AISC Night School

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We will begin shortly. Please standby.

Modern Methods for Learning the Basics of Structural Stability: From Behavior to Practice

Session 3: Behavior of Flexural Members – The Fundamentals

October 20, 2020

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Course Description

Behavior of Flexural Members – The Fundamentals
October 20, 2020

Using an approach similar to that employed in Session 1, this lecture will provide an overview of the strengths and limitations of the solution to the differential equation that defines the elastic lateral-torsional buckling (LTB) strength of beams. Related flexural and torsional concepts, including the benefits of warping resistance, will be briefly reviewed. The assumption of elastic behavior will then be relaxed to define the inelastic LTB and plastic moment capacities of flexural members. The strength of beams without slender elements will be covered and ultimately presented in the form of beam resistance curves. The speakers will conclude by introducing the second learning module.

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Learning Objectives

- Explain the limit state of full yielding for flexural members.
- Describe lateral-torsional buckling behavior of beams.
- Explain how the length between brace points of the compression flange of a member in flexure affects lateral-torsional buckling.
- Explain the application of the lateral-torsional buckling moment gradient factor, C_b.

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Modern Methods for Learning The Basics of Structural Stability: From Behavior to Practice

Session 3: Behavior of Flexural Members – The Fundamentals
October 20, 2020

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Course Overview

- Topics
  - Compression Members (Weeks 1 & 2)
  - Flexural Members (Weeks 3 & 4)
  - Beam-Columns (Weeks 5 & 6)
  - Systems (Weeks 7 & 8)
- "Active" learning! Weekly virtual lab experiences…
- Case studies from the real world…

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Course Overview (2)

Course Overview (3)

- Focus of the course is on fundamentals!
- Better understanding of behavior will result in improved design
- Key Definitions
  - **Stability:** Under load, component returns to current state after applying a small disturbance such as a deflection
  - **Bifurcation (critical load)**: Theoretical point at which loading a component results in an instantaneous change from current state to significant deflection – two options: not buckled or buckled
  - **Instability:** Loading a component results in a realistic transition from small deflection to significant deflection – buckling preceded by deflection

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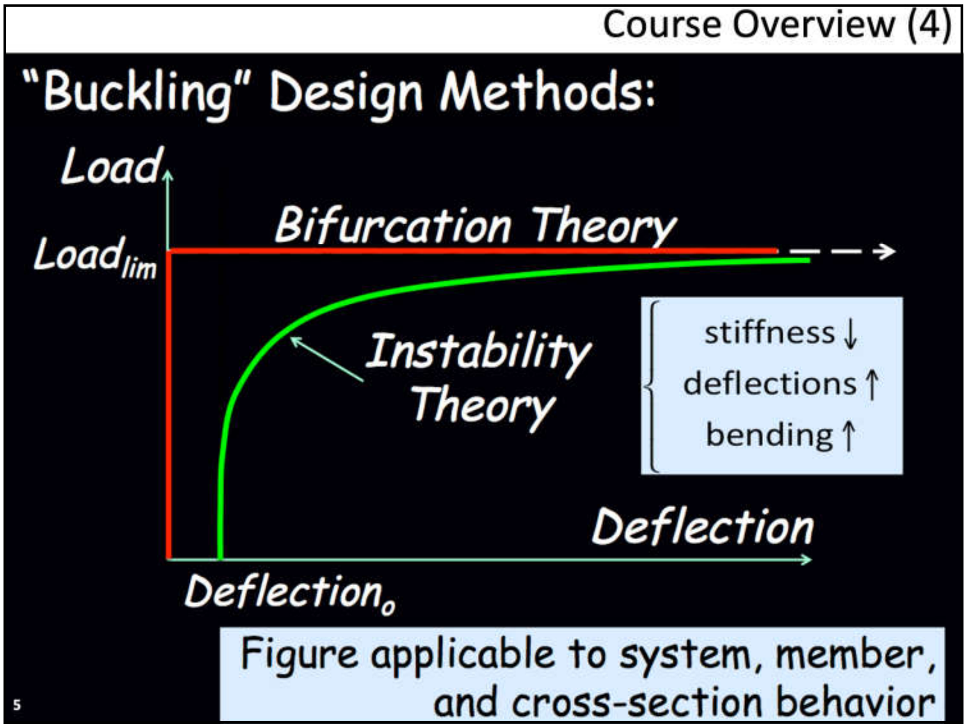
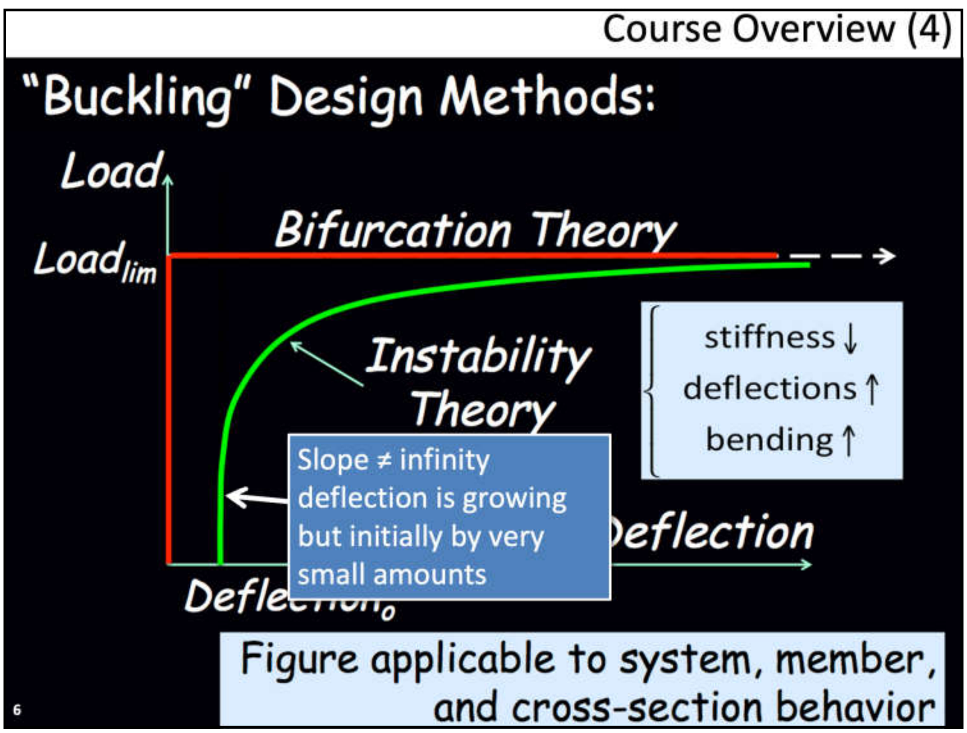
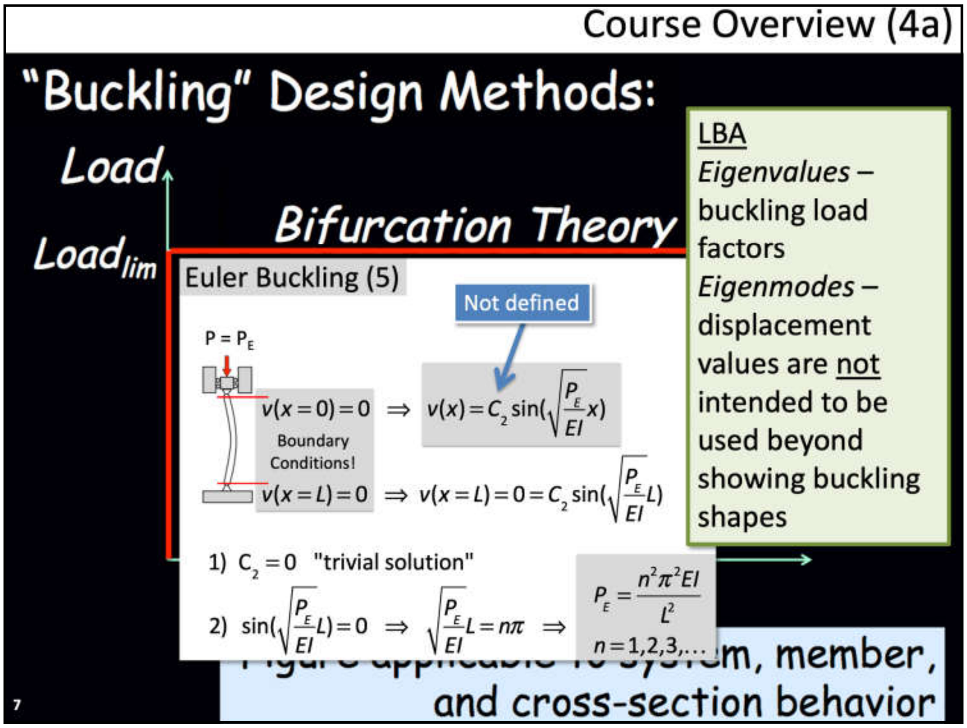

Course Overview (4)
"Buckling" Design Methods:
Load
Bifurcation Theory
Load$_{lim}$
Instability Theory
stiffness ↓
deflections ↑
bending ↑
Deflection
Deflection$_o$
Figure applicable to system, member, and cross-section behavior
5


Course Overview (4)
"Buckling" Design Methods:
Load
Bifurcation Theory
Load$_{lim}$
Instability Theory
stiffness ↓
deflections ↑
bending ↑
Slope ≠ infinity
deflection is growing
but initially by very
small amounts
Figure applicable to system, member, and cross-section behavior
6



Course Overview (5)

Analysis acronyms:

LBA: linear buckling analysis; elastic critical load analysis; elastic eigenvalue analysis; assumes bifurcation theory

GNA: geometric nonlinear analysis; 2nd-order elastic analysis; assumes equilibrium on the deformed shape and linear elastic material, with no initial imperfections

GNIA: same as GNA, but includes initial imperfections

MNA: material nonlinear analysis; 1st-order inelastic analysis; assumes equilibrium on the undeformed shape and accounts for yielding, with no initial imperfections

GMNIA: geometric and material nonlinear analysis; 2nd-order inelastic analysis; assumes equilibrium on the deformed shape, accounts for yielding, and includes initial imperfections

hmmm....one resource that may be of use is the textbook available at www.mastan2.com

Modern Methods for Learning The Basics of Structural Stability: From Behavior to Practice

Course Introduction

Compression Members

Flexural Members

Beam-Columns

Systems

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Limit States of Flexural Members

- Full yielding (today!)
- Instability
  - Along the member length (today!)
    - Lateral torsional buckling
      - elastic
      - inelastic
  - At the cross section
    - local buckling

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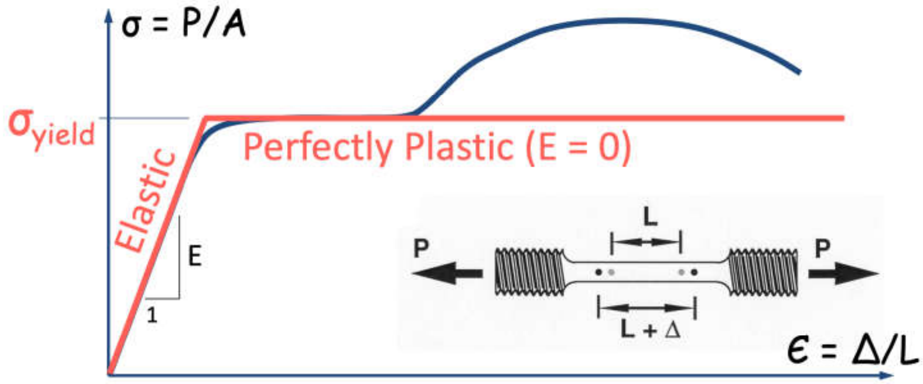
Full Yielding

- Tensile test



- Assume same response for compression
  - $\sigma_{y,compression} = \sigma_{y,tension} = \sigma_{yield}$
  - Neglect strain hardening (assume elastic-plastic)

