AISC

Night School

Thank you for joining our live webinar today.
We will begin shortly. Please standby.

Modern Methods for Learning the Basics of Structural Stability: From Behavior to Practice

Session 1: Behavior of Compression Members – The Fundamentals

October 6, 2020

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Stronger.
Steel.

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Course Description

Behavior of Compression Members – The Fundamentals
October 6, 2020

This session will begin with a brief overview of the 8-lecture course, and its unique format. The behavior of compression members will then be covered. The assumptions in the solution to the Euler column problem will be used as a basis for systematically moving from the theoretical solution presented in 1757 to the modern day methods of design and analysis of compression members. Emphasis will be placed on the effects of material yielding accentuated by the presence of residual stresses, initial imperfections, and end conditions. The flexural buckling strength of members without slender elements will be covered and ultimately presented in the form of column curves. The speakers will conclude by introducing the first learning module.

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Learning Objectives

- List the assumptions in the Euler column solution.
- Describe how bending is produced on a column.
- Describe the aspects that are taken into account to define a column's strength in the AISC Specification's column curve.
- Explain how column end restraint affects the behavior of a column.

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Modern Methods for Learning The Basics of Structural Stability: From Behavior to Practice

Session 1: Behavior of Compression Members – The Fundamentals
October 6, 2019

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Modern Methods for Learning The Basics of Structural Stability: From Behavior to Practice

- Course Introduction
- Compression Members
- Flexural Members
- Beam-Columns
- Systems

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Course Overview

- Topics
  - Compression Members (Weeks 1 & 2)
  - Flexural Members (Weeks 3 & 4)
  - Beam-Columns (Weeks 5 & 6)
  - Systems (Weeks 7 & 8)
- "Active" learning! Weekly virtual lab experiences…
- Case studies from the real world…

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Course Overview (2)

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Course Overview (3)

- Focus of the course is on fundamentals!
- Better understanding of behavior will result in improved design
- Key Definitions
  - **Stability:** Under load, component returns to current state after applying a small disturbance such as a deflection
  - **Bifurcation (critical load)**: Theoretical point at which loading a component results in an instantaneous change from current state to significant deflection – two options: not buckled or buckled
  - **Instability:** Loading a component results in a realistic transition from small deflection to significant deflection – buckling preceded by deflection

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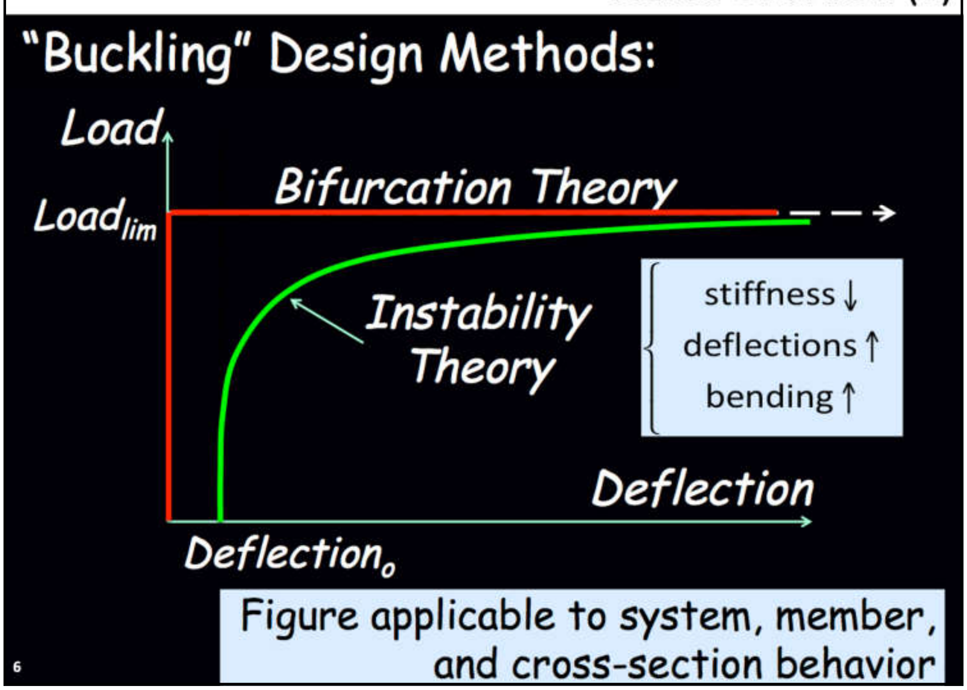
Course Overview (4)



Course Overview (5)

Analysis acronyms:

LBA: linear buckling analysis; elastic critical load analysis; elastic eigenvalue analysis; assumes bifurcation theory

GNA: geometric nonlinear analysis; 2nd-order elastic analysis; assumes equilibrium on the deformed shape and linear elastic material, with no initial imperfections

GNIA: same as GNA, but includes initial imperfections

MNA: material nonlinear analysis; 1st-order inelastic analysis; assumes equilibrium on the undeformed shape and accounts for yielding, with no initial imperfections

GMNIA: geometric and material nonlinear analysis; 2nd-order inelastic analysis; assumes equilibrium on the deformed shape, accounts for yielding, and includes initial imperfections

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Modern Methods for Learning The Basics of Structural Stability: From Behavior to Practice

Course Introduction

Compression Members

Flexural Members

Beam-Columns

Systems

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Limit States of Compression Members

- Full yielding (today)
- Instability
  - Along the member length
    - Flexural buckling (today's emphasis!)
    - Torsional buckling
    - Flexural-torsional buckling
  - At the cross section
    - local buckling

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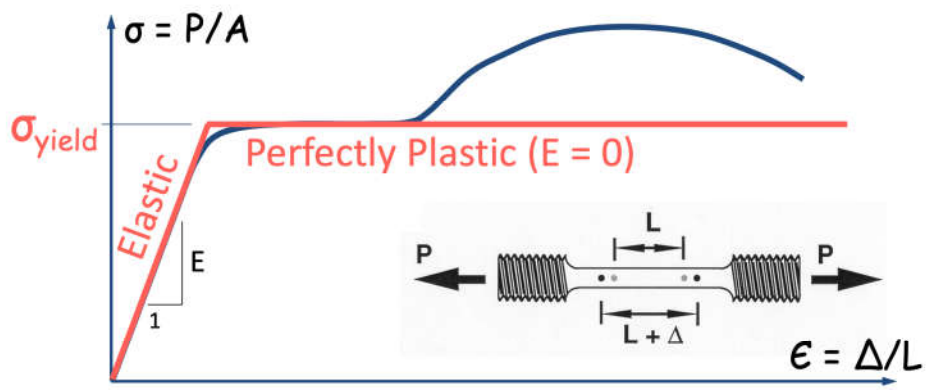
Full Yielding

- Tensile test



- Assume same response for compression
  - $\sigma_{y,compression} = \sigma_{y,tension} = \sigma_{yield}$
  - Neglect strain hardening (assume elastic-plastic)

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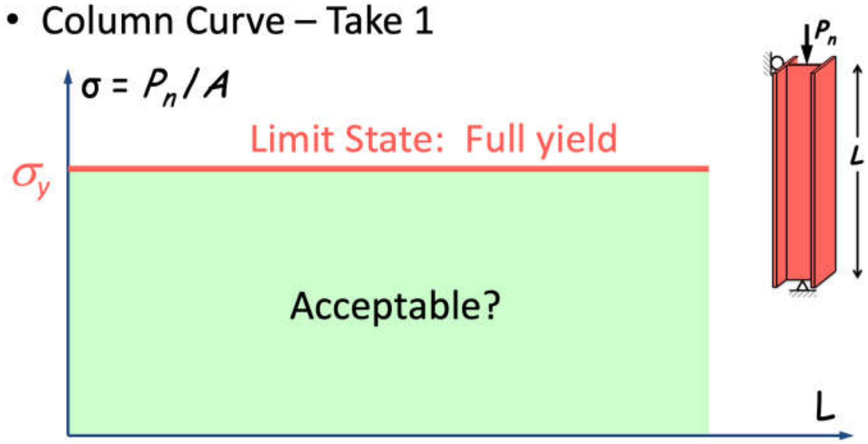
Full Yielding (2)

- Column Curve – Take 1



- What about:
  - member instability ??? (today!)
  - cross section instability (local buckling) ???

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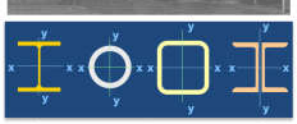
Types of Member Instability

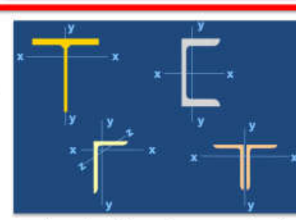
Flexural Buckling



(centroid = shear center)

Torsional Buckling

Flexural-torsional Buckling



(centroid ≠ shear center)

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Flexural Buckling

- Euler's column
  - solution
  - assumptions
- Undoing Euler's assumptions (approaching reality)
  - bending before bifurcation
  - not fully elastic (partial yielding)
  - support conditions
- Column curves
  - AISC
  - others

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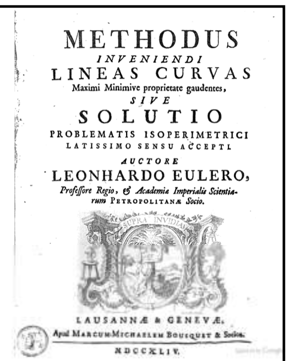
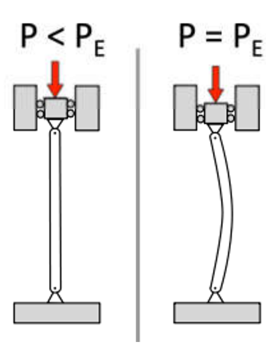
Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions!
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)


METHODUS
INVENIENDI
LINEAS CURVAS
Maximi Minimive proprietate gaudentes,
SIVE
SOLUTIO
PROBLEMATIS ISOPERIMETRICI
LATISSIMO SENSU ACCEPTI
AUCTORE
LEONHARDO EULERO,
Professore Regio, & Academiæ Imperialis Scientiarum Petropolitanæ Socio.
LAUSANNÆ & GENEVÆ,
Apud Marcum-Michaelem Bousquet & Socios.
MDCCXLIV.



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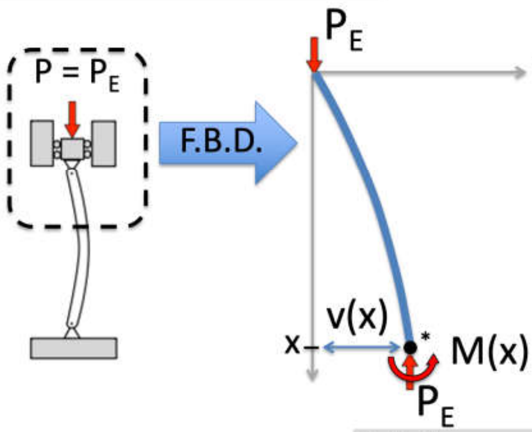
Euler Buckling (3)



Equilibrium on deformed shape:

$$\Sigma M_* = 0$$

$$M(x) + P_E v(x) = 0$$

Moment-curvature:

$$M(x) = EI\frac{d^2v(x)}{dx^2}$$

$$EI\frac{d^2v}{dx^2} + P_E v = 0 \Rightarrow$$

Solution:

$$v(x) = C_1\cos\left(\sqrt{\frac{P_E}{EI}}x\right) + C_2\sin\left(\sqrt{\frac{P_E}{EI}}x\right)$$

wolframalpha.com
a2*y"(x)+a1*y(x)=0

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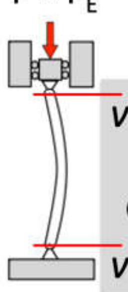
Euler Buckling (5)



$$v(x=0)=0 \Rightarrow v(x) = C_2\sin\left(\sqrt{\frac{P_E}{EI}}x\right)$$

Boundary Conditions!

$$v(x=L)=0 \Rightarrow v(x=L) = 0 = C_2\sin\left(\sqrt{\frac{P_E}{EI}}L\right)$$

1) $C_2 = 0$ "trivial solution"

2) $\sin\left(\sqrt{\frac{P_E}{EI}}L\right) = 0 \Rightarrow \sqrt{\frac{P_E}{EI}}L = n\pi \Rightarrow$

$$P_E = \frac{n^2\pi^2 EI}{L^2}$$

$$n = 1, 2, 3, \ldots$$

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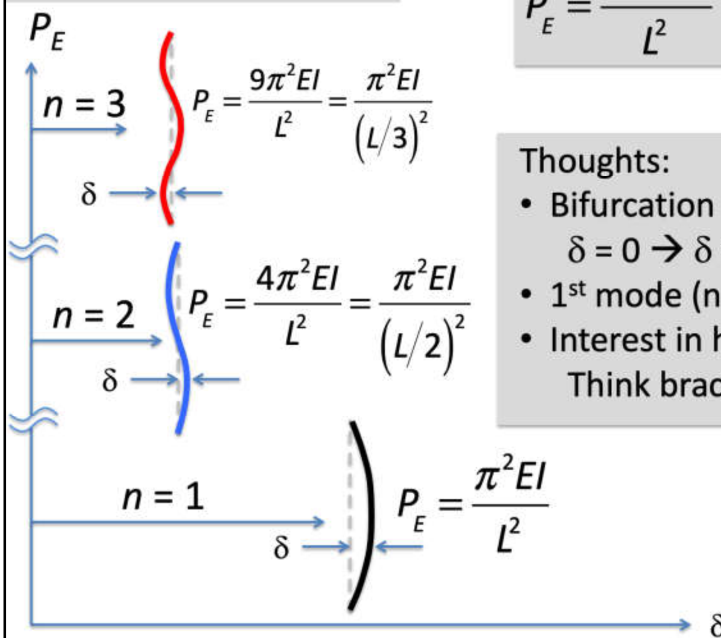
Euler Buckling (6)

$$P_E = \frac{n^2 \pi^2 EI}{L^2} \qquad n = 1,2,3,\ldots$$



Thoughts:

- Bifurcation
  δ = 0 → δ = unbounded
- 1st mode (n = 1) controls!
- Interest in higher modes?
  Think bracing!

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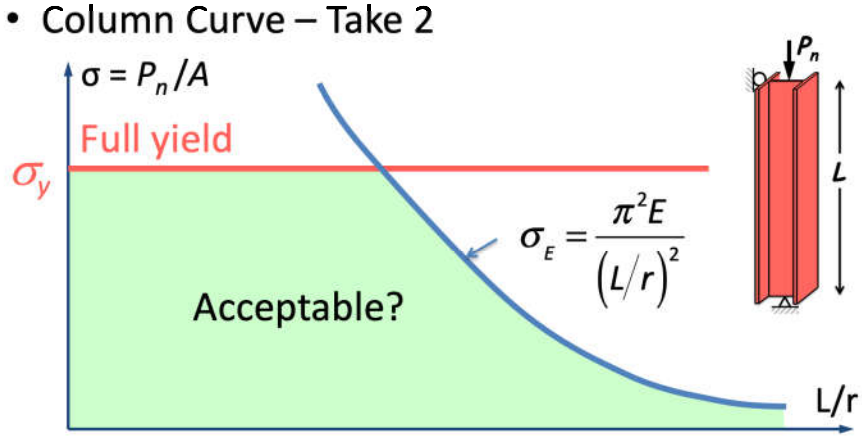
Euler Buckling (7)

- Euler Buckling Stress

$$P_E = \frac{\pi^2 EI}{L^2} \Rightarrow \sigma_E = \frac{P_E}{A} = \frac{\pi^2 E}{(L/r)^2} \quad \text{with } r = \sqrt{\frac{I}{A}}$$

- Column Curve – Take 2



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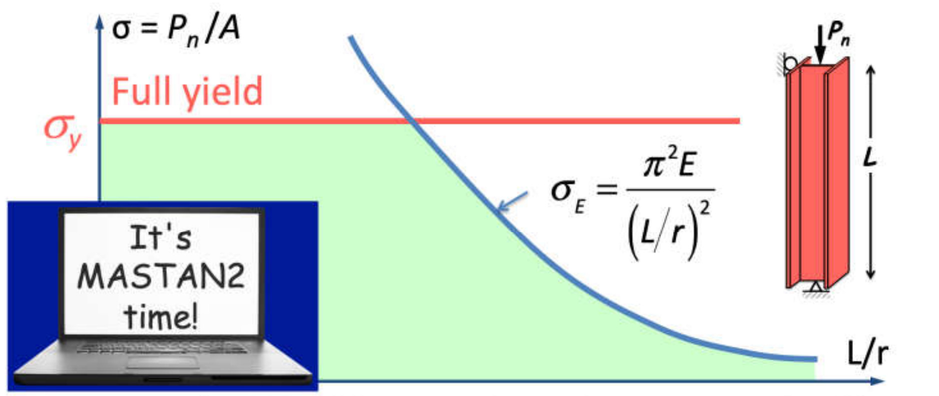
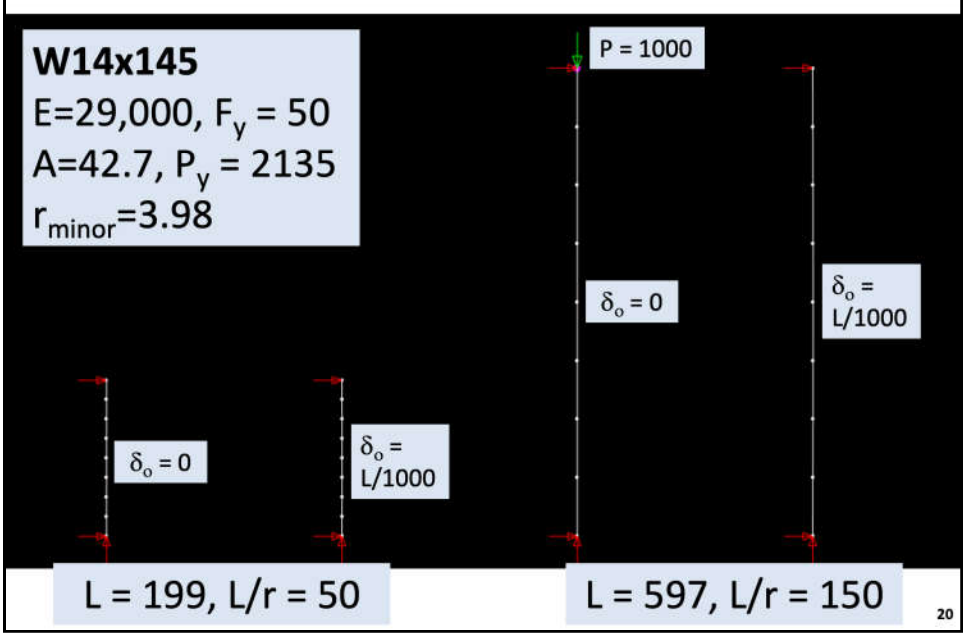


W14x145 (E=29,000, F_y = 50, A=42.7, r_{minor}=3.98)

