Basic Steel Design -- Session 5:
Compression + Bending

Louis F. Geschwindner

Smarter.
Stronger.
Steel.

AISC Night School

Welcome to today's webinar.

Today's audio will be broadcast through the internet. Please be sure and turn up the volume on your speakers.

AISC Night School

Today's live webinar will begin shortly.
Please standby.

Today's audio will be broadcast through the internet.

Please type any questions or comments through the Chat feature on the left portion of your screen.

AISC Night School

AIA Credit

AISC is a Registered Provider with The American Institute of Architects Continuing Education Systems (AIA/CES). Credit(s) earned on completion of this program will be reported to AIA/CES for AIA members. Certificates of Completion for both AIA members and non-AIA members are available upon request.

This program is registered with AIA/CES for continuing professional education. As such, it does not include content that may be deemed or construed to be an approval or endorsement by the AIA of any material of construction or any method or manner of handling, using, distributing, or dealing in any material or product.

Questions related to specific materials, methods, and services will be addressed at the conclusion of this presentation.

AISC Night School

Copyright Materials

AISC Night School

Session Description

22.5 Compression + Bending
March 3, 2020

This lecture will discuss the behavior and design of beam-columns. The session will review elastic and plastic interaction principles, AISC interaction equations and design rules of thumb. The session will explore the design of members in single axis bending as well as the design of single angles for bending plus compression.

Learning Objectives:

- Describe the behavior and design of steel beam-column members.
- Apply the AISC Specification interaction equations for the design of members with bending plus compression.
- List the design aids for beam-columns and demonstrate how to apply in design.
- Describe the design process for unsymmetric shapes with combined stress.

Basic Steel Design: A review of the principles of steel design according to ANSI/AISC 360-16

Night School 22

Lesson 5

Compression + Bending

Smarter.
Stronger.
Steel.

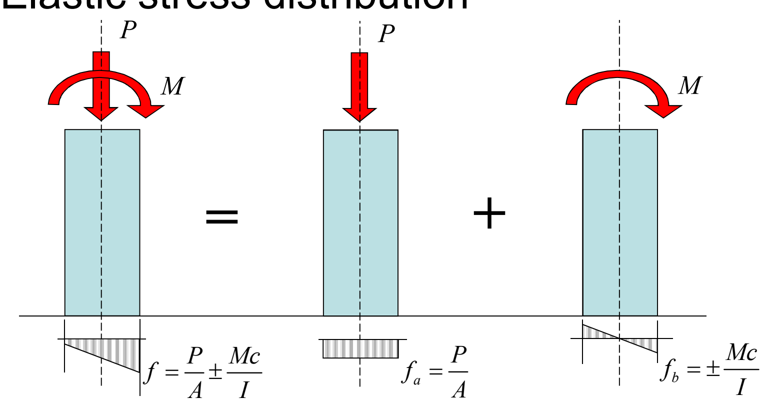
Lesson 5 – Compression + Bending

- Combined force members
  - Interaction with elastic stress distribution
  - Interaction with plastic stress distribution
  - Specification interaction equations
  - Design aids for beam-columns
  - Initial beam-column selection
  - Single axis bending with axial load
  - Unsymmetric shapes with combined stress

5.9

Compression + Bending

- Elastic stress distribution



5.10

Compression + Bending

- Elastic stress distribution
  - Could limit bending stress to a specific value, F_b
  - Could limit axial stress to a specific value, F_a
  - But these limits are likely not the same value so what we really need is a way to limit the combination

$$f = \frac{P}{A} \pm \frac{Mc}{I} \leq ?$$

5.11

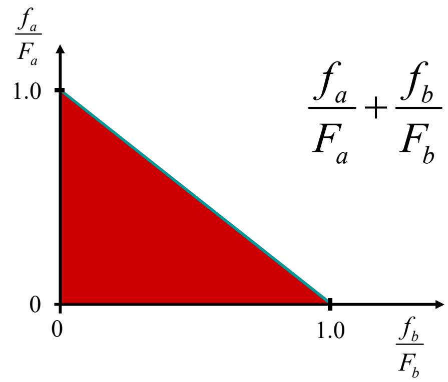
Compression + Bending

- Elastic stress distribution
  - The usual way to apply these limits is through an interaction equation
  - The ratio of applied stress to the stress limit for axial, f_a/F_a, and bending, f_b/F_b, are added
  - The sum is limited to 1.0
  - Thus, you can never use more than 100% of the available stress

5.12

Compression + Bending

- Elastic stress interaction

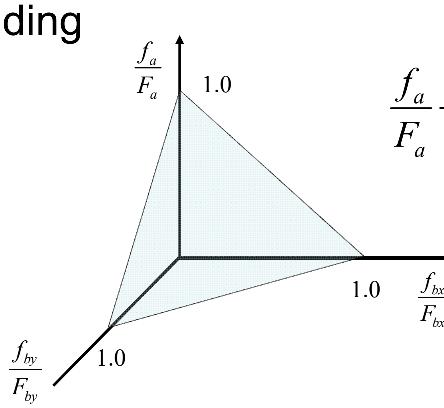


$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \le 1.0$$

5.13

Compression + Bending

- Elastic stress interaction for two axis bending



$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \le 1.0$$

5.14

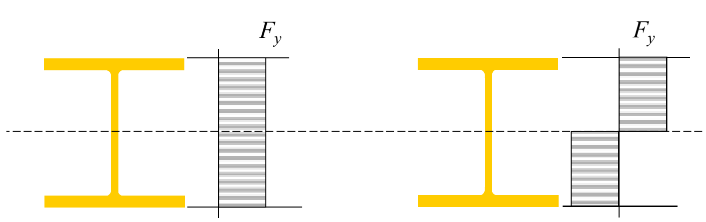
Compression + Bending

- But there is a problem with all this.
  - We know that we are not looking at elastic behavior.
  - Columns may buckle elastically but they may also buckle inelastically. They also have yielding as their upper limit.
  - Beams may behave plastically, inelastically, or elastically.

5.15

Compression + Bending

- Consider a stub column with bending, a member in which length plays no part.

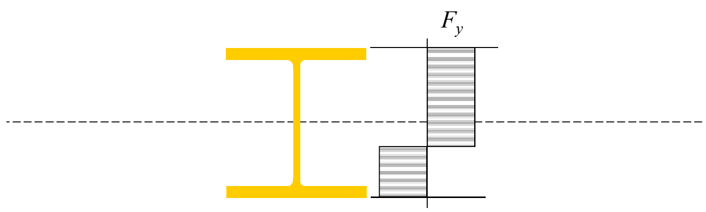


Pure Compression, P_y Pure Bending, M_{px}

5.16

Compression + Bending

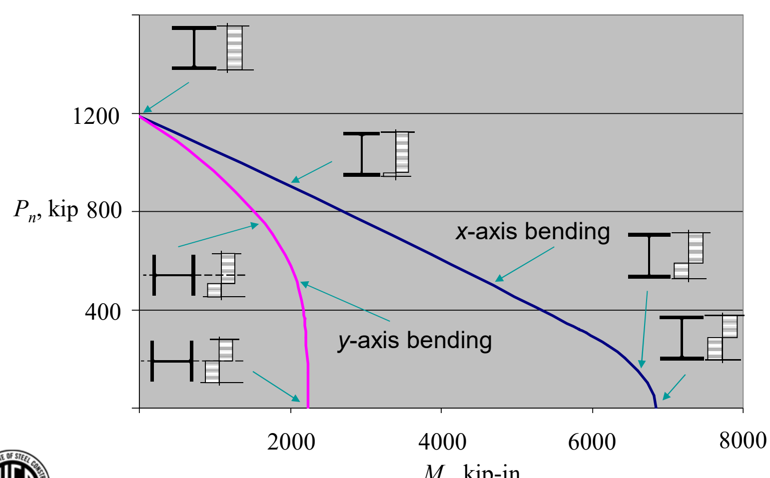
- What might the stress distribution look like if the column carried both axial compression and bending?



Combined Compression and Bending, P_n, M_{nx}

5.17

For a W14x82



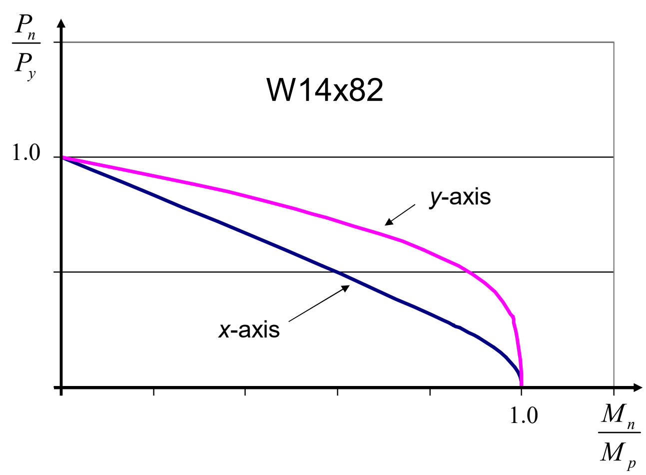
5.18

Compression + Bending

- To nondimensionalize
  - Divide the axial force by the pure axial strength, P_y
  - Divide the *x*-axis moment by the pure *x*-axis bending strength, M_{px}
  - Divide the *y*-axis moment by the pure *y*-axis bending strength, M_{py}

5.19

Compression + Bending



5.20

Compression + Bending

- To follow this approach for design
  - Each shape requires its own interaction diagrams for *x*- and *y*-axis bending.
  - Each material with different yield stress will require its own set of diagrams.
  - Shapes other than W-shapes are quite complex to deal with.
  - Thus, the Specification makes a simplification.

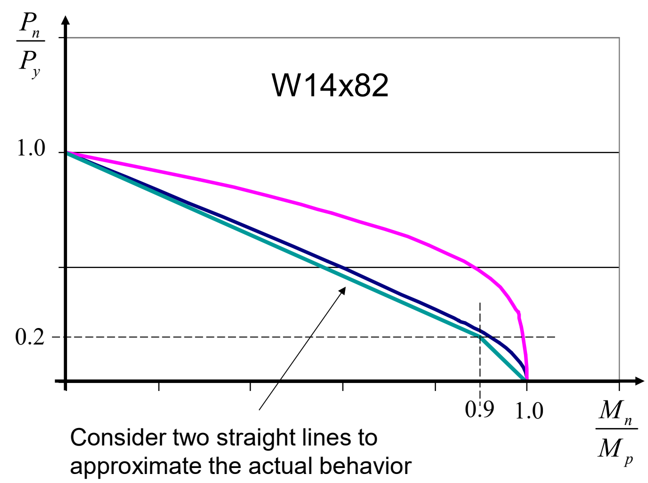
5.21

Compression + Bending

- After studying the full set of W-shapes, two straight line segments with a kink were selected to represent the interaction diagram.

5.22

Compression + Bending



5.23

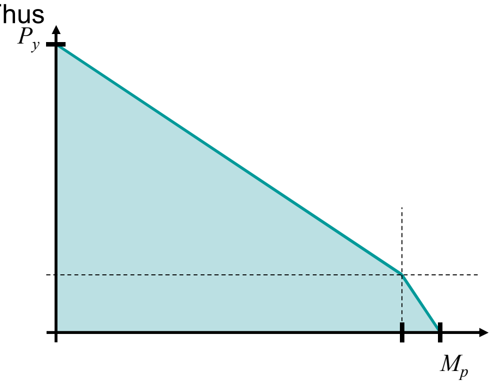
Compression + Bending

- Notice that
  - The proposed straight lines are quite accurate, yet conservative, for *x*-axis bending of this W14x82
  - They are not very accurate for *y*-axis bending but are very conservative
    - Since the magnitude of moments for *y*-axis bending are relatively small, compared to *x*-axis bending, this error is not considered a critical shortcoming.