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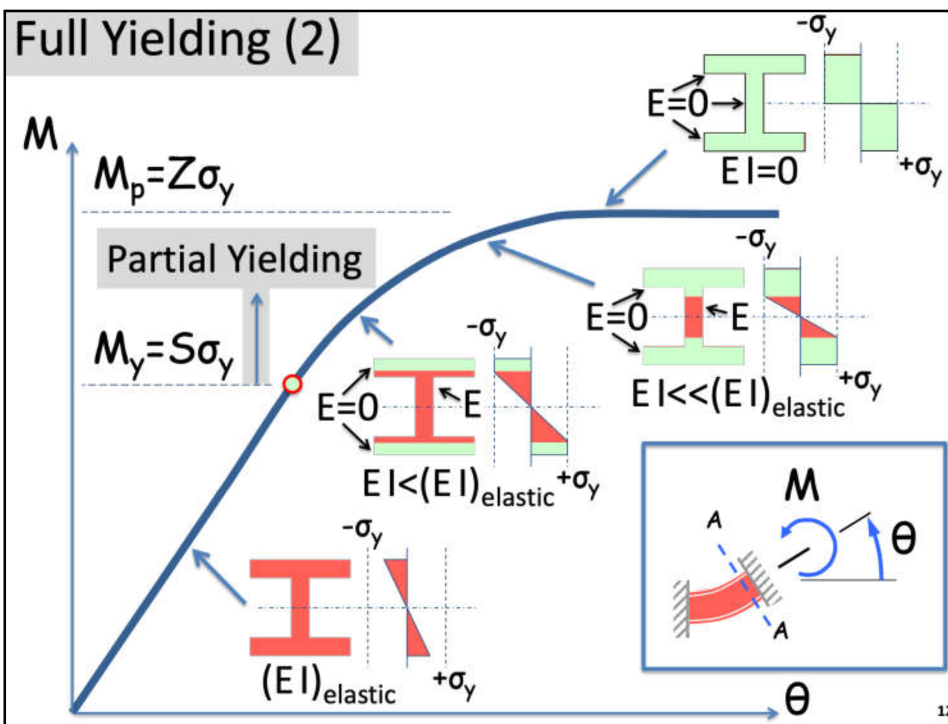
Full Yielding (2)



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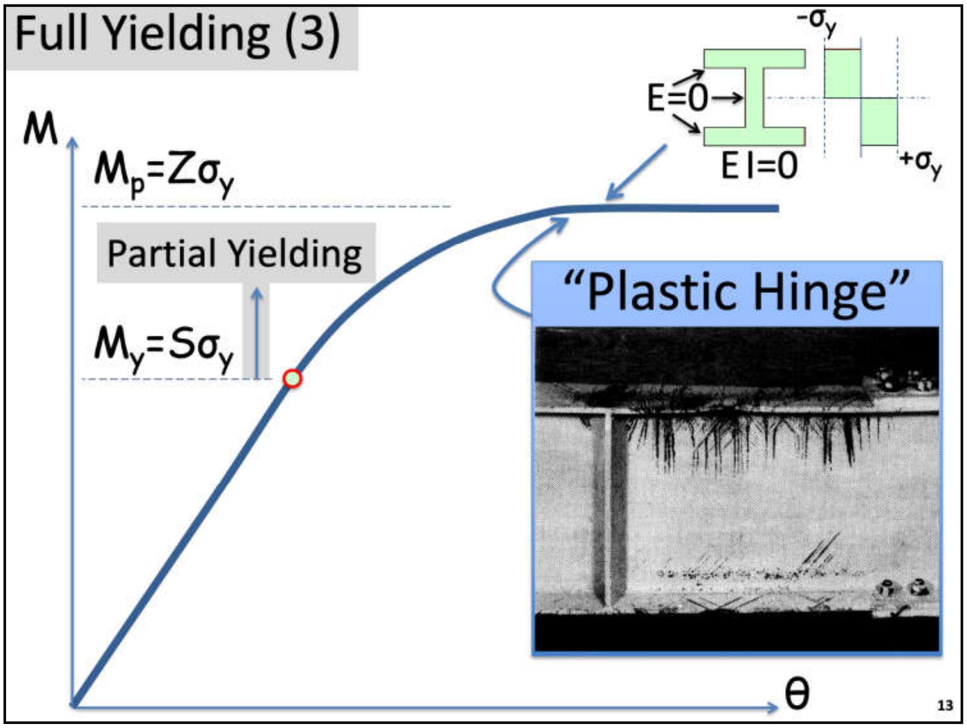
Full Yielding (3)
M
M_p=Zσ_y
Partial Yielding
M_y=Sσ_y
E=0
E I=0
-σ_y
+σ_y
"Plastic Hinge"
θ
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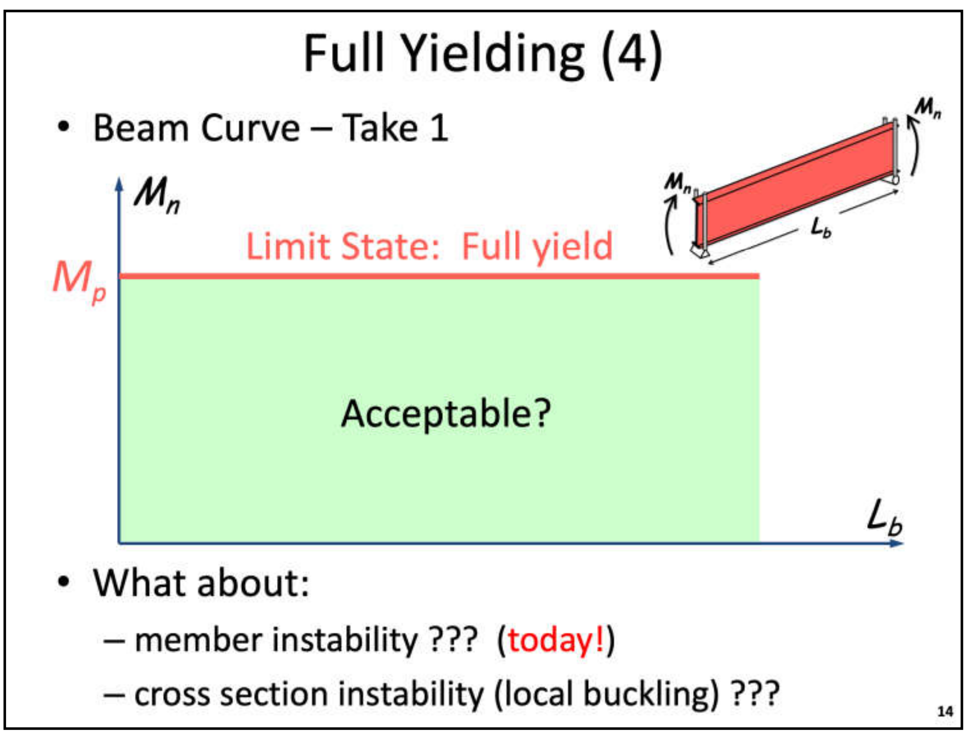
Full Yielding (4)
• Beam Curve – Take 1
M_n
M_p
Limit State: Full yield
Acceptable?
L_b
• What about:
– member instability ??? (today!)
– cross section instability (local buckling) ???
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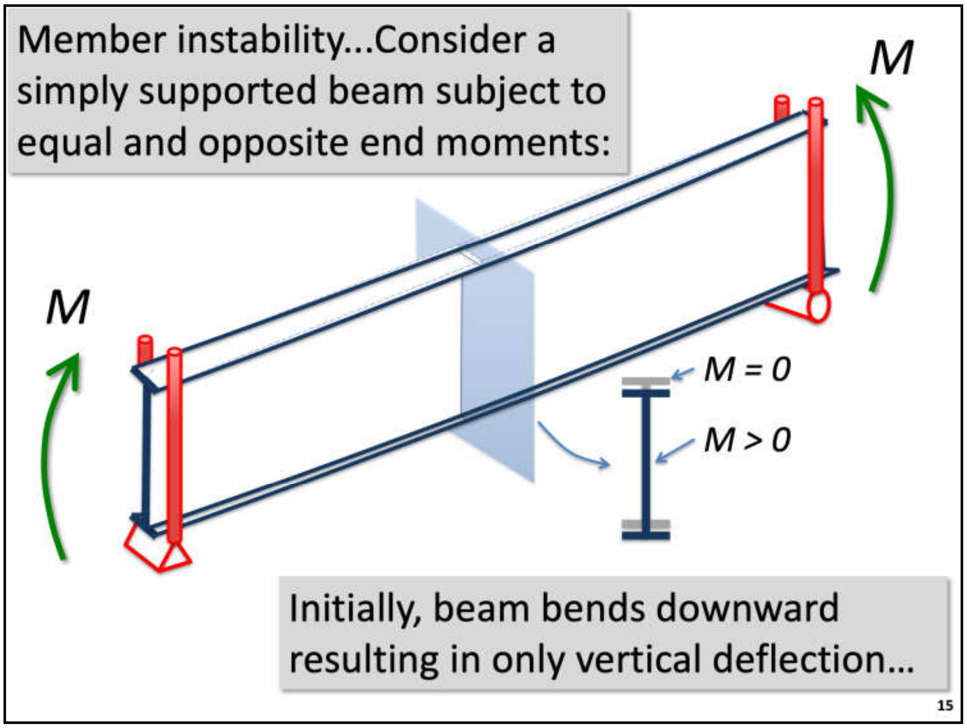
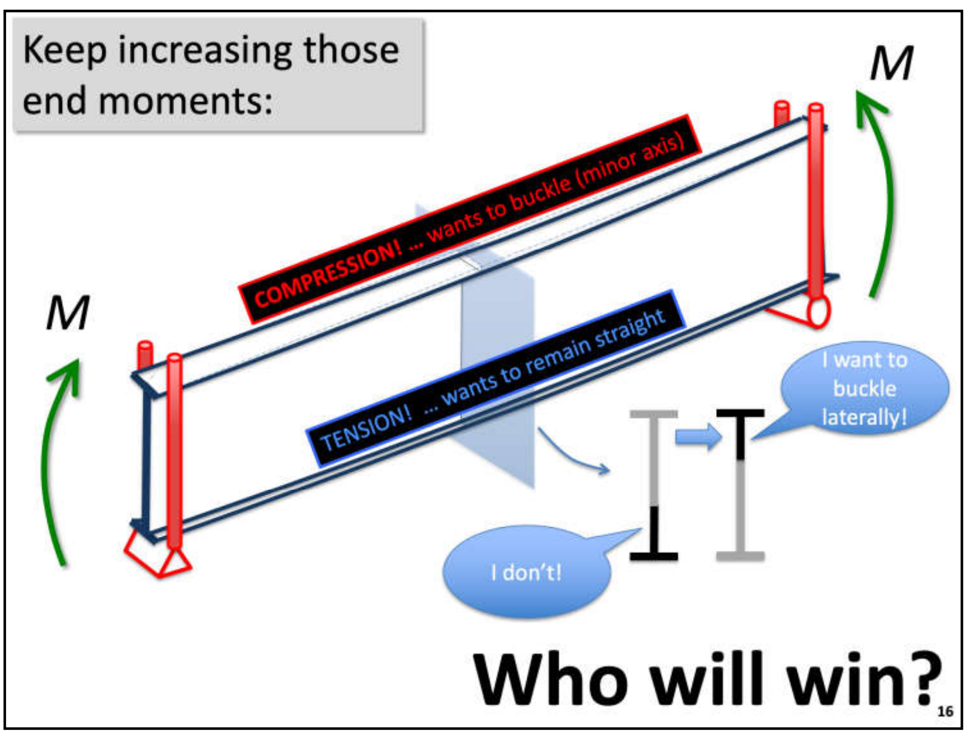

Member instability...Consider a simply supported beam subject to equal and opposite end moments:
M
M
M = 0
M > 0
Initially, beam bends downward resulting in only vertical deflection...
15