L/r	L	$P_{cr,LBA}$	$P_{cr,GNA}$	$P_{cr,MNA}$	$P_{cr,min}/A$
50	199				
150	597				

NS24_L1_Example_1.mat



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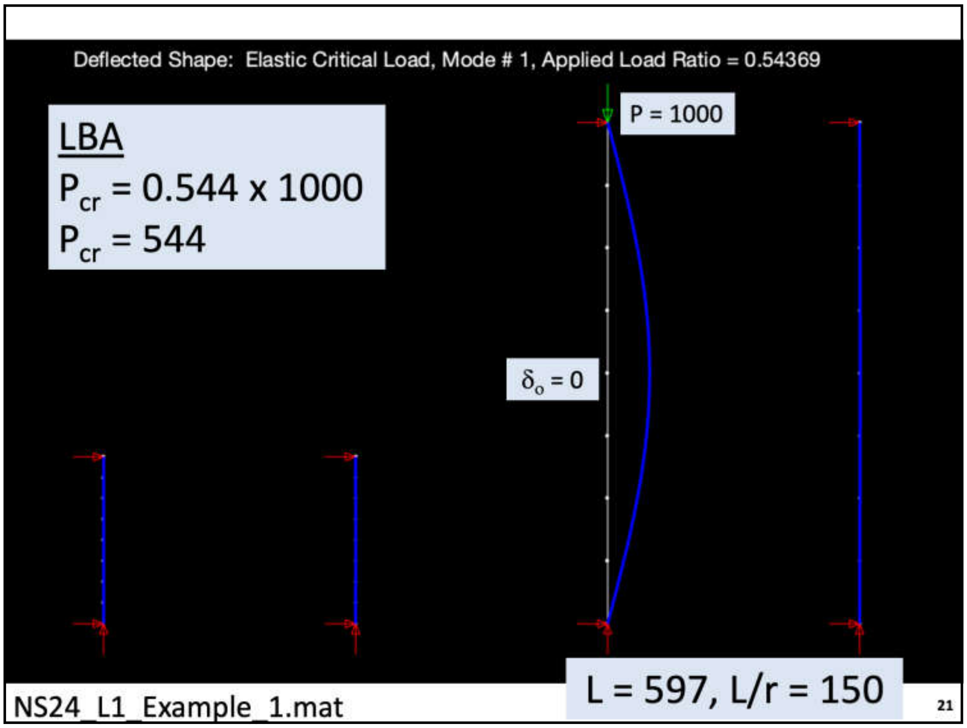
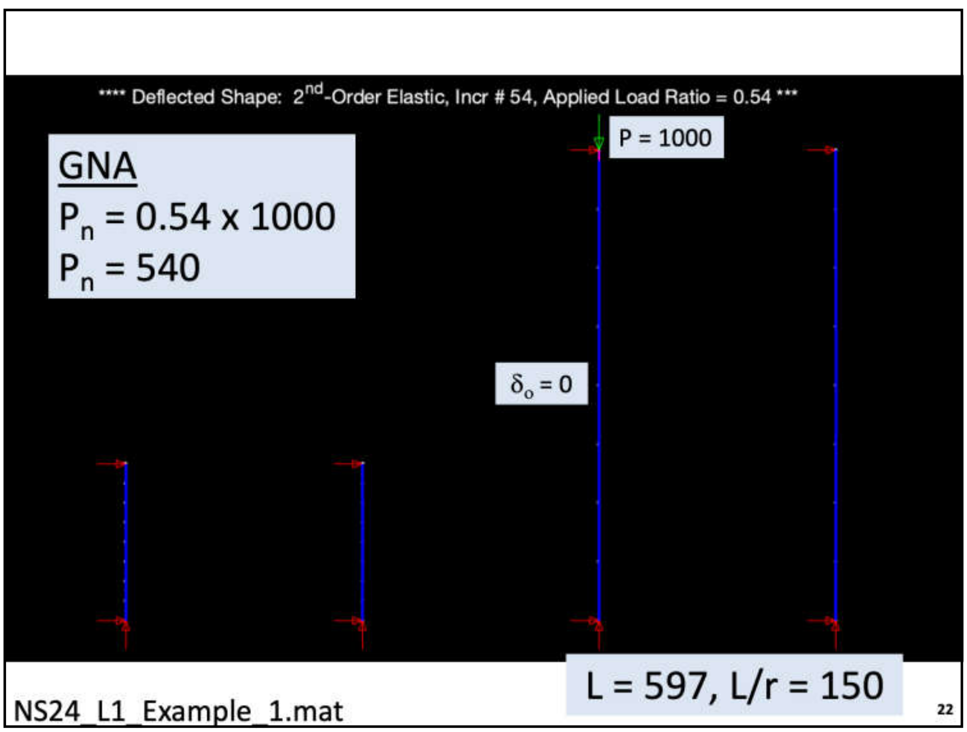
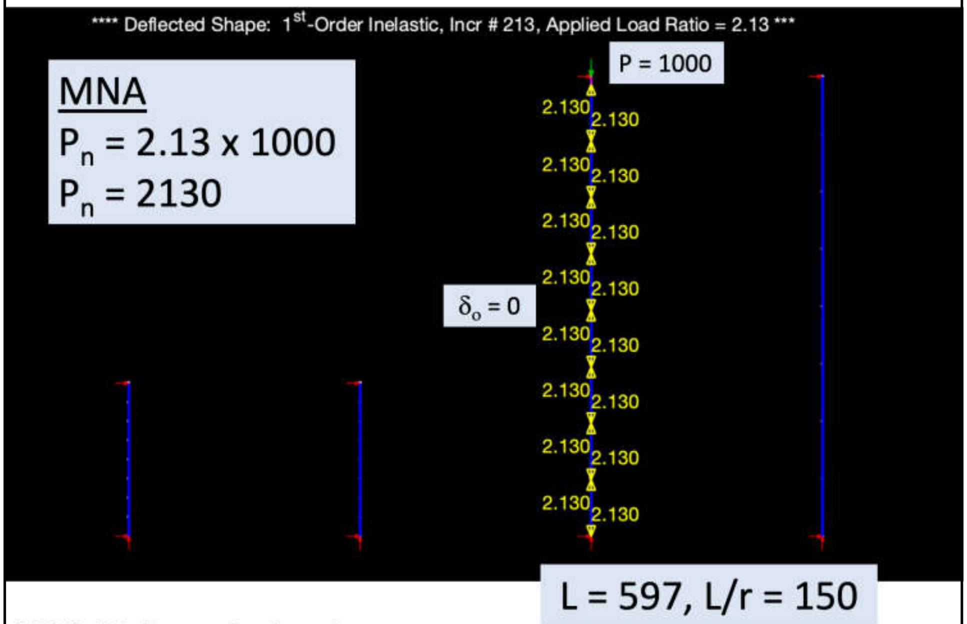

Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 0.54369
LBA
P_{cr} = 0.544 x 1000
P_{cr} = 544
P = 1000
$\delta_o = 0$
NS24_L1_Example_1.mat
L = 597, L/r = 150
21


**** Deflected Shape: 2nd-Order Elastic, Incr # 54, Applied Load Ratio = 0.54 ***
GNA
P_n = 0.54 x 1000
P_n = 540
P = 1000
$\delta_o = 0$
NS24_L1_Example_1.mat
L = 597, L/r = 150
22



NS24_L1_Example_1.mat

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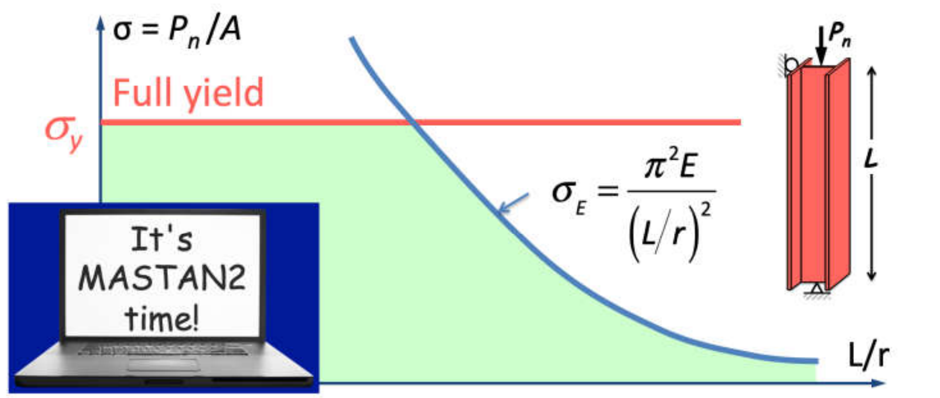


W14x145 (E=29,000, F_y = 50, A=42.7, r_{minor}=3.98)

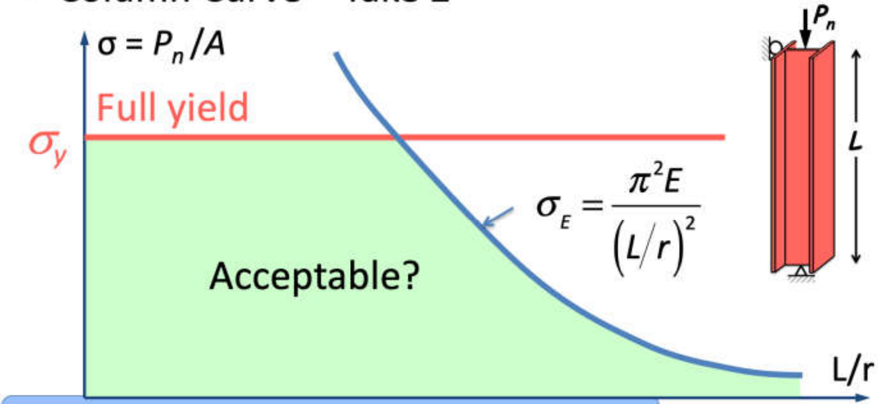
L/r	L	$P_{cr,LBA}$	$P_{n,GNA}$	$P_{n,MNA}$	$P_{n,min}/A$
50	199	4890	4930	2130	49.9
150	597	544	540	2130	12.6

Euler Buckling (7)

- Euler Buckling Stress

$$P_E = \frac{\pi^2 EI}{L^2} \Rightarrow \sigma_E = \frac{P_E}{A} = \frac{\pi^2 E}{(L/r)^2} \quad \text{with } r = \sqrt{\frac{I}{A}}$$

- Column Curve – Take 2



- What about those assumptions?

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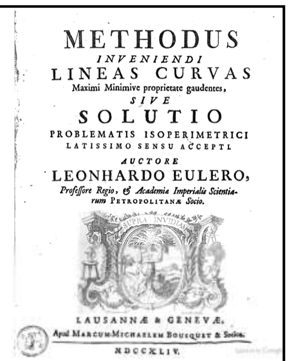
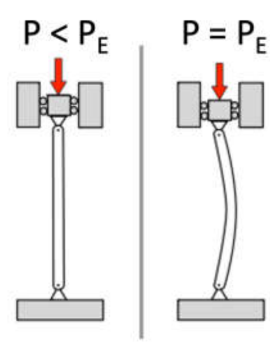
Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)


METHODUS INVENIENDI LINEAS CURVAS Maximi Minimive proprietate gaudentes, SIVE SOLUTIO PROBLEMATIS ISOPERIMETRICI LATISSIMO SENSU ACCEPTI AUCTORE LEONHARDO EULERO, Professore Regio, & Academiæ Imperialis Scientiarum Petropolitanæ Socio.

LAUSANNÆ & GENEVÆ, Apud Marcum-Michaelem Bousquet & Socios.

MDCCXLIV.



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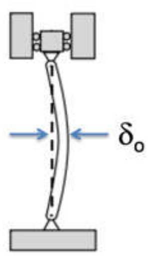
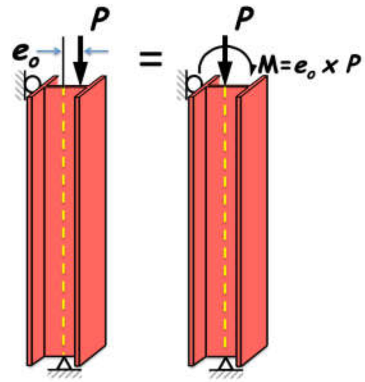
Bending

- Bending can be produced by:

1. Prior to loading, column is not perfectly straight



2. Axial load not concentrically applied (e_o is small, but not zero!)



Reality: Some combination of above exists…

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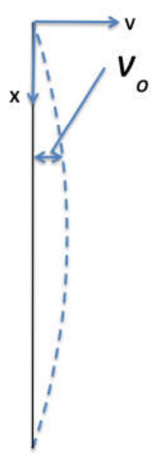
Bending (2)

Let's consider a column with initial out-of-straightness:



$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

Initial imperfection at mid-length
e.g. δ_o = L/1000

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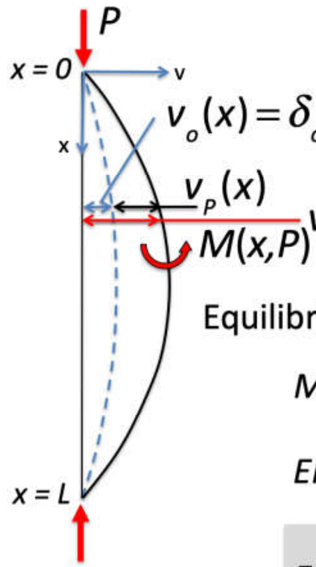
Bending (3)

Column with initial out-of-straightness:



Equilibrium → Differential Equation:

$$M(x,P) + Pv(x) = 0$$

$$EI\frac{d^2 v_P}{dx^2} + P\left(v_o(x) + v_P(x)\right) = 0$$

$$EI\frac{d^2 v_P}{dx^2} + Pv_P(x) = -Pv_o(x) = -P\delta_o \sin\frac{\pi x}{L}$$

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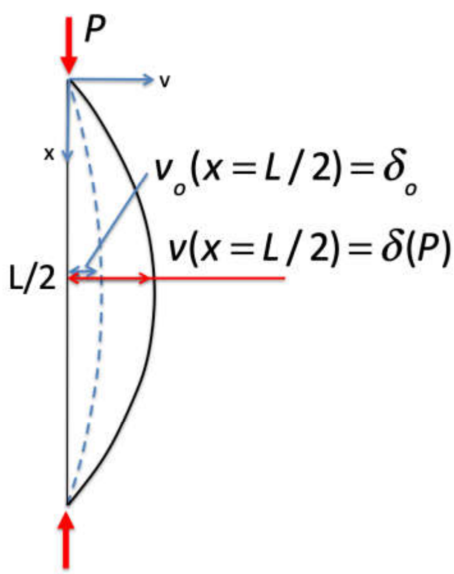
Bending (4)

Column with initial out-of-straightness:



$$v(x) = \frac{1}{1 - \frac{P}{P_E}} v_o(x)$$

$$\delta(P) = \frac{1}{1 - \frac{P}{P_E}} \times \delta_o$$

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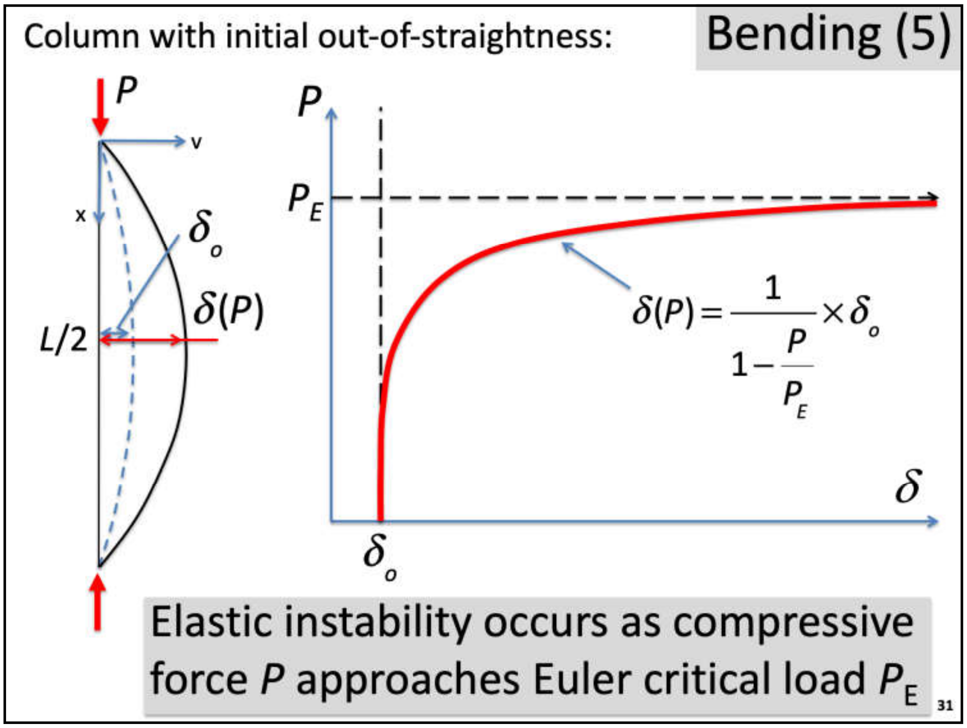
Column with initial out-of-straightness:
Bending (5)
P
v
x
δ_o
$\delta(P)$
L/2
P
P_E
$\delta(P) = \frac{1}{1 - \frac{P}{P_E}} \times \delta_o$
δ
δ_o
Elastic instability occurs as compressive force P approaches Euler critical load P_E
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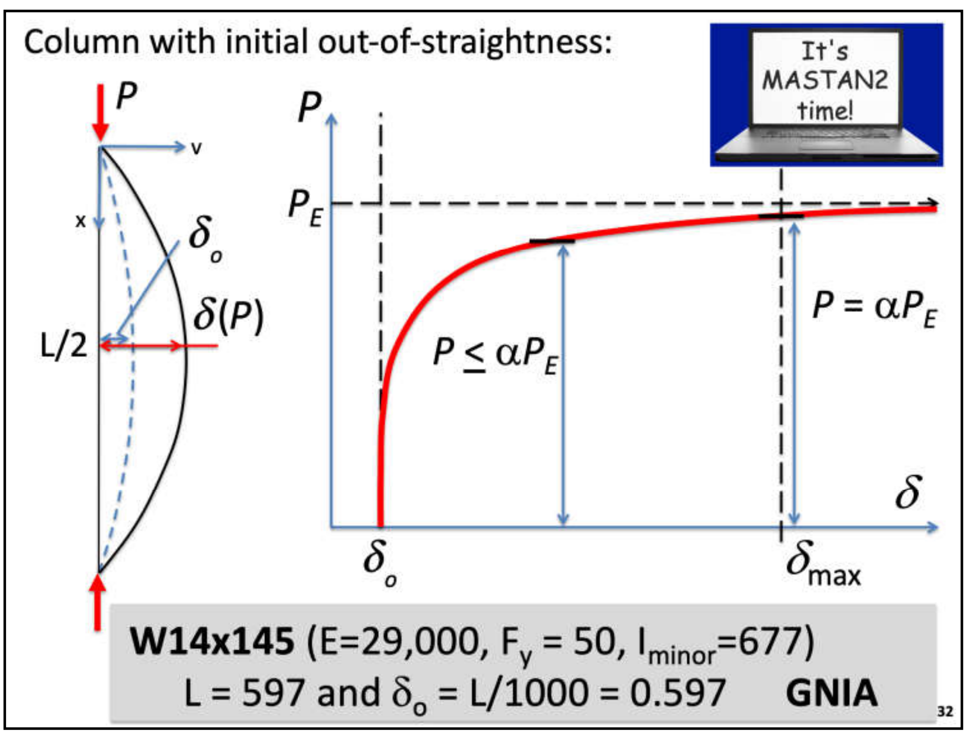
Column with initial out-of-straightness:
It's MASTAN2 time!
P
v
x
δ_o
$\delta(P)$
L/2
P
P_E
$P \leq \alpha P_E$
$P = \alpha P_E$
δ
δ_o
δ_{max}
W14x145 (E=29,000, F_y = 50, I_{minor}=677)
L = 597 and δ_o = L/1000 = 0.597 **GNIA**
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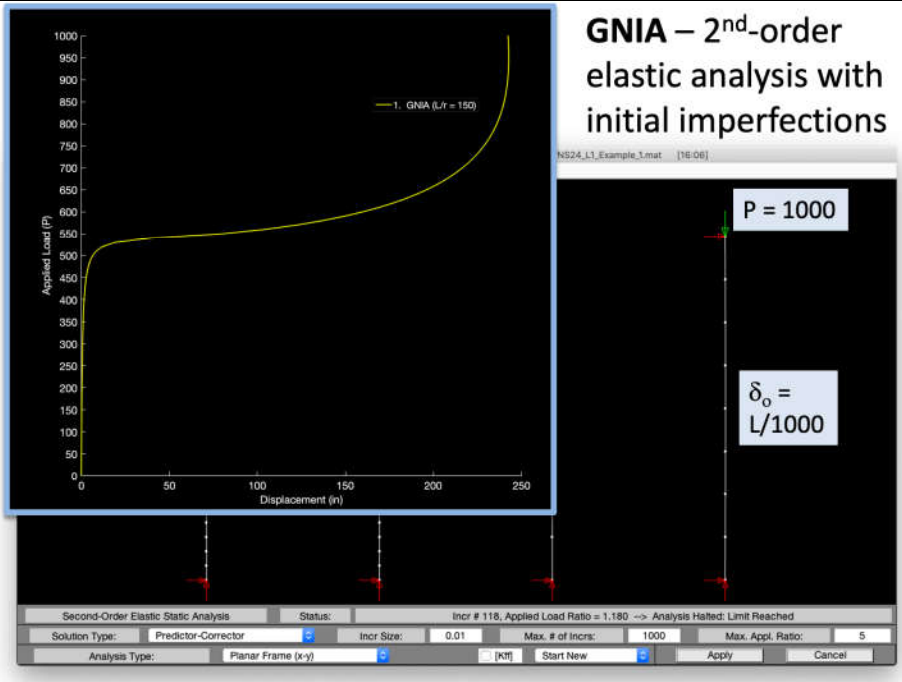
GNIA – 2nd-order elastic analysis with initial imperfections



