  - The critical stress is the same, regardless of element slenderness (E3-2, E3-3)
  - Slender element comes into play through the effective area

$$P_n = F_{cr} A_e \quad (E7-1)$$



3.58

## Chapter E

- E7. Members with Slender Elements

– when  $\frac{h}{t_w} = \lambda \leq \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$

Web of an I-shape

$$\lambda_r = 0.56 \sqrt{\frac{E}{F_y}}$$

$$b_e = b \quad (E7-2)$$



3.59

## Chapter E

- E7. Members with Slender Elements

– when  $\frac{h}{t_w} = \lambda > \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$

Web of an I-shape

$$\lambda_r = 0.56 \sqrt{\frac{E}{F_y}}$$

$$b_e = b \left( 1 - c_1 \sqrt{\frac{F_{el}}{F_{cr}}} \right) \sqrt{\frac{F_{el}}{F_{cr}}} \quad (E7-3)$$



Elastic local buckling stress

$$F_{el} = \left( c_2 \frac{\lambda_r}{\lambda} \right)^2 F_y \quad (E7-5)$$

3.60

# Chapter E

Table E7.1  
Effective Width Imperfection Adjustment Factor,  $c_1$   
and  $c_2$  Factor.

Case	Slender Element	$c_1$	$c_2$
(a)	Stiffened elements except walls of square and rectangular HSS	0.18	1.31
(b)	Walls of square and rectangular HSS	0.20	1.38
(c)	All other elements	0.22	1.49

Round HSS are treated differently

$$c_2 = \frac{1 - \sqrt{1 - 4c_1}}{2c_1}$$



3.61

## Example 3

- Determine the compressive strength of a built-up slender flange I-shape.  $L_c = KL = 20$  ft



Flange: 24 x 0.5 in.  
Web: 24 x 0.75 in.  
 $r_y = 5.24$  in.

Web slenderness, Case 5

$$h/t_w = 24.0/0.75 = 32$$

$$\lambda_r = 1.49\sqrt{E/F_y} = 35.9$$

Thus, the web is not slender

Flange slenderness, Case 2

$$k_c = \frac{4}{\sqrt{h/t_w}} = \frac{4}{\sqrt{24.0/0.75}} = 0.707$$

$$\begin{aligned} \lambda_{rf} &= 0.64\sqrt{k_c E/F_y} \\ &= 0.64\sqrt{0.707(29,000)/50} \\ &= 13.0 < b_f/2t_f = 24 \end{aligned}$$

Thus, the flange is slender



3.62

## Example 3

- Determine the compressive strength of a built-up slender flange I-shape.  $L_c = KL = 20$  ft



Flange: 24 x 0.5 in.  
Web: 24 x 0.75 in.  
 $r_y = 5.24$  in.

$$\frac{L_c}{r_y} = \frac{20(12)}{5.24} = 45.8$$

$$F_e = \frac{\pi^2 E}{(L_c/r)^2} = 136 \text{ ksi}$$

$$\frac{F_y}{F_e} = \frac{50}{136} = 0.368 < 2.25$$

$$F_{cr} = 0.658^{(0.368)} (50) = 42.9 \text{ ksi}$$



3.63

## Example 3

Table E7.1 Effective Width Imperfection Adjustment Factor,  $c_1$  and  $c_2$  Factor.

Case	Slender Element	$c_1$	$c_2$
(a)	Stiffened elements except walls of square and rectangular HSS	0.18	1.31
(b)	Walls of square and rectangular HSS	0.20	1.38
(c)	All other elements	0.22	1.49

We know that the flange will act as a slender element if  $F_{cr} = F_y$ .

But, at the actual compression stress will it act as a slender element?



3.64

## Example 3

- Determine if the flange will actually act slender.

$$\lambda_{rf} = 13.0 \quad \text{and} \quad F_{cr} = 42.9 \text{ ksi}$$

$$b_f/2t_f = 24.0 > \lambda_{rf} \sqrt{F_y/F_{cr}} = 13.0 \sqrt{50/42.9} = 14.0$$

Thus, the flange will behave as a slender element for a column with a stress of 42.9 ksi

- Determine the effective width

$$F_{el} = \left( c_2 \frac{\lambda_r}{\lambda} \right)^2 F_y \quad (\text{E7-5})$$

$$= \left( 1.49 \left( \frac{13.0}{24.0} \right) \right)^2 (50)$$

$$= 32.6$$

$$b_e = b \left( 1 - c_1 \sqrt{\frac{F_{el}}{F_{cr}}} \right) \sqrt{\frac{F_{el}}{F_{cr}}} \quad (\text{E7-3})$$

$$= 12 \left( 1 - 0.22 \sqrt{\frac{32.6}{42.9}} \right) \sqrt{\frac{32.6}{42.9}} = 8.45$$