Keep increasing those end moments:
M
M
COMPRESSION! ... wants to buckle (minor axis)
TENSION! ... wants to remain straight
I want to buckle laterally!
I don't!
Who will win?
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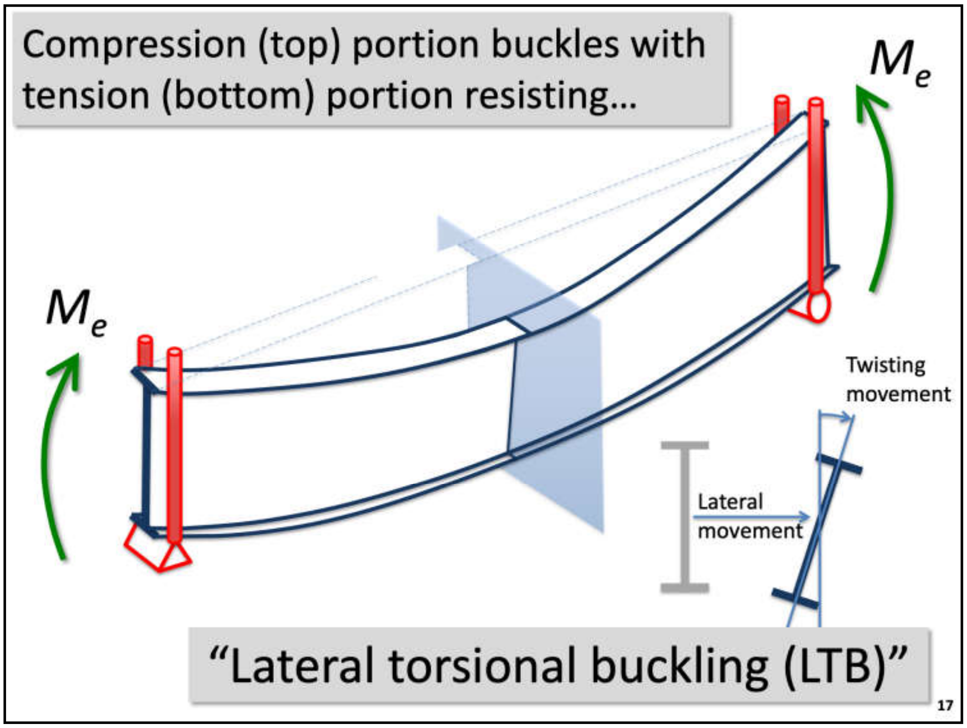
Compression (top) portion buckles with
tension (bottom) portion resisting…
M_e
M_e
Twisting
movement
Lateral
movement
"Lateral torsional buckling (LTB)"
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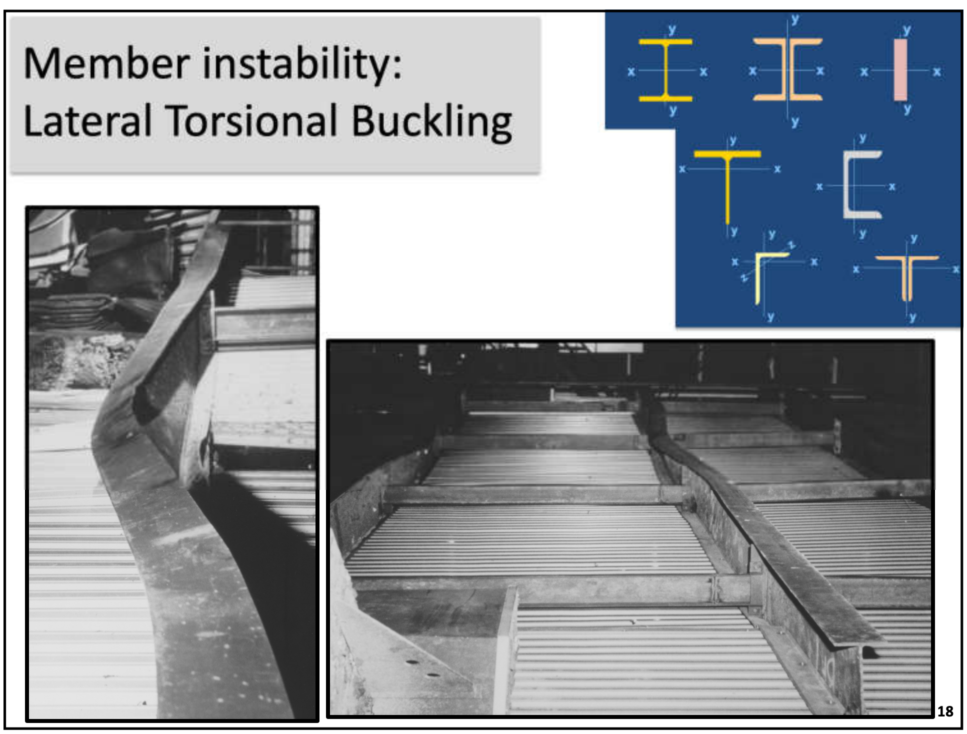
Member instability:
Lateral Torsional Buckling
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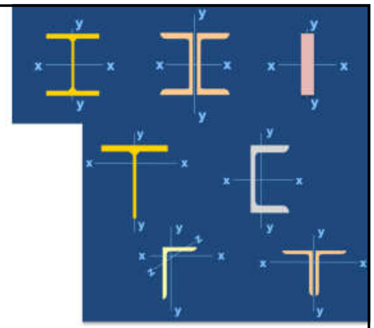
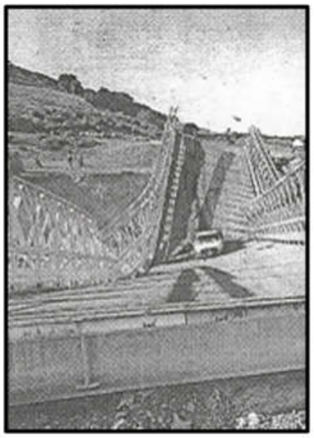
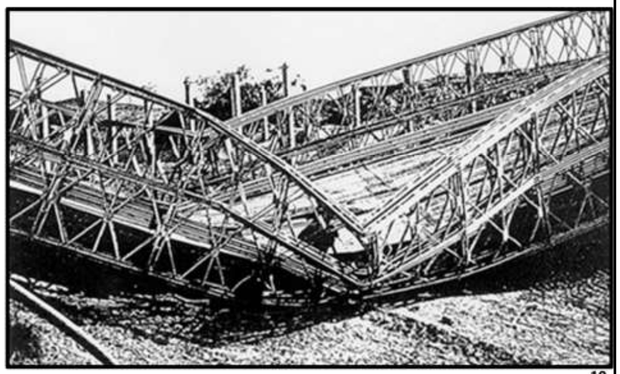
Member instability:
Lateral Torsional Buckling







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Lateral Torsional Buckling

- Theoretical bifurcation
  - solution
  - assumptions
- Undoing those assumptions (approaching reality)
  - not fully elastic, partial yielding
  - alternative loading and support conditions
- Beam curves
  - AISC
  - others

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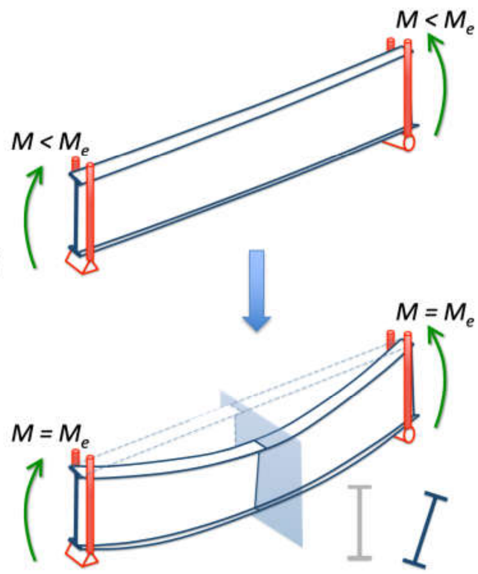
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



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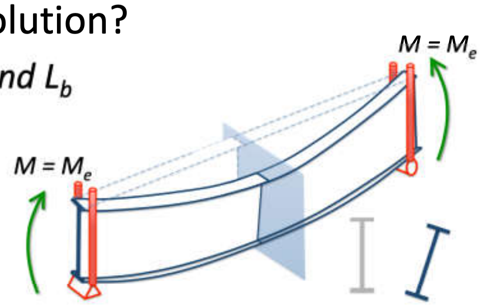
LTB (2)

- Before obtaining the theoretical solution for M_e, let's do a "parametric" analysis…
- Terms expected in the solution?
  - Minor axis buckling: EI_y and L_b
  - Torsion
    - St. Venant: GJ and L_b
    - Warping: EC_w and L_b
  - Others? π (of course!)



- What's their impact?

Terms in numerator → Material: $E\uparrow, G\uparrow \Rightarrow M_e\uparrow$

Terms in numerator → Section: $I_y\uparrow, J\uparrow, C_w\uparrow \Rightarrow M_e\uparrow$

Term in denominator → Unbraced length: $L_b\uparrow \Rightarrow M_e\downarrow$

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Wait…

- Minor axis buckling, I recall from earlier slides

$$P_E = \frac{\pi^2 EI}{L^2}$$

- But, I need a quick refresher on torsion!

St. Venant ?
Warping ????

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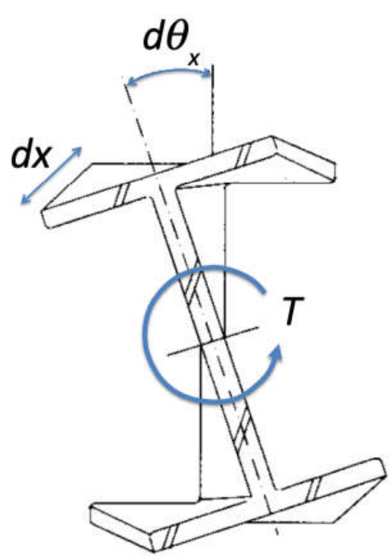
St. Venant Torsion

Consider a portion of the member of length *dx* subject to a torque *T*. If we consider only St. Venant (uniform) torsion, the rotation per unit length is:

$$\frac{d\theta_x}{dx} = \frac{T}{GJ}$$



$\theta = TL/GJ$

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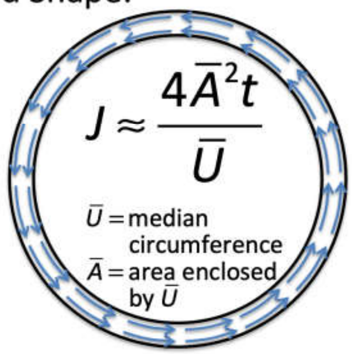
St. Venant Torsion (2)

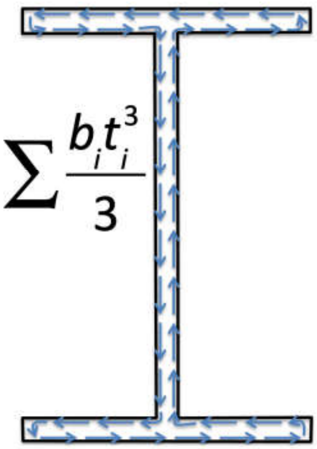
Closed Shape:



$$J \approx \frac{4\bar{A}^2 t}{\bar{U}}$$

$\bar{U}$ = median circumference
$\bar{A}$ = area enclosed by $\bar{U}$

Open Shape:



$$J \approx \sum \frac{b_i t_i^3}{3}$$

Circular Hollow Shape:
t = 0.25", A = 3.84 in^2
$D = A/(\pi t)$ = 4.90"

$$J = \frac{4(\pi \bar{D}^2/4)^2 t}{\pi \bar{D}} = 22.95 \text{ in}^4$$

W8x13 (t_w=0.23", t_f=0.26"):
A = 3.84 in^2
J = 0.0871 in^4

Factor of 264…closed sections rule in torsion!

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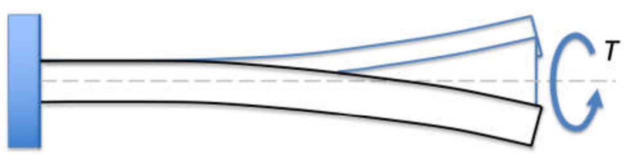
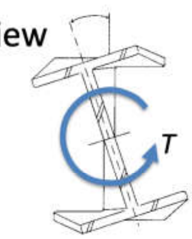
Warping Torsion (your new best friend!)

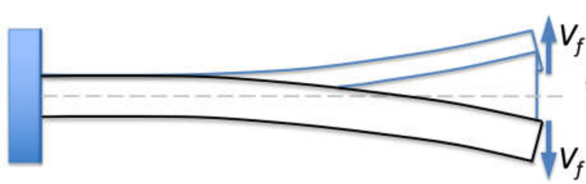
Top View



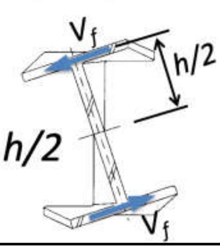
End View



Notice that this torque T causes the flanges to bend in opposite directions. The resistance to this "cross flange" bending opposes the applied torque (yeah!)