NS24_L1_Example_1.mat

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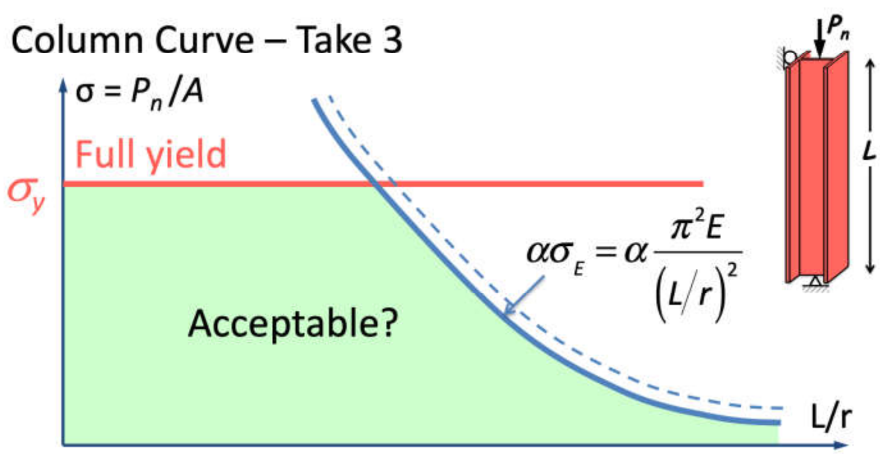
Bending (7)

- Limit elastic bending deflections

$$P \le \alpha P_E \;\Rightarrow\; \frac{P}{A} \le \alpha \frac{P_E}{A} \;\Rightarrow\; \sigma \le \alpha \sigma_E$$

- Column Curve – Take 3



- Consider yielding due to bending plus axial force?

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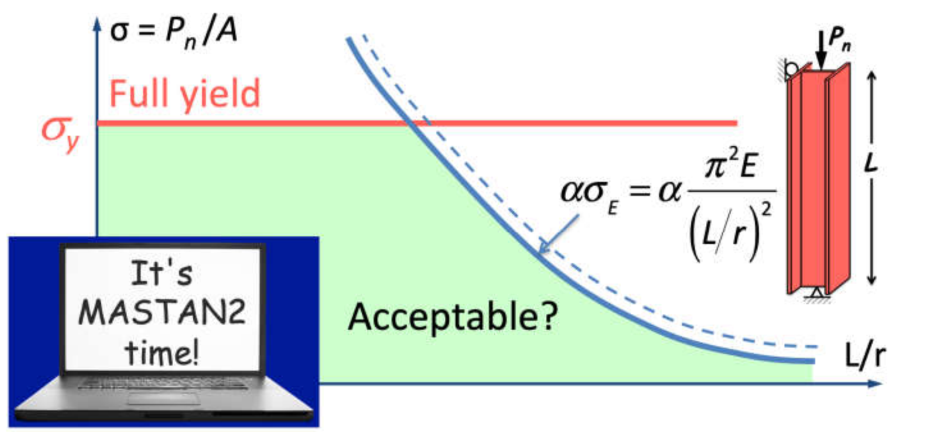

W14x145 (E=29,000, F_y = 50, A=42.7, r_{minor}=3.98)

L/r	L	$P_{n,GNA}/A$	$P_{n,GNIA}$	$P_{n,min}/A$
150	597	12.6		

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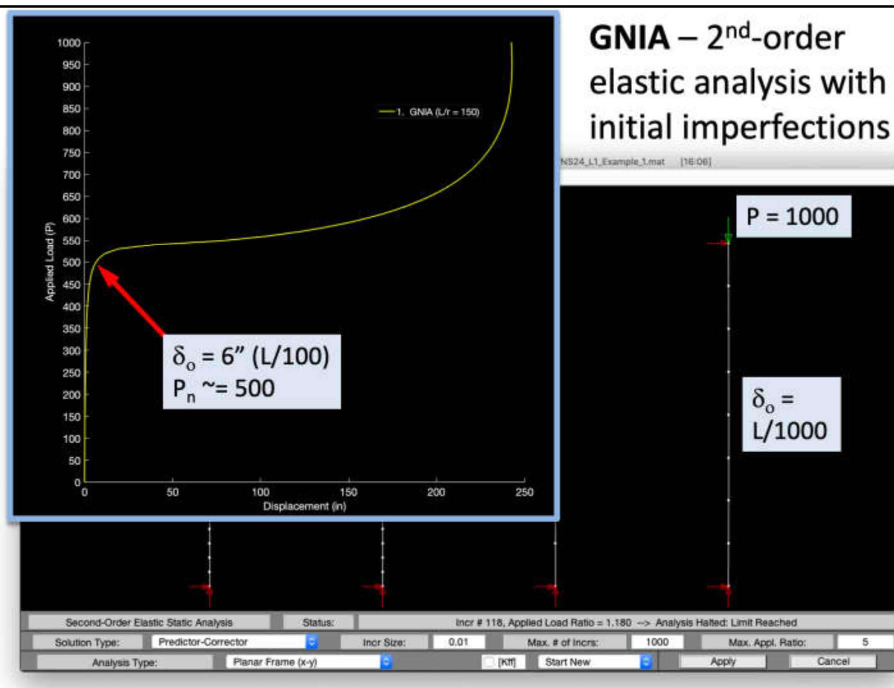
GNIA – 2nd-order elastic analysis with initial imperfections



NS24_L1_Example_1.mat

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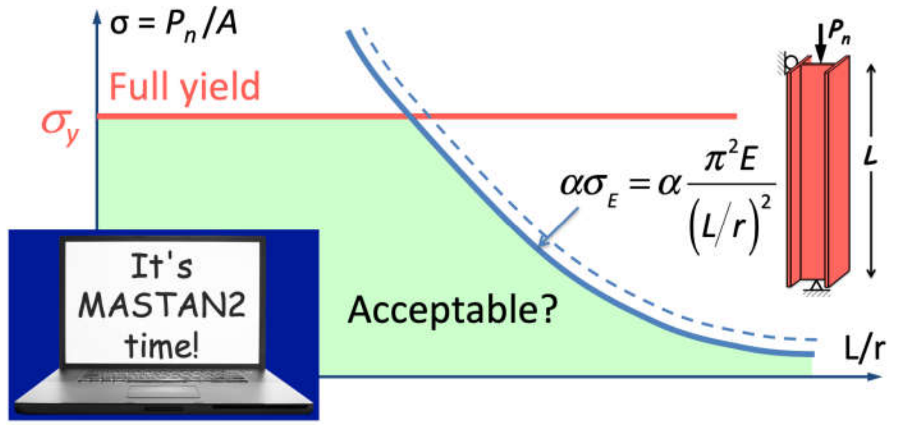

W14x145 (E=29,000, F_y = 50, A=42.7, r_{minor}=3.98)

L/r	L	$P_{n,GNA}/A$	$P_{n,GNIA}$	$P_{n,min}/A$
150	597	12.6	~500	~11.7

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Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions!
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
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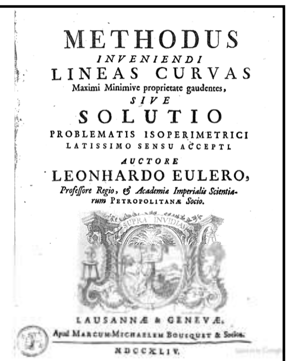

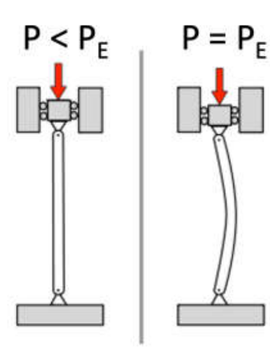

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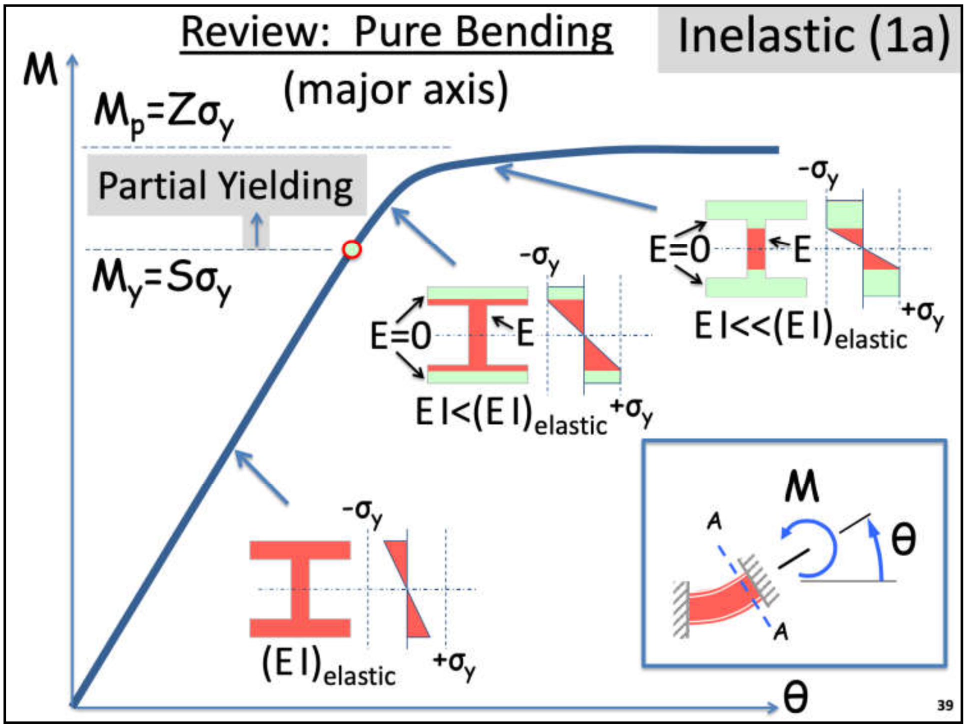
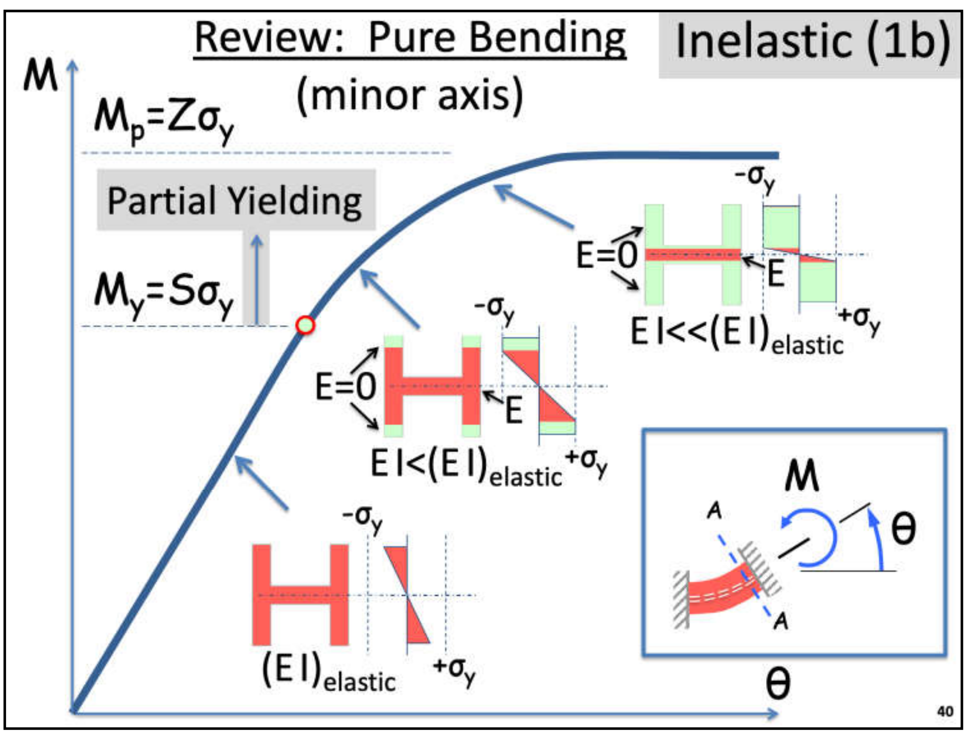
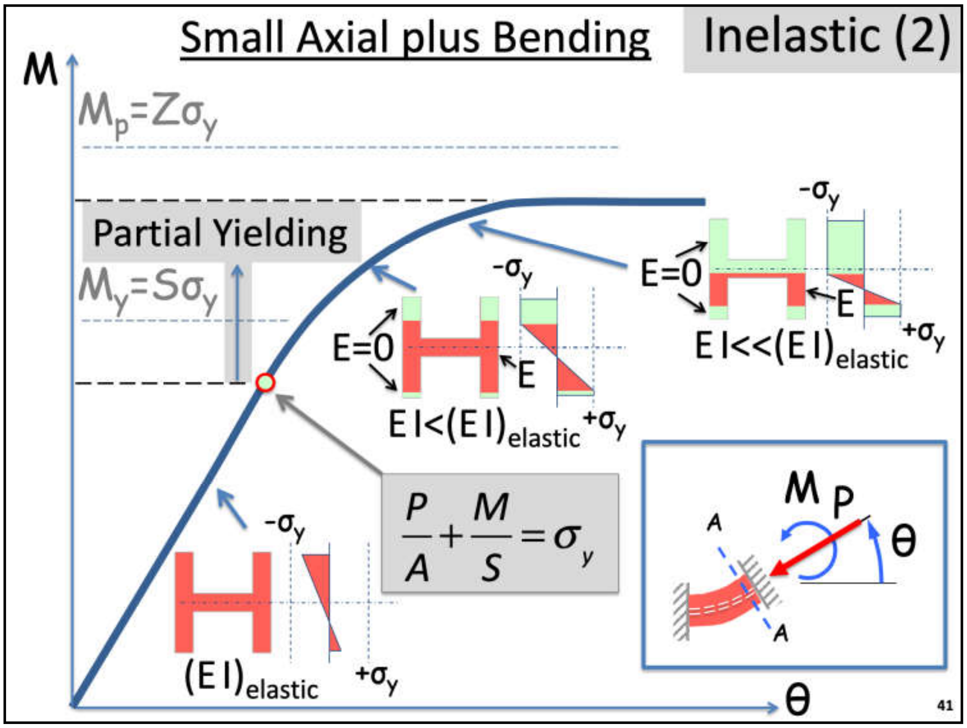
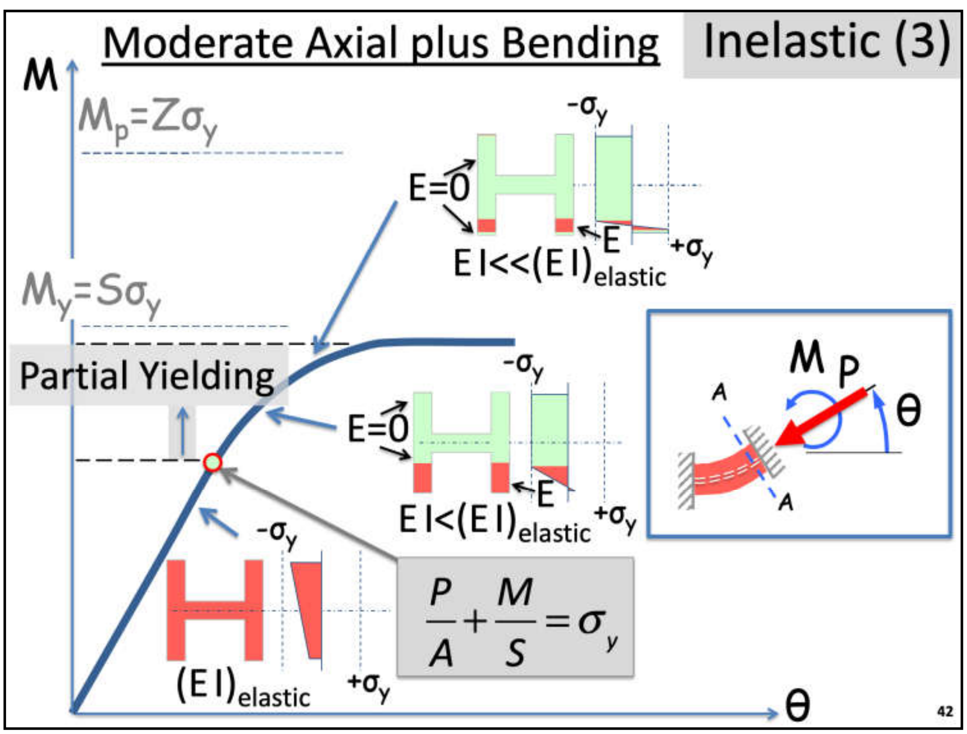
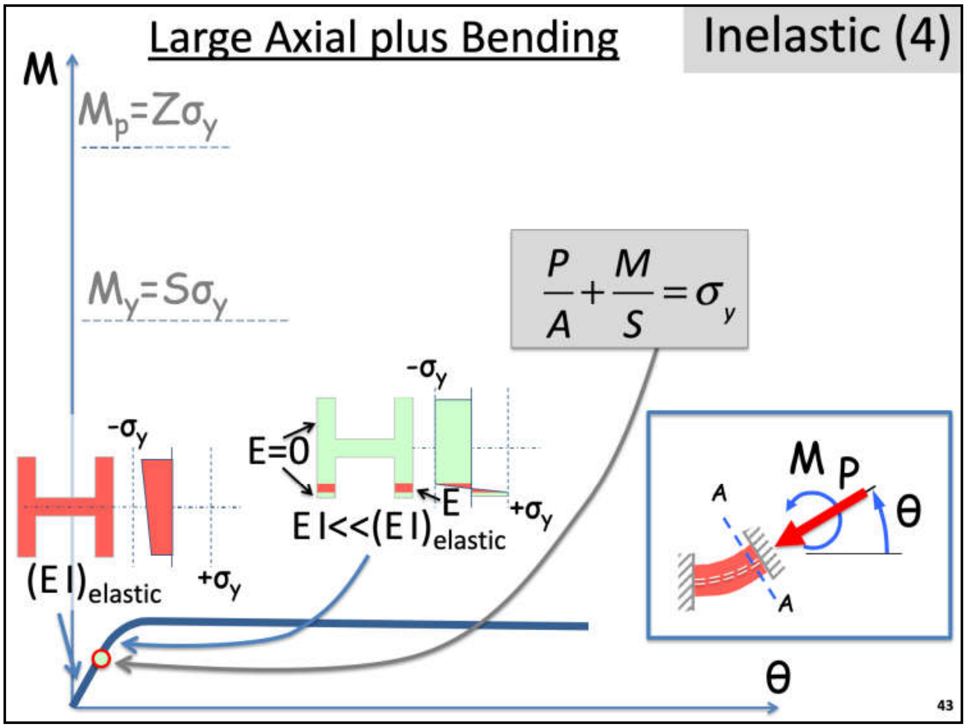
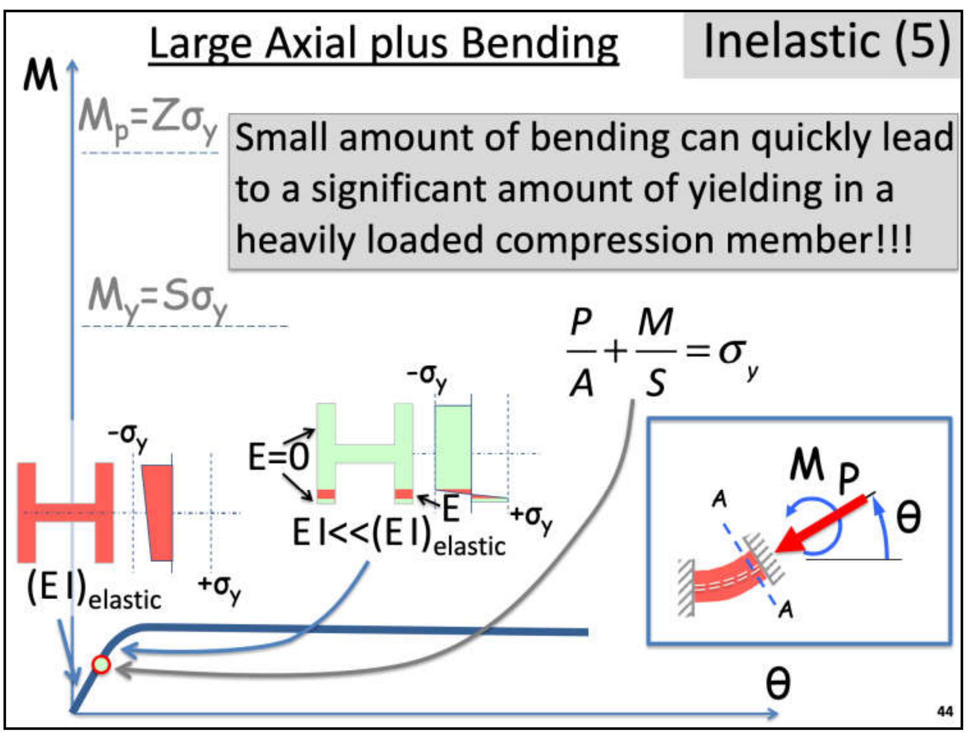