Note there is no upper limit on  $b_e$  since it will always be less than  $b$

3.65

## Example 3

- Then determine the effective area

$$A_g = 24(0.75) + 2(12.0)(0.5) + 2(12.0)(0.5) = 42.0 \text{ in.}^2$$

web                      flange                      flange

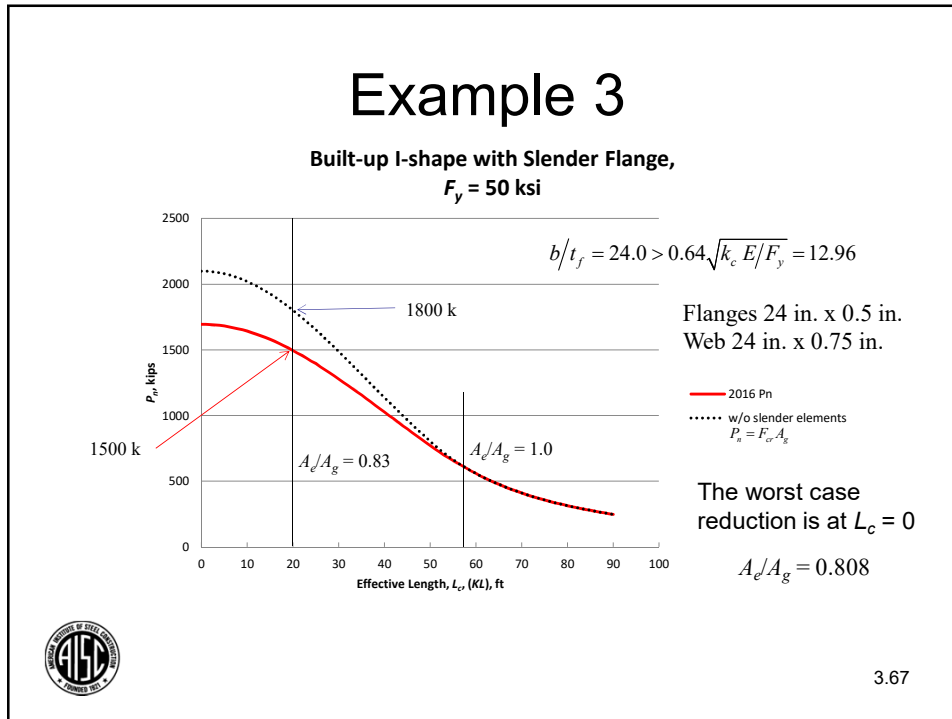
$$A_e = 24(0.75) + 2(8.45)(0.5) + 2(8.45)(0.5) = 34.9 \text{ in.}^2$$

- Using the effective area, determine the nominal strength

$$P_n = 42.9(34.9) = 1500 \text{ kips}$$



3.66



### Example 3

- At what effective length,  $L_c$ , will there be no reduction in effective area?
- when  $\frac{b_f}{2t_f} = \lambda \leq \lambda_r \sqrt{\frac{F_y}{F_{cr}}}$ ,  $b_e = b$
- therefore

$$\frac{b_f}{2t_f} = 24 = 13 \sqrt{\frac{50}{F_{cr}}} \quad \text{thus, } F_{cr} = 14.7 \text{ ksi}$$

We could solve for  $L_c/r$  but we will use Manual Table 4-14.

3.68

### Example 3

$$\phi F_{cr} = 0.9(14.7) = 13.2 \text{ ksi}$$

$$\frac{L_c}{r} = 131$$

thus,

$$L_c = 131r_y = 131(5.24) = 686 \text{ in.} \\ = 57.2 \text{ ft}$$



Table 4-14 (continued)  
Available Critical Stress for  
Compression Members

$\frac{L_c}{r}$	$F_y = 35 \text{ ksi}$		$F_y = 36 \text{ ksi}$		$F_y = 46 \text{ ksi}$		$F_y = 50 \text{ ksi}$		$F_y = 65 \text{ ksi}$		$F_y = 70 \text{ ksi}$	
	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$	$F_{cr}/\Omega_c$	$\phi_c F_{cr}$
	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi	ksi
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
121	9.91	14.9	10.0	15.0	10.3	15.4	10.3	15.4	10.3	15.4	10.3	15.4
122	9.79	14.7	9.85	14.8	10.1	15.2	10.1	15.2	10.1	15.2	10.1	15.2
123	9.67	14.5	9.72	14.6	9.94	14.9	9.94	14.9	9.94	14.9	9.94	14.9
124	9.55	14.3	9.59	14.4	9.78	14.7	9.78	14.7	9.78	14.7	9.78	14.7
125	9.43	14.2	9.47	14.2	9.62	14.5	9.62	14.5	9.62	14.5	9.62	14.5
126	9.31	14.0	9.35	14.0	9.47	14.2	9.47	14.2	9.47	14.2	9.47	14.2
127	9.19	13.8	9.22	13.9	9.32	14.0	9.32	14.0	9.32	14.0	9.32	14.0
128	9.07	13.6	9.10	13.7	9.17	13.8	9.17	13.8	9.17	13.8	9.17	13.8
129	8.95	13.4	8.98	13.5	9.03	13.6	9.03	13.6	9.03	13.6	9.03	13.6
130	8.83	13.3	8.86	13.3	8.89	13.4	8.89	13.4	8.89	13.4	8.89	13.4
131	8.71	13.1	8.73	13.1	8.76	13.2	8.76	13.2	8.76	13.2	8.76	13.2
132	8.60	12.9	8.61	12.9	8.63	13.0	8.63	13.0	8.63	13.0	8.63	13.0
133	8.48	12.7	8.49	12.8	8.50	12.8	8.50	12.8	8.50	12.8	8.50	12.8
134	8.37	12.6	8.37	12.6	8.37	12.6	8.37	12.6	8.37	12.6	8.37	12.6
135	8.25	12.4	8.25	12.4	8.25	12.4	8.25	12.4	8.25	12.4	8.25	12.4
136	8.13	12.2	8.13	12.2	8.13	12.2	8.13	12.2	8.13	12.2	8.13	12.2

3.69

### Torsional Buckling

- Doubly symmetric members may exhibit buckling in a torsional mode.



These shapes are arranged in order of increasing torsional strength

Strength of the cruciform is very likely to be controlled by the limit state of torsional buckling while strength of the closed shapes will not.