$T \approx 2V_f h/2$



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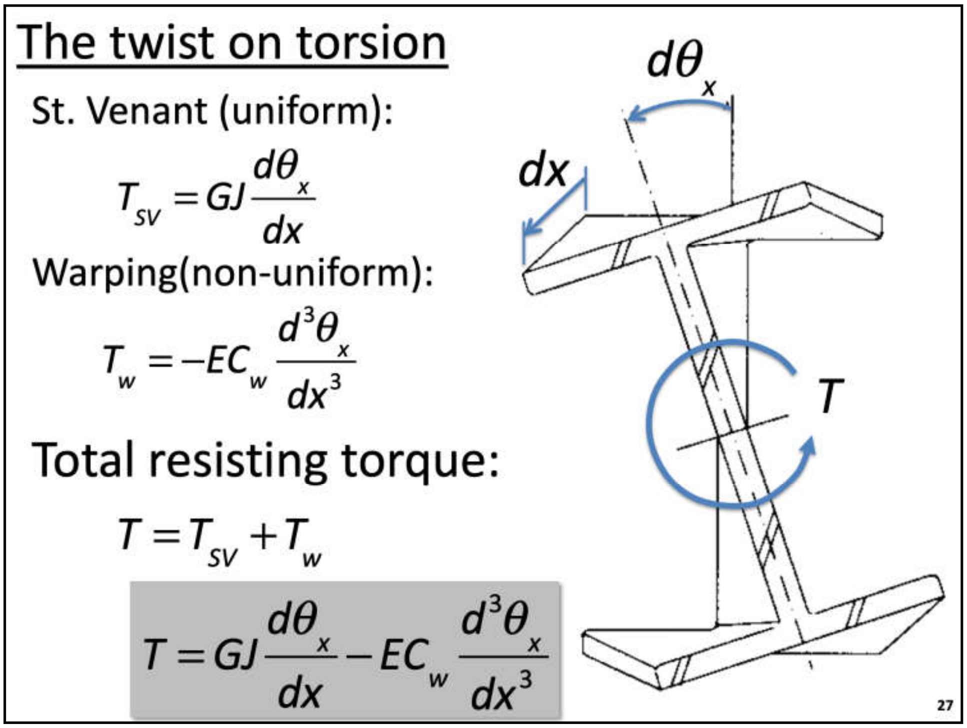
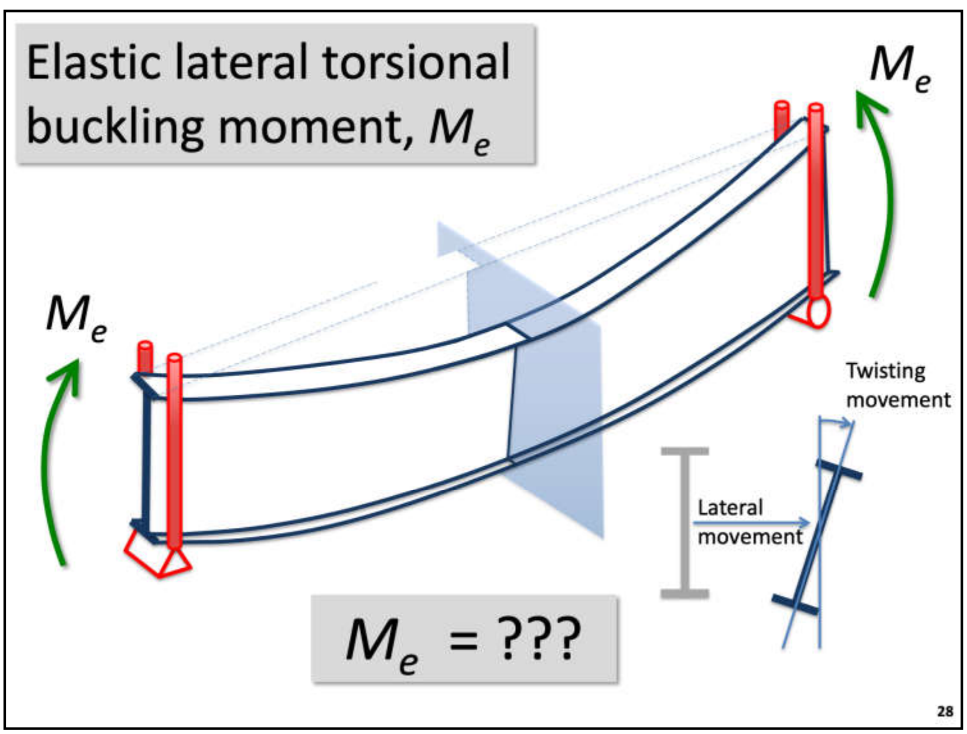

The twist on torsion
St. Venant (uniform):
$T_{SV} = GJ\frac{d\theta_x}{dx}$
Warping(non-uniform):
$T_w = -EC_w\frac{d^3\theta_x}{dx^3}$
Total resisting torque:
$T = T_{SV} + T_w$
$T = GJ\frac{d\theta_x}{dx} - EC_w\frac{d^3\theta_x}{dx^3}$
$d\theta_x$
dx
T
27


Elastic lateral torsional buckling moment, M_e
M_e
M_e
Twisting movement
Lateral movement
M_e = ???
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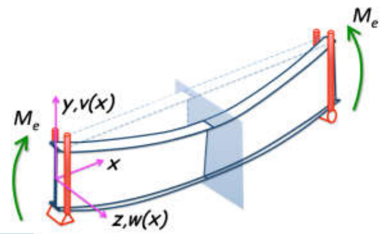
Summary



Equilibrium on deformed shape:

"applied" torque — "resisting" torque

$$T = M_e \frac{dw}{dx} = GJ\frac{d\theta_x}{dx} - EC_w \frac{d^3\theta_x}{dx^3}$$

$$\frac{d}{dx}\left(M_e \frac{dw}{dx}\right) = \frac{d}{dx}\left(GJ\frac{d\theta_x}{dx} - EC_w \frac{d^3\theta_x}{dx^3}\right)$$

$$M_e \frac{d^2w}{dx^2} = GJ\frac{d^2\theta_x}{dx^2} - EC_w \frac{d^4\theta_x}{dx^4}$$

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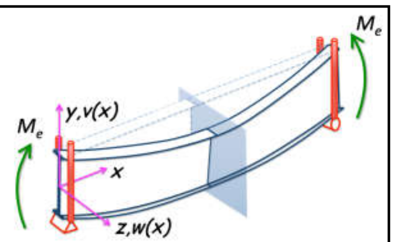
Summary (2)



$$M_e \frac{d^2w}{dx^2} = GJ\frac{d^2\theta_x}{dx^2} - EC_w \frac{d^4\theta_x}{dx^4}$$

$$M_{y'} = M_e\theta_x = -EI_{y'}\frac{d^2w}{dx^2} \quad \Rightarrow \quad \frac{d^2w}{dx^2} = -\frac{M_e}{EI_{y'}}\theta_x$$

$$-\frac{M_e^2}{EI_{y'}}\theta_x = GJ\frac{d^2\theta_x}{dx^2} - EC_w \frac{d^4\theta_x}{dx^4}$$

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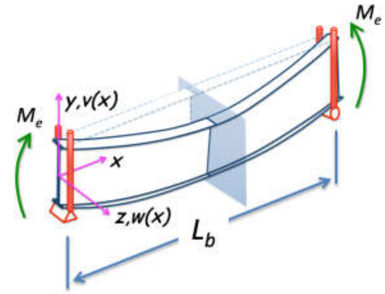
Summary (3)

Solve differential equation



$$EC_w \frac{d^4\theta_x}{dx^4} - GJ\frac{d^2\theta_x}{dx^2} - \frac{M_e^2}{EI_{y'}}\theta_x = 0$$

and apply boundary conditions

$$\theta_x(x=0)=0,\ \theta_x(x=L_b)=0$$

$$\theta_x^{"}(x=0)=0,\ \theta_x^{"}(x=L_b)=0$$

Results in

$$M_e^2 = \left(\frac{\pi^2 EI_y}{L_b^2}\right)\left(GJ + \frac{\pi^2}{L_b^2}EC_w\right)$$

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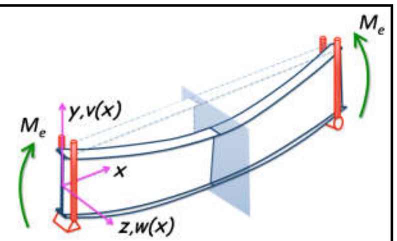
Summary (4)

This sort of makes sense!



$$M_e^2 = \left(\frac{\pi^2 EI_y}{L_b^2}\right)\left(GJ + \frac{\pi^2}{L_b^2}EC_w\right)$$

Top flange in compression trying to produce minor axis buckling

Bottom flange in tension resisting this minor axis buckling by creating a resisting torque, which includes both St. Venant and Warping components

which simplifies to:

$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

Also note that our earlier parametric study was spot on!

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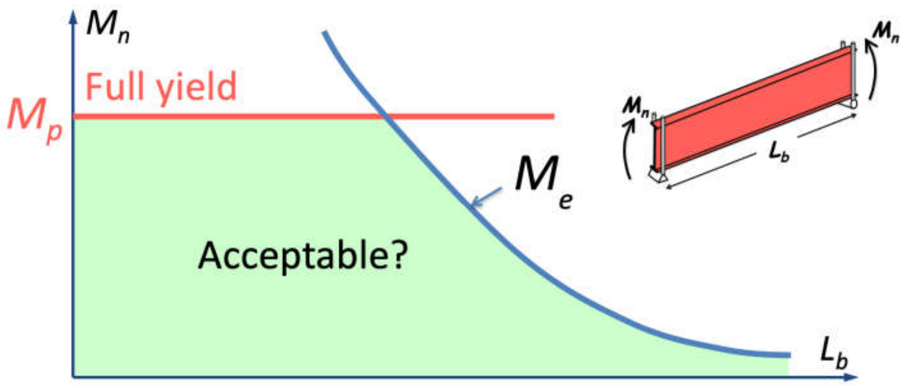
- Elastic lateral-torsional buckling

$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

- Beam Curve – Take 2



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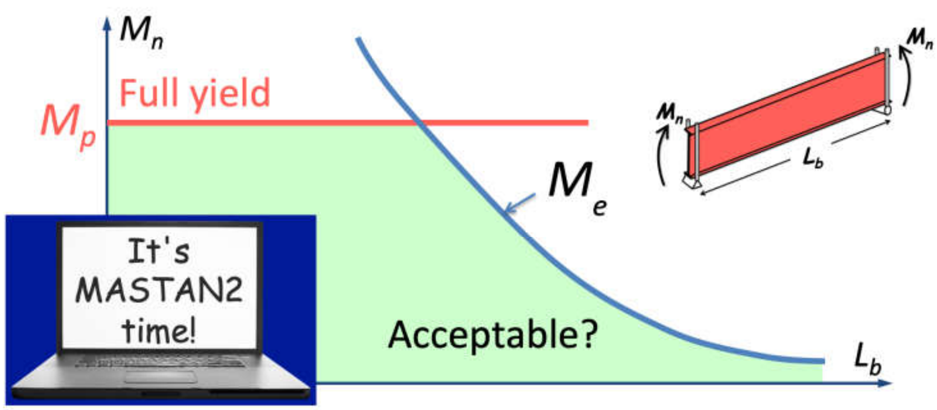

W24x68 (E=29,000, F_y = 50, Z=177, include warping)

L_b	$M_{n,LBA}$	$M_{n,GNA}$	$M_{n,MNA}$	$M_{n,min}$
100				
300				

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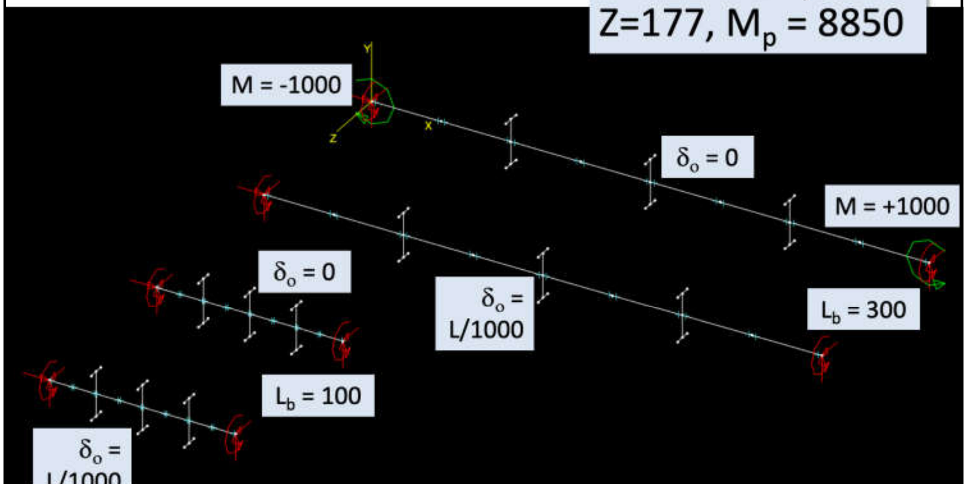
NS24_L3_Example_1.mat