5.24

Compression + Bending

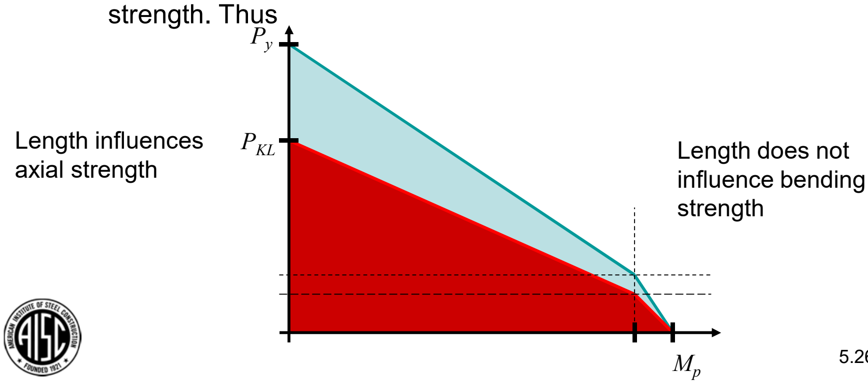
- How can we account for column length effects?
  - Look at plot for strength rather than nondimensionalized strength. Thus



5.25

Compression + Bending

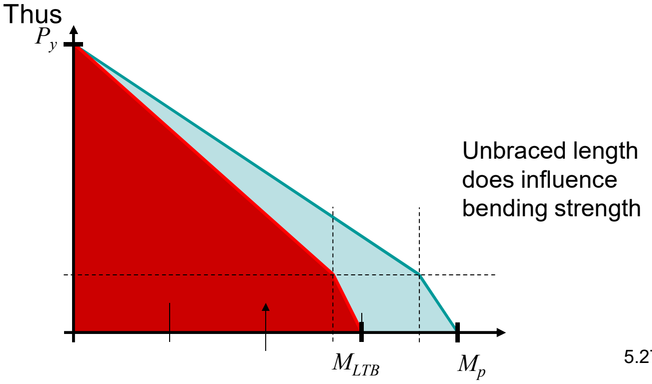
- How can we account for column length effects?
  - Look at plot for strength rather than nondimensionalized strength. Thus



5.26

Compression + Bending

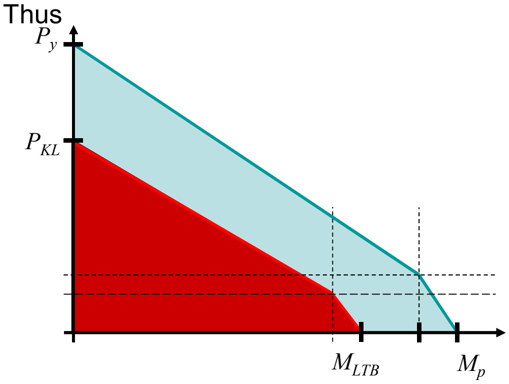
- How can we account for other bending limit states?
  – Look at plot for strength rather than nondimensionalized strength. Thus



5.27

Compression + Bending

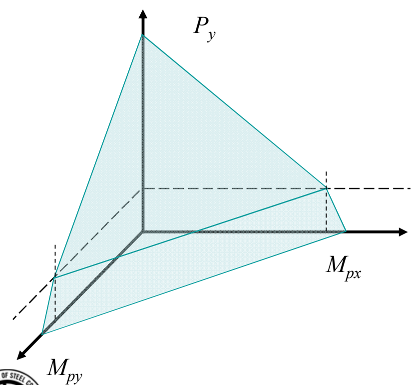
- Combine reductions in axial and flexural strengths.
  – Look at plot for strength rather than nondimensionalized strength. Thus



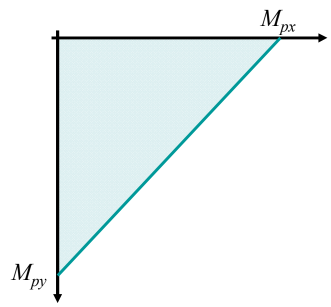
5.28

Compression + Bending

Now look at this same column with axial and bending about both axes.



And with bending only about both axes.



5.29

Compression + Bending

- Now we can look at the Specification equations and see that they are nondimensionalized with the available axial strength and the available bending strength.

$$\frac{P_r}{P_c} \quad \text{and} \quad \frac{M_r}{M_c}$$

5.30

Design for Combined Forces

H1.1. Doubly and Singly Symmetric Members subject to Flexure and Axial Force

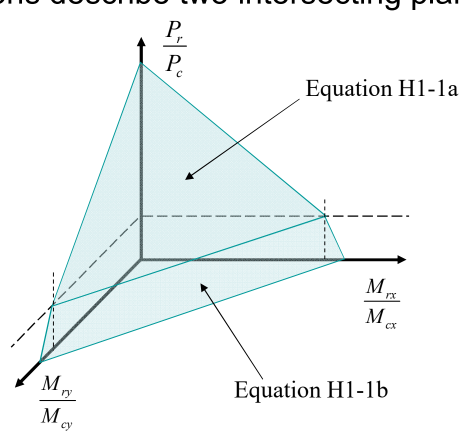
When $\frac{P_r}{P_c} \geq 0.2$ $$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad \text{(H1-1a)}$$

When $\frac{P_r}{P_c} < 0.2$ $$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad \text{(H1-1b)}$$

5.31

Compression + Bending

Thus, the Specification interaction equations describe two intersecting planes



5.32

Compression + Bending

- Definitions (ASD)

$P_r =$ required compressive strength (ASD)

$P_c = P_n / \Omega_c =$ allowable compressive strength

$M_r =$ required flexural strength (ASD)

$M_c = M_n / \Omega_b =$ allowable flexural strength

$\Omega_c = 1.67$

$\Omega_b = 1.67$

Determine required strength according to Chapter C

5.33

Compression + Bending

- Definitions (LRFD)

$P_r =$ required compressive strength (LRFD)

$P_c = \phi_c P_n =$ design compressive strength

$M_r =$ required flexural strength (LRFD)

$M_c = \phi_b M_n =$ design flexural strength

$\phi_c = 0.90$

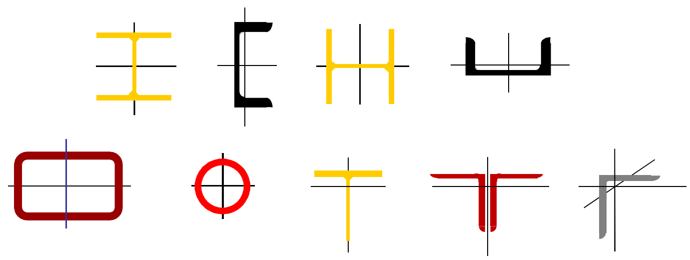
$\phi_b = 0.90$

Determine required strength according to Chapter C

5.34

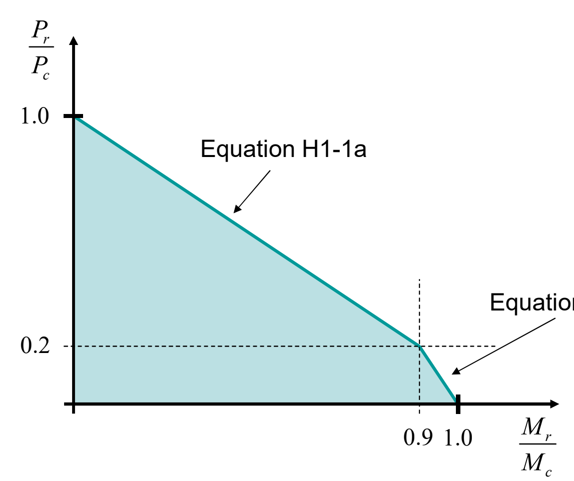
Compression + Bending

- Equations H1-1a and H1-1b apply to all doubly and singly symmetric members



5.35

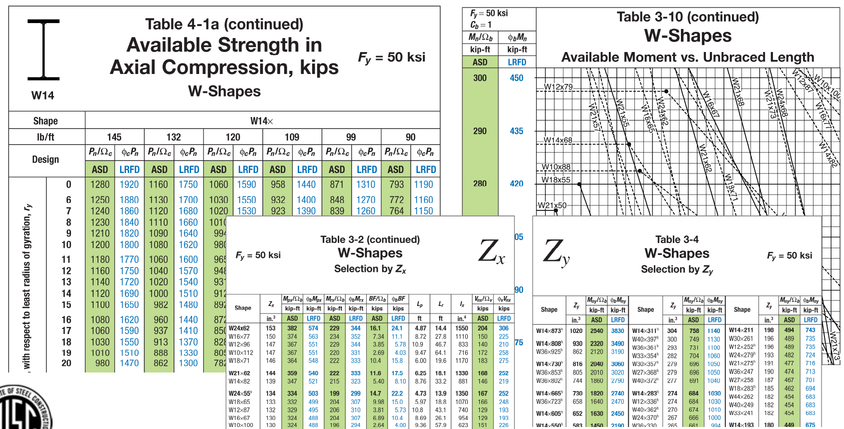
Compression + Bending



5.36

Compression + Bending

- Beam-Column Design using Manual Tables



Compression + Bending

- Beam-Column Design using Manual Tables
  - Part 6 of the Manual contains a single table to assist in the design of members for combined forces.
  - Table entries included strength for all W-shapes.
  - Can be used to design for pure bending, pure compression, and pure tension.
  - Available for W-shapes only .

Compression + Bending

- These values can be found in Table 6-2 for all W-shapes, unlike the tables in Parts 3 and 4.
- Table 6-2 considers all appropriate limit states, including effective length for column buckling and unbraced length for beam lateral-torsional buckling.
- Table 6-2 also includes values for tension yield and tension rupture.
- All limit states considered in Lessons 2 through 4 are included.

5.39

Available Compressive Strength

The left half of Table 6-2 is the same as Table 4-1a.

However, it does include all W-shapes, thus more than Table 4-1a.