3.70

## Torsional Buckling

- The elastic torsional buckling stress for doubly symmetric members is a function of two types of torsion, pure torsion and warping torsion. The Specification gives:

$$F_e = \left[ \frac{\pi^2 EC_w}{L_{cz}^2} + GJ \right] \frac{1}{I_x + I_y} \quad (E4-2)$$

Warping Torsion      Pure Torsion

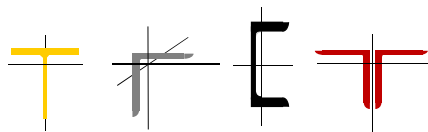
- This elastic torsional buckling stress is then used, like the elastic flexural buckling stress, to obtain the critical stress for this limit state.



3.71

## Flexural-Torsional Buckling

- Singly symmetric members can buckle in a mode that combines torsional and flexural buckling which we call flexural-torsional buckling.



3.72

## Flexural-Torsional Buckling

- For singly symmetric members the elastic flexural-torsional buckling stress.

$$F_e = \left( \frac{F_{ey} + F_{ez}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}} \right] \quad (\text{E4-3})$$

The y-axis is the axis of symmetry and the z-axis represents the torsional axis



3.73

## Flexural-Torsional Buckling

- The elastic torsional buckling stress for a singly symmetric member is given by

$$F_{ez} = \left[ \frac{\pi^2 EC_w}{L_{cz}^2} + GJ \right] \frac{1}{A_g \bar{r}_o^2} \quad (\text{E4-7})$$

- The elastic flexural buckling stress is given by

$$F_{ey} = \frac{\pi^2 E}{\left( \frac{L_{cy}}{r_y} \right)^2} \quad (\text{E4-6})$$

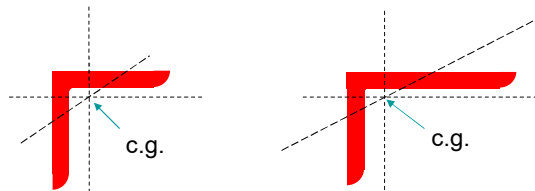


We will look at an example when we address built-up members

3.74

## Single Angle Compression Members

- These members may be singly symmetric (equal legs) or non-symmetric (unequal legs)
- To be axially loaded, they must be loaded at the centroid (unlikely)



3.75

## Single Angle Compression Members

- E5. Single-Angle Compression Members
  - May consider only flexural buckling, if
$$b/t \leq 0.71\sqrt{E/F_y}$$
This is  $b/t = 20$  for  $F_y = 36$  ksi
    - This limit is met by all currently produced angles.
  - Otherwise must consider flexural-torsional buckling
  - There is also a special case described in Section E5 for when the single angle is not loaded at the centroid but eccentricity may be neglected.



3.76

## Single Angle Compression Members

- If the member is
  - loaded at its ends through same leg
  - attached by welding or a minimum of two bolts
  - has no intermediate transverse loads
  - $L_c/r$  determined here does not exceed 200
  - long leg/short leg  $\leq 1.7$
- Then  
Use the modified slenderness ratio and ignore eccentricity



3.77

## Single Angle Compression Members

- As an example, for equal leg angles that are individual members or webs of planer trusses

$$\text{when } \frac{L}{r_a} \leq 80: \quad \frac{L_c}{r} = 72 + 0.75 \frac{L}{r_a} \quad (\text{E5-1})$$

$$\text{when } \frac{L}{r_a} > 80: \quad \frac{L_c}{r} = 32 + 1.25 \frac{L}{r_a} \quad (\text{E5-2})$$



3.78

## Single Angle Compression Members

- For the same equal leg angle that is part of a box or space truss

$$\text{when } \frac{L}{r_a} \leq 75: \quad \frac{L_c}{r} = 60 + 0.8 \frac{L}{r_a} \quad (\text{E5-3})$$

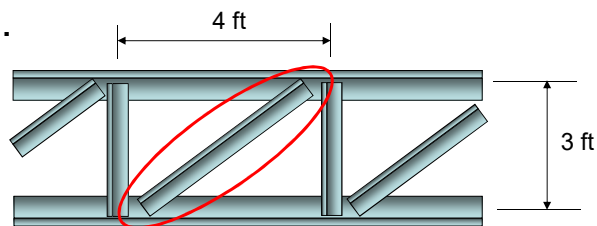
$$\text{when } \frac{L}{r_a} > 75: \quad \frac{L_c}{r} = 45 + \frac{L}{r_a} \quad (\text{E5-4})$$



3.79

## Example 4

- Determine the available compressive strength of a 5 x 3 x 1/2 A36 angle used as a web member of a truss. The web member is 5 ft long and welded to the chords.



3.80

## Example 4

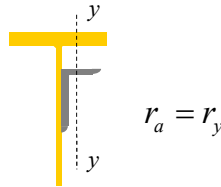
- The angle is attached through its 5 in. leg at each end. There are no intermediate transverse loads. It satisfies the requirements of Section E5

$$A_g = 3.75 \text{ in.}^2$$

$$r_x = 1.58 \text{ in.}$$

$$r_y = 0.824 \text{ in.}$$

$$r_z = 0.642 \text{ in.}$$



3.81

## Example 4

- Determine the effective slenderness

$$\frac{L}{r_a} = \frac{L}{r_y} = \frac{5.0(12)}{0.824} = 72.8 < 80$$

- Therefore use Eq. E5-1

$$\frac{L_c}{r} = 72 + 0.75 \left( \frac{L}{r_a} \right)$$

$$\frac{L_c}{r} = 72 + 0.75(72.8) = 127 < 4.71 \sqrt{\frac{E}{F_y}} = 134 \quad \text{Use Eq. E3-2}$$