W24x68
E=29,000, F_y = 50
Z=177, M_p = 8850



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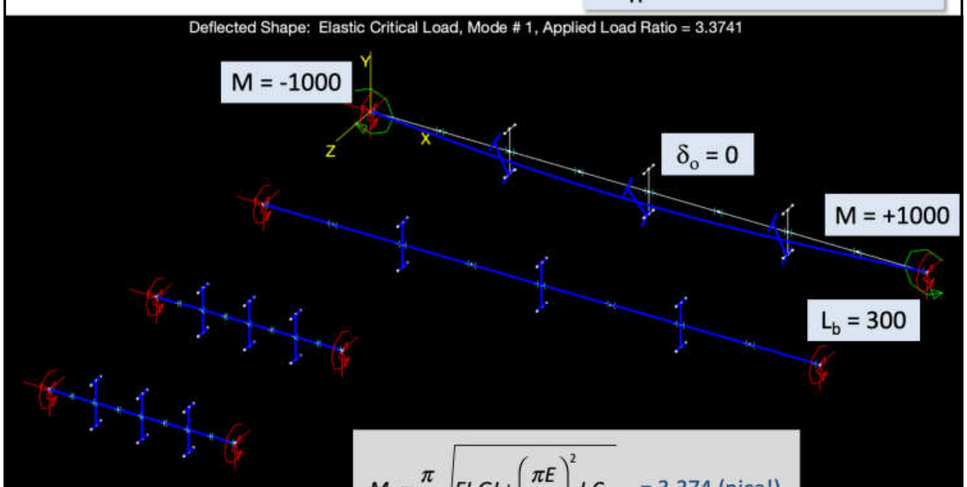
NS24_L3_Example_1.mat

<u>LBA</u>
M_n = 3.374 x 1000
M_n = 3374



$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$ = 3,374 (nice!)

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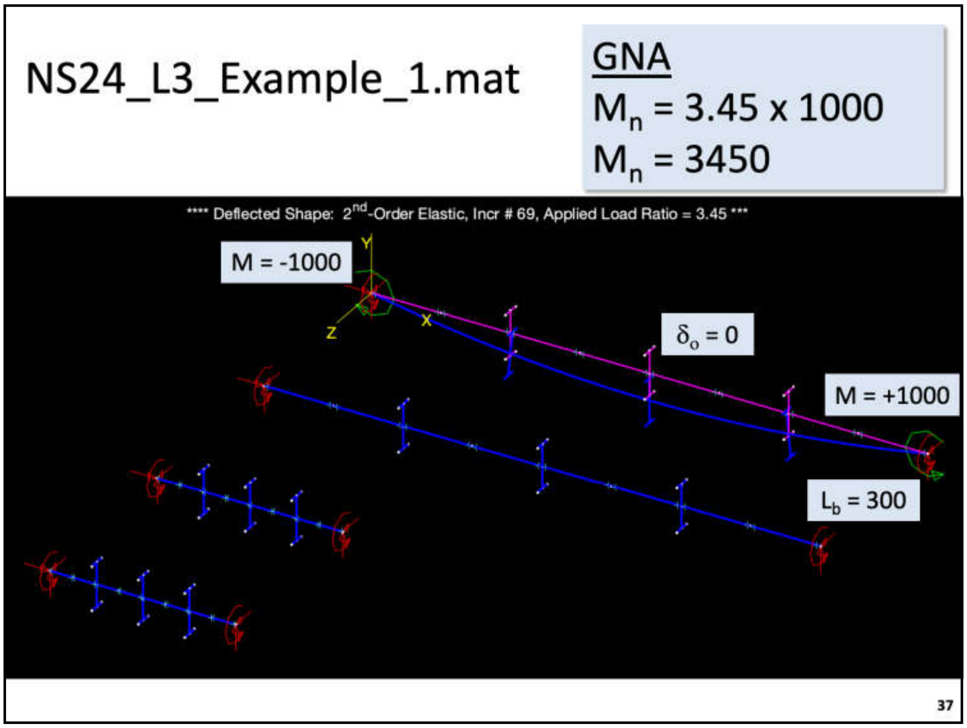
NS24_L3_Example_1.mat
GNA
Mn = 3.45 x 1000
Mn = 3450
**** Deflected Shape: 2nd-Order Elastic, Incr # 69, Applied Load Ratio = 3.45 ***
M = -1000
δo = 0
M = +1000
Lb = 300
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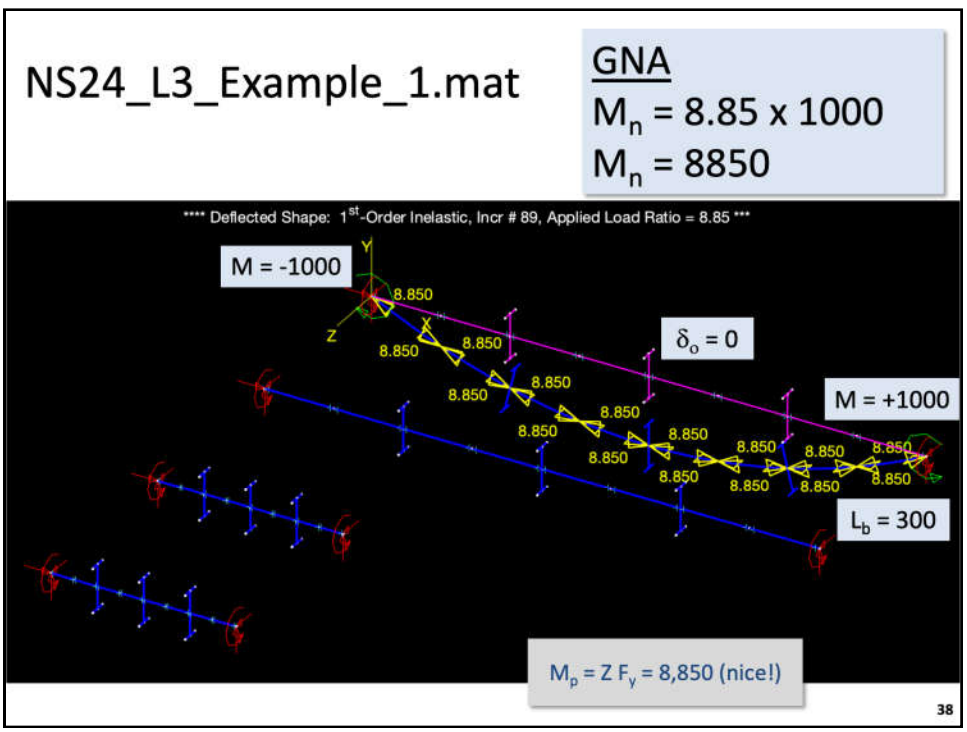
NS24_L3_Example_1.mat
GNA
Mn = 8.85 x 1000
Mn = 8850
**** Deflected Shape: 1st-Order Inelastic, Incr # 89, Applied Load Ratio = 8.85 ***
M = -1000
δo = 0
M = +1000
Lb = 300
8.850
Mp = Z Fy = 8,850 (nice!)
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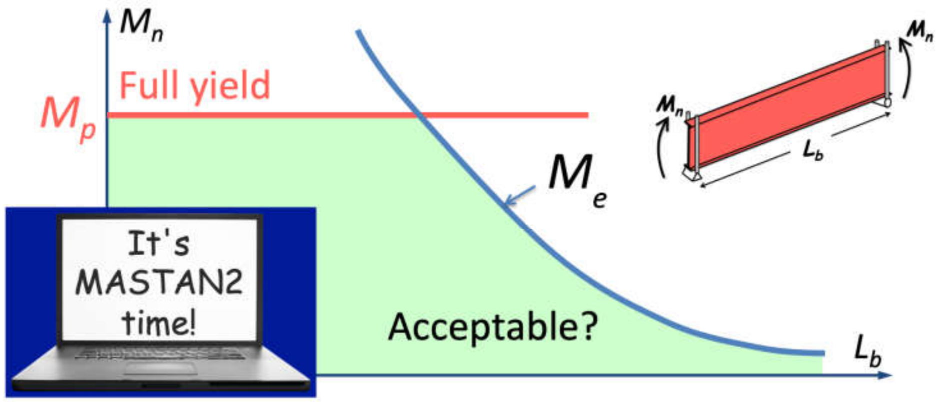

W24x68 (E=29,000, F_y = 50, Z=177, include warping)

L_b	$M_{n,LBA}$	$M_{n,GNA}$	$M_{n,MNA}$	$M_{n,min}$
100	24206	24600	8850	8850
300	3374	3450	8850	3374

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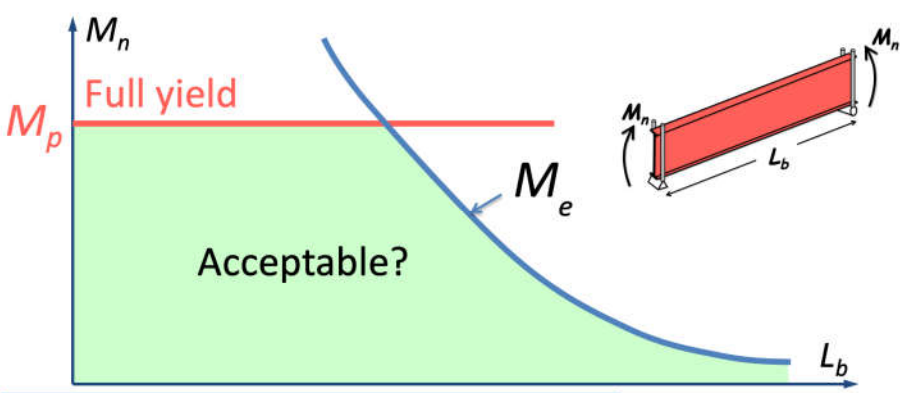
- Elastic lateral-torsional buckling

$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

- Beam Curve – Take 2



- What about those assumptions?