The length given in the middle column are the Effective Lengths with respect to the y-axis, just as for Table 4-1a

Table 6-2 (continued)
Available Strength for Members Subject to Axial, Shear, Flexural and Combined Forces
W-Shapes

$F_y = 50$ ksi
$F_u = 65$ ksi

W14

W14× 109 P_n/Ω_c ASD	109 $\phi_c P_n$ LRFD	99 P_n/Ω_c ASD	99 $\phi_c P_n$ LRFD	90 P_n/Ω_c ASD	90 $\phi_c P_n$ LRFD	Shape lb/ft / Design: Effective length, L_c, ft, with respect to least radius of gyration, r_y, or unbraced length, L_b, ft, for X-X axis bending	W14× 109 M_{nx}/Ω_b ASD	109 $\phi_b M_{nx}$ LRFD	99[f] M_{nx}/Ω_b ASD	99[f] $\phi_b M_{nx}$ LRFD	90[f] M_{nx}/Ω_b ASD	90[f] $\phi_b M_{nx}$ LRFD
958	1440	871	1310	793	1190	0	479	720	430	646	382	574
932	1400	848	1270	772	1160	6	479	720	430	646	382	574
923	1390	839	1260	764	1150	7	479	720	430	646	382	574
913	1370	830	1250	755	1140	8	479	720	430	646	382	574
901	1350	819	1230	745	1120	9	479	720	430	646	382	574
888	1340	807	1210	735	1100	10	479	720	430	646	382	574
874	1310	794	1190	723	1090	11	479	720	430	646	382	574
859	1290	780	1170	710	1070	12	479	720	430	646	382	574
843	1270	766	1150	697	1050	13	479	720	430	646	382	574
826	1240	750	1130	682	1030	14	475	714	427	642	382	574
808	1210	733	1100	667	1000	15	470	706	422	635	382	574
789	1190	716	1080	652	979	16	465	699	417	627	378	568
770	1160	698	1050	635	955	17	460	691	413	620	373	560
750	1130	680	1020	618	929	18	455	684	408	613	368	553
729	1100	661	994	601	903	19	450	676	403	605	363	546
708	1060	642	964	583	877	20	445	669	398	598	358	539
664	998	602	904	547	822	22	435	654	388	583	349	524
620	931	561	843	509	766	24	425	639	378	569	339	510
574	863	519	781	472	709	26	415	623	369	554	329	495
529	796	478	719	434	653	28	405	608	359	539	320	481
485	729	438	658	397	597	30	395	593	349	524	310	466
441	663	398	598	361	543	32	385	578	339	510	300	452
399	600	360	541	326	490	34	375	563	329	495	291	437
359	539	323	485	292	439	36	365	548	320	480	281	423
322	484	290	435	262	394	38	355	533	310	466	271	408
290	437	261	393	237	356	40	345	518	300	451	262	394
263	396	237	356	215	323	42	335	503	290	436	252	379
240	361	216	325	196	294	44	325	488	280	422	239	359
220	330	198	297	179	269	46	315	473	269	404	226	339
202	303	181	273	164	247	48	305	458	255	384	214	322
186	279	167	251	151	228	50	291	438	243	365	204	306

Properties

	109 ASD	109 LRFD	99 ASD	99 LRFD	90 ASD	90 LRFD
Available Strength in Tensile Yielding, kips (P_n/Ω_t, $\phi_t P_n$)	958	1440	871	1310	793	1190
Available Strength in Tensile Rupture ($A_e = 0.75A_g$), kips (P_n/Ω_t, $\phi_t P_n$)	780	1170	709	1060	647	970
Available Strength in Shear, kips (V_n/Ω_v, $\phi_v V_n$)	150	225	138	207	123	185
Available Strength in Flexure about Y-Y Axis, kip-ft (M_{ny}/Ω_b, $\phi_b M_{ny}$)	231	348	207	311	181	273

	109		99		90	
Limiting Unbraced Lengths, ft	L_p = 13.2	L_r = 48.5	L_p = 13.5	L_r = 45.3	L_p = 15.1	L_r = 42.5
Area, in.²	32.0		29.1		26.5	
Moment of Inertia, in.⁴	I_x = 1240	I_y = 447	I_x = 1110	I_y = 402	I_x = 999	I_y = 362
r_y, in.	3.73		3.71		3.70	
r_x/r_y	1.67		1.66		1.66	

[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

5.40

Available Flexural Strength

The right half of Table 6-2 does not have a comparable table in Part 3.

However, it provides the same information as given in Table 3-10. But again, for all W-shapes

The length given in the middle column are the Unbraced Lengths of the compression flange.

Table 6-2 (continued)
Available Strength for Members Subject to Axial, Shear, Flexural and Combined Forces
W-Shapes

$F_y = 50$ ksi
$F_u = 65$ ksi

W14

Effective length, L_c, ft, with respect to least radius of gyration, r_y, or unbraced length, L_b, ft, for X-X axis bending	W14×109 P_n/Ω_c ASD	W14×109 $\phi_c P_n$ LRFD	W14×99 P_n/Ω_c ASD	W14×99 $\phi_c P_n$ LRFD	W14×90 P_n/Ω_c ASD	W14×90 $\phi_c P_n$ LRFD	W14×109 M_{nx}/Ω_b ASD	W14×109 $\phi_b M_{nx}$ LRFD	W14×99[f] M_{nx}/Ω_b ASD	W14×99[f] $\phi_b M_{nx}$ LRFD	W14×90[f] M_{nx}/Ω_b ASD	W14×90[f] $\phi_b M_{nx}$ LRFD
	Available Compressive Strength, kips						Available Flexural Strength, kip-ft					
0	958	1440	871	1310	793	1190	479	720	430	646	382	574
6	932	1400	848	1270	772	1160	479	720	430	646	382	574
7	923	1390	839	1260	764	1150	479	720	430	646	382	574
8	913	1370	830	1250	755	1140	479	720	430	646	382	574
9	901	1350	819	1230	745	1120	479	720	430	646	382	574
10	888	1340	807	1210	735	1100	479	720	430	646	382	574
11	874	1310	794	1190	723	1090	479	720	430	646	382	574
12	859	1290	780	1170	710	1070	479	720	430	646	382	574
13	843	1270	766	1150	697	1050	479	720	430	646	382	574
14	826	1240	750	1130	682	1030	475	714	427	642	382	574
15	808	1210	733	1100	667	1000	470	706	422	635	382	574
16	789	1190	716	1080	652	979	465	699	417	627	378	568
17	770	1160	698	1050	635	955	460	691	413	620	373	560
18	750	1130	680	1020	618	929	455	684	408	613	368	553
19	729	1100	661	994	601	903	450	676	403	605	363	546
20	708	1060	642	964	583	877	445	669	398	598	358	539
22	664	998	602	904	547	822	435	654	388	583	349	524
24	620	931	561	843	509	766	425	639	378	569	339	510
26	574	863	519	781	472	709	415	623	369	554	329	495
28	529	796	478	719	434	653	405	608	359	539	320	481
30	485	729	438	658	397	597	395	593	349	524	310	466
32	441	663	398	598	361	543	385	578	339	510	300	452
34	399	600	360	541	326	490	375	563	329	495	291	437
36	359	539	323	485	292	439	365	548	320	480	281	423
38	322	484	290	435	262	394	355	533	310	466	271	408
40	290	437	261	393	237	356	345	518	300	451	262	394
42	263	396	237	356	215	323	335	503	290	436	252	379
44	240	361	216	325	196	294	325	488	280	422	239	359
46	220	330	198	297	179	269	315	473	269	404	226	339
48	202	303	181	273	164	247	305	458	255	384	214	322
50	186	279	167	251	151	228	291	438	243	365	204	306

Properties

	W14×109		W14×99		W14×90	
Available Strength in Tensile Yielding, kips	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$
	958	1440	871	1310	793	1190
Available Strength in Tensile Rupture ($A_e = 0.75A_g$), kips	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$
	780	1170	709	1060	647	970
Available Strength in Shear, kips	V_n/Ω_v	$\phi_v V_n$	V_n/Ω_v	$\phi_v V_n$	V_n/Ω_v	$\phi_v V_n$
	150	225	138	207	123	185
Available Strength in Flexure about Y-Y Axis, kip-ft	M_{ny}/Ω_b	$\phi_b M_{ny}$	M_{ny}/Ω_b	$\phi_b M_{ny}$	M_{ny}/Ω_b	$\phi_b M_{ny}$
	231	348	207	311	181	273
Limiting Unbraced Lengths, ft	L_p	L_r	L_p	L_r	L_p	L_r
	13.2	48.5	13.5	45.3	15.1	42.5
Area, in.^{2}	32.0		29.1		26.5	
Moment of Inertia, in.^{4}	I_x	I_y	I_x	I_y	I_x	I_y
	1240	447	1110	402	999	362
r_y, in.	3.73		3.71		3.70	
r_x/r_y	1.67		1.66		1.66	

[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

5.41

Other Available Strengths

Strengths not a function of length:

Tensile Yield and Tensile Rupture, the same as Table 5-1.

Shear, the same as Tables 3-2 and 3-6.

Flexure about the y-axis, the same as Table 3-4.

Table 6-2 (continued)
Available Strength for Members Subject to Axial, Shear, Flexural and Combined Forces
W-Shapes

$F_y = 50$ ksi
$F_u = 65$ ksi

W14

Effective length, L_c, ft, with respect to least radius of gyration, r_y, or unbraced length, L_b, ft, for X-X axis bending	W14×109 P_n/Ω_c ASD	W14×109 $\phi_c P_n$ LRFD	W14×99 P_n/Ω_c ASD	W14×99 $\phi_c P_n$ LRFD	W14×90 P_n/Ω_c ASD	W14×90 $\phi_c P_n$ LRFD	W14×109 M_{nx}/Ω_b ASD	W14×109 $\phi_b M_{nx}$ LRFD	W14×99[f] M_{nx}/Ω_b ASD	W14×99[f] $\phi_b M_{nx}$ LRFD	W14×90[f] M_{nx}/Ω_b ASD	W14×90[f] $\phi_b M_{nx}$ LRFD
	Available Compressive Strength, kips						Available Flexural Strength, kip-ft					
0	958	1440	871	1310	793	1190	479	720	430	646	382	574
6	932	1400	848	1270	772	1160	479	720	430	646	382	574
7	923	1390	839	1260	764	1150	479	720	430	646	382	574
8	913	1370	830	1250	755	1140	479	720	430	646	382	574
9	901	1350	819	1230	745	1120	479	720	430	646	382	574
10	888	1340	807	1210	735	1100	479	720	430	646	382	574
11	874	1310	794	1190	723	1090	479	720	430	646	382	574
12	859	1290	780	1170	710	1070	479	720	430	646	382	574
13	843	1270	766	1150	697	1050	479	720	430	646	382	574
14	826	1240	750	1130	682	1030	475	714	427	642	382	574
15	808	1210	733	1100	667	1000	470	706	422	635	382	574
16	789	1190	716	1080	652	979	465	699	417	627	378	568
17	770	1160	698	1050	635	955	460	691	413	620	373	560
18	750	1130	680	1020	618	929	455	684	408	613	368	553
19	729	1100	661	994	601	903	450	676	403	605	363	546
20	708	1060	642	964	583	877	445	669	398	598	358	539
22	664	998	602	904	547	822	435	654	388	583	349	524
24	620	931	561	843	509	766	425	639	378	569	339	510
26	574	863	519	781	472	709	415	623	369	554	329	495
28	529	796	478	719	434	653	405	608	359	539	320	481
30	485	729	438	658	397	597	395	593	349	524	310	466
32	441	663	398	598	361	543	385	578	339	510	300	452
34	399	600	360	541	326	490	375	563	329	495	291	437
36	359	539	323	485	292	439	365	548	320	480	281	423
38	322	484	290	435	262	394	355	533	310	466	271	408
40	290	437	261	393	237	356	345	518	300	451	262	394
42	263	396	237	356	215	323	335	503	290	436	252	379
44	240	361	216	325	196	294	325	488	280	422	239	359
46	220	330	198	297	179	269	315	473	269	404	226	339
48	202	303	181	273	164	247	305	458	255	384	214	322
50	186	279	167	251	151	228	291	438	243	365	204	306