The limit for determining use of Eq. E3-2 or E3-3



3.82

## Example 4

- Determine the elastic buckling stress from Eq. E3-4

$$F_e = \frac{\pi^2 E}{\left(\frac{L_c}{r}\right)^2} = \frac{\pi^2 (29000)}{(127)^2} = 17.7$$

and the critical stress from Eq. E3-2

$$F_{cr} = (0.658)^{\frac{36}{17.7}} (36) = 15.4 \text{ ksi}$$



3.83

## Example 4

- Nominal strength

$$P_n = F_{cr} A_g = 15.4 (3.75) = 57.8 \text{ kips}$$

- **ASD**  $\frac{P_n}{\Omega_c} = \frac{57.8}{1.67} = 34.6 \text{ kips}$

- **LRFD**  $\phi_c P_n = 0.9 (57.8) = 52.0 \text{ kips}$



3.84

## Example 4

- To use the concentrically loaded single angle tables, determine the effective  $L_c$  with respect to the z-axis based on the slenderness ratio already determined

$$L_{c \text{ eff}} = \left( \frac{L_c}{r} \right) r_z = \frac{127(0.642)}{12} = 6.79 \text{ ft}$$



3.85

## Example 4

Table 4-11 (continued)  
Available Strength in Axial Compression, kips  $F_y = 36 \text{ ksi}$   
Concentrically Loaded Single Angles

Shape	L5 × 3 1/2 ×				L5 × 3 ×							
	1/4"		1/2"		7/16"		3/8"		5/16"		1/4"	
lb/ft	7.00		12.8		11.3		9.80		8.20		6.60	
Design	$P_n/\phi_c \Omega_c$		$\phi_c P_n$		$P_n/\phi_c \Omega_c$		$\phi_c P_n$		$P_n/\phi_c \Omega_c$		$\phi_c P_n$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	37.0	55.6	80.8	122	71.4	107	60.8	91.4	47.8	71.8	35.1	52.8
1	36.7	55.1	79.4	119	70.1	105	59.9	90.0	47.1	70.8	34.6	52.0
2	35.7	53.7	75.1	113	66.3	99.7	57.3	86.0	45.1	67.8	33.2	49.9
3	34.2	51.4	68.5	103	60.5	91.0	52.4	78.7	41.9	63.0	30.9	46.5
4	32.0	48.1	60.2	90.5	53.3	80.0	46.1	69.3	37.9	56.9	28.0	42.1
5	29.1	43.8	51.0	76.7	45.2	67.9	39.1	58.8	33.1	49.8	24.6	37.0
6	25.9	38.9	41.7	62.7	37.0	55.5	32.1	48.2	27.2	40.8	21.0	31.6
7	22.5	33.8	32.8	49.3	29.1	43.8	25.3	38.0	21.5	32.3	17.4	26.1
8	19.1	28.7	25.2	37.9	22.4	33.7	19.5	29.3	16.6	24.9	13.5	20.2
9	15.4	23.2	19.9	29.9	17.7	26.6	15.4	23.1	13.1	19.7	10.6	16.0
10	12.5	18.8	16.1	24.2	14.3	21.5	12.5	18.7	10.6	15.9	8.61	12.9
11	10.3	15.5										
12	8.69	13.1										

Interpolating  
 $P_a = 34.7 \text{ kips}$   
 $P_u = 52.1 \text{ kips}$



3.86

## Single Angle

- If the requirements of Section E5 are not met, eccentricity must be considered

Eccentricity will induce bending moments. Combined axial force and bending will be addressed in Lesson 5.



Table 4-12 (continued)  
Available Strength in Axial Compression, kips  
Eccentrically Loaded Single Angles  $F_y = 36$  ksi

Shape	L5 x 3 1/2 x				L5 x 3 x							
	1/4 <sup>cl</sup>		1/2		7/16		3/8 <sup>cl</sup>		3/16 <sup>cl</sup>		1/4 <sup>cl</sup>	
lb/ft	7.00		12.8		11.3		9.80		8.20		6.60	
Design	$P_u/\phi_c P_n$		$\phi_c P_n$		$P_u/\phi_c P_n$		$\phi_c P_n$		$P_u/\phi_c P_n$		$\phi_c P_n$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	32.1	48.3	36.2	54.4	34.7	52.2	34.2	51.4	31.9	47.9	30.0	45.1
1	32.3	48.5	35.3	53.2	34.0	51.1	33.4	50.3	31.1	46.8	29.1	43.9
2	32.4	48.6	33.0	49.8	31.8	48.0	31.2	47.1	28.9	43.7	26.8	40.6
3	33.1	49.1	29.0	45.3	28.6	43.4	27.8	42.2	25.7	39.0	23.5	35.6
4	30.0	46.0	26.3	40.1	25.0	38.1	24.0	36.5	22.1	33.8	20.0	30.5
5	25.0	38.5	22.7	34.7	21.4	32.7	20.3	31.0	18.7	28.6	16.6	25.6
6	20.6	31.9	19.3	29.6	18.1	27.7	16.9	26.0	15.4	23.7	13.7	21.2
7	17.0	26.5	16.3	25.0	15.1	23.2	14.0	21.6	12.7	19.6	11.3	17.5
8	14.1	22.0	13.6	20.9	12.6	19.4	11.6	17.9	10.4	16.1	9.23	14.2
9	11.7	18.2	11.5	17.8	10.6	16.4	9.79	15.0	8.72	13.4	7.64	11.8
10	9.79	15.2	9.95	15.2	9.12	14.0	8.33	12.8	7.38	11.3	6.43	9.94
11	8.31	12.9										
12	7.14	11.0										

sect to least radius of gyration,  $r_x$

3.87

## Built-Up Members

- Built-up members are composed of two shapes interconnected with bolts or welds or with at least one open side interconnected by plates or lacing.
- The key to determining the strength of built-up members is determining the correct slenderness ratio.