40

Lateral Torsional Buckling

- Theoretical bifurcation
  - solution
  - assumptions
- Undoing those assumptions (approaching reality)
  - not fully elastic, partial yielding
  - alternative loading and support conditions
- Beam curves
  - AISC
  - others

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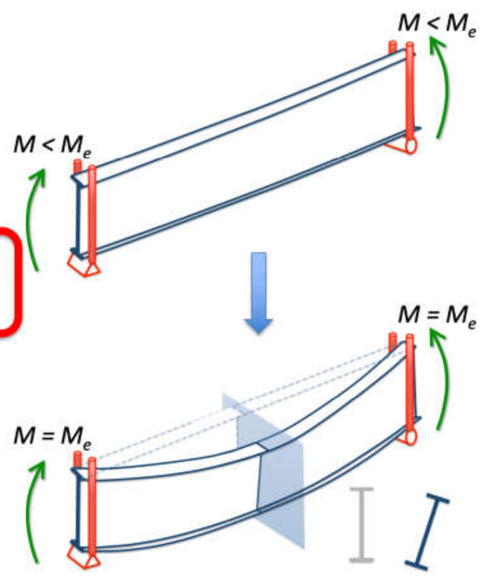
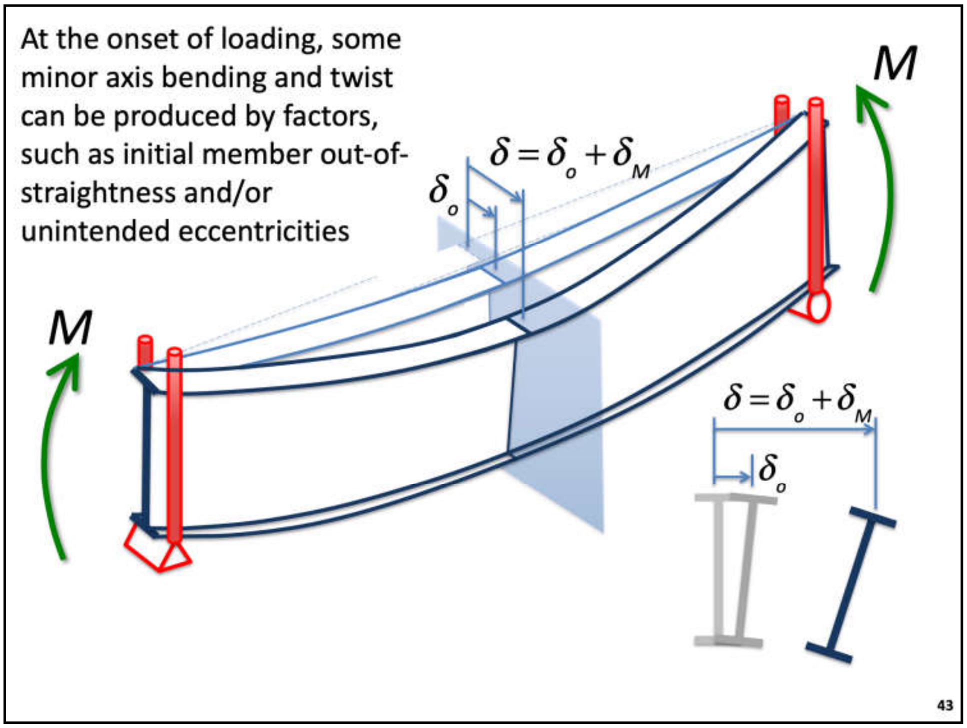
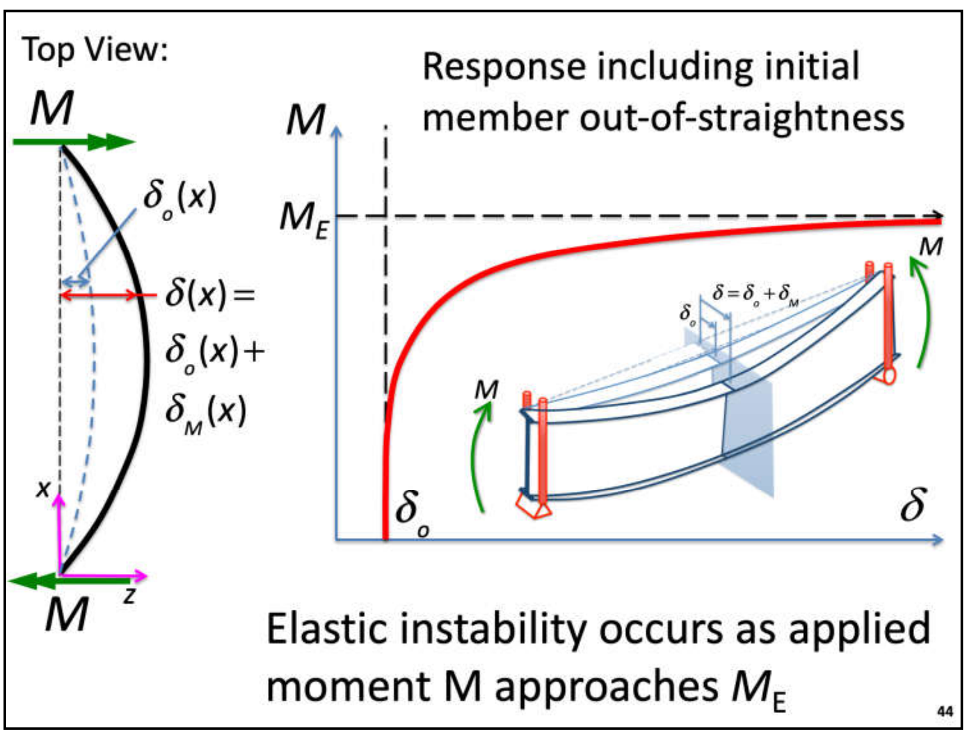
Lateral Torsional Buckling (LTB)

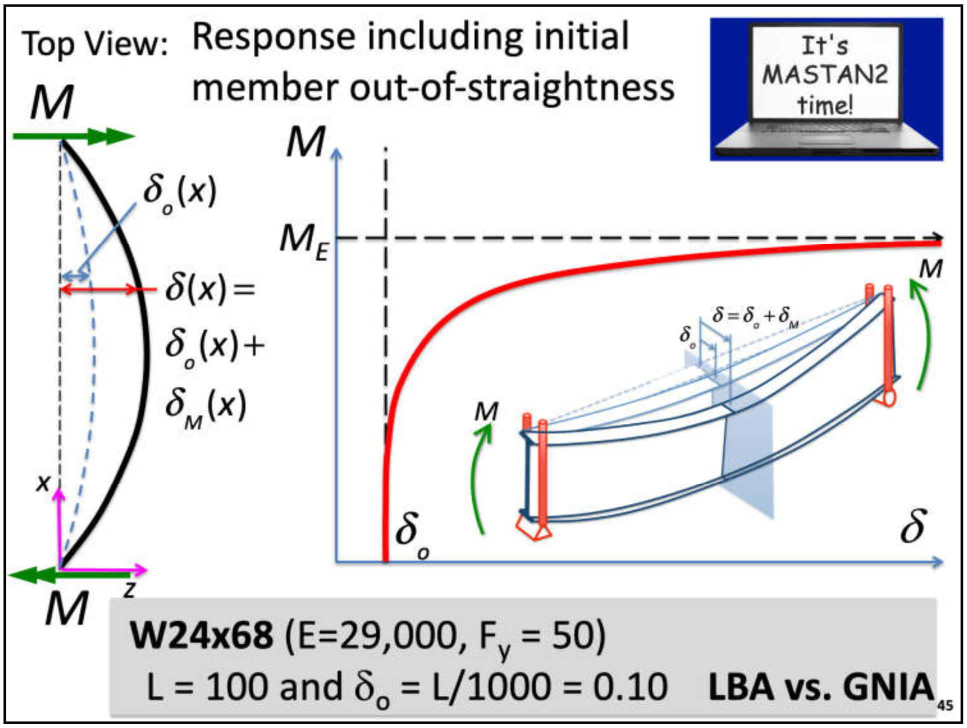
- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)

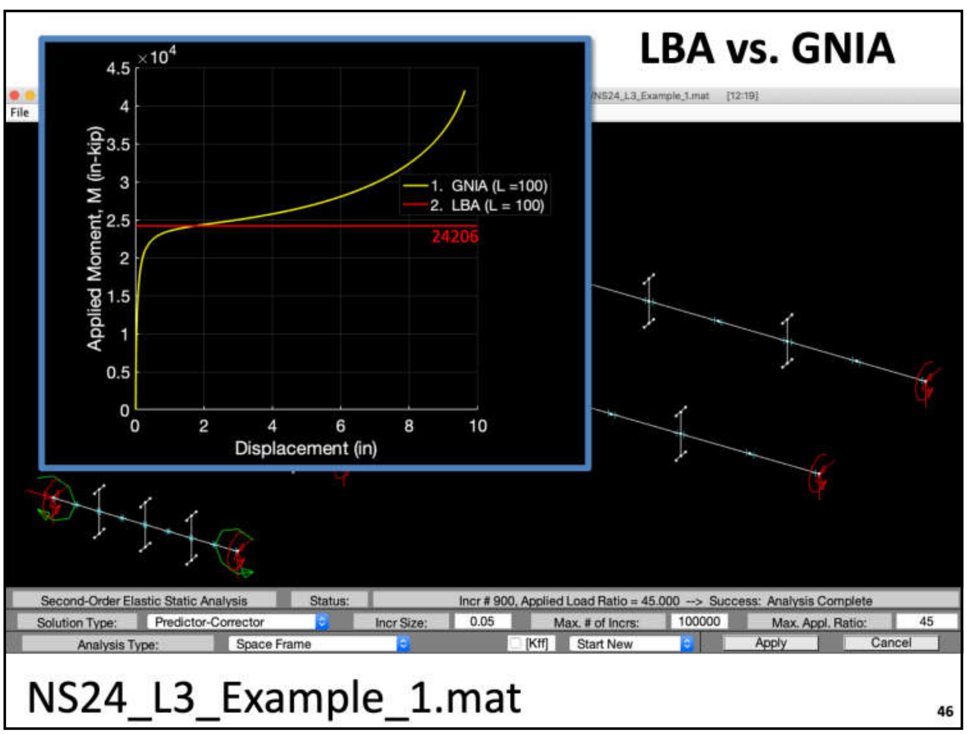


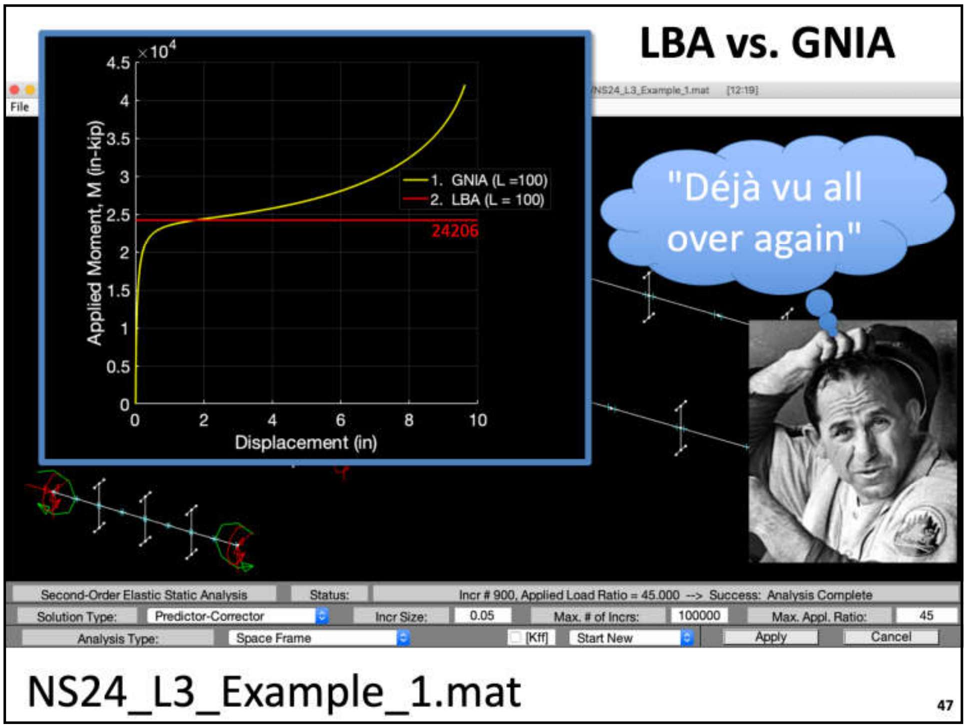
42


At the onset of loading, some minor axis bending and twist can be produced by factors, such as initial member out-of-straightness and/or unintended eccentricities
M
M
$\delta = \delta_o + \delta_M$
δ_o
$\delta = \delta_o + \delta_M$
δ_o
43


Top View:
M
$\delta_o(x)$
$\delta(x) = \delta_o(x) + \delta_M(x)$
x
z
M
Response including initial member out-of-straightness
M
M_E
δ_o
δ
Elastic instability occurs as applied moment M approaches M_E
44

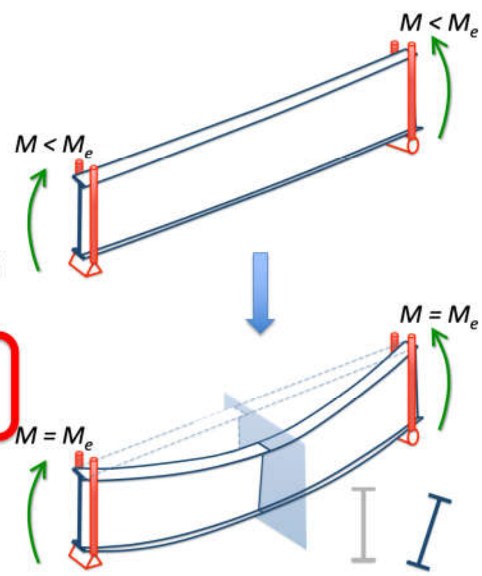

Top View:
Response including initial member out-of-straightness
It's MASTAN2 time!
M
$\delta_o(x)$
$\delta(x) = \delta_o(x) + \delta_M(x)$
M_E
$\delta = \delta_o + \delta_M$
δ_o
δ
x
z
W24x68 (E=29,000, F_y = 50)
L = 100 and δ_o = L/1000 = 0.10
LBA vs. GNIA
45


LBA vs. GNIA
$\times 10^4$
Applied Moment, M (in-kip)
Displacement (in)
1. GNIA (L =100)
2. LBA (L = 100)
24206
File
Second-Order Elastic Static Analysis
Status:
Incr # 900, Applied Load Ratio = 45.000 --> Success: Analysis Complete
Solution Type:
Predictor-Corrector
Incr Size:
0.05
Max. # of Incrs:
100000
Max. Appl. Ratio:
45
Analysis Type:
Space Frame
[Kff]
Start New
Apply
Cancel
NS24_L3_Example_1.mat
46



Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



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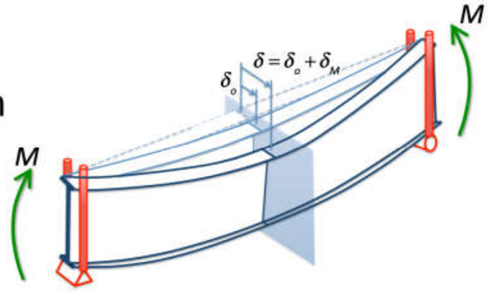
Partial Yielding

- As loading is applied, cross section may begin to yield due to
  - major axis bending
  - minor axis bending
  - torsion (warping stresses)



- Yielding is accentuated by presence of residual stresses
- Yielding results in loss of stiffness, which may result in inelastic lateral torsional buckling.