Review: Pure Bending
(major axis)
Inelastic (1a)
M
$M_p=Z\sigma_y$
Partial Yielding
$M_y=S\sigma_y$
$-\sigma_y$
E=0
E
$EI<<(EI)_{elastic}$
$+\sigma_y$
$EI<(EI)_{elastic}$
$(EI)_{elastic}$
M
A
θ
θ
39


Review: Pure Bending
(minor axis)
Inelastic (1b)
M
$M_p=Z\sigma_y$
Partial Yielding
$M_y=S\sigma_y$
$-\sigma_y$
E=0
E
$EI<<(EI)_{elastic}$
$+\sigma_y$
$EI<(EI)_{elastic}$
$(EI)_{elastic}$
M
A
θ
θ
40


Small Axial plus Bending
Inelastic (2)
M
$M_p=Z\sigma_y$
Partial Yielding
$M_y=S\sigma_y$
$-\sigma_y$
E=0
E
EI<<(EI)elastic
$+\sigma_y$
$-\sigma_y$
E=0
E
EI<(EI)elastic
$+\sigma_y$
$\frac{P}{A}+\frac{M}{S}=\sigma_y$
M P
θ
A
A
$-\sigma_y$
(EI)elastic
$+\sigma_y$
θ
41


Moderate Axial plus Bending
Inelastic (3)
M
$M_p=Z\sigma_y$
$-\sigma_y$
E=0
E
EI<<(EI)elastic
$+\sigma_y$
$M_y=S\sigma_y$
Partial Yielding
$-\sigma_y$
E=0
E
EI<(EI)elastic
$+\sigma_y$
M P
θ
A
A
$-\sigma_y$
$\frac{P}{A}+\frac{M}{S}=\sigma_y$
(EI)elastic
$+\sigma_y$
θ
42

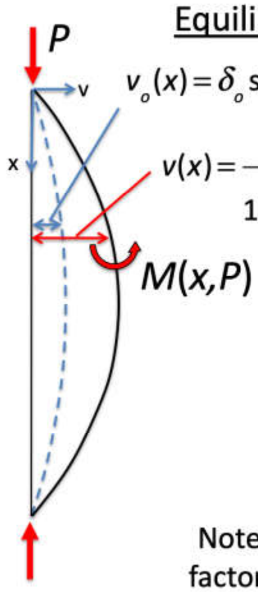

Large Axial plus Bending
Inelastic (4)
M
$M_p = Z\sigma_y$
$M_y = S\sigma_y$
$\frac{P}{A} + \frac{M}{S} = \sigma_y$
$-\sigma_y$
$-\sigma_y$
E=0
E
$EI << (EI)_{elastic}$
$+\sigma_y$
$(EI)_{elastic}$
$+\sigma_y$
M P
θ
A
A
θ
43


Large Axial plus Bending
Inelastic (5)
M
$M_p = Z\sigma_y$
Small amount of bending can quickly lead to a significant amount of yielding in a heavily loaded compression member!!!
$M_y = S\sigma_y$
$\frac{P}{A} + \frac{M}{S} = \sigma_y$
$-\sigma_y$
$-\sigma_y$
E=0
E
$EI << (EI)_{elastic}$
$+\sigma_y$
$(EI)_{elastic}$
$+\sigma_y$
M P
θ
A
A
θ
44

Inelastic (6)

Closer look at that bending...

Equilibrium on deformed shape:



$$v_o(x) = \delta_o \sin\frac{\pi x}{L}$$

$$v(x) = \frac{1}{1-\frac{P}{P_E}} v_o(x)$$

$$M(x,P) + Pv(x) = 0$$

$$M(x,P) = -Pv(x)$$

$$M(x,P) = -P\frac{1}{1-\frac{P}{P_E}}\delta_o \sin\frac{\pi x}{L}$$

$$M(x,P) = \frac{1}{1-\frac{P}{P_E}} \cdot \left(-P\delta_o \sin\frac{\pi x}{L}\right)$$

Note: amplification factor to account for 2nd-order effects

$$M(x,P) = \frac{1}{1-\frac{P}{P_E}} \cdot M(x,P)^{1st-order}$$

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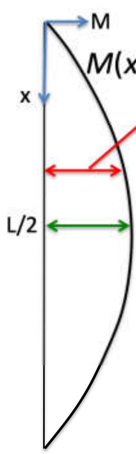
Inelastic (7)

Closer look at that bending:

Elastic M-diagram:



$$M(x,P) = \frac{-P}{1-\frac{P}{P_E}}\delta_o \sin\frac{\pi x}{L}$$

$$M(\frac{L}{2},P) = \frac{-P}{1-\frac{P}{P_E}}\delta_o$$

All is good…as long as all is elastic, i.e. no yielding!

$$\left|\frac{P}{A}\right| + \left|\frac{M(x,P)}{S}\right| < \sigma_y$$

But, yielding will occur when

$$\left|\frac{P}{A}\right| + \left|\frac{M(L/2,P)}{S}\right| = \sigma_y$$

or, an axial load *P* that satisfies:

Note: relatively simple equation to compute axial force that produces first yield (excludes σ_{res})

$$\frac{P}{A} + \frac{1}{\left(1-\frac{P}{P_E}\right)}\frac{P\delta_o}{S} = \sigma_y$$

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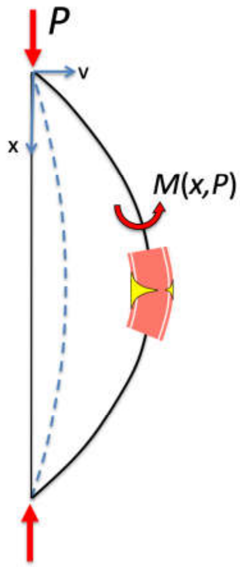
And, once yielding occurs (ouch!): **Inelastic (8)**



1. Yielded portion loses stiffness, $EI \downarrow$
2. Increases in deflection, $v(x) \uparrow$
3. Increases moment, $M(x) = P \cdot v(x) \uparrow$
4. Resulting in more yielding…

5. If equilibrium, apply more P
6. Repeat above steps 1 to 4
7. Apply more P repeating steps 1 to 6 until instability!

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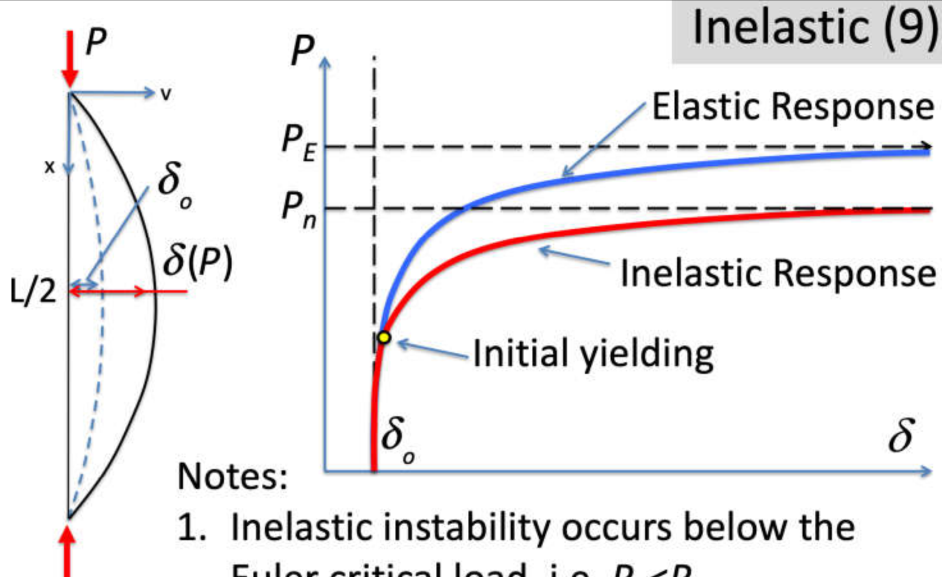
Inelastic (9)



Notes:

1. Inelastic instability occurs below the Euler critical load, i.e. $P_n < P_E$
2. The smaller the column slenderness L/r, the further P_n is below P_E $(L/r \downarrow \;\; P_E \Uparrow)$

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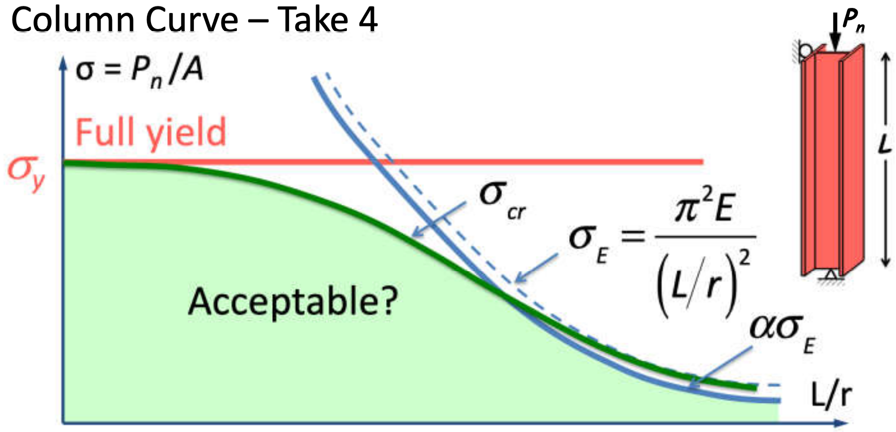
Inelastic (10)

- Axial plus bending may cause yielding

$$\sigma_{cr} = \frac{P_n}{A} \qquad L/r \rightarrow 0,\ \sigma_{cr} = \sigma_y$$

$$L/r \uparrow,\ \sigma_{cr} < \sigma_y \ \text{ and } \ \sigma_{cr} < \sigma_E$$

- Column Curve – Take 4



- What about residual stresses?

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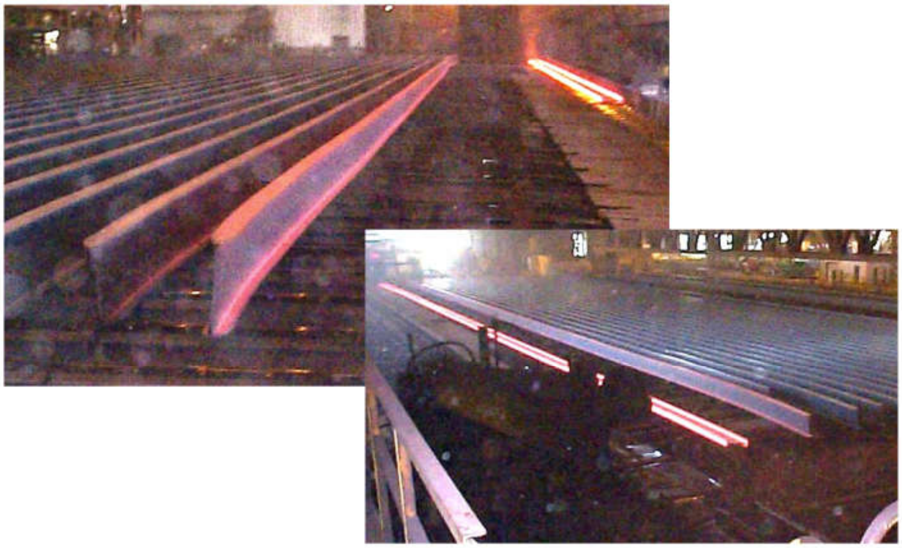
Residual Stresses

- Occurs in structural shapes
  - Uneven cooling of hot-rolled shape after rolling
  - Welding of plates for fabricated or built-up shapes
  - Cold bending during fabrication
- Magnitude and distribution of residual stresses depend on the cross-sectional shape and dimensions
- Residual stresses are usually <u>independent</u> of F_y
- Thermal residual stresses occur in rolled wide flange shapes because locations with high surface area (e.g., flange tips) cool well before locations with smaller surface area (flange-to-web intersections)
- But, what about the impact of rotary straightening? And, today's production via near net sections?

50

Residual Stresses (2)



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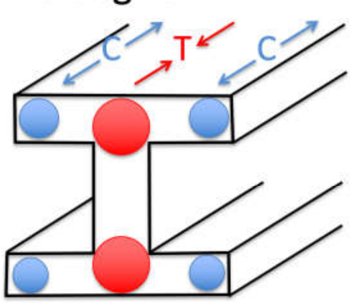
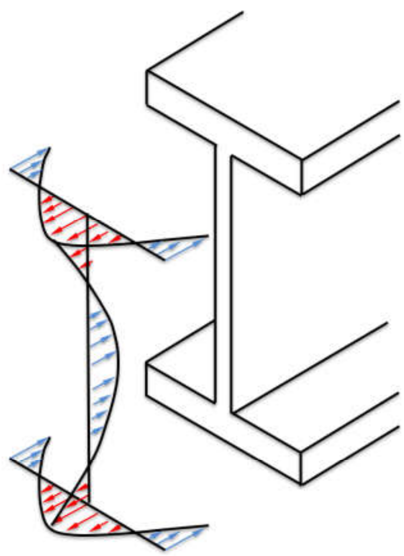
Residual Stresses (3)

Because areas with large surface area cool first and stiffen, the push-me pull-me begins…





Closer to actual distribution

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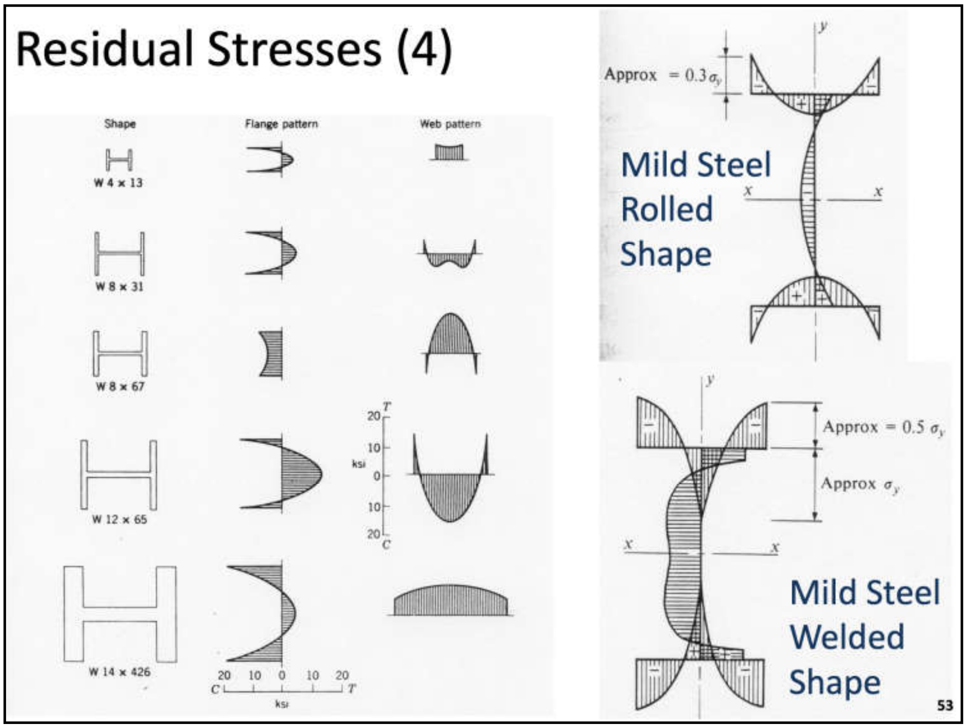
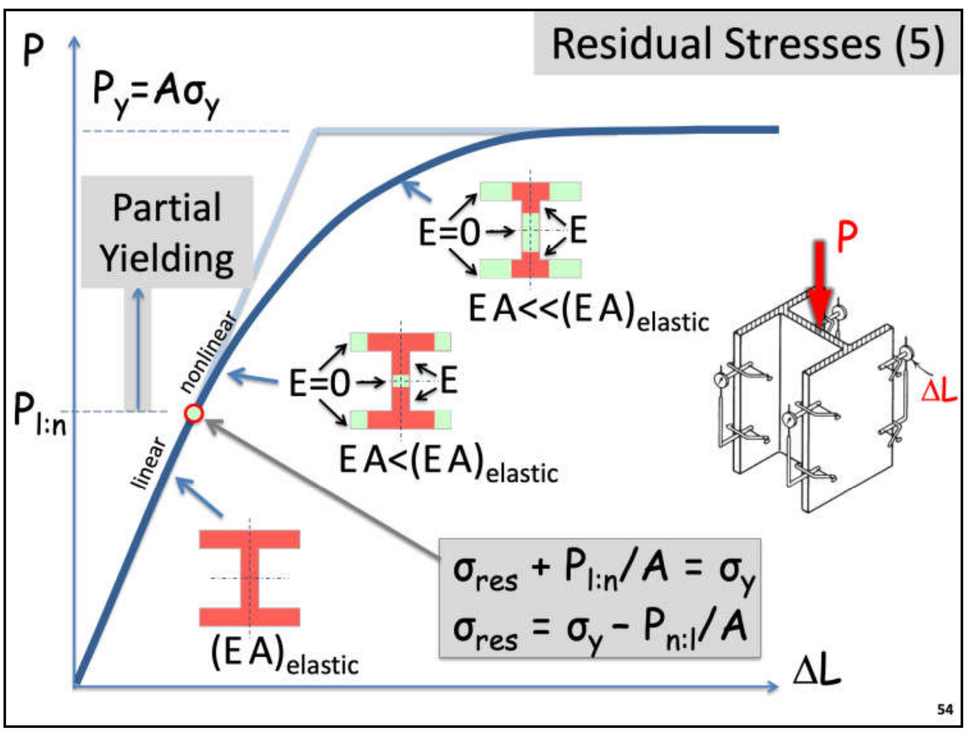
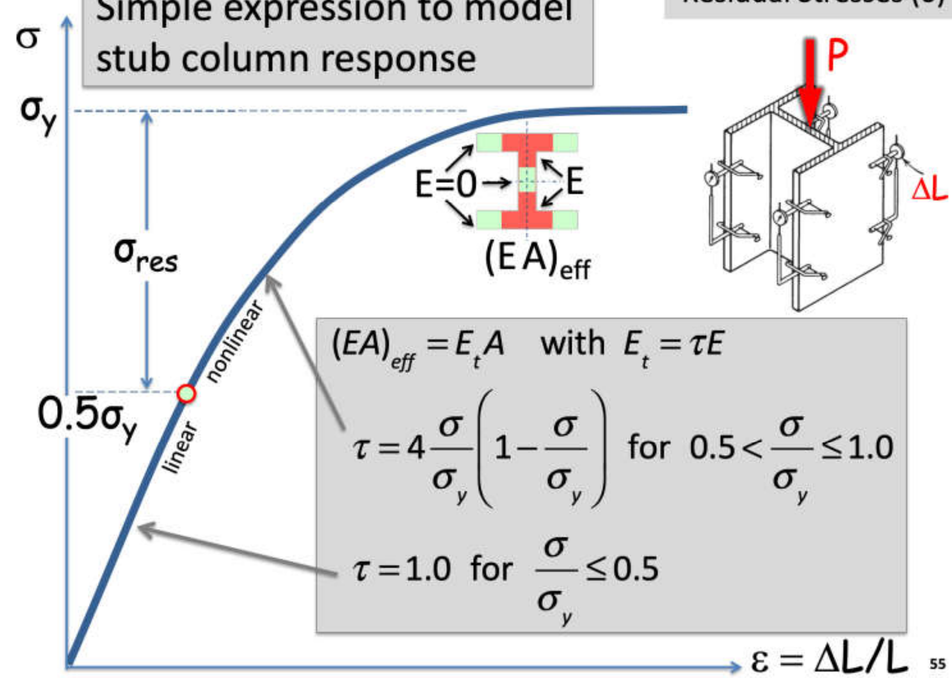