3.88

## Built-Up Members

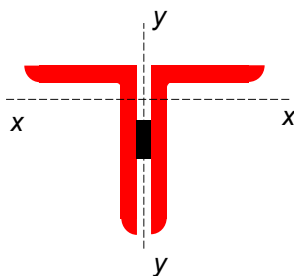
- Section E6 requires
  - “The end connection shall be welded or connected by means of pretensioned bolts with Class A or B faying surfaces.”
- Slip in the end connection could cause the built-up member to lose strength and behave as individual members.



3.89

## Built-Up Members

- Nominal strength is determined using a modified slenderness ratio if connectors are in shear in the buckling mode.



Buckling about the x-axis  
treat as two single angles

Buckling about y-axis treat  
as a built-up member



3.90

## Built-Up Members

- When intermediate connectors are snug-tight bolts,

$$\left(\frac{L_c}{r}\right)_m = \sqrt{\left(\frac{L_c}{r}\right)_o^2 + \left(\frac{a}{r_i}\right)^2} \quad (\text{E6-1})$$

$\left(\frac{L_c}{r}\right)_m$  = modified slenderness ratio

$\left(\frac{L_c}{r}\right)_o$  = built-up member acting as a unit

$a$  = distance between connectors

$r_i$  = minimum radius of gyration of component



3.91

## Built-Up Members

- When intermediate connectors are welds or pretensioned bolts

for  $\frac{a}{r_i} \leq 40$   $\left(\frac{L_c}{r}\right)_m = \left(\frac{L_c}{r}\right)_o$  (E6-2a)

for  $\frac{a}{r_i} > 40$   $\left(\frac{L_c}{r}\right)_m = \sqrt{\left(\frac{L_c}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2}$  (E6-2b)



3.92

## Built-Up Members

- Definitions:

$\left(\frac{L_c}{r}\right)_m$  = modified slenderness ratio

$\left(\frac{L_c}{r}\right)_o$  = slenderness ratio of built-up member acting as a unit

$K_i = 0.50$  for angles back-to-back

= 0.75 for channels back-to-back

= 0.86 for all other cases

$a$  = distance between connectors

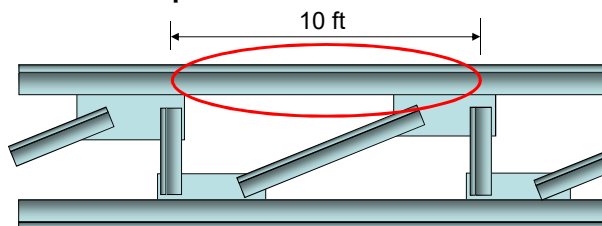
$r_i$  = minimum radius of gyration of component



3.93

## Example 5

- Determine the available compressive strength of 2-L5 x 3 x 1/2 LLBB A36 angles used as the top chord of a truss. The angles are attached with welds at two intermediate points and at the ends.



3.94

## Example 5

- Determine the available compressive strength of 2-L5 x 3 x 1/2 LLBB A36 angles used as the top chord of a truss.

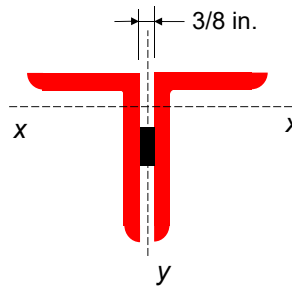
Single Angle Table 1-7

$$A_g = 3.75 \text{ in.}^2 \quad I_x = 9.43 \text{ in.}^4$$

$$r_x = 1.58 \text{ in.} \quad I_y = 2.55 \text{ in.}^4$$

$$r_y = 0.824 \text{ in.} \quad J = 0.322 \text{ in.}^4$$

$$r_z = 0.642 \text{ in.} \quad C_w = 0.444 \text{ in.}^6$$



3.95

## Example 5

- Combined properties, Table 1-15 in red here

$$A_g = 2(3.75) = 7.50 \text{ in.}^2 \quad H = 0.646$$

$$I_x = 2(9.43) = 18.9 \text{ in.}^4 \quad \bar{r}_o = 2.51 \text{ in.}$$

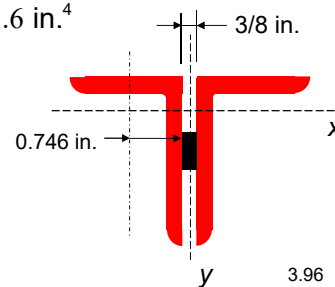
$$r_x = 1.58 \text{ in.}$$

$$I_y = 2 \left( 2.55 + 3.75 \left( 0.746 + \frac{3}{16} \right)^2 \right) = 11.6 \text{ in.}^4$$

$$r_y = \sqrt{\frac{11.6}{7.50}} = 1.24 \text{ in.}$$

$$J = 2(0.322) = 0.644 \text{ in.}^4$$

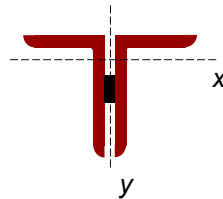
$$C_w = 2(0.444) = 0.888 \text{ in.}^6$$



3.96

## Example 5

- For a singly symmetric compression member we will see that the applicable limit states are flexural-torsional buckling about the axis of symmetry,  $y$ -axis, and flexural buckling about the other axis,  $x$ -axis.



97

## Example 5

- Since the double angle compression member is a built-up member,
  - if the buckling mode involves relative deformation that produces shear forces in the connectors between individual shapes, the modified slenderness ratio as a function of connector spacing must be determined according to Section E6.
  - If the buckling mode does not involve relative deformation, the slenderness ratio using the actual effective length and radius of gyration is used.