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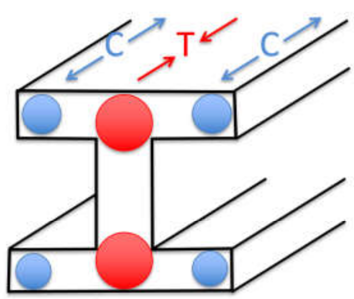
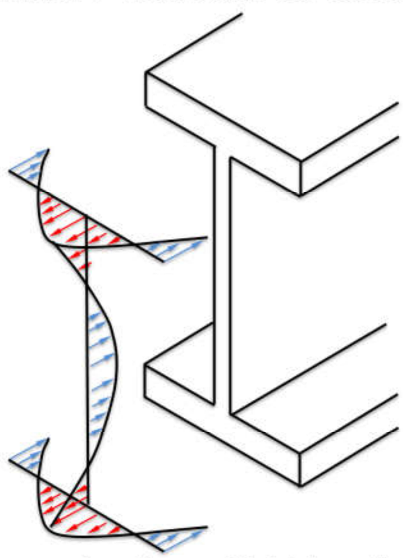
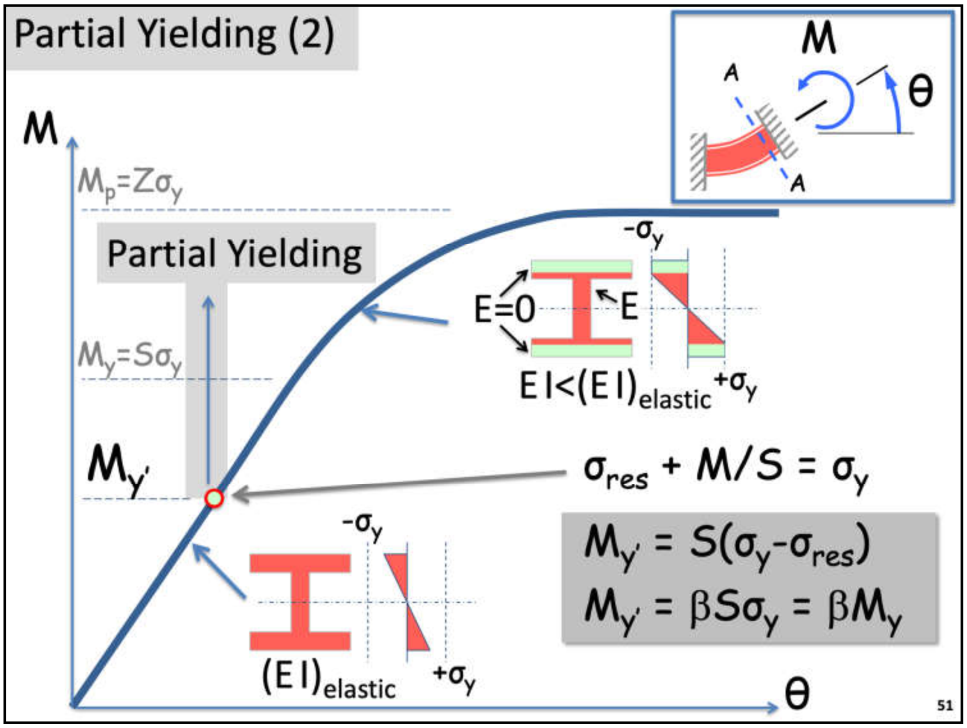
Reminder: Residual Stresses

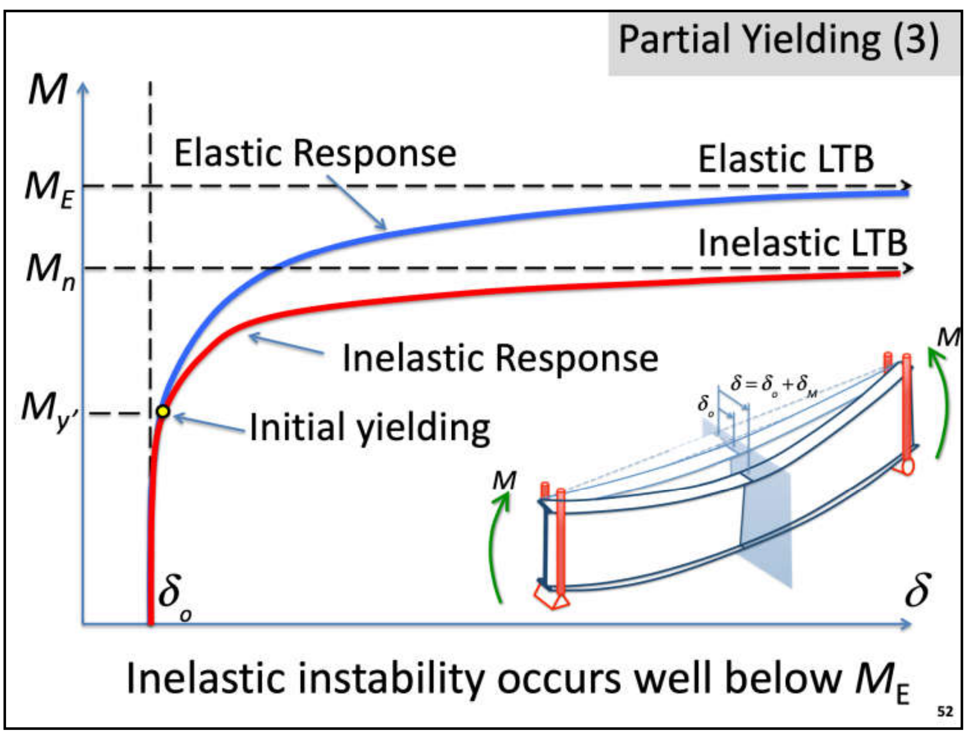




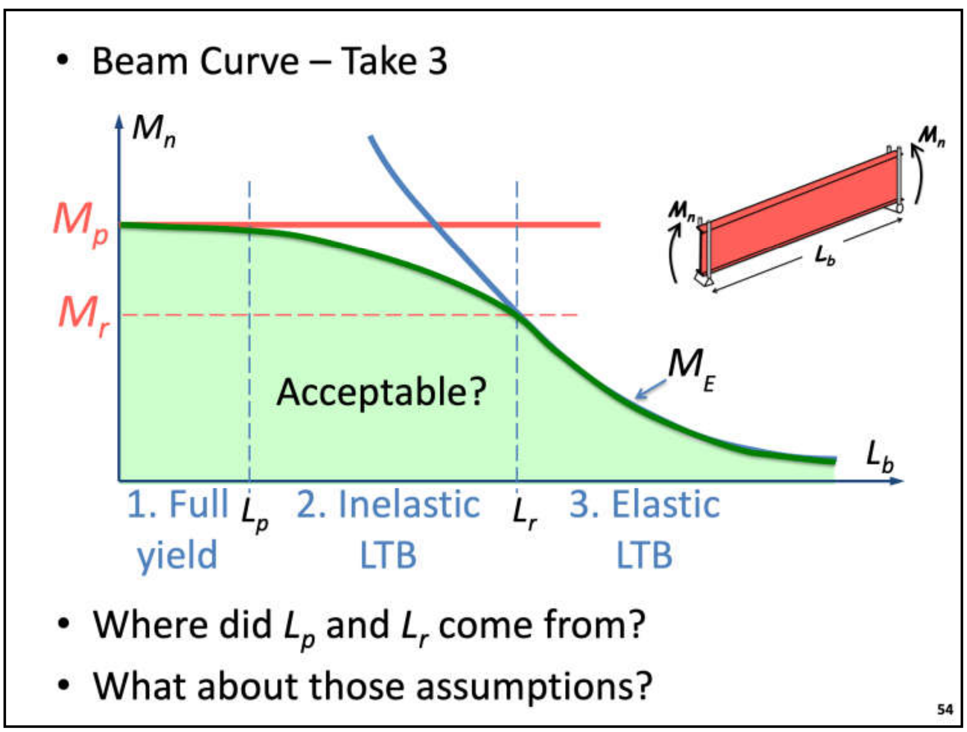
Occur in rolled wide flange shapes because locations with high surface area (e.g., flange tips) cool well before locations with smaller surface area (flange-to-web intersections)

50


Partial Yielding (2)
M
M
A
A
θ
$M_p = Z\sigma_y$
Partial Yielding
$M_y = S\sigma_y$
$-\sigma_y$
E=0
E
$EI<(EI)_{elastic}$
$+\sigma_y$
$M_{y'}$
$\sigma_{res} + M/S = \sigma_y$
$M_{y'} = S(\sigma_y - \sigma_{res})$
$M_{y'} = \beta S\sigma_y = \beta M_y$
$-\sigma_y$
$(EI)_{elastic}$
$+\sigma_y$
θ
51


Partial Yielding (3)
M
Elastic Response
Elastic LTB
M_E
Inelastic LTB
M_n
Inelastic Response
$M_{y'}$
Initial yielding
$\delta = \delta_o + \delta_M$
δ_o
M
M
δ_o
δ
Inelastic instability occurs well below M_E
52


Progress of the test
Force in kN
Vertical displacement in mm
Horizontal displacement in mm
1:28 / 3:31
Lateral Torsional Buckling Test
26,053 views • Jan 18, 2017
223
2
SHARE
SAVE
Civil Engineering RWTH Aachen University
5.29K subscribers
https://www.youtube.com/watch?v=8vb-duSRZAI


• Beam Curve – Take 3
M_n
M_p
M_r
Acceptable?
M_E
L_b
1. Full yield
L_p
2. Inelastic LTB
L_r
3. Elastic LTB
• Where did L_p and L_r come from?
• What about those assumptions?
54

- Where did L_r come from?
  - L_r is unbraced length at transition from inelastic to elastic LTB
  - Equate M_r (moment to produce first yield including residual stresses) with M_e (elastic LTB moment)…and solve for L_b

$$S(\sigma_y - \sigma_{res}) = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$$

Specification for Structural Steel Buildings, July 7, 2016
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

L_r, the limiting unbraced length for the limit state of inelastic lateral-torsional buckling, in. (mm), is:

$$L_r = 1.95r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_xh_o} + \sqrt{\left(\frac{Jc}{S_xh_o}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}} \quad \text{(F2-6)}$$

where

r_y = radius of gyration about y-axis, in. (mm)

$$r_{ts}^2 = \frac{\sqrt{I_yC_w}}{S_x} \quad \text{(F2-7)}$$

Yikes!!!