Properties

	W14×109		W14×99		W14×90	
Available Strength in Tensile Yielding, kips	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$
	958	1440	871	1310	793	1190
Available Strength in Tensile Rupture ($A_e = 0.75A_g$), kips	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$
	780	1170	709	1060	647	970
Available Strength in Shear, kips	V_n/Ω_v	$\phi_v V_n$	V_n/Ω_v	$\phi_v V_n$	V_n/Ω_v	$\phi_v V_n$
	150	225	138	207	123	185
Available Strength in Flexure about Y-Y Axis, kip-ft	M_{ny}/Ω_b	$\phi_b M_{ny}$	M_{ny}/Ω_b	$\phi_b M_{ny}$	M_{ny}/Ω_b	$\phi_b M_{ny}$
	231	348	207	311	181	273
Limiting Unbraced Lengths, ft	L_p	L_r	L_p	L_r	L_p	L_r
	13.2	48.5	13.5	45.3	15.1	42.5
Area, in.^{2}	32.0		29.1		26.5	
Moment of Inertia, in.^{4}	I_x	I_y	I_x	I_y	I_x	I_y
	1240	447	1110	402	999	362
r_y, in.	3.73		3.71		3.70	
r_x/r_y	1.67		1.66		1.66	

[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

5.42

Other Available Data

Unbraced lengths, L_p and L_r

Area

Moment of Inertia, I_x and I_y

Radius of gyration, r_y and r_x/r_y

Table 6-2 (continued)
Available Strength for Members Subject to Axial, Shear, Flexural and Combined Forces
W-Shapes

F_y = 50 ksi
F_u = 65 ksi

W14

Shape W14× 109 P_n/Ω_c ASD	109 $\phi_c P_n$ LRFD	99 P_n/Ω_c ASD	99 $\phi_c P_n$ LRFD	90 P_n/Ω_c ASD	90 $\phi_c P_n$ LRFD	Design: Effective length, L_c, ft, with respect to least radius of gyration, r_y, or unbraced length, L_b, ft, for X-X axis bending	109 M_{nx}/Ω_b ASD	109 $\phi_b M_{nx}$ LRFD	99[f] M_{nx}/Ω_b ASD	99[f] $\phi_b M_{nx}$ LRFD	90[f] M_{nx}/Ω_b ASD	90[f] $\phi_b M_{nx}$ LRFD
Available Compressive Strength, kips							Available Flexural Strength, kip-ft					
958	1440	871	1310	793	1190	0	479	720	430	646	382	574
932	1400	848	1270	772	1160	6	479	720	430	646	382	574
923	1390	839	1260	764	1150	7	479	720	430	646	382	574
913	1370	830	1250	755	1140	8	479	720	430	646	382	574
901	1350	819	1230	745	1120	9	479	720	430	646	382	574
888	1340	807	1210	735	1100	10	479	720	430	646	382	574
874	1310	794	1190	723	1090	11	479	720	430	646	382	574
859	1290	780	1170	710	1070	12	479	720	430	646	382	574
843	1270	766	1150	697	1050	13	479	720	430	646	382	574
826	1240	750	1130	682	1030	14	475	714	427	642	382	574
808	1210	733	1100	667	1000	15	470	706	422	635	382	574
789	1190	716	1080	652	979	16	465	699	417	627	378	568
770	1160	698	1050	635	955	17	460	691	413	620	373	560
750	1130	680	1020	618	929	18	455	684	408	613	368	553
729	1100	661	994	601	903	19	450	676	403	605	363	546
708	1060	642	964	583	877	20	445	669	398	598	358	539
664	998	602	904	547	822	22	435	654	388	583	349	524
620	931	561	843	509	766	24	425	639	378	569	339	510
574	863	519	781	472	709	26	415	623	369	554	329	495
529	796	478	719	434	653	28	405	608	359	539	320	481
485	729	438	658	397	597	30	395	593	349	524	310	466
441	663	398	598	361	543	32	385	578	339	510	300	452
399	600	360	541	326	490	34	375	563	329	495	291	437
359	539	323	485	292	439	36	365	548	320	480	281	423
322	484	290	435	262	394	38	355	533	310	466	271	408
290	437	261	393	237	356	40	345	518	300	451	262	394
263	396	237	356	215	323	42	335	503	290	436	252	379
240	361	216	325	196	294	44	325	488	280	422	239	359
220	330	198	297	179	269	46	315	473	269	404	226	339
202	303	181	273	164	247	48	305	458	255	384	214	322
186	279	167	251	151	228	50	291	438	243	365	204	306

Properties

109		99		90	
Available Strength in Tensile Yielding, kips					
P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$
958	1440	871	1310	793	1190
Available Strength in Tensile Rupture ($A_e = 0.75A_g$), kips					
P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$
780	1170	709	1060	647	970
Available Strength in Shear, kips					
V_n/Ω_v	$\phi_v V_n$	V_n/Ω_v	$\phi_v V_n$	V_n/Ω_v	$\phi_v V_n$
150	225	138	207	123	185
Available Strength in Flexure about Y-Y Axis, kip-ft					
M_{ny}/Ω_b	$\phi_b M_{ny}$	M_{ny}/Ω_b	$\phi_b M_{ny}$	M_{ny}/Ω_b	$\phi_b M_{ny}$
231	348	207	311	181	273

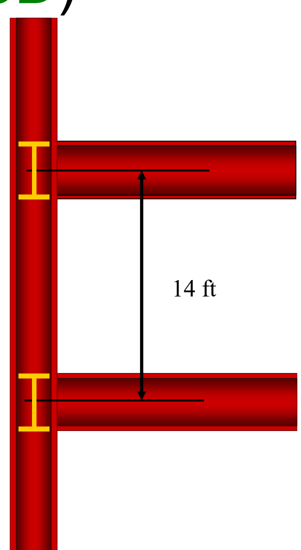
109		99		90	
Limiting Unbraced Lengths, ft					
L_p	L_r	L_p	L_r	L_p	L_r
13.2	48.5	13.5	45.3	15.1	42.5
Area, in.^{2}					
32.0		29.1		26.5	
Moment of Inertia, in.^{4}					
I_x	I_y	I_x	I_y	I_x	I_y
1240	447	1110	402	999	362
r_y, in.					
3.73		3.71		3.70	
r_x/r_y					
1.67		1.66		1.66	

[f] Shape exceeds compact limit for flexure with F_y = 50 ksi.

5.43

Example 1 (ASD)

- An ASTM A992 W14x90 column must carry an axial force of 333 kips, an *x*-axis bending moment of 169 ft-kips, and a *y*-axis bending moment of 20 ft-kips.
- These results are from a second-order Direct Analysis. (Considered in Lessons 6 and 7)
- Will this column adequately support these loads?



The column is 14 ft long, is bending about both axes, has a length of 14 ft about the *x*- and *y*-axis and an unbraced length of the compression flange of 14 ft.

5.44

Example 1 (ASD)

The use of the direct analysis method means that we may use an effective length equal to the actual length, thus K = 1.0

Also, since we used a second-order analysis there is no need to amplify forces or moments. Again, this will be covered in Lessons 6 and 7

$$P_n/\Omega_c = 682 \text{ kips}$$

$$M_{nx}/\Omega_b = 382 \text{ ft-kips}$$

$$M_{ny}/\Omega_b = 181 \text{ ft-kips}$$

Table 6-2 (continued)
Available Strength for Members Subject to Axial, Shear, Flexural and Combined Forces
W-Shapes

F_y = 50 ksi
F_u = 65 ksi

W14

W14×						Shape	W14×					
109		99		90		lb/ft	109		99[f]		90[f]	
P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	Design	M_{nx}/Ω_b	$\phi_b M_{nx}$	M_{nx}/Ω_b	$\phi_b M_{nx}$	M_{nx}/Ω_b	$\phi_b M_{nx}$
Available Compressive Strength, kips							Available Flexural Strength, kip-ft					
ASD	LRFD	ASD	LRFD	ASD	LRFD	...st radius of gyration, r_y, ...X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD
958	1440	871	1310	793	1190	0	479	720	430	646	382	574
932	1400	848	1270	772	1160	6	479	720	430	646	382	574
923	1390	839	1260	764	1150	7	479	720	430	646	382	574
913	1370	830	1250	755	1140	8	479	720	430	646	382	574
901	1350	819	1230	745	1120	9	479	720	430	646	382	574
888	1340	807	1210	735	1100	10	479	720	430	646	382	574
874	1310	794	1190	723	1090	11	479	720	430	646	382	574
859	1290	780	1170	710	1070	12	479	720	430	646	382	574
843	1270	766	1150	697	1050	13	479	720	430	646	382	574
826	1240	750	1130	682	1030	14	475	714	427	642	382	574
808	1210	733	1100	667	1000	15	470	706	422	635	382	574
789	1190	716	1080	652	979	16	465	699	417	627	378	568
202	303	181	273	164	247	48	305	458	255	384	214	322
186	279	167	251	151	228	50	291	438	243	365	204	306

Properties

Available Strength in Tensile Yielding, kips					
P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$
958	1440	871	1310	793	1190
Available Strength in Tensile Rupture ($A_e = 0.75A_g$), kips					
P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$
780	1170	709	1060	647	970
Available Strength in Shear, kips					
V_n/Ω_v	$\phi_v V_n$	V_n/Ω_v	$\phi_v V_n$	V_n/Ω_v	$\phi_v V_n$
150	225	138	207	123	185
Available Strength in Flexure about Y-Y Axis, kip-ft					
M_{ny}/Ω_b	$\phi_b M_{ny}$	M_{ny}/Ω_b	$\phi_b M_{ny}$	M_{ny}/Ω_b	$\phi_b M_{ny}$
231	348	207	311	181	273

Limiting Unbraced Lengths, ft					
L_p	L_r	L_p	L_r	L_p	L_r
13.2	48.5	13.5	45.3	15.1	42.5
Area, in.^{2}					
32.0		29.1		26.5	
Moment of Inertia, in.^{4}					
I_x	I_y	I_x	I_y	I_x	I_y
1240	447	1110	402	999	362
r_y, in.					
3.73		3.71		3.70	
r_x/r_y					
1.67		1.66		1.66	

[f] Shape exceeds compact limit for flexure with F_y = 50 ksi.

5.45

Example 1 (ASD)

Determine which equation to use:

$$P_r/P_c = 333/682 = 0.49 > 0.2$$

Therefore use Eq. H1-1a

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \le 1.0$$

$$\frac{333}{682} + \frac{8}{9}\left(\frac{169}{382} + \frac{20}{181}\right) = 0.98 \le 1.0$$

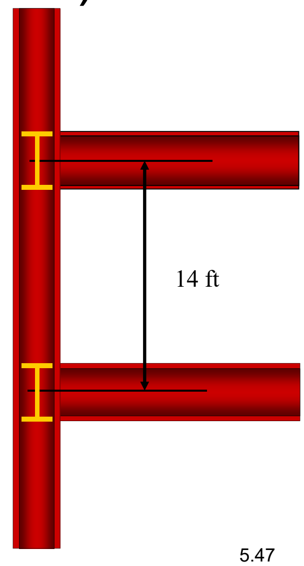
Therefore the W14x90 is adequate

5.46

Example 1 (LRFD)

- An ASTM A992 W14x90 column must carry an axial force of 500 kips, an *x*-axis bending moment of 253 ft-kips, and a *y*-axis bending moment of 30 ft-kips.
- These results are from a second-order Direct Analysis. (Considered in Lessons 6 and 7)
- Will this column adequately support these loads?

The column is 14 ft long, is bending about both axes, has a length of 14 ft about the *x*- and *y*-axis and an unbraced length of the compression flange of 14 ft.