98

## Example 5

- For the x-axis (no connector shear)
  - The slenderness ratio is

$$\frac{L_c}{r_x} = \frac{10(12)}{1.58} = 75.9$$

- and the elastic buckling stress is

$$F_{ex} = \frac{\pi^2 E}{\left(\frac{L_c}{r_x}\right)^2} = \frac{\pi^2 (29,000)}{(75.9)^2} = 49.7 \text{ ksi} \quad (\text{E3-4})$$



99

## Example 5

- For the y-axis (connectors in shear)
  - Buckling produces shear forces in the connectors between individual shapes.
  - For our example, place pretensioned connectors at the 1/3 points of the column

$$a = 40.0 \text{ in.}$$



100

## Example 5

- For the y-axis (connectors in shear)

– As a single unit  $\left(\frac{L_c}{r_y}\right)_o = \frac{10(12)}{1.24} = 96.8$

- Between connectors

$$\frac{a}{r_i} = \frac{a}{r_z} = \frac{40}{0.642} = 62.3 > 40$$

- Thus,

$$\left(\frac{L_c}{r}\right)_m = \sqrt{\left(\frac{L_c}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} = \sqrt{(96.8)_o^2 + \left(\frac{0.5(40)}{0.642}\right)^2} = 102 \quad \text{E6-2b}$$



101

## Example 5

- For the y-axis (connectors in shear)

- With the slenderness ratio

$$\frac{L_c}{r_y} = \left(\frac{L_c}{r}\right)_m = 102$$

- the elastic buckling stress is

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{L_c}{r_y}\right)^2} = \frac{\pi^2 (29,000)}{(102)^2} = 27.5 \text{ ksi} \quad \text{(E3-4)}$$



102

## Example 5

- E4. Torsional and Flexural-torsional Buckling
  - E4.(b) for singly symmetric members twisting about the shear center where  $y$  is the axis of symmetry

$$F_e = \left( \frac{F_{ey} + F_{ez}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}} \right] \quad (\text{E4-3})$$

If  $x$  is the axis of symmetry replace  $F_{ey}$  by  $F_{ex}$



103

## Example 5

- For torsional buckling

$$F_{ez} = \left[ \frac{\pi^2 EC_w}{L_{cz}^2} + GJ \right] \frac{1}{A_g \bar{r}_o^2} \quad (\text{E4-7})$$

$$\begin{aligned} F_{ez} &= \left[ \frac{\pi^2 E (0.888)}{(12(10))^2} + 11,200(0.644) \right] \frac{1}{7.50(2.51)^2} \\ &= [17.5 + 7213] \frac{1}{47.3} = 153 \text{ ksi} \end{aligned}$$



3.104

## Example 5

- For flexural-torsional buckling

$$F_e = \left( \frac{F_{ey} + F_{ez}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}} \right] \quad (E4-3)$$

$$F_e = \left( \frac{27.5 + 153}{2(0.646)} \right) \left[ 1 - \sqrt{1 - \frac{4(27.5)(153)(0.646)}{(27.5 + 153)^2}} \right]$$

$$= 25.7 \text{ ksi}$$

This is less than  $F_{ey}$ , thus flexural-torsional buckling will control.



3.105

## Example 5

- Determine the critical stress

$$\frac{F_y}{F_e} = \frac{36.0}{25.7} = 1.40 < 2.25$$

This is the "other way" to determine which equation to use in determining  $F_{cr}$ .

- Therefore use Eq. E3-2

$$F_{cr} = (0.658)^{\frac{36}{25.7}} (36) = 20.0 \text{ ksi}$$



3.106

## Example 5

- The nominal strength is then,

$$P_n = F_{cr} A_g = 20.0(7.50) = 150 \text{ kips}$$

- The available strength is

For **LRFD**

$$\phi P_n = 0.9(150) = 135 \text{ kips}$$

For **ASD**

$$\frac{P_n}{\Omega} = \frac{150}{1.67} = 89.8 \text{ kips}$$



3.107

## Example 5

If back-to-back spacing is greater than 3/8 in., the table values are conservative.

$$\phi P_n = 136 \text{ kips}$$

$$\frac{P_n}{\Omega} = 90.2 \text{ kips}$$

Note 2 intermediate connectors required.



Table 4-9 (continued)  
Available Strength in Axial Compression, kips  
Double Angles—LLBB

$F_y = 36 \text{ ksi}$

Shape	2L5 × 3 ×									
	1/2		7/16		3/8		3/8		1/2	
lb/ft	25.6		22.6		19.6		16.4		13.2	
Design	$P_n/\Omega_c$		$\phi P_n$		$P_n/\Omega_c$		$\phi P_n$		$P_n/\Omega_c$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	162	243	143	214	122	183	95.6	144	70.2	106
4	140	211	121	181	99.9	150	76.0	114	50.6	76.0
6	129	194	111	167	92.8	140	72.0	108	48.2	72.5
8	109	165	94.7	142	79.4	119	63.4	95.2	43.4	65.3
10	90.2	138	78.0	117	65.4	98.3	52.4	78.8	37.5	56.3
12	71.1	107	61.3	92.1	51.3	77.1	41.2	61.9	30.1	45.3
14	53.8	80.8	46.3	69.6	38.8	58.4	31.4	47.2	23.4	35.3
16	41.5	62.4	35.9	53.9	30.2	45.4	24.9	36.9	18.5	27.6
18	33.0	49.6	28.5	42.9	24.1	36.2	19.7	29.6	15.0	22.5
20	26.8	40.3	23.3	35.9	19.6	29.5	16.1	24.2		

Properties of 2 angles—3/8 in. back to back

$A_g$ , in. <sup>2</sup>	7.50	6.62	5.72	4.82	3.88
$r_x$ , in.	1.58	1.59	1.60	1.61	1.62
$r_y$ , in.	1.24	1.23	1.22	1.21	1.19

Properties of single angle

$r_x$ , in.	0.642	0.644	0.646	0.649	0.652
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ASD  $\phi_c = 0.90$

\* For Y-Y axis, welded or pretensioned bolted intermediate connectors with Class A or B faying surfaces must be used.  
\* For required number of intermediate connectors, see the discussion of Table 4-8.  
\* Shape is slender for compression with  $F_y = 36 \text{ ksi}$ ; tabulated values have been adjusted accordingly.  
Note: Heavy line indicates  $\lambda_c \geq 1.0$  or greater than 200.