Residual Stresses (4)
Shape
Flange pattern
Web pattern
W 4 × 13
W 8 × 31
W 8 × 67
W 12 × 65
W 14 × 426
ksi
20 10 0 10 20
C
T
Approx = 0.3σy
Mild Steel Rolled Shape
Approx = 0.5 σy
Approx σy
Mild Steel Welded Shape
53


Residual Stresses (5)
P
Py=Aσy
Partial Yielding
nonlinear
linear
Pl:n
E=0→ E
EA<<(EA)elastic
EA<(EA)elastic
(EA)elastic
P
ΔL
σres + Pl:n/A = σy
σres = σy – Pn:l/A
ΔL
54

Residual Stresses (6)

Simple expression to model stub column response

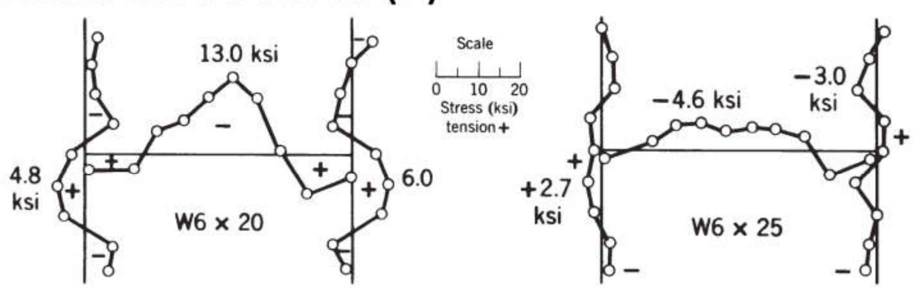


$$(EA)_{eff} = E_t A \quad \text{with} \quad E_t = \tau E$$

$$\tau = 4\frac{\sigma}{\sigma_y}\left(1 - \frac{\sigma}{\sigma_y}\right) \quad \text{for} \quad 0.5 < \frac{\sigma}{\sigma_y} \le 1.0$$

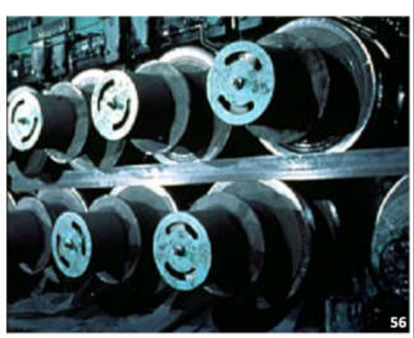
$$\tau = 1.0 \quad \text{for} \quad \frac{\sigma}{\sigma_y} \le 0.5$$

Residual Stresses (7)



<u>Rotary Straightening</u>

1. Standard practice at rolling mills
2. Sections up to W24x370
3. Reduces residual stresses in the flange tips
4. Result: Actual residual stress pattern in as-delivered rolled sections is quite variable



Residual Stresses (8)

- Euler -to- Inelastic Buckling Stress

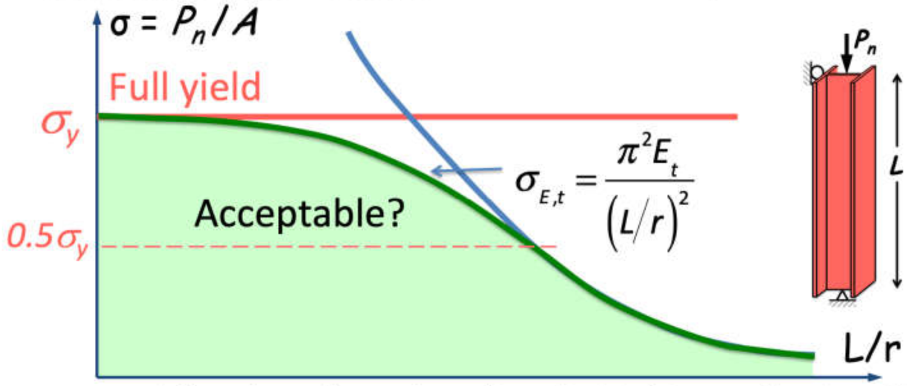
$$\sigma_E = \frac{\pi^2 E}{(L/r)^2} \Rightarrow \sigma_{E,t} = \frac{\pi^2 E_t}{(L/r)^2}$$

$E_t = \tau E$

$\tau = 4\frac{\sigma}{\sigma_y}\left(1 - \frac{\sigma}{\sigma_y}\right)$ for $0.5 < \frac{\sigma}{\sigma_y} \le 1.0$

$\tau = 1.0$ for $\frac{\sigma}{\sigma_y} \le 0.5$

- Column Curve – Take 5



- But wait! What about bending (initial imperfections)?

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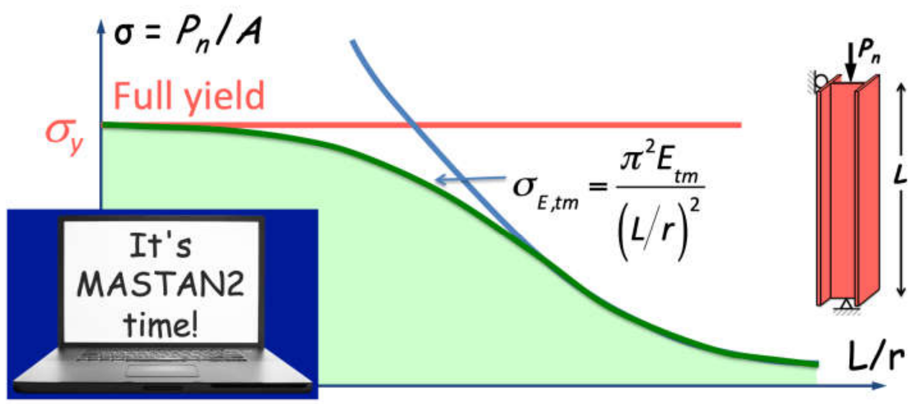

W14x145 (E=29,000, F_y = 50, A=42.7, r_{minor}=3.98)

L/r	L	$P_{n,MNA}$	$P_{n,GNA}$	$P_{n,GMNIA}$	$P_{n,min}/A$
50	199	2,135	4,930		
150	597	2,135	540		

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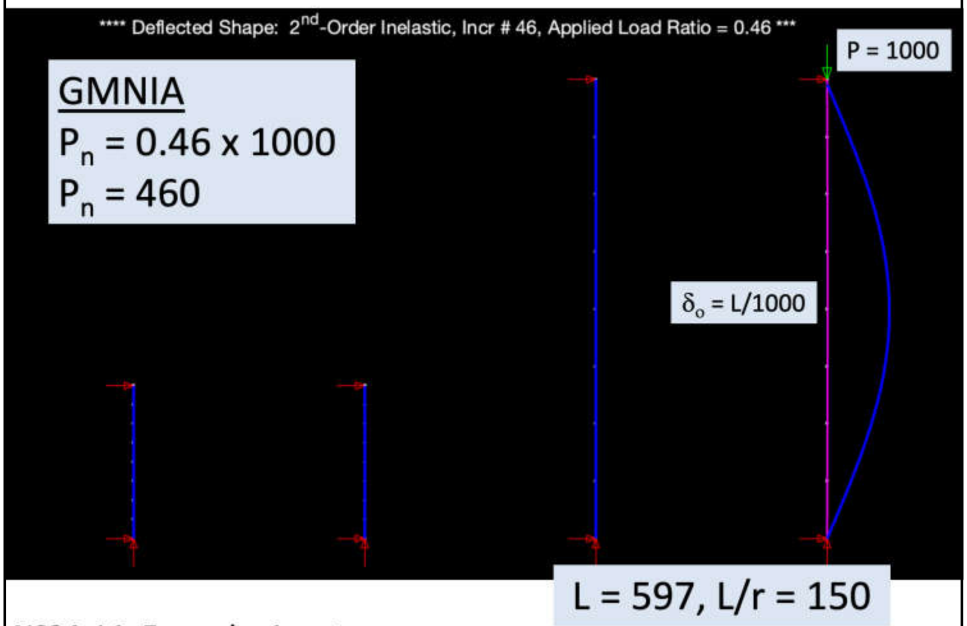

NS24_L1_Example_1.mat

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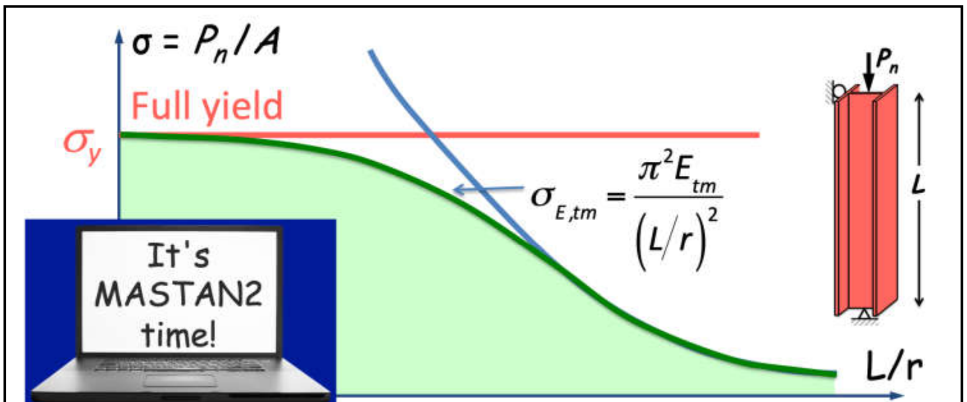
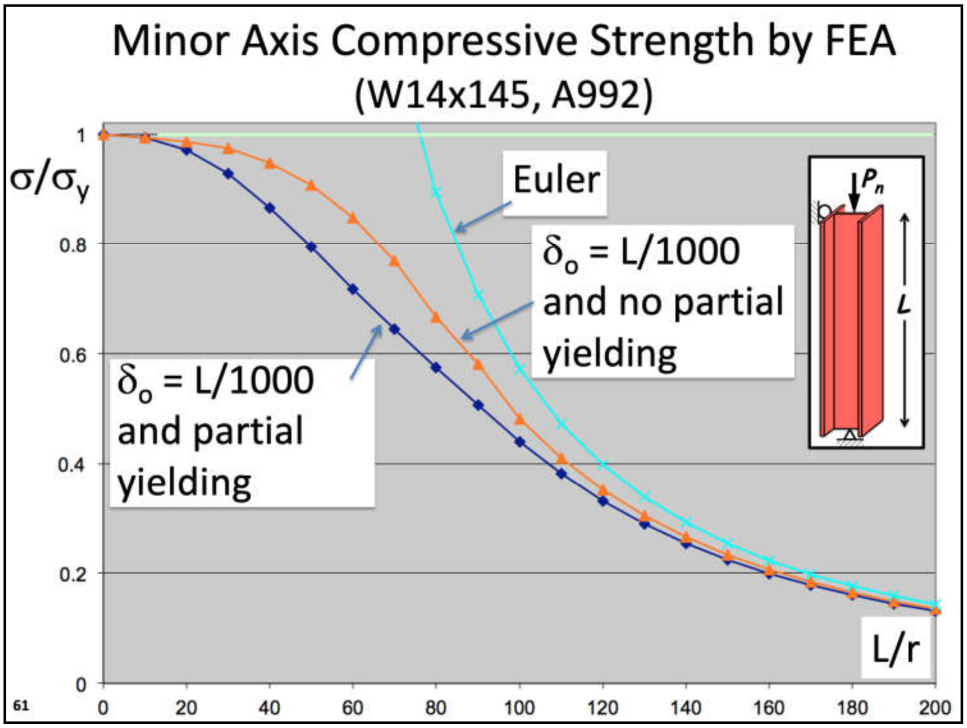


W14x145 (E=29,000, F_y = 50, A=42.7, r_{minor}=3.98)

L/r	L	$P_{n,MNA}$	$P_{n,GNA}$	$P_{n,GMNIA}$	$P_{n,min}/A$
50	199	2,135	4,930	1710	40.0
150	597	2,135	540	460	10.8

60



Compressive Strength Curves

- Key observations from FEA
  - Strength reduced for initial imperfection and further reduced for residual stresses
  - All curves approach Euler, but are slightly below
  - Partial yielding accentuated by residual stresses impact minor axis strength more than major axis strength
  - Different strength curves for major and minor axis bending
- Additional thoughts
  - Strength curves for W-shapes are function of dimensions, and thus will vary depending on W-shape
  - Other shapes (e.g., HSS, C’s, and built-up shapes) will also have different compressive strength curves

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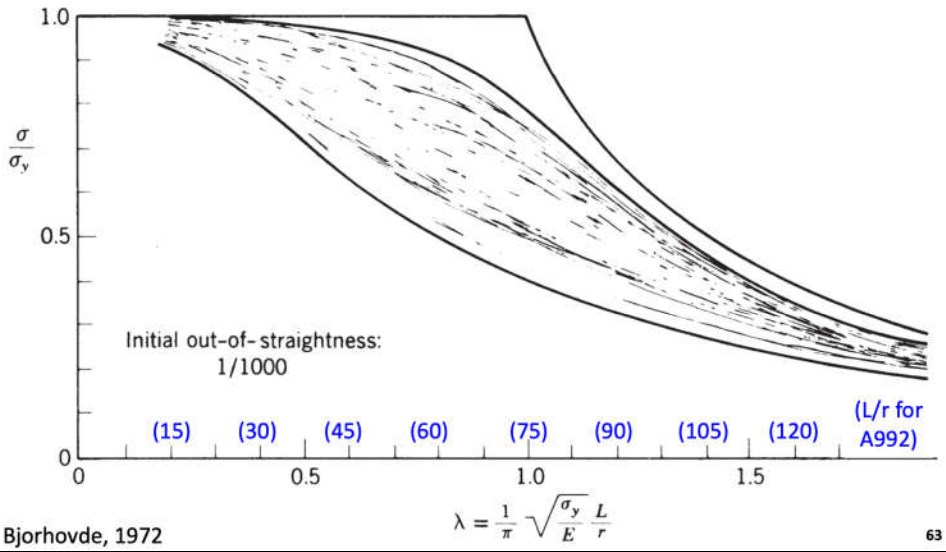
Maximum Compressive Strength Curves for Many Different Column Types



Bjorhovde, 1972

63

Column Curves for Design

- AISC employs a single curve "fit" to experimental and analytical data. Other codes use multiple curves.
- Background to AISC curve:
  - Bjorhovde, R. (1972), "Deterministic and Probabilistic Approaches to the Strength of Steel Columns," Ph.D. Dissertation, Lehigh University, Bethlehem, PA.
  - Tide, R.H.R. (2001), "A Technical Note: Derivation of the LRFD Column Design Equations," Engineering Journal, AISC, Vol. 38, No. 3, 3rd Quarter, pp. 137–139.
  - Ziemian, R.D. (ed.) (2010), *Guide to Stability Design Criteria for Metal Structures*, 6th Ed., John Wiley & Sons, Inc., Hoboken, NJ.