5.47

Example 1 (LRFD)

The use of the direct analysis method means that we may use an effective length equal to the actual length, thus *K* = 1.0

Also, since we used a second-order analysis there is no need to amplify forces or moments. Again, this will be covered in Lessons 6 and 7

$$\phi_c P_n = 1030 \text{ kips}$$
$$\phi_b M_{nx} = 574 \text{ ft-kips}$$
$$\phi_b M_{ny} = 273 \text{ ft-kips}$$

Table 6-2 (continued)
Available Strength for Members Subject to Axial, Shear, Flexural and Combined Forces
W-Shapes

$F_y = 50$ ksi
$F_u = 65$ ksi

W14

W14× 109 P_n/Ω_c ASD	109 $\phi_c P_n$ LRFD	99 P_n/Ω_c ASD	99 $\phi_c P_n$ LRFD	90 P_n/Ω_c ASD	90 $\phi_c P_n$ LRFD	Design: Effective length KL (ft) with respect to least radius of gyration, r_y, or L_b for X-X axis bending	W14× 109 M_{nx}/Ω_b ASD	109 $\phi_b M_{nx}$ LRFD	99[f] M_{nx}/Ω_b ASD	99[f] $\phi_b M_{nx}$ LRFD	90[f] M_{nx}/Ω_b ASD	90[f] $\phi_b M_{nx}$ LRFD
958	1440	871	1310	793	1190	0	479	720	430	646	382	574
932	1400	848	1270	772	1160	6	479	720	430	646	382	574
923	1390	839	1260	764	1150	7	479	720	430	646	382	574
913	1370	830	1250	755	1140	8	479	720	430	646	382	574
901	1350	819	1230	745	1120	9	479	720	430	646	382	574
888	1340	807	1210	735	1100	10	479	720	430	646	382	574
874	1310	794	1190	723	1090	11	479	720	430	646	382	574
859	1290	780	1170	710	1070	12	479	720	430	646	382	574
843	1270	766	1150	697	1050	13	479	720	430	646	382	574
826	1240	750	1130	682	1030	14	475	714	427	642	382	574
808	1210	733	1100	667	1000	15	470	706	422	635	382	574
789	1190	716	1080	652	979	16	465	699	417	627	378	568
202	303	181	273	164	247	48	305	458	255	384	214	322
186	279	167	251	151	228	50	291	438	243	365	204	306

Properties

	109		99		90	
Available Strength in Tensile Yielding, kips	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$
	958	1440	871	1310	793	1190
Available Strength in Tensile Rupture ($A_e = 0.75A_g$), kips	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$	P_n/Ω_t	$\phi_t P_n$
	780	1170	709	1060	647	970
Available Strength in Shear, kips	V_n/Ω_v	$\phi_v V_n$	V_n/Ω_v	$\phi_v V_n$	V_n/Ω_v	$\phi_v V_n$
	150	225	138	207	123	185
Available Strength in Flexure about Y-Y Axis, kip-ft	M_{ny}/Ω_b	$\phi_b M_{ny}$	M_{ny}/Ω_b	$\phi_b M_{ny}$	M_{ny}/Ω_b	$\phi_b M_{ny}$
	231	348	207	311	181	273
Limiting Unbraced Lengths, ft	L_p	L_r	L_p	L_r	L_p	L_r
	13.2	48.5	13.5	45.3	15.1	42.5
Area, in.^{2}	32.0		29.1		26.5	
Moment of Inertia, in.^{4}	I_x	I_y	I_x	I_y	I_x	I_y
	1240	447	1110	402	999	362
r_y, in.	3.73		3.71		3.70	
r_x/r_y	1.67		1.66		1.66	

[f] Shape exceeds compact limit for flexure with $F_y = 50$ ksi.

5.48

Example 1 (LRFD)

Determine which equation to use:

$$P_r / P_c = 500/1030 = 0.49 > 0.2$$

Therefore use Eq. H1-1a

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$$

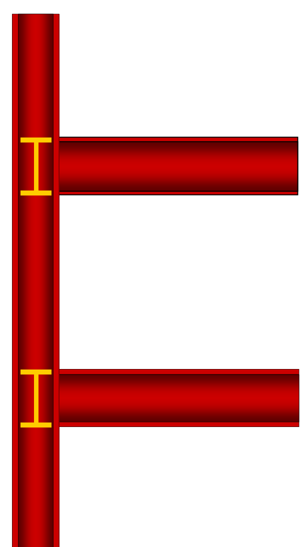
$$\frac{500}{1030} + \frac{8}{9}\left(\frac{253}{574} + \frac{30}{273}\right) = 0.97 \leq 1.0$$

Therefore the W14x90 is adequate

5.49

Initial Beam-Column Selection

- How would we start a design if we had the force and moments from Example 1 but did not know what size column we were using?



5.50

Initial Beam-Column Selection

- Start with Equation H1-1a

$$\frac{P_r}{P_c}+\frac{8}{9}\frac{M_{rx}}{M_{cx}}+\frac{8}{9}\frac{M_{ry}}{M_{cy}}\leq 1.0$$

- Multiply both sides by P_c

$$P_r+\frac{8}{9}\frac{M_{rx}P_c}{M_{cx}}+\frac{8}{9}\frac{M_{ry}P_c}{M_{cy}}\leq P_c$$

5.51

Initial Beam-Column Selection

- Multiply last term by M_{cx}/M_{cx} and reorganize

$$P_r+\frac{8P_c}{9M_{cx}}M_{rx}+\frac{8P_c}{9M_{cx}}\frac{M_{cx}}{M_{cy}}M_{ry}\leq P_c$$

- Let

$$m=\frac{8P_c}{9M_{cx}}$$

$$U=\frac{M_{cx}}{M_{cy}}$$

5.52

Initial Beam-Column Selection

- Substitute these new terms back into the equation and

$$P_r + \frac{8P_c}{9M_{cx}} M_{rx} + \frac{8P_c}{9M_{cx}} \frac{M_{cx}}{M_{cy}} M_{ry} \leq P_c$$

becomes

$$P_r + mM_{rx} + mUM_{ry} \leq P_c$$

5.53

Initial Beam-Column Selection

- This can be thought of as

Effective required strength $\leq$ Available compressive strength

$$P_{eff} = P_r + mM_{rx} + mUM_{ry} \leq P_c$$

So what can we do about *m* and *U*?

5.54

Initial Beam-Column Selection

- Ignore the influence of length, thus no column buckling or lateral-torsional buckling
- Assume both axial and flexural stresses are F_y.

$$m = \frac{8P_c}{9M_{cx}} = \frac{8F_y A}{9F_y Z_x} = \frac{8A}{9Z_x}$$

$$U = \frac{M_{cx}}{M_{cy}} = \frac{F_y Z_x}{F_y Z_y} = \frac{Z_x}{Z_y}$$

5.55

Initial Beam-Column Selection

- Evaluate $m = \frac{8A}{9Z_x}$ and $U = \frac{Z_x}{Z_y}$ for all W-shapes

Shape	m_{avg}	U_{avg}
W6	4.41	3.01
W8	3.25	3.11
W10	2.62	3.62
W12	2.08	3.47
W14	1.72	2.86

Calculated *m* was multiplied by 12 to permit working in kips and ft.

5.56

Initial Beam-Column Selection

- At this stage in design, the apparent level of accuracy of these numbers is unnecessary.
- Even these values are not all that accurate since they represent an average.
- These will be simplified in order to make them easy to remember and use.

5.57

Initial Beam-Column Selection

- Consider U
  - Only the smallest of the shapes in each group have U values appreciably greater than 3.
  - Thus, a reasonable value will be taken as U = 3.0 for all W-shapes up to W14.
- Consider m
  - Assume a moment arm of 0.89d for determination of the plastic section modulus.
  - Then m = 24/d (include the 12 for unit conversion)

5.58

Initial Beam-Column Selection

- The simplified multipliers become

Shape	m_{avg}	m=24/d	U_{avg}	U
W6	4.41	4.0	3.01	3.0
W8	3.25	3.0	3.11	3.0
W10	2.62	2.4	3.62	3.0
W12	2.08	2.0	3.47	3.0
W14	1.72	1.71	2.86	3.0

Remember, the inaccuracy inherent here is not a concern since any final design must ultimately satisfy the interaction equations.

5.59

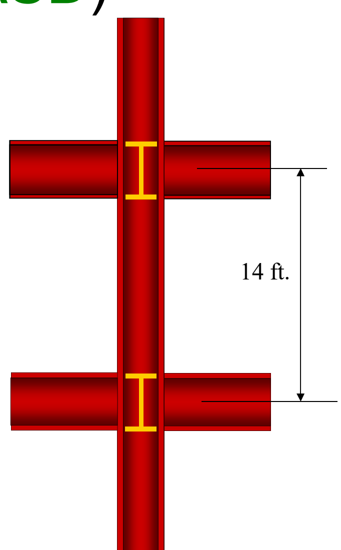
Example 2 (ASD)

An ASTM A992 column must carry an ASD axial force of 333 kips, an x-axis bending moment of 169 ft-kips, and a y-axis bending moment of 20 ft-kips. These results are from a second-order direct analysis.

The column is 14 ft long. Try a W14

$$m = 1.71$$

$$U = 3.0$$



5.60

Example 2 (ASD)

- Determine the effective axial force.

$$P_{eff} = 333 + 1.71(169) + 1.71(3.0)(20) = 725 \text{ kips}$$

- From Manual Table 4-1a, a W14x99 will support 750 kips and a W14x90 will support 682 kips.
  - We already know from Example 1 that the W14x90 works.

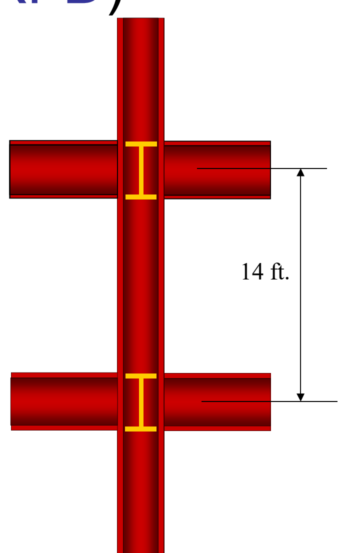
5.61

Example 2 (LRFD)

An ASTM A992 column must carry an LRFD axial force of 500 kips, an *x*-axis bending moment of 253 ft-kips, and a *y*-axis bending moment of 30 ft-kips. These results are from a second-order direct analysis.

The column is 14 ft long. Try a W14

$$m = 1.71$$
$$U = 3.0$$



5.62

Example 2 (LRFD)

- Determine the effective axial force.

$$P_{eff} = 500 + 1.71(253) + 1.71(3.0)(30) = 1090 \text{ kips}$$

- From Manual Table 4-1, a W14x99 will support 1130 kips and a W14x90 will support 1030 kips.
  - We already know from Example 1 that the W14x90 works.

5.63

Single Axis Bending

- Up to this point we have combined worst case column buckling with worst case flexure.
- However, it is possible to separate beam-column behavior into the in-plane effects and the out-of-plane effects.
- The Specification provides for the special case of doubly symmetric rolled compact members subject to single axis flexure and compression

5.64

Single Axis Bending

H1.3. Doubly Symmetric Rolled Compact Members Subject to Single-Axis Flexure and Compression

"For doubly symmetric rolled compact members with the effective length for torsional buckling less than or equal to the effective length for flexural-buckling, $L_{cz} \leq L_{cy}$, subjected to flexure and compression with moments primarily about their major axis, it is permissible to address the two independent limit states, in-plane instability and out-of-plane buckling or lateral-torsional buckling, separately in lieu of the combined approach provided in Section H1.1."