3.108

# Example 5

x-axis strength is independent of intermediate connectors

$$\phi P_n = 179 \text{ kips}$$

$$\frac{P_n}{\Omega} = 119 \text{ kips}$$

y-axis strength includes flexural-torsional buckling strength

$$\phi P_n = 136 \text{ kips}$$

$$\frac{P_n}{\Omega} = 90.2 \text{ kips}$$



Table 4-9 (continued)  
Available Strength in Axial Compression, kips  
Double Angles—LLBB

$F_y = 36 \text{ ksi}$

Shape	2L5 x 3 x 1/2										No. of connectors
	1/2		7/16		3/8		3/4		1 1/4		
Design	$\phi P_n$	$P_n/\Omega$	$\phi P_n$	$P_n/\Omega$	$\phi P_n$	$P_n/\Omega$	$\phi P_n$	$P_n/\Omega$	$\phi P_n$	$P_n/\Omega$	
0	182	243	143	214	122	183	95.6	144	70.2	106	
2	190	240	141	212	120	181	94.7	142	69.6	105	
4	164	231	136	204	117	176	92.1	138	67.7	102	
6	145	218	128	193	111	167	87.8	132	64.7	97.2	
8	109	165	94.7	142	78.4	119	63.4	95.2	45.4	65.3	
10	119	179	106	159	91.7	138	75.4	113	55.9	84.0	
12	104	157	92.7	139	80.5	121	67.9	102	50.5	75.9	
14	89.2	134	79.3	119	69.0	104	58.6	88.0	44.7	66.2	
16	74.3	112	66.2	98.5	57.8	86.8	45.1	73.9	33.9	54.4	
18	60.3	90.7	53.9	81.0	47.2	70.9	40.3	60.5	32.8	49.0	
20	48.9	73.4	43.7	65.6	38.2	57.4	32.6	49.0	26.6	39.9	
22	40.4	60.7	36.1	54.2	31.6	47.5	26.9	40.5	22.0	33.0	
24	33.9	51.0	30.3	45.6	26.5	39.9	22.8	34.0	18.5	27.7	
0	182	243	143	214	122	183	95.6	144	70.2	106	
2	145	218	125	187	103	155	77.8	117	51.7	77.7	
4	140	211	121	181	99.9	150	76.0	114	50.6	76.0	
6	129	194	111	167	92.8	140	72.0	108	48.2	72.5	
8	109	165	94.7	142	78.4	119	63.4	95.2	45.4	65.3	
10	90.2	136	78.0	117	65.4	98.3	52.4	78.8	37.5	56.3	
12	71.1	107	61.3	82.1	51.3	77.1	41.2	61.9	30.1	45.2	
14	53.8	80.6	46.3	69.6	38.8	58.4	31.4	47.2	23.4	35.1	
16	41.6	62.4	35.8	53.9	30.2	45.4	24.6	36.8	18.5	27.0	
18	33.0	49.6	28.5	42.9	24.1	36.2	19.7	29.6	15.0	22.5	
20	26.8	40.3	23.2	34.9	19.6	29.5	16.1	24.2			

Properties of 2 angles—1/2 in. back to back

$A_g$ , in. <sup>2</sup>	7.50	6.62	5.72	4.82	3.88
$r_x$ , in.	1.58	1.50	1.40	1.41	1.42
$r_y$ , in.	1.24	1.23	1.22	1.21	1.19

Properties of single angle

$r_x$ , in.	0.642	0.644	0.646	0.649	0.652
-------------	-------	-------	-------	-------	-------

ASD LRFD

$\Omega_c = 1.67$   $\phi_c = 0.90$

\* For Y-Y axis, welded or pretensioned bolted intermediate connectors with Class A or B flaying surfaces must be used.  
† For required number of intermediate connectors, see the discussion of Table 4-6.  
‡ Shape is slender for compression with  $F_y = 36$  ksi; tabulated values have been adjusted accordingly.  
§ Bold heavy line indicates  $L_c/r$  equal to or greater than 200.

3.109

# Summary

- Looked at the limit states for compression members
- Addressed flexural buckling
- Considered design of compression members
- Treated members with slender elements
- Discussed torsional and flexural-torsional buckling
- Treated the special case of single angles
- Addressed built-up members



3.110

## Lesson 4

- The next lesson will look at the principles of design for flexural members, including shear
- We will look at the material in Chapters F and G of the *Specification* and Part 3 of the Manual



3.111



## Thank You

American Institute of Steel Construction  
130 East Randolph St., Suite 2000  
Chicago, IL 60601



3.112

## Individual Session Registrants

### PDH Certificates

- You will receive an email on how to report attendance from: [registration@aisc.org](mailto:registration@aisc.org).
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!



## Individual Session Registrants

### PDH Certificates

- Reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.



## 8-Session Registrants

### PDH Certificates

One certificate will be issued at the conclusion of all 8 sessions.



## 8-Session Registrants

### Access to the quiz

Information for accessing the quiz will be emailed to you by Thursday. It will contain a link to access the quiz. EMAIL COMES FROM [NIGHTSCHOOL@AISC.ORG](mailto:NIGHTSCHOOL@AISC.ORG).