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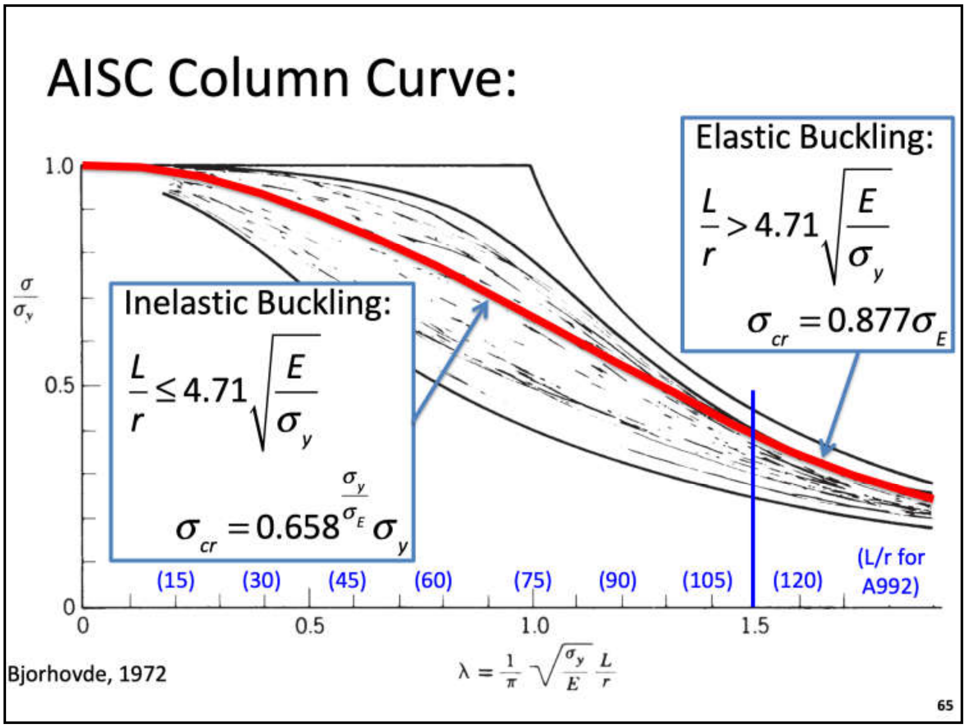
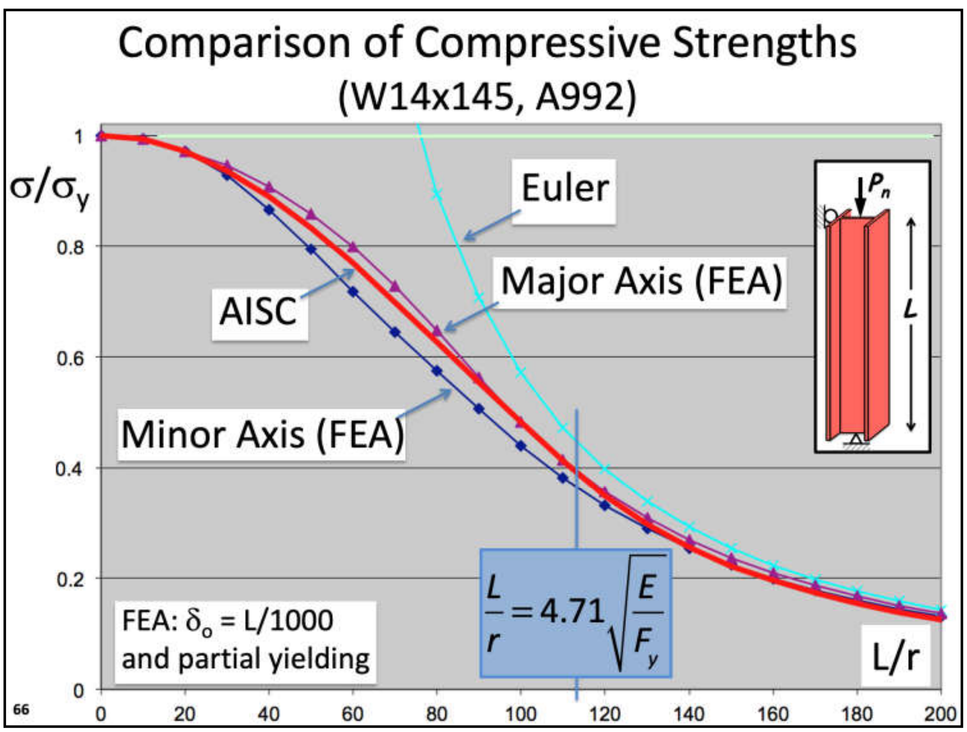
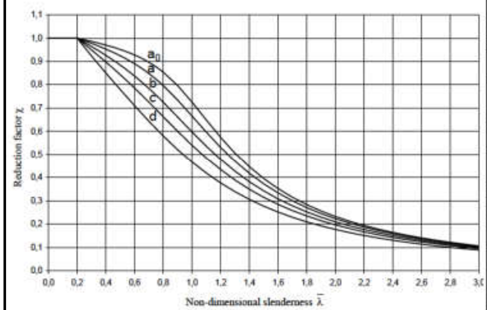

AISC Column Curve:
Elastic Buckling:
$\frac{L}{r} > 4.71\sqrt{\frac{E}{\sigma_y}}$
$\sigma_{cr} = 0.877\sigma_E$
Inelastic Buckling:
$\frac{L}{r} \leq 4.71\sqrt{\frac{E}{\sigma_y}}$
$\sigma_{cr} = 0.658^{\frac{\sigma_y}{\sigma_E}}\sigma_y$
$\frac{\sigma}{\sigma_y}$
(15) (30) (45) (60) (75) (90) (105) (120) (L/r for A992)
$\lambda = \frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}}\frac{L}{r}$
Bjorhovde, 1972
65


Comparison of Compressive Strengths
(W14x145, A992)
σ/σ_y
Euler
Major Axis (FEA)
AISC
Minor Axis (FEA)
P_n
L
FEA: δ_o = L/1000
and partial yielding
$\frac{L}{r} = 4.71\sqrt{\frac{E}{F_y}}$
L/r
66

Multiple Column Curves:

Eurocode 3



Cross section		Limits	Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	S 460
Rolled sections	h/b > 1,2	$t_f \le 40$ mm	y – y z – z	a b	a_0 a_0
		40 mm < $t_f \le 100$	y – y z – z	b c	a a
	h/b ≤ 1,2	$t_f \le 100$ mm	y – y z – z	b c	a a
		$t_f > 100$ mm	y – y z – z	d d	c c
Welded I-sections		$t_f \le 40$ mm	y – y z – z	b c	b c
		$t_f > 40$ mm	y – y z – z	c d	c d
Hollow sections		hot finished	any	a	a_0
		cold formed	any	c	c
Welded box sections		generally (except as below)	any	b	b
		thick welds: $a > 0,5t_f$ $b/t_f < 30$ $h/t_w < 30$	any	c	c
U-, T- and solid sections			any	c	c
L-sections			any	b	b

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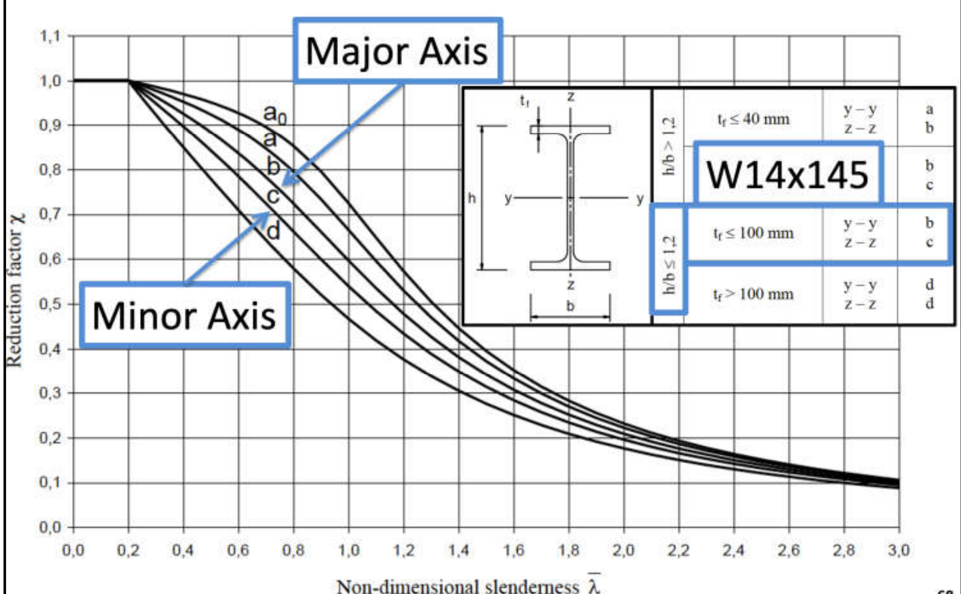
Multiple Column Curves: Eurocode 3



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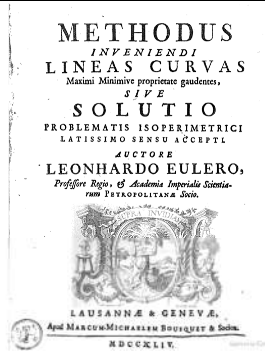
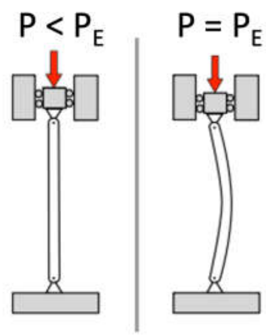
Euler Buckling

- Leonhard Euler, 1744 and 1757
- Assumptions
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded
  - linear elastic behavior (E = constant)
  - pinned-roller supports (frictionless)





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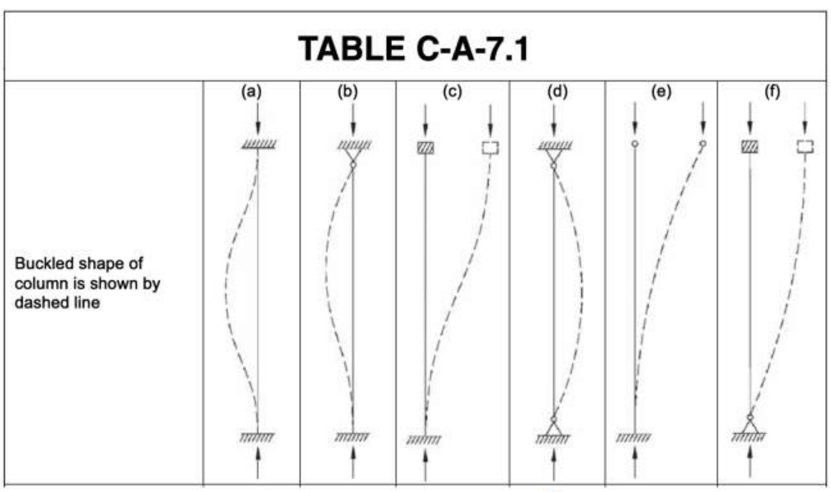
Support Conditions



Euler Buckling

What about the others?

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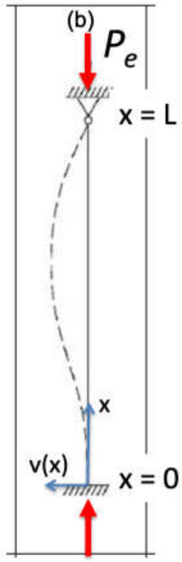
Support Conditions (2)



Equilibrium → Differential Equation:

$$M(x=0)=M_o \Rightarrow EI\frac{d^2v}{dx^2}+P_e v=\frac{M_o x}{L}$$

Solution:

$$v(x)=C_1\cos\left(\sqrt{\frac{P_e}{EI}}x\right)+C_2\sin\left(\sqrt{\frac{P_e}{EI}}x\right)+\frac{M_o x}{P_e L}$$

Boundary Conditions:

$$v(x=0)=0,\ v'(x=0)=0,\ v(x=L)=0$$

$$P_e=\frac{\pi^2 EI}{(0.70L)^2} \Rightarrow \sigma_e=\frac{P_e}{A}=\frac{\pi^2 E}{(KL/r)^2} \text{ with } K=0.70$$

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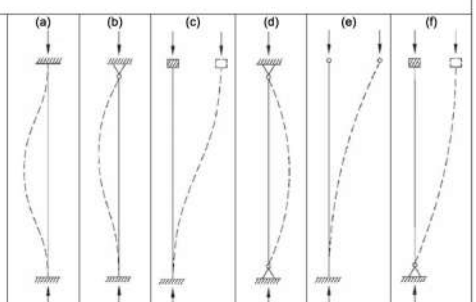
Support Conditions (3)

TABLE C-A-7.1
Approximate Values of Effective Length Factor, *K*

	(a)	(b)	(c)	(d)	(e)	(f)
Buckled shape of column is shown by dashed line						
Theoretical *K* value	0.5	0.7	1.0	1.0	2.0	2.0



Elastic Buckling Stress:

$$\sigma_e=\frac{\pi^2 E}{(KL/r)^2}$$

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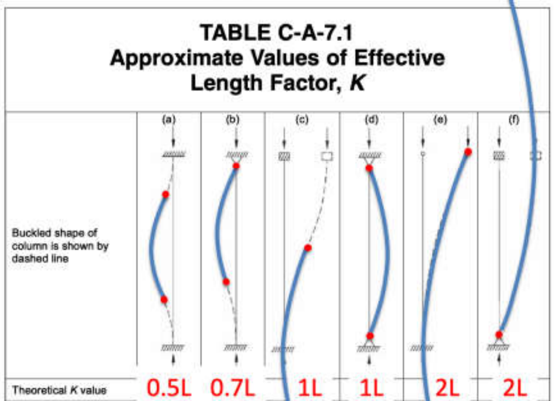
Support Conditions (4)



Elastic Buckling Stress:

$$\sigma_e = \frac{\pi^2 E}{\left(KL/r\right)^2}$$

Notes on "effective length" *KL*:

- Find the Euler column?!
- Distance between inflection points (M=0)

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Overview of Learning Modules

✓Objectives

- Have "fun" focusing on the stability of members and systems
- Alternative to experimental laboratory
- Hands-on ("active learning"); what if scenarios?
- Software independent

- Consistent format
  - Overview
  - Learning Objectives
  - Method
  - Hints
  - Questions

- MASTAN2 details
- More fun!
- Resources:
  spreadsheets
  tutorial videos (~140 min)

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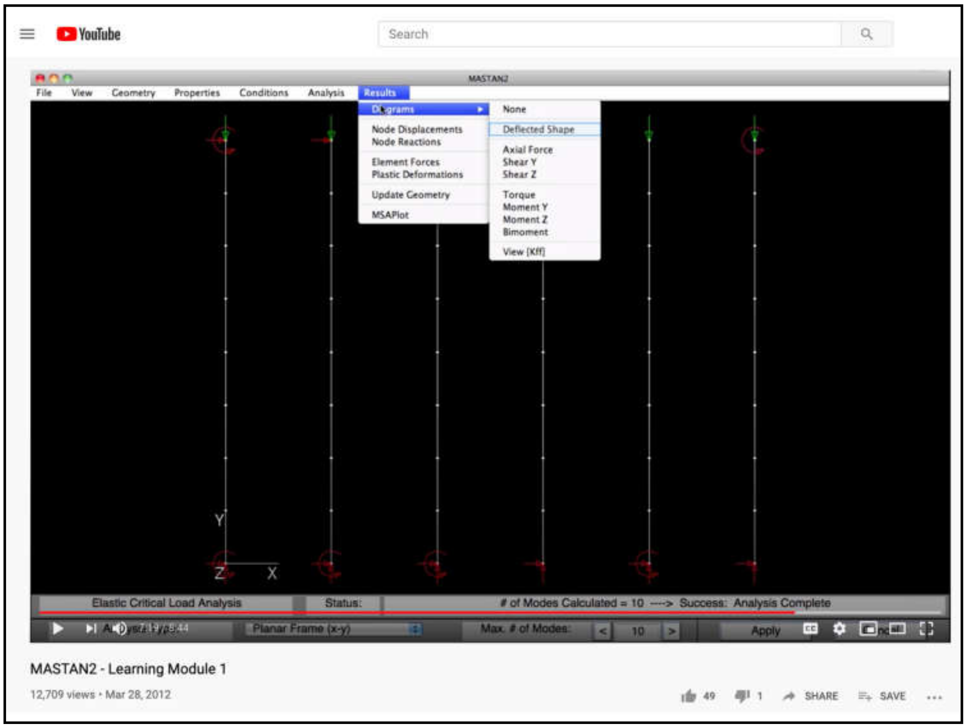
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Node Reactions
Element Forces
Plastic Deformations
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MSAPlot
None
Deflected Shape
Axial Force
Shear Y
Shear Z
Torque
Moment Y
Moment Z
Bimoment
View [Kff]
Y
Z
X
Elastic Critical Load Analysis
Status:
# of Modes Calculated = 10 ---> Success: Analysis Complete
Planar Frame (x-y)
Max. # of Modes:
10
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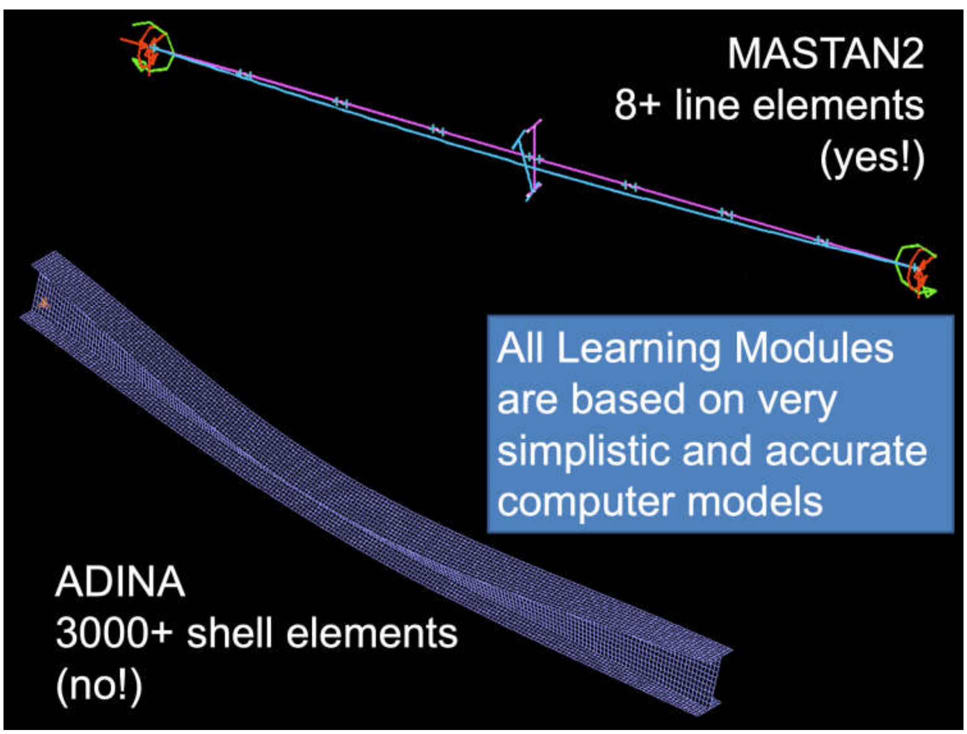
MASTAN2
8+ line elements
(yes!)
All Learning Modules are based on very simplistic and accurate computer models
ADINA
3000+ shell elements
(no!)

Available Learning Modules

1. Elastic Column Buckling and the Effect of End Restraint (Now!)
2. Factors Influencing the Flexural Buckling Strength of Compression Members (Your virtual laboratory assignment!)
3. Effective Length *K*-factors for Frame Members
4. Factors Influencing the Strength of Flexural Members
5. Lateral-Torsional Buckling of Beams with Moment Gradient
6. Beam Design by Elastic and Inelastic Analyses
7. Second-order (P-Δ and P-δ) Effects
8. Strength of Beam-Columns
9. Design by the Direct Analysis Method