5.65

Single Axis Bending

- (a) For the limit-state of in-plane instability, Equations H1-1a and H1-1b are used with P_c and M_{cx} determined in the plane of bending.
  - This means the column strength is determined for *x*-axis buckling
  - The bending strength is M_p. (no consideration of lateral-torsional buckling)

5.66

Single Axis Bending

- (b) For the limit-state of out-of-plane plane buckling and lateral-torsional buckling:

$$\frac{P_r}{P_{cy}}\left(1.5-0.5\frac{P_r}{P_{cy}}\right)+\left(\frac{M_{rx}}{C_b M_{cx}}\right)^2 \leq 1.0 \qquad \text{(H1-3)}$$

where

P_{cy} = available compressive strength out of the plane of bending

C_b = lateral-torsional buckling moment gradient factor

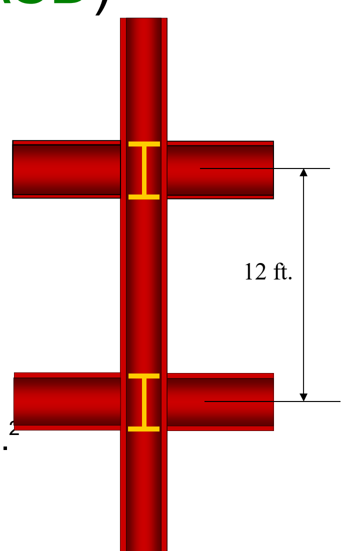
M_{cx} = available lateral-torsional strength for strong axis flexure with $C_b = 1.0$

5.67

Example 3 (ASD)

- Check the adequacy of a W16x57 column in single axis bending using the alternate provisions of Section H1.3
- Compare the results to a solution if the alternate provisions are not used.

$r_x = 6.72$ in. $r_y = 1.60$ in. $A = 16.8$ in.^{2}



5.68

Example 3 (ASD)

- The column must carry an axial load, P_a = 188 kips and moment about the strong axis, M_a = 100 ft-kips, at each end bending the column in reverse curvature.
- The column has a length of 12 ft about the *x*- and *y*-axis and an unbraced length of the compression flange of 12 ft.
- Results are from a second-order Direct Analysis, thus use K = 1.0.

5.69

Example 3 (ASD)

- For the limit-state of in-plane instability, P_c and M_{cx} are determined in the plane of bending.

$$\frac{L_c}{r_x} = \frac{12(12)}{6.72} = 21.4 \leq 113$$

$$F_{ex} = \frac{\pi^2(29{,}000)}{(21.4)^2} = 625 \text{ ksi}$$

Note that there are no column tables for the W16's and Table 6-2 does not include x-axis strength.

5.70

Example 3 (ASD)

Since $$\frac{L_c}{r} \le 113$$

Therefore

$$F_{cr} = (0.658)^{\left(\frac{50}{625}\right)}(50) = 48.4 \text{ ksi}$$

and $$P_n = 48.4(16.8) = 813 \text{ kips}$$

$$\frac{P_n}{\Omega} = \frac{813}{1.67} = 487 \text{ kips}$$

5.71

Example 3 (ASD)

- In-plane bending strength is M_p. From Table 3-2

$$\frac{M_p}{\Omega} = 262 \text{ ft-kips}$$

- Thus, in the plane of bending

$$\frac{188}{487} + \frac{8}{9}\left(\frac{100}{262}\right) = 0.73 < 1.0 \qquad \text{(H1-1a)}$$

5.72

Example 3 (ASD)

For out-of-plane,

$$\frac{L_c}{r_y} = \frac{12(12)}{1.60} = 90.0 \leq 113$$

$$F_{ey} = \frac{\pi^2(29{,}000)}{(90.0)^2} = 35.3 \text{ ksi}$$

5.73

Example 3 (ASD)

Therefore

$$F_{cr} = (0.658)^{\left(\frac{50}{35.3}\right)}(50) = 27.6 \text{ ksi}$$

and

$$P_n = 27.6(16.8) = 464 \text{ kips}$$

$$\frac{P_n}{\Omega} = \frac{464}{1.67} = 278 \text{ kips}$$

This could have been found in Table 6-2

5.74

Example 3 (ASD)

- From the beam curves (Table 3-10), with C_b = 1.0 (or from Table 6-2)

$$M_{cx} = \frac{M_n}{\Omega} = 211 \text{ ft-kips}$$

- and, from Eq. F1-1, with equal and opposite end moments,

$$C_b = 2.27$$

5.75

Example 3 (ASD)

For out-of-plane,

$$\frac{P_r}{P_{cy}}\left(1.5 - 0.5\frac{P_r}{P_{cy}}\right) + \left(\frac{M_{rx}}{C_b M_{cx}}\right)^2 \leq 1.0 \qquad \text{(H1-2)}$$

$$\frac{188}{278}\left(1.5 - 0.5\left(\frac{188}{278}\right)\right) + \left(\frac{100}{2.27(211)}\right)^2 = 0.83 \leq 1.0$$

Eq. H1-1a = 0.73 < 1.0 and Eq. H1-2 = 0.83 < 1.0
Thus, the column is adequate

5.76

Example 3 (ASD)

- Without using the alternate provisions
- Eq H1-1a

$$\left(\frac{188}{278}\right)+\frac{8}{9}\left(\frac{100}{262}\right)=1.02>1.0$$

Bending strength, *x*-axis, using C_b

$$M_{cx}=2.27(211)=479>\frac{M_p}{\Omega}=262$$

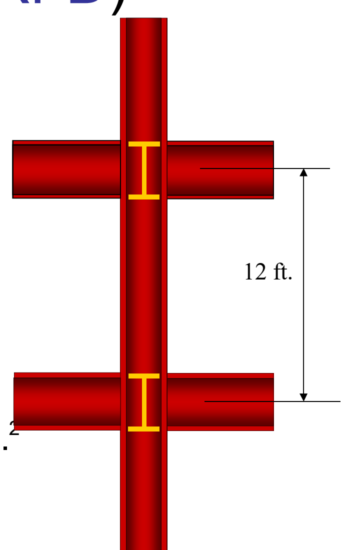
Compressive strength for buckling about *y*-axis

5.77

Example 3 (LRFD)

- Check the adequacy of a W16x57 column in single axis bending using the alternate provisions of Section H1.3
- Compare the results to a solution if the alternate provisions are not used.

$r_x = 6.72$ in. $r_y = 1.60$ in. $A = 16.8$ in.^{2}



5.78

Example 3 (LRFD)

- The column must carry an axial load, P_u = 282 kips and moment about the strong axis, M_u = 150 ft-kips, at each end bending the column in reverse curvature.
- The column has a length of 12 ft about the *x*- and *y*-axis and an unbraced length of the compression flange of 12 ft.
- Results are from a second-order Direct Analysis, thus use K = 1.0.

5.79

Example 3 (LRFD)

- For the limit-state of in-plane instability, P_c and M_{cx} are determined in the plane of bending.

$$\frac{L_c}{r_x} = \frac{12(12)}{6.72} = 21.4 \leq 113$$

$$F_{ex} = \frac{\pi^2(29{,}000)}{(21.4)^2} = 625 \text{ ksi}$$

Note that there are no column tables for the W16's and Table 6-2 does not include x-axis strength.

5.80

Example 3 (LRFD)

Since $\frac{L_c}{r} \leq 113$

Therefore

$$F_{cr} = \left(0.658\right)^{\left(\frac{50}{625}\right)}\left(50\right) = 48.4 \text{ ksi}$$

and

$$P_n = 48.4\left(16.8\right) = 813 \text{ kips}$$

$$\phi P_n = 0.9\left(813\right) = 732 \text{ kips}$$

5.81

Example 3 (LRFD)

- In-plane bending strength is M_p. From Table 3-2

$$\phi M_p = 394 \text{ ft-kips}$$

- Therefore, in the plane of bending

$$\frac{282}{732} + \frac{8}{9}\left(\frac{150}{394}\right) = 0.72 < 1.0 \qquad \text{(H1-1a)}$$

5.82

Example 3 (LRFD)

For out-of-plane,

$$\frac{L_c}{r_y} = \frac{12(12)}{1.60} = 90.0 \leq 113$$

$$F_{ey} = \frac{\pi^2(29{,}000)}{(90.0)^2} = 35.3 \text{ ksi}$$

5.83

Example 3 (LRFD)

Therefore

$$F_{cr} = (0.658)^{\left(\frac{50}{35.3}\right)}(50) = 27.6 \text{ ksi}$$

and

$$P_n = 27.6(16.8) = 464 \text{ kips}$$

$$\phi P_n = 0.9(464) = 418 \text{ kips}$$

This could have been found in Table 6-2

5.84

Example 3 (LRFD)

- From the beam curves (Table 3-10), with C_b = 1.0 (or from Table 6-2)

$$M_{cx} = \phi M_n = 318 \text{ ft-kips}$$

- and, from Eq. F1-1, with equal and opposite end moments,

$$C_b = 2.27$$

5.85

Example 3 (LRFD)

For out-of-plane,

$$\frac{P_r}{P_{cy}}\left(1.5 - 0.5\frac{P_r}{P_{cy}}\right) + \left(\frac{M_{rx}}{C_b M_{cx}}\right)^2 \leq 1.0 \qquad \text{(H1-2)}$$

$$\frac{282}{418}\left(1.5 - 0.5\left(\frac{282}{418}\right)\right) + \left(\frac{150}{2.27(318)}\right)^2 = 0.83 \leq 1.0$$

Eq. H1-1a = 0.72 < 1.0 and Eq. H1-2 = 0.83 < 1.0
Thus, the column is adequate

5.86

Example 3 (LRFD)

- Without using the alternate provisions

$$\left(\frac{282}{418}\right)+\frac{8}{9}\left(\frac{150}{394}\right)=1.01>1.0$$

Bending strength, *x*-axis, using C_b

$$M_{cx}=2.27(318)=722>\phi M_p=394$$

Compressive strength for buckling about *y*-axis

5.87

Compression + Bending

H2. Unsymmetric and Other Members Subject to Flexure and Axial Force

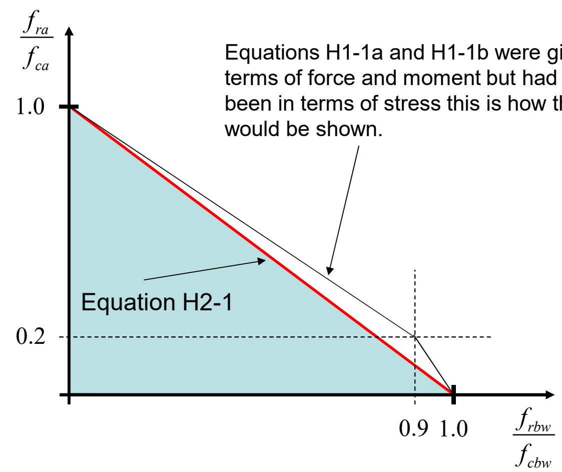
- For all unsymmetric members

$$\left|\frac{f_{ra}}{F_{ca}}+\frac{f_{rbw}}{F_{cbw}}+\frac{f_{rbz}}{F_{cbz}}\right|\leq 1.0 \qquad \text{(H2-1)}$$

- This may be used for any member in place of the equations in Section H1.
- It requires the superposition of the stresses at critical points in the cross section

5.88

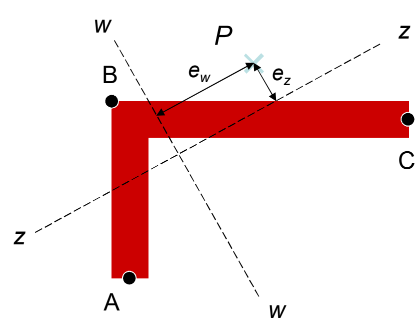
Compression + Bending



5.89

Single Angle

- Consider a single angle loaded as shown



z and *w* represent the major and minor principal axes.

e_z and e_w represent the eccentricity of load, *P*, from those axes.