### Quiz and attendance records

Posted Thursday mornings. [www.aisc.org/nightschool](http://www.aisc.org/nightschool) -- Click on Current Course Details.

### Reasons for quiz

- EEU – You must take all quizzes and the final exam to receive EEU.
- PDHs – If you watch a recorded session, you must pass quiz for PDHs.
- REINFORCEMENT – Reinforce what you learn tonight. Get more out of the course.



*Note: If you attend the live presentation, you do not have to take the quizzes to receive PDHs*

## 8-Session Registrants

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### Access to the recording

Information for accessing the recording will be emailed to you by Thursday. The recording will be available for four weeks. (For 8-session registrants only.)  
EMAIL COMES FROM [NIGHTSCHOOL@AISC.ORG](mailto:NIGHTSCHOOL@AISC.ORG).

### PDHs via recording

If you watch a recorded session, you must take *and pass* the quiz for PDHs.



## 8-Session Registrants

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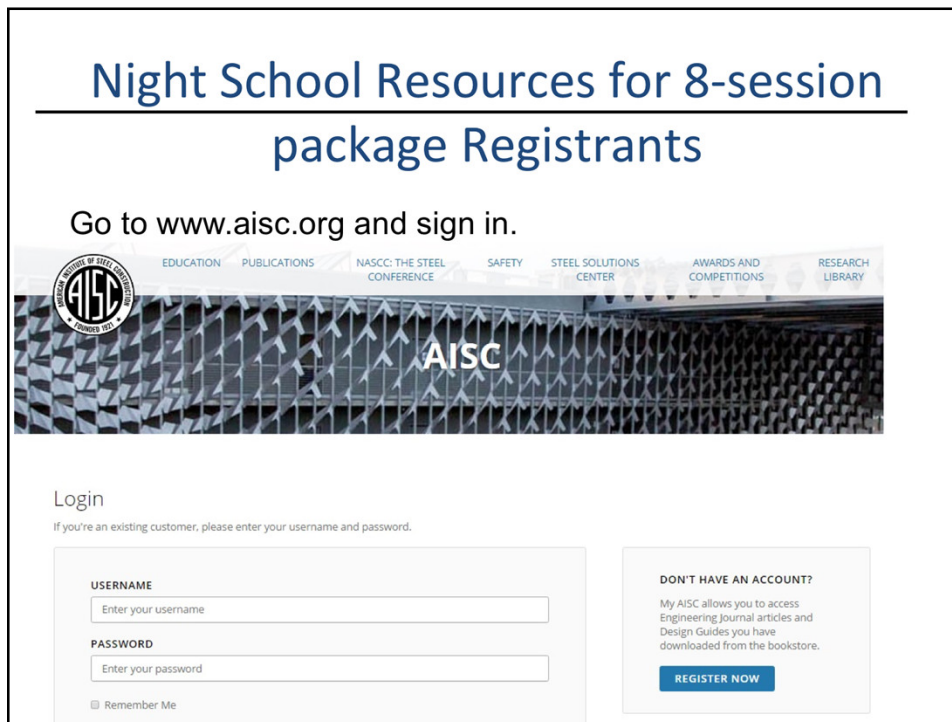
### Night School Resources

Find all your handouts, quizzes and quiz scores, recording access, and attendance information all in one place!



## Night School Resources for 8-session package Registrants

Go to [www.aisc.org](http://www.aisc.org) and sign in.



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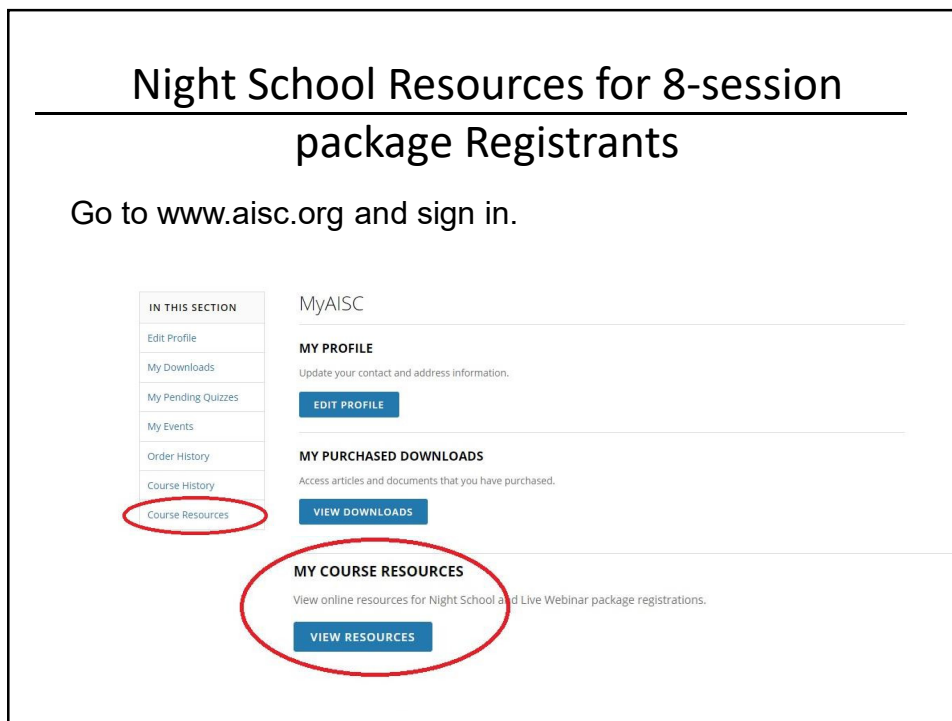
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