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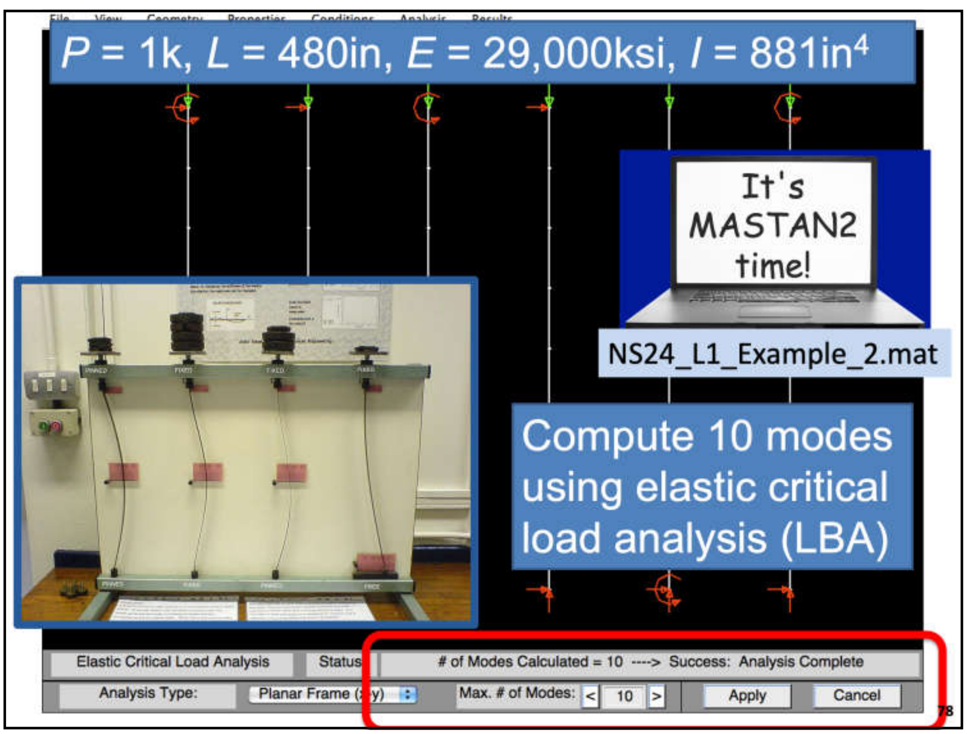
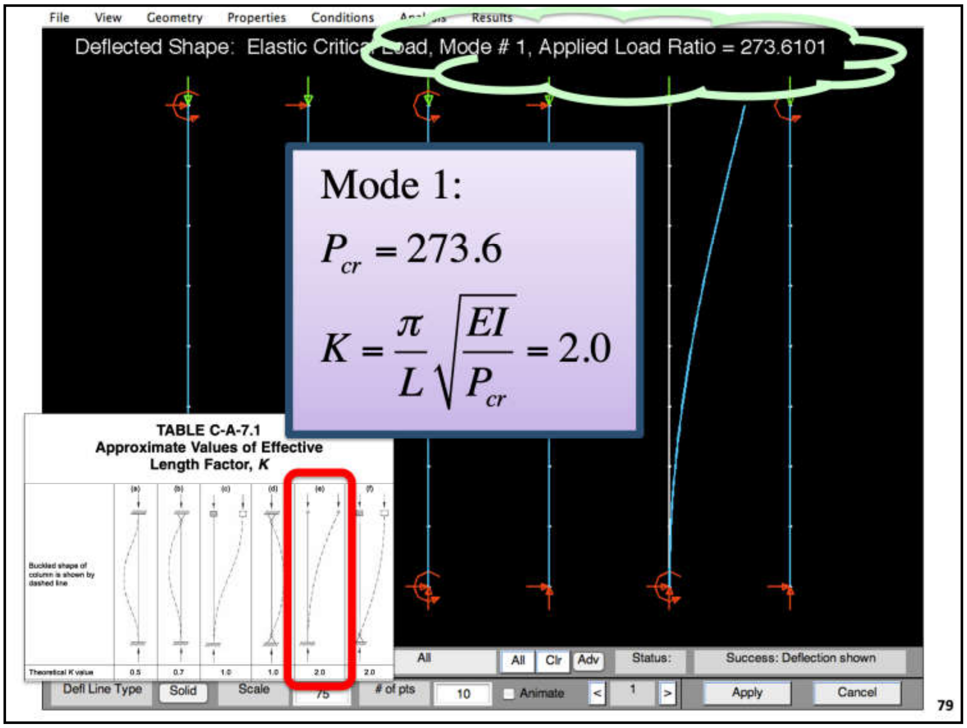
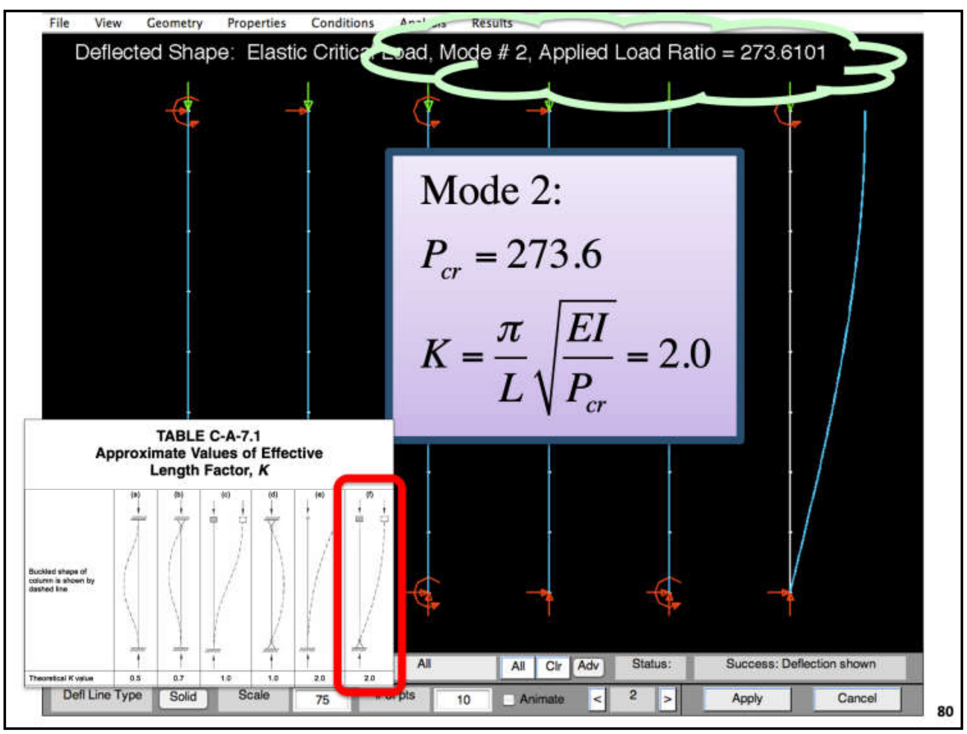
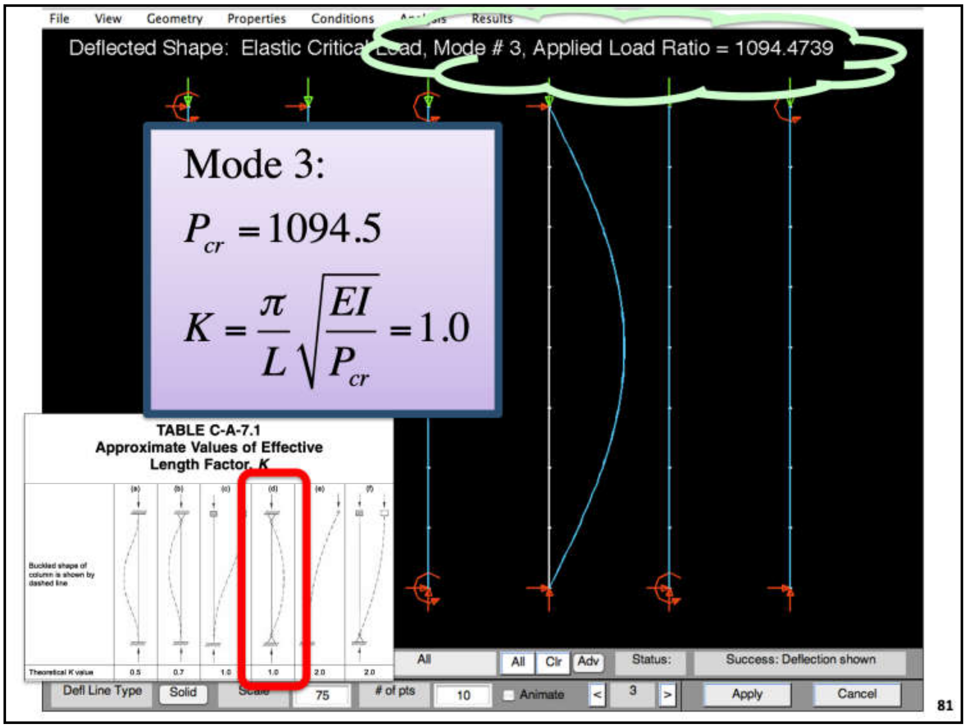
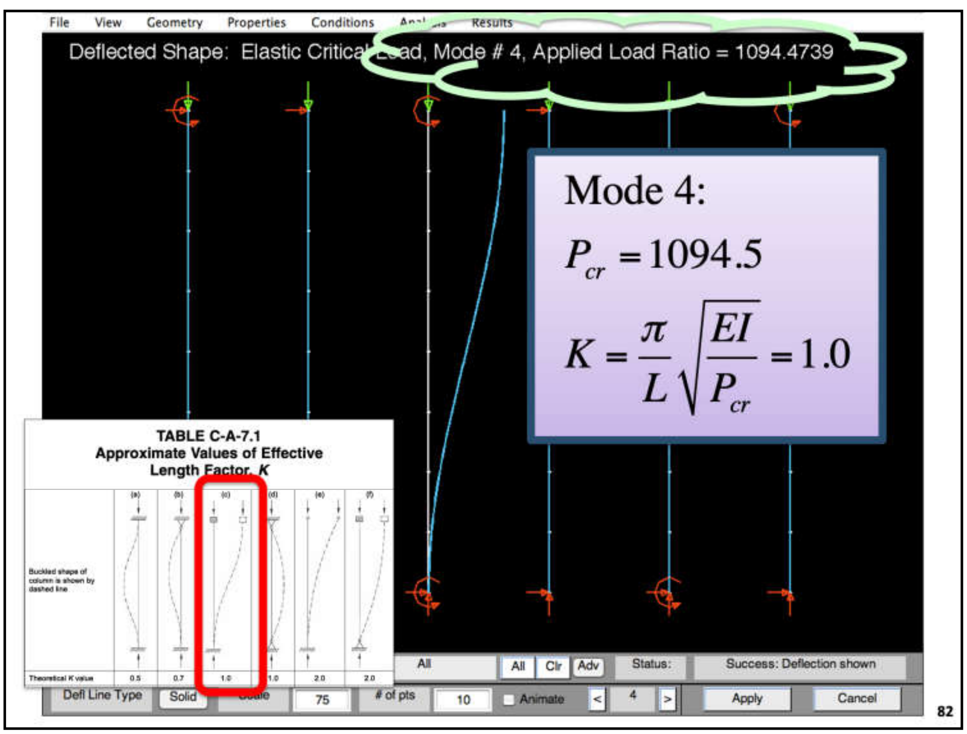
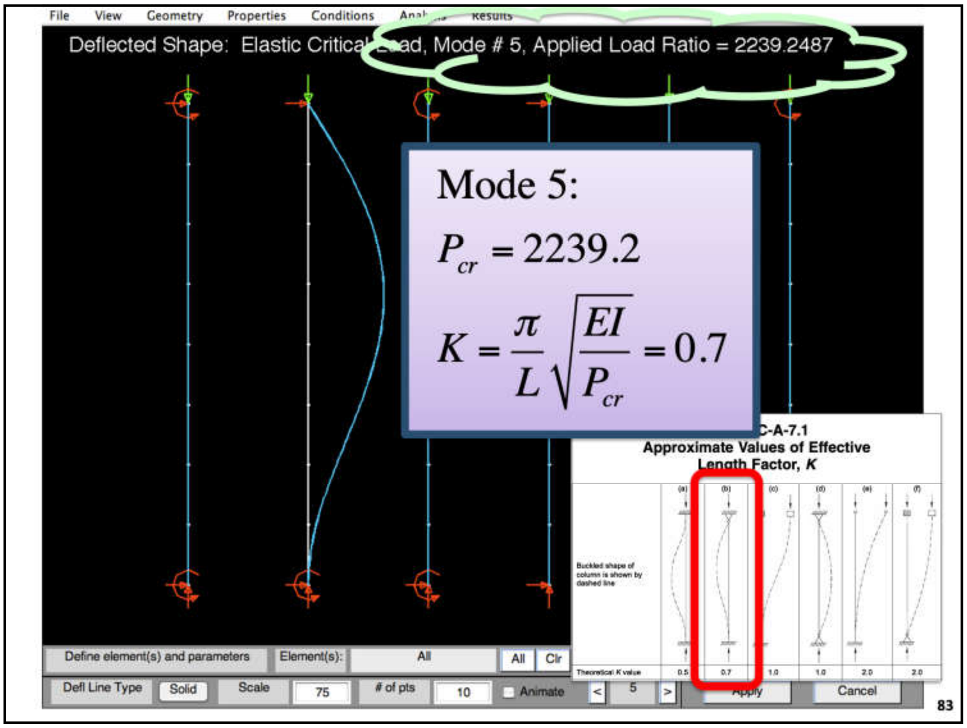


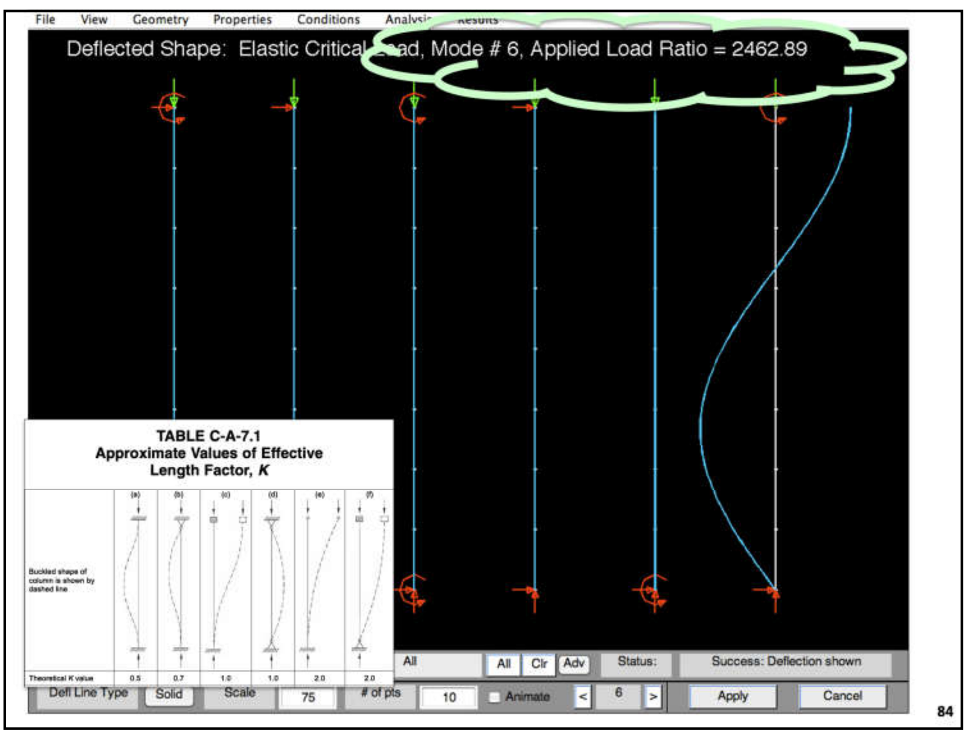

File View Geometry Properties Conditions Results
Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 273.6101
Mode 1:
$P_{cr} = 273.6$
$K = \frac{\pi}{L}\sqrt{\frac{EI}{P_{cr}}} = 2.0$
TABLE C-A-7.1
Approximate Values of Effective Length Factor, K
Buckled shape of column is shown by dashed line
Theoretical K value 0.5 0.7 1.0 1.0 2.0 2.0
All All Clr Adv Status: Success: Deflection shown
Defl Line Type Solid Scale 75 # of pts 10 Animate 1 Apply Cancel
79

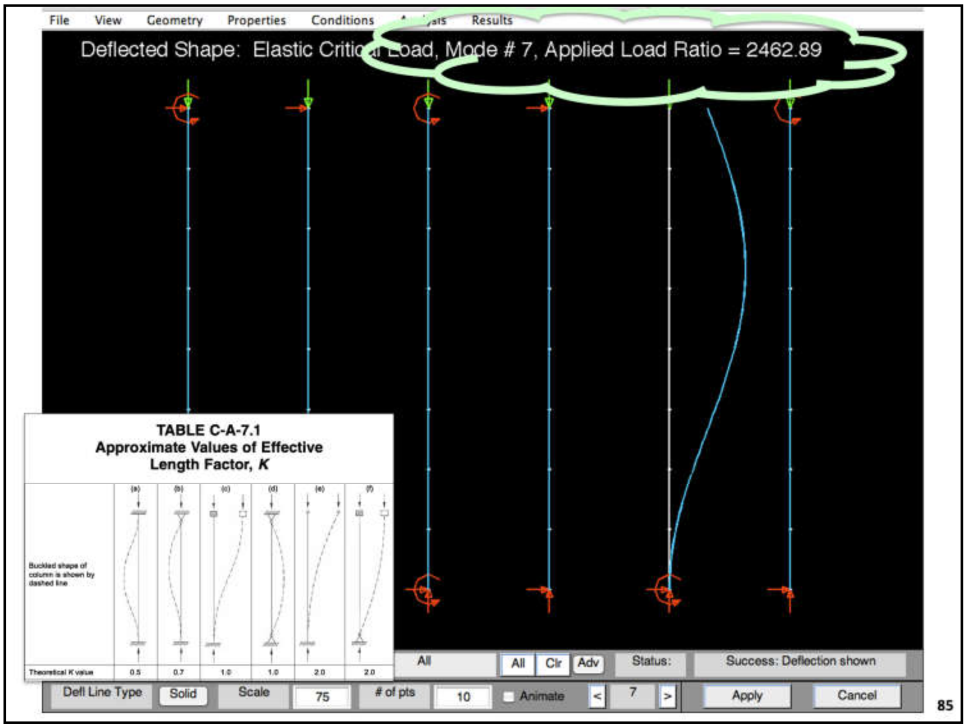

File View Geometry Properties Conditions Results
Deflected Shape: Elastic Critical Load, Mode # 2, Applied Load Ratio = 273.6101
Mode 2:
$P_{cr} = 273.6$
$K = \frac{\pi}{L}\sqrt{\frac{EI}{P_{cr}}} = 2.0$
TABLE C-A-7.1
Approximate Values of Effective Length Factor, K
Buckled shape of column is shown by dashed line
Theoretical K value 0.5 0.7 1.0 1.0 2.0 2.0
All All Clr Adv Status: Success: Deflection shown
Defl Line Type Solid Scale 75 10 Animate 2 Apply Cancel
80

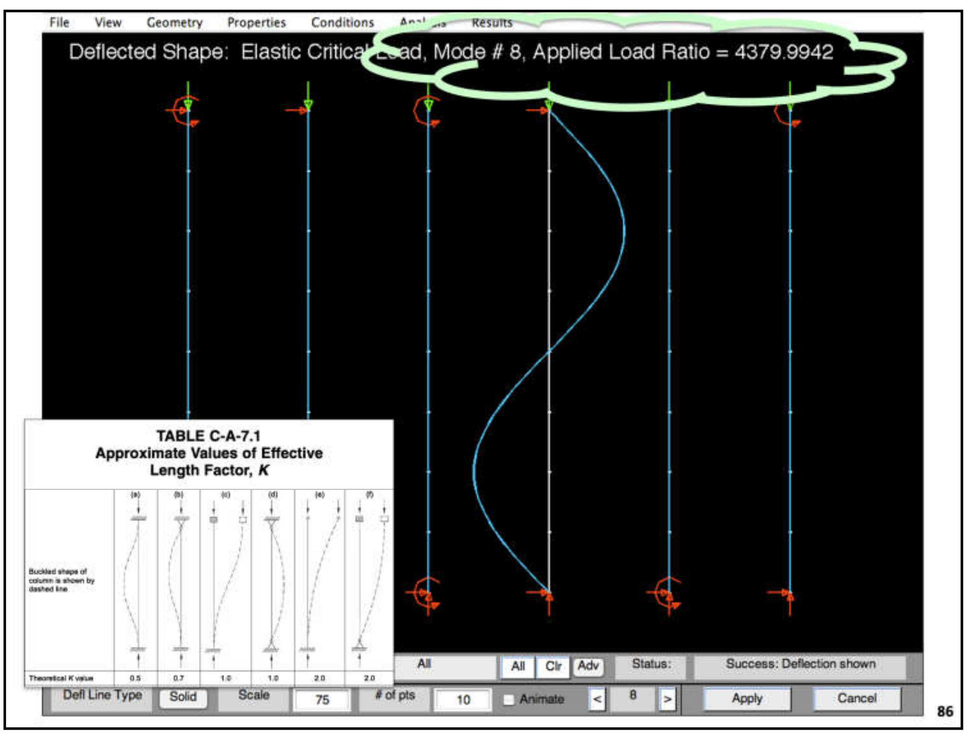

File View Geometry Properties Conditions Results
Deflected Shape: Elastic Critical Load, Mode # 3, Applied Load Ratio = 1094.4739
Mode 3:
$P_{cr} = 1094.5$
$K = \frac{\pi}{L}\sqrt{\frac{EI}{P_{cr}}} = 1.0$
TABLE C-A-7.1
Approximate Values of Effective
Length Factor, K
Buckled shape of column is shown by dashed line
Theoretical K value 0.5 0.7 1.0 1.0 2.0 2.0
All All Clr Adv Status: Success: Deflection shown
Defl Line Type Solid 75 # of pts 10 Animate < 3 > Apply Cancel
81