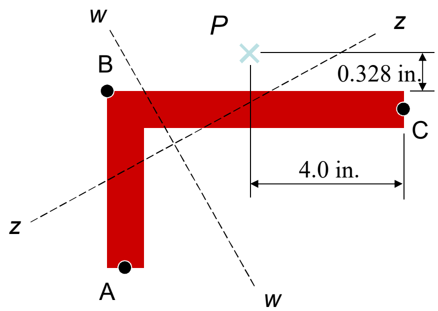
Points A, B, and C are the points on the section that must be checked through the interaction equation.

5.90

Example 4 (ASD)

- Will a 5 ft long L8x4x7/16 A36 angle support a compressive load of P_a = 40 kips at an eccentricity of 0.328 in. from the back face of the leg?

This is similar to Example E.14 in the Design Examples V15



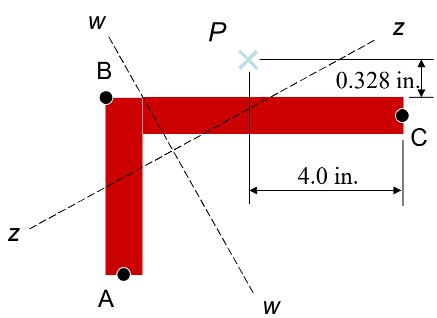
Determine sense of bending stress

Point	M_w	M_z
A	T	T
B	T	C
C	C	T

5.91

Example 4 (ASD)

- Interaction equations at the 3 critical points



Compression is positive

$$\left|\frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}}\right|_A \le 1.0$$

$$\left|\frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}}\right|_B \le 1.0$$

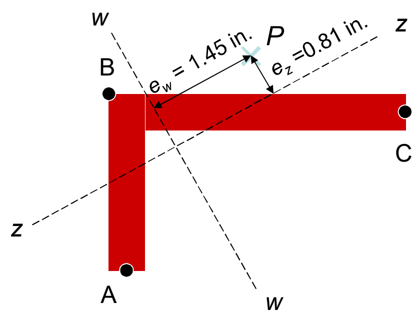
$$\left|\frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}}\right|_C \le 1.0$$

Eq. H2-1 applied at each point

5.92

Example 4 (ASD)

- Required strength



$$P_a = 40.0 \text{ kips}$$

$$M_{aw} = B_{1w}(e_w)(P_a)$$
$$= 1.02(40)(1.45)$$
$$= 59.2 \text{ in.-kips}$$

$$M_{az} = B_{1z}(e_z)(P_a)$$
$$= 1.27(40)(0.810)$$
$$= 41.1 \text{ in.-kips}$$

B_{1w} and B_{1z} are second order amplifiers covered in Lesson 6

5.93

Example 4 (ASD)

- Determine the required stresses at points A, B, and C.
- These calculations will use the section modulus referred to each point about each axis.

$S_{wA} = 10.9 \text{ in.}^3$ $S_{zA} = 1.61 \text{ in.}^3$

$S_{wB} = 14.6 \text{ in.}^3$ $S_{zB} = 2.51 \text{ in.}^3$

$S_{wC} = 7.04 \text{ in.}^3$ $S_{zC} = 5.07 \text{ in.}^3$

These section properties are available in the AISC Shapes Database V15

and for axial stress, $A = 5.11 \text{ in.}^2$

5.94

Example 4 (ASD)

- Determine the required stresses at points A, B, and C.

	$P_a = 40.0$ kips		$M_{aw} = 59.2$ in.-kips		$M_{az} = 41.1$ in.-kips	
Point	A	f_a (ksi)	S_w	f_{bw} (ksi)	S_z	f_{bz} (ksi)
A	5.11	**7.83**	10.9	**- 5.43**	1.61	**- 25.5**
B	5.11	**7.83**	14.6	**- 4.05**	2.51	**16.4**
C	5.11	**7.83**	7.04	**8.41**	5.07	**- 8.11**

5.95

Example 4 (ASD)

- Determine available strength
  - Flexural buckling about the z-axis

$$\frac{P_n}{\Omega} = 78.4 \text{ kips}$$

  - Lateral-torsional buckling about the *w*-axis

$$\frac{M_{nw}}{\Omega} = 166 \text{ in.-kips}$$

  - Yielding about the *z*-axis

$$\frac{M_{nz}}{\Omega} = 52.1 \text{ in.-kips}$$

5.96

Example 4 (ASD)

- Determine the available stresses at points A, B, and C.

	$\frac{P_n}{\Omega} = 78.4$ kips		$\frac{M_{nw}}{\Omega} = 166$ in.-kips		$\frac{M_{nz}}{\Omega} = 52.1$ in.-kips	
Point	A	F_a (ksi)	S_w	F_{bw}(ksi)	S_z	F_{bz} (ksi)
A	5.11	**15.3**	10.9	**15.2**	1.61	**32.4**
B	5.11	**15.3**	14.6	**11.4**	2.51	**20.8**
C	5.11	**15.3**	7.04	**23.6**	5.07	**10.3**

5.97

Example 4 (ASD)

- Determine the results of Eq H2-1 at points A, B, and C.

$$\left|\frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}}\right|_A \leq 1.0$$

Point	f_a/F_a	f_{bw}/F_{bw}	f_{bz}/F_{bz}		≤ 1.0
A	**+ 0.512**	**- 0.357**	**- 0.778**	=	**- 0.623**
B	**+ 0.512**	**- 0.355**	**+ 0.788**	=	**0.945**
C	**+ 0.512**	**+ 0.356**	**- 0.787**	=	**0.081**

Point B is the critical point on the angle and the column will support this load at this location.

5.98

Example 4 (ASD)

- Note that regardless of the point under consideration, the value of the ratio came out essentially the same, except for sign.
- This is because the same section modulus or area occurs in the numerator and denominator.
- Thus, all this could be simplified by taking just ratios of moment or force.

5.99

Example 4 (ASD)

- Look at point B with equation H2-1 in terms of force and moment.

$$\frac{P_a}{P_n/\Omega} - \frac{M_{aw}}{M_{nw}/\Omega} + \frac{M_{az}}{M_{nz}/\Omega} \leq 1.0$$

$$\frac{40.0}{78.4} - \frac{59.2}{166} + \frac{41.1}{52.1} =$$

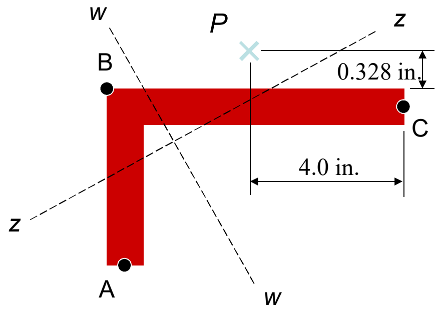
$$0.510 - 0.357 + 0.789 = 0.942 \leq 1.0$$

So why is equation H2-1 given in terms of stress?
To capture signs for tension and compression

5.100

Example 4 (ASD)

- Look again at the problem we solved



The location of the load was not selected by accident.

It is located at the midpoint of the 8 in. leg and at ¾ the thickness of the angle from the back of the angle.

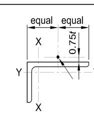
Manual Table 4-12 uses this location in tabulating the available strength of eccentrically loaded single angles

5.101

Example 4 (ASD)

F_y = 36 ksi

Table 4-12 (continued)
Available Strength in Axial Compression, kips
Eccentrically Loaded Single Angles



L8–L7

Shape		L8×4×						L7×4×					
		9/16[c]		1/2[c,f]		7/16[c,f]		3/4		5/8		1/2[c]	
lb/ft		21.9		19.6		17.2		26.2		22.1		17.9	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
	0	60.0	90.2	57.8	87.0	55.5	83.4	65.1	97.9	62.5	94.0	59.5	89.4
	1	59.4	89.3	57.2	86.0	54.8	82.5	64.4	96.8	61.8	93.0	58.8	88.4
	2	57.6	86.7	55.3	83.4	52.9	79.7	62.1	93.5	59.8	90.0	56.7	85.4
	3	54.7	82.5	52.4	79.2	50.0	75.5	58.7	88.6	56.5	85.3	53.4	80.6
	4	51.0	77.2	48.7	73.8	46.2	70.1	54.8	82.9	52.3	79.3	49.2	74.6
	5	46.7	70.9	44.6	67.7	42.1	64.0	50.3	76.4	47.7	72.4	44.4	67.5
Properties													
A_g, in.^{2}		6.49		5.80		5.11		7.74		6.50		5.26	
r_z, in.		0.859		0.863		0.867		0.855		0.860		0.866	
ASD		LRFD											
$\Omega_c = 1.67$		$\phi_c = 0.90$											

[c] Shape is slender for compression with F_y = 36 ksi; tabulated values have been adjusted accordingly.
[f] Shape exceeds compact limit for flexure with F_y = 36 ksi.
Note: Heavy line indicates L_c/r_z equal to or greater than 200.

For L_c with respect to the *z*-axis equal to 5.0 ft

$$\frac{P_n}{\Omega} = 42.1 \text{ kips}$$

Since

$$P_a = 40.0 \text{ kips} < \frac{P_n}{\Omega}$$

the column will carry this load at the given eccentricities.

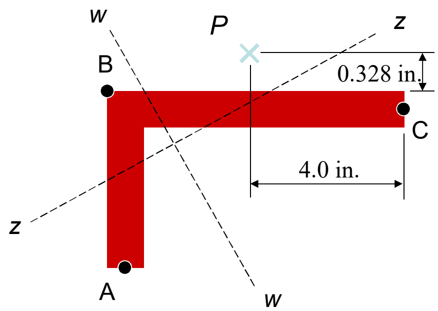
This was a lot less work!

5.102

Example 4 (LRFD)

- Will a 5 ft long L8x4x7/16 A36 angle support a compressive load of P_u = 60 kips at an eccentricity of 0.328 in. from the back face of the leg?

This is similar to Example E.14 in the Design Examples V15



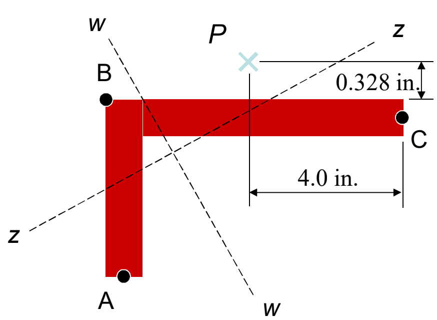
Determine sense of bending stress

Point	M_w	M_z
A	T	T
B	T	C
C	C	T

5.103

Example 4 (LRFD)

- Interaction equations at the 3 critical points



Compression is positive

$$\left|\frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}}\right|_A \le 1.0$$

$$\left|\frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}}\right|_B \le 1.0$$

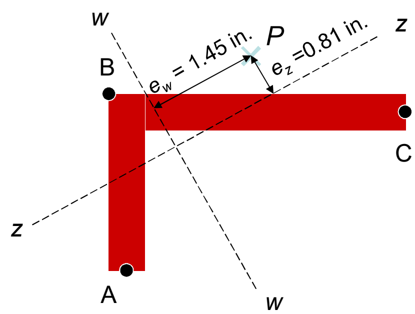
$$\left|\frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}}\right|_C \le 1.0$$

Eq. H2-1 applied at each point

5.104

Example 4 (LRFD)

- Required strength



$$P_u = 60.0 \text{ kips}$$

$$M_{uw} = B_{1w}(e_w)(P_u)$$
$$= 1.02(60)(1.45)$$
$$= 88.7 \text{ in.-kips}$$

$$M_{uz} = B_{1z}(e_z)(P_u)$$
$$= 1.24(60)(0.810)$$
$$= 60.3 \text{ in.-kips}$$

B_{1w} and B_{1z} are second order amplifiers covered in Lesson 6

5.105

Example 4 (LRFD)

- Determine the required stresses at points A, B, and C.
- These calculations will use the section modulus referred to each point about each axis.