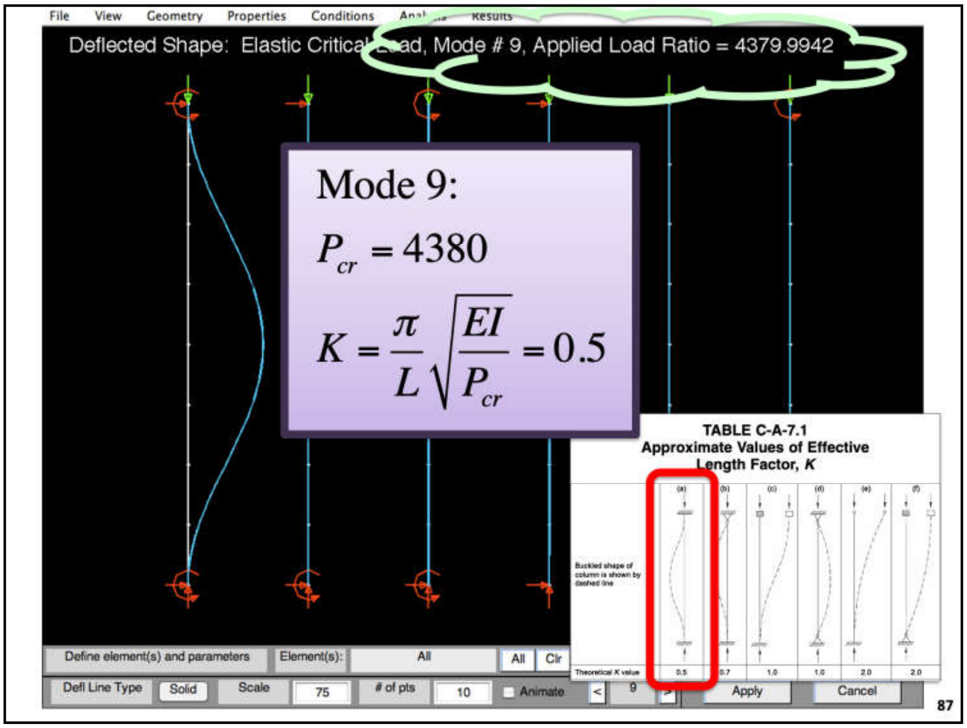

File View Geometry Properties Conditions Results
Deflected Shape: Elastic Critical Load, Mode # 4, Applied Load Ratio = 1094.4739
Mode 4:
$P_{cr} = 1094.5$
$K = \frac{\pi}{L}\sqrt{\frac{EI}{P_{cr}}} = 1.0$
TABLE C-A-7.1
Approximate Values of Effective
Length Factor, K
Buckled shape of column is shown by dashed line
Theoretical K value 0.5 0.7 1.0 1.0 2.0 2.0
All All Clr Adv Status: Success: Deflection shown
Defl Line Type Solid 75 # of pts 10 Animate < 4 > Apply Cancel
82


File View Geometry Properties Conditions
Deflected Shape: Elastic Critical Load, Mode # 5, Applied Load Ratio = 2239.2487
Mode 5:
$P_{cr} = 2239.2$
$K = \frac{\pi}{L}\sqrt{\frac{EI}{P_{cr}}} = 0.7$
C-A-7.1
Approximate Values of Effective
Length Factor, K
Define element(s) and parameters
Element(s):
All
All
Clr
Defl Line Type
Solid
Scale
75
# of pts
10
Animate
5
Apply
Cancel
83


File View Geometry Properties Conditions
Deflected Shape: Elastic Critical Load, Mode # 6, Applied Load Ratio = 2462.89
TABLE C-A-7.1
Approximate Values of Effective
Length Factor, K
All
All
Clr
Adv
Status:
Success: Deflection shown
Defl Line Type
Solid
Scale
75
# of pts
10
Animate
6
Apply
Cancel
84


File View Geometry Properties Conditions Results
Deflected Shape: Elastic Critical Load, Mode # 7, Applied Load Ratio = 2462.89
TABLE C-A-7.1
Approximate Values of Effective
Length Factor, K
Status: Success: Deflection shown
Defl Line Type Solid Scale 75 # of pts 10 Animate 7 Apply Cancel
85


File View Geometry Properties Conditions Results
Deflected Shape: Elastic Critical Load, Mode # 8, Applied Load Ratio = 4379.9942
TABLE C-A-7.1
Approximate Values of Effective
Length Factor, K
Status: Success: Deflection shown
Defl Line Type Solid Scale 75 # of pts 10 Animate 8 Apply Cancel
86



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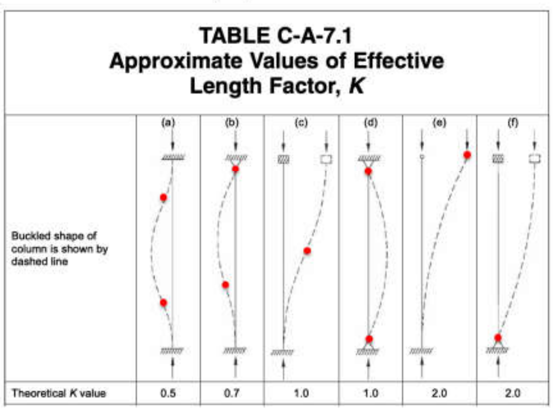
Support Conditions (5)



Elastic Buckling Stress:

$$\sigma_e = \frac{\pi^2 E}{\left(KL/r\right)^2}$$

Notes on "effective length" *KL*:

- Distance between inflection points (M=0)
- Function of degree of column end-restraint
- Degree of column end-restraint can be difficult to compute accurately in real structures (hmmm…)

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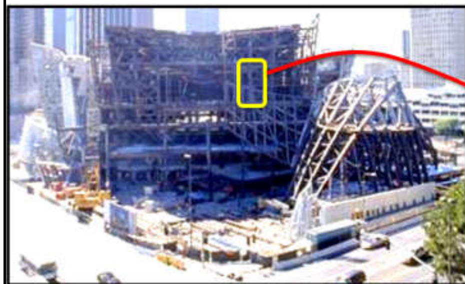
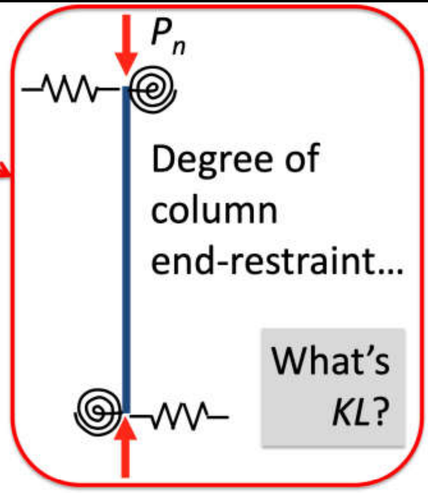
Support Conditions (6)





Possible solutions:

- Diff. Eq./Eigenvalue FEA
- Alignment charts (careful!)
- Direct Analysis Method!

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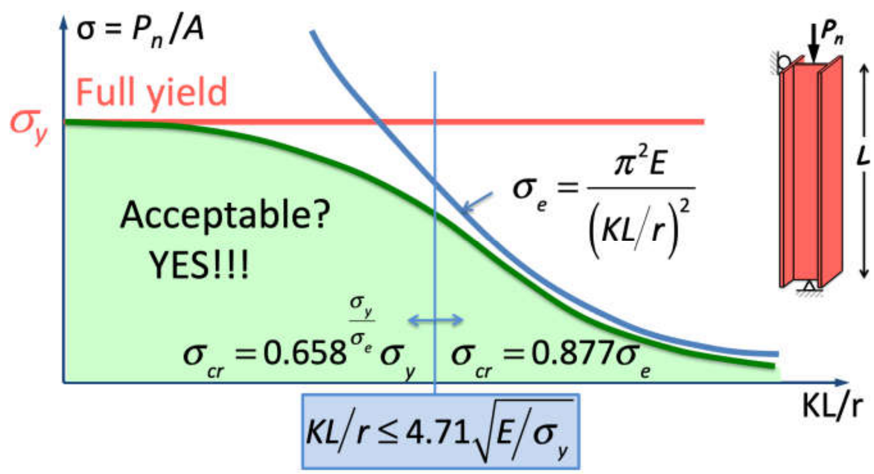
Support Conditions (7)

- Degree of column end restraint accounted for by use of "effective length" KL (i.e., $\sigma_E \rightarrow \sigma_e$)
- AISC Column Curve – Final Take!



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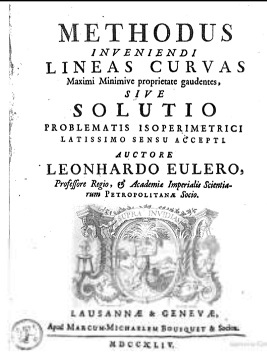
Euler Buckling

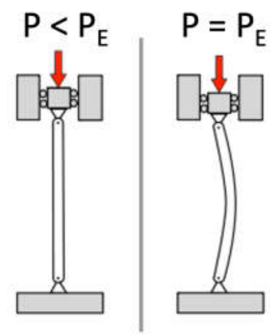
- Leonhard Euler, 1744 and 1757
- Assumptions!
  - prismatic member (I = constant)
  - small deflections after buckling
  - no bending prior to bifurcation
    - perfectly straight
    - concentrically loaded

    Undone
  - linear elastic behavior (E = constant)

    Undone
  - pinned-roller supports (frictionless)

    Undone





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Summary – Compression

- Course introduction and stability concepts
- Limit states of compression members with focus on flexural buckling
- Euler Buckling → Maximum Compressive Strength Column Curve
- Column curve accounts for:
  - full yielding
  - bending due to initial imperfection (out-of-straightness)
  - partial yielding accentuated by presence of residual stresses
  - degree of end restraint

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Summary – Compression (cont.)

- AISC and other column curves
- Other ideas introduced, including
  - moment amplification factor (2nd-order effects)
  - stiffness reduction τ-factor
  - Difficulty in computing K-factors…

- Your virtual laboratory assignment…
  - Try to recreate examples from this lecture
  - Checkout Learning Module 1 (look familiar?)
  - Try to complete a portion of Learning Module 2

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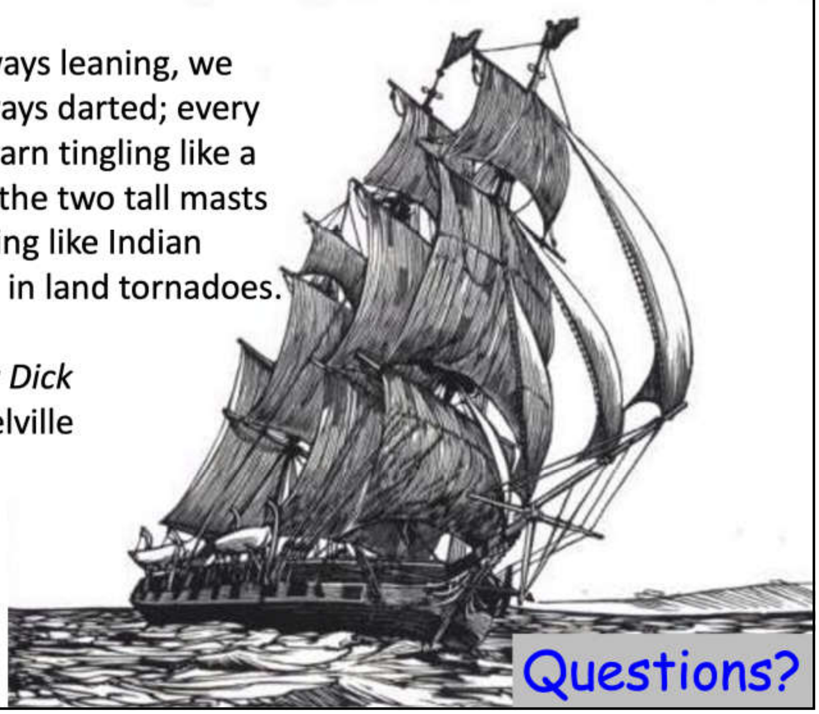
Sideways leaning, we sideways darted; every ropeyarn tingling like a wire; the two tall masts buckling like Indian canes in land tornadoes.

Moby Dick
H. Melville
1851



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Thank You!

Hope you enjoyed this lecture!

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AISC | Questions?

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Stronger.
Steel.

Single-Session Registrants

CEU / PDH Certificates

- You will receive an email on how to report attendance from: registration@aisc.org.
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!

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Single-Session Registrants

CEU / PDH Certificates

- Reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.

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8-Session Registrants

CEU / PDH Certificates

One certificate will be issued at the conclusion of the course.

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8-Session Registrants

Attendance and PDH Certificates

- You have two options to receive credit for a given session.
  - Option 1: Watch the live session. Credit for live attendance will be displayed on the Course Resources table within two days of the session.
  - Option 2: Watch the recording and pass the associated quiz.

Videos and Quizzes

- For each session, find access within two business days after the live air date. (An email will be sent from nightschool@aisc.org.)
- Reasons for quiz:
  - EEU – You must take all quizzes and the final exam to receive EEU.
  - PDHs – If you watch a recorded session, you must pass quiz for PDHs.
  - Reinforce what you learn in the lectures and get more out of the course!

Distribution of Certificates

All certificates will be issued after the course is completed. Only the registrant will receive a certificate for the course.

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8-Session Registrants

Course Resources

Find all your handouts, quizzes and quiz scores, recording access, and attendance information in one place!

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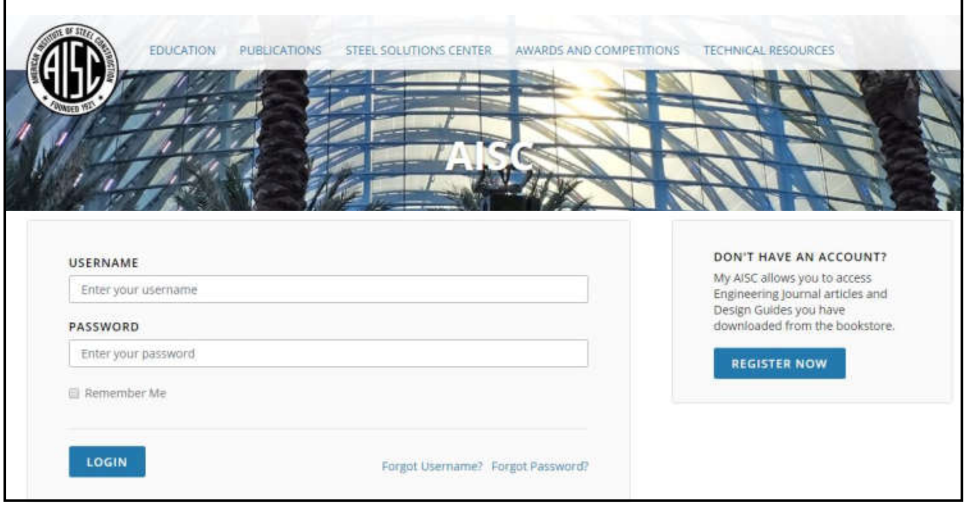
8-Session Registrants

Course Resources

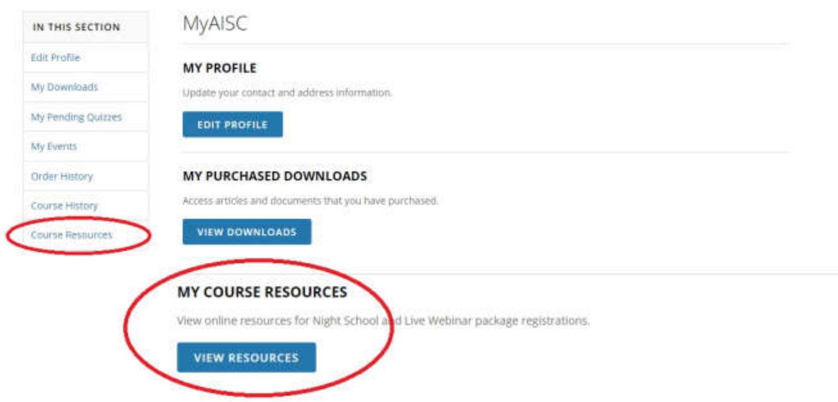
Go to www.aisc.org and sign in.



8-Session Registrants

Course Resources

Go to www.aisc.org and sign in.



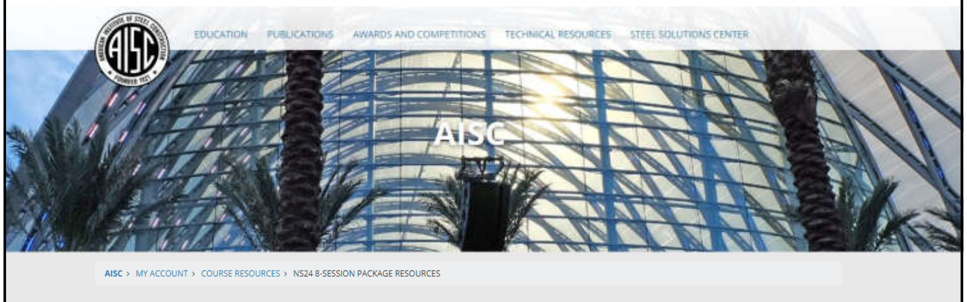
8-Session Registrants

Course Resources

Event	Start Date
Seismic Design in Steel	1/1/1900 12:00:00 AM
4-Session Package-Design of Facade Attachments	5/9/2019 1:30:00 PM
NS 15 8-Session Package-Night School 15 - Fundamentals of Connection Design	10/3/2017 7:00:00 PM
NS 16 8-Session Package-Night School 16 - Seismic Design in Steel	2/5/2018 7:00:00 PM
NS 17 4-Session Package-Night School 17: Design of Facade Attachments	7/16/2018 7:00:00 PM
NS 18 8-Session Package-Night School 18: Steel Construction: Mill To Topping Out	10/15/2018 7:00:00 PM
NS 19 8-Session Package-Night School 19: Connection Design	2/4/2019 7:00:00 PM
NS 20 8-Session Package-Night School 20: Classical Methods of Structural Analysis	6/3/2019 7:00:00 PM
8-Session Package-Seismic Design in Steel - Concepts & Examples	7/16/2018 1:30:00 PM

8-Session Registrants

Course Resources



Night School 24: Modern Methods for Learning Structural Stability

8-SESSION PACKAGE RESOURCES

Event	Date	Handouts	Video	Quiz	Attendance
NS24.1 - Compression Members - The Fundamentals	Oct 6 2020 7:00PM EDT	Handouts	Available 10/08/2020 5:00PM EDT	Available 10/08/2020 5:00PM EDT	Pending
NS24.2 - Compression Members - Practical Considerations	Oct 13 2020 7:00PM EDT	Handouts	Available 10/15/2020 5:00PM EDT	Available 10/15/2020 5:00PM EDT	Pending
NS24.3 - Behavior of Flexural Members - The Fundamentals	Oct 20 2020 7:00PM EDT	Handouts	Available 10/22/2020 5:00PM EDT	Available 10/22/2020 5:00PM EDT	Pending
NS24.4 - Flexural Members - Practical Considerations	Oct 27 2020 7:00PM EDT	Handouts	Available 10/29/2020 5:00PM EDT	Available 10/29/2020 5:00PM EDT	Pending
NS24.5 - Stability of Beam-Columns - The Fundamentals	Nov 10 2020 7:00PM EST	Handouts	Available 11/12/2020 5:00PM EST	No longer available	Pending
NS24.6 - Stability of Beam-Columns - Practical Consideration	Nov 17 2020 7:00PM EST	Handouts	Available 11/19/2020 5:00PM EST	No longer available	Pending
NS24.7 - Behavior of Structural Systems - The Fundamentals	Dec 1 2020 7:00PM EST	Handouts	Available 12/03/2020 5:00PM EST	No longer available	Pending
NS24.8 - Structural Systems - Practical Considerations	Dec 8 2020 7:00PM EST	Handouts	Available 12/10/2020 5:00PM EST	No longer available	Pending
NS24 - Final Exam	N/A			No longer available	

AISC | Thank you.

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