$S_{wA} = 10.9$ in.^{3} $\quad S_{zA} = 1.61$ in.^{3}

$S_{wB} = 14.6$ in.^{3} $\quad S_{zB} = 2.51$ in.^{3}

$S_{wC} = 7.04$ in.^{3} $\quad S_{zC} = 5.07$ in.^{3}

These section properties are available in the AISC Shapes Database V15

and for axial stress, $A = 5.11$ in.^{2}

5.106

Example 4 (LRFD)

- Determine the required stresses at points A, B, and C.

	$P_u = 60.0$ kips		$M_{uw} = 88.7$ in.-kips		$M_{uz} = 60.3$ in.-kips	
Point	A	f_a (ksi)	S_w	f_{bw}(ksi)	S_z	f_{bz} (ksi)
A	5.11	**11.7**	10.9	**- 8.14**	1.61	**- 37.5**
B	5.11	**11.7**	14.6	**- 6.08**	2.51	**24.0**
C	5.11	**11.7**	7.04	**12.6**	5.07	**- 11.9**

5.107

Example 4 (LRFD)

- Determine available strength
  - Flexural buckling about the z-axis

$$\phi P_n = 118 \text{ kips}$$

  - Lateral-torsional buckling about the *w*-axis

$$\phi M_{nw} = 249 \text{ in.-kips}$$

  - Yielding about the *z*-axis

$$\phi M_{nz} = 78.3 \text{ in.-kips}$$

5.108

Example 4 (LRFD)

- Determine the available stresses at points A, B, and C.

	$\phi P_n = 118$ kips		$\phi M_{nw} = 249$ in.-kips		$\phi M_{nz} = 78.3$ in.-kips	
Point	A	F_a (ksi)	S_w	F_{bw}(ksi)	S_z	F_{bz} (ksi)
A	5.11	**23.1**	10.9	**22.8**	1.61	**48.6**
B	5.11	**23.1**	14.6	**17.1**	2.51	**31.2**
C	5.11	**23.1**	7.04	**35.4**	5.07	**15.4**

5.109

Example 4 (LRFD)

- Determine the results of Eq H2-1 at points A, B, and C.

$$\left| \frac{f_{ra}}{F_{ca}} - \frac{f_{rbw}}{F_{cbw}} - \frac{f_{rbz}}{F_{cbz}} \right|_A \leq 1.0$$

Point	f_a/F_a	f_{bw}/F_{bw}	f_{bz}/F_{bz}		≤ 1.0
A	**+ 0.506**	**- 0.357**	**- 0.772**	=	**- 0.623**
B	**+ 0.506**	**- 0.356**	**+ 0.769**	=	**0.919**
C	**+ 0.506**	**+ 0.356**	**- 0.773**	=	**0.089**

Point B is the critical point on the angle and the column will support this load at this location.

5.110

Example 4 (LRFD)

- Note that regardless of the point under consideration, the value of the ratio came out essentially the same, except for sign.
- This is because the same section modulus or area occurs in the numerator and denominator.
- Thus, all this could be simplified by taking just ratios of moment or force.

5.111

Example 4 (LRFD)

- Look at point B with equation H2-1 in terms of force and moment.

$$\frac{P_u}{\phi_c P_n} - \frac{M_{uw}}{\phi_b M_{nw}} + \frac{M_{uz}}{\phi_b M_{nz}} \leq 1.0$$

$$\frac{60.0}{118} - \frac{88.7}{249} + \frac{60.3}{78.3} =$$

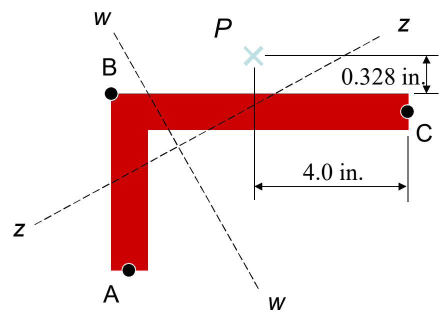
$$0.508 - 0.356 + 0.770 = 0.922 \leq 1.0$$

So why is equation H2-1 given in terms of stress?
To capture signs for tension and compression

5.112

Example 4 (LRFD)

- Look again at the problem we solved



The location of the load was not selected by accident.

It is located at the midpoint of the 8 in. leg and at ¾ the thickness of the angle from the back of the angle.

Manual Table 4-12 uses these locations in tabulating the available strength of eccentrically loaded single angles

5.113

Example 4 (LRFD)

F_y = 36 ksi

Table 4-12 (continued)
Available Strength in Axial Compression, kips
Eccentrically Loaded Single Angles

L8–L7

Shape		L8×4×						L7×4×					
		9/16[c]		1/2[c,f]		7/16[c,f]		3/4		5/8		1/2[c]	
lb/ft		21.9		19.6		17.2		26.2		22.1		17.9	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
	0	60.0	90.2	57.8	87.0	55.5	83.4	65.1	97.9	62.5	94.0	59.5	89.4
	1	59.4	89.3	57.2	86.0	54.8	82.5	64.4	96.8	61.8	93.0	58.8	88.4
	2	57.6	86.7	55.3	83.4	52.9	79.7	62.1	93.5	59.8	90.0	56.7	85.4
	3	54.7	82.5	52.4	79.2	50.0	75.5	58.7	88.6	56.5	85.3	53.4	80.6
	4	51.0	77.2	48.7	73.8	46.2	70.1	54.8	82.9	52.3	79.3	49.2	74.6
	5	46.7	70.9	44.6	67.7	42.1	64.0	50.3	76.4	47.7	72.4	44.4	67.5
Properties													
A_g, in.^{2}		6.49		5.80		5.11		7.74		6.50		5.26	
r_z, in.		0.859		0.863		0.867		0.855		0.860		0.866	

ASD	LRFD
$\Omega_c = 1.67$	$\phi_c = 0.90$

[c] Shape is slender for compression with $F_y = 36$ ksi; tabulated values have been adjusted accordingly.
[f] Shape exceeds compact limit for flexure with $F_y = 36$ ksi.
Note: Heavy line indicates L_c/r_z equal to or greater than 200.

For L_c with respect to the *z*-axis equal to 5.0 ft

$$\phi P_n = 64.0 \text{ kips}$$

Since

$$P_u = 60.0 \text{ kips} < \phi P_n$$

the column will carry this load at the given eccentricities.

This was a lot less work!

5.114

Summary

- Looked at development of elastic and plastic approaches to the interaction.
- Used a single Manual table to determine all required strengths for combined forces.
- Derived a simple approach for initial selection of beam-column members
- Investigated a special approach when bending is only about the *x*-axis.
- Addressed the approach for unsymmetric members

5.115

Lesson 6

- The next lesson will begin the discussion of stability analysis and design
- This part 1 will look at determination of required strength and second-order analysis
- We will concentrate on Chapter C and Appendix 8.

5.116

Thank You

American Institute of Steel Construction
130 East Randolph St., Suite 2000
Chicago, IL 60601

3.117

Individual Session Registrants

PDH Certificates

- You will receive an email on how to report attendance from: registration@aisc.org.
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!

Individual Session Registrants

PDH Certificates

- Reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.

8-Session Registrants

PDH Certificates

One certificate will be issued at the conclusion of all 8 sessions.

8-Session Registrants

Access to the quiz

Information for accessing the quiz will be emailed to you by Thursday. It will contain a link to access the quiz. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG.

Quiz and attendance records

Posted Thursday mornings. www.aisc.org/nightschool -- Click on Current Course Details.

Reasons for quiz

- EEU – You must take all quizzes and the final exam to receive EEU.
- PDHs – If you watch a recorded session, you must pass quiz for PDHs.
- REINFORCEMENT – Reinforce what you learn tonight. Get more out of the course.

Note: If you attend the live presentation, you do not have to take the quizzes to receive PDHs

8-Session Registrants

Access to the recording

Information for accessing the recording will be emailed to you by Thursday. The recording will be available for four weeks. (For 8-session registrants only.) EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG.

PDHs via recording

If you watch a recorded session, you must take *and pass* the quiz for PDHs.

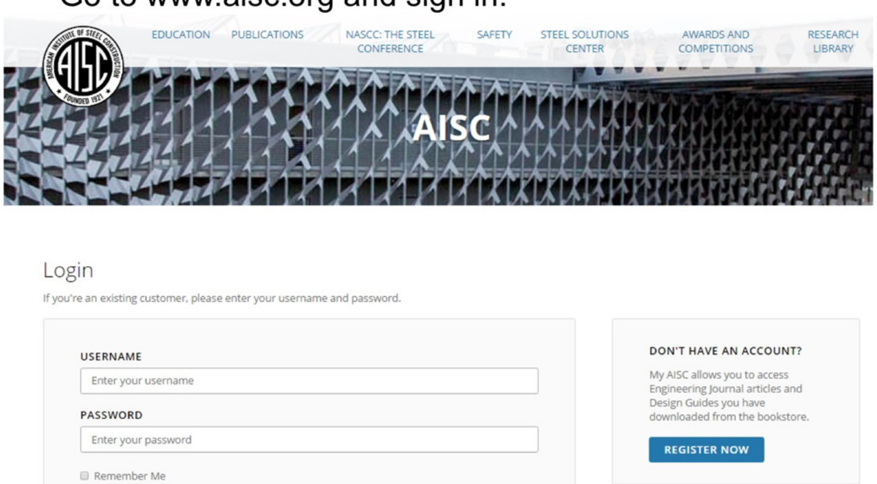
8-Session Registrants

Night School Resources

Find all your handouts, quizzes and quiz scores, recording access, and attendance information all in one place!

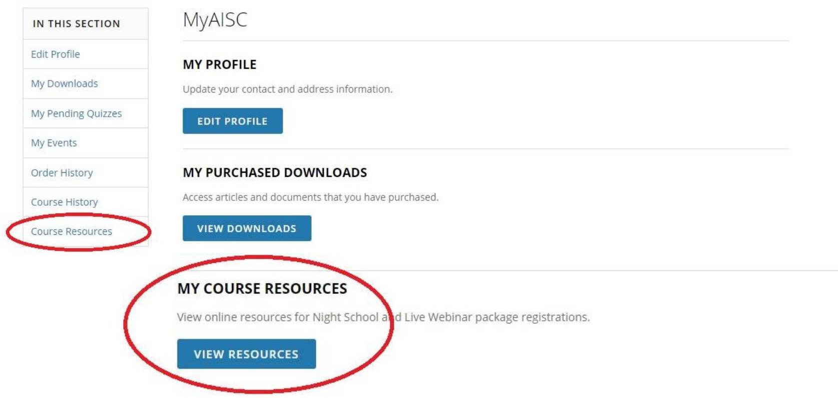
Night School Resources for 8-session package Registrants

Go to www.aisc.org and sign in.



Night School Resources for 8-session package Registrants

Go to www.aisc.org and sign in.



Please fill out our brief survey at the conclusion of the webinar. We greatly appreciate your feedback.

AISC | Thank you

Smarter.
Stronger.
Steel.