55

- Where did L_p come from?
  - L_p is unbraced length at transition from full yielding to inelastic LTB
  - Game of darts in an AISC cigar filled room… …not quite!
  - Varies from code to code and is based on analytical and experimental studies
  - For a compact I-shaped member, AISC gives

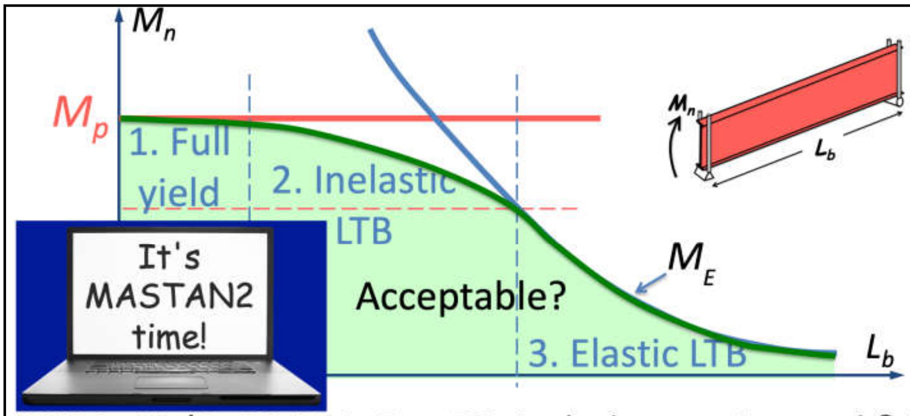
$$L_p = 1.76r_y\sqrt{E/F_y}$$

Specification for Structural Steel Buildings, July 7, 2016
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

L_p, the limiting laterally unbraced length for the limit state of yielding, in. (mm), is:

$$L_p = 1.76r_y\sqrt{\frac{E}{F_y}} \quad \text{(F2-5)}$$

56



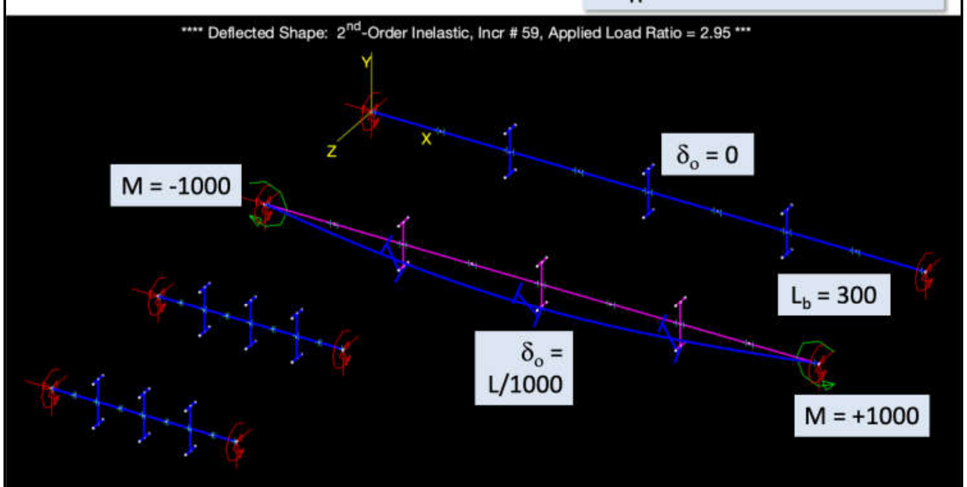
W24x68 (E=29,000, F_y = 50, include warping and δ_o)

L_b	$M_{n,LBA}$	$M_{n,MNA}$	$M_{n,GMNIA}$	$M_{n,min}$
100	24206	8850		
300	3374	8850		

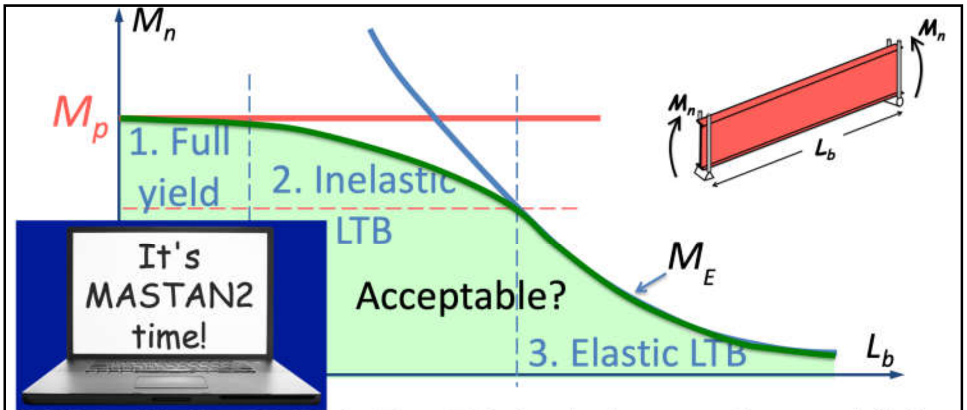
57

NS24_L3_Example_1.mat

GMNIA
M_n = 2.95 x 1000
M_n = 2950



58



W24x68 (E=29,000, F_y = 50, include warping and δ_o)

L_b	$M_{n,LBA}$	$M_{n,MNA}$	$M_{n,GMNIA}$	$M_{n,min}$
100	24206	8850	8200	8200
300	3374	8850	2950	2950

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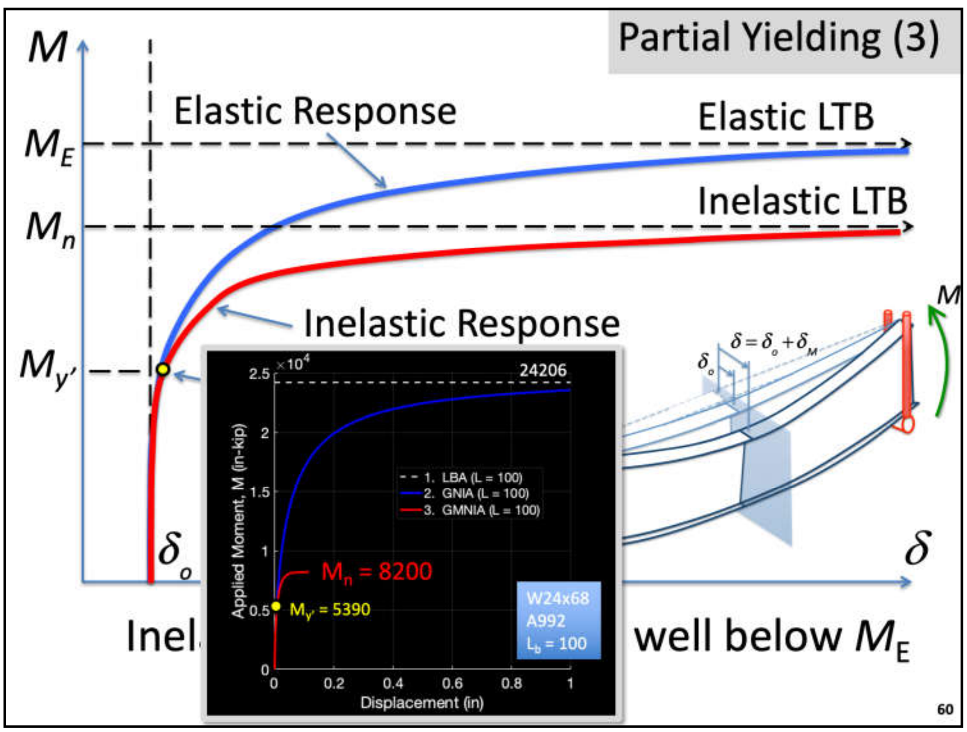
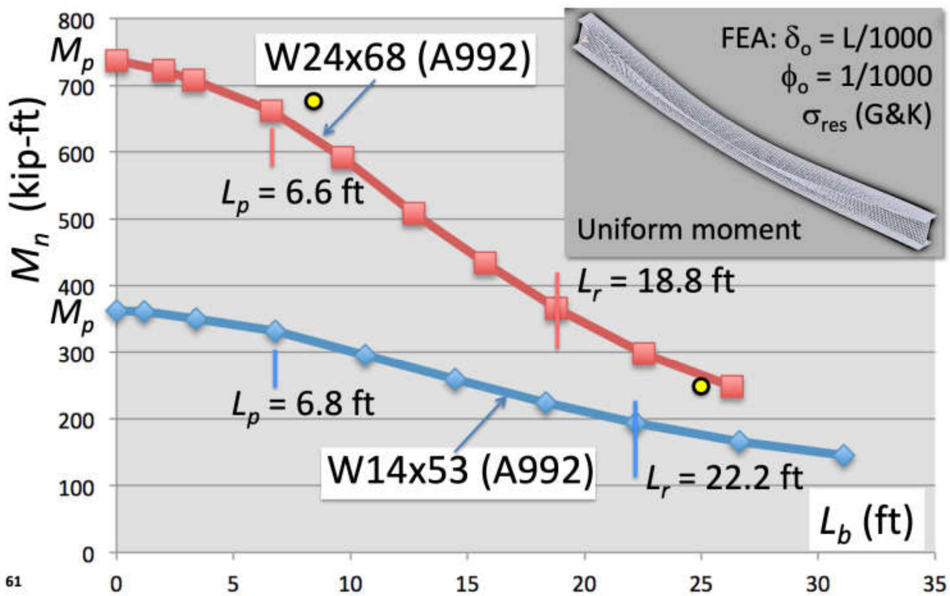


60

Flexural Strengths per FEA



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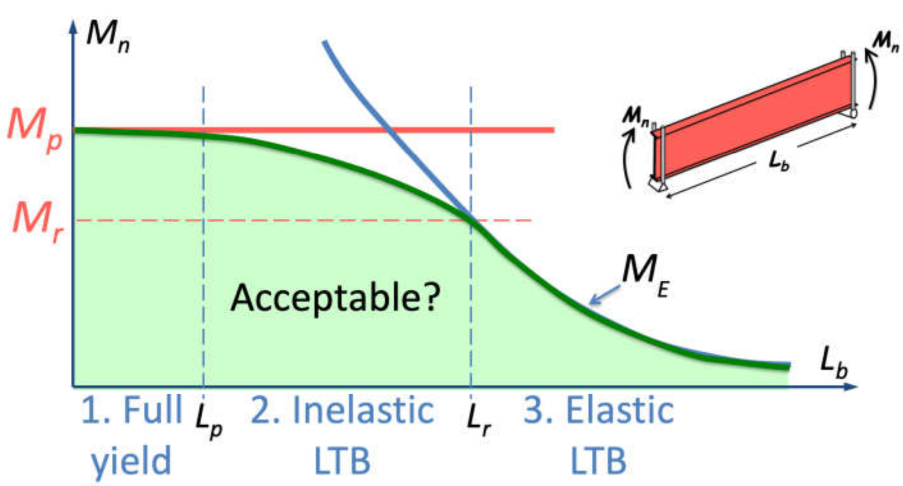
- Beam Curve – Take 3



- Where did L_p and L_r come from? (done!)
- What about those assumptions? (hmmm…)

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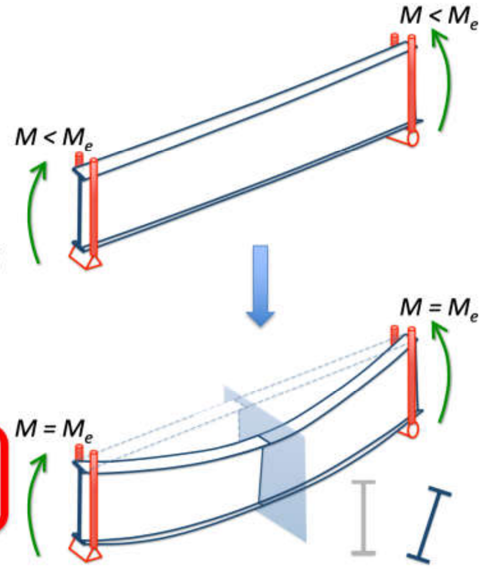
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



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Uniform Moment Distribution

- Provides for "simplest" differential equation and corresponding solution to the elastic LTB problem.
- Most conservative case
  - $M(x)$ = constant
  - maximum compressive stress occurs along entire unbraced length
- In place of formulating and solving for other moment $M(x)$ distributions, results can be adequately approximated by scaling the uniform moment in/elastic LTB solution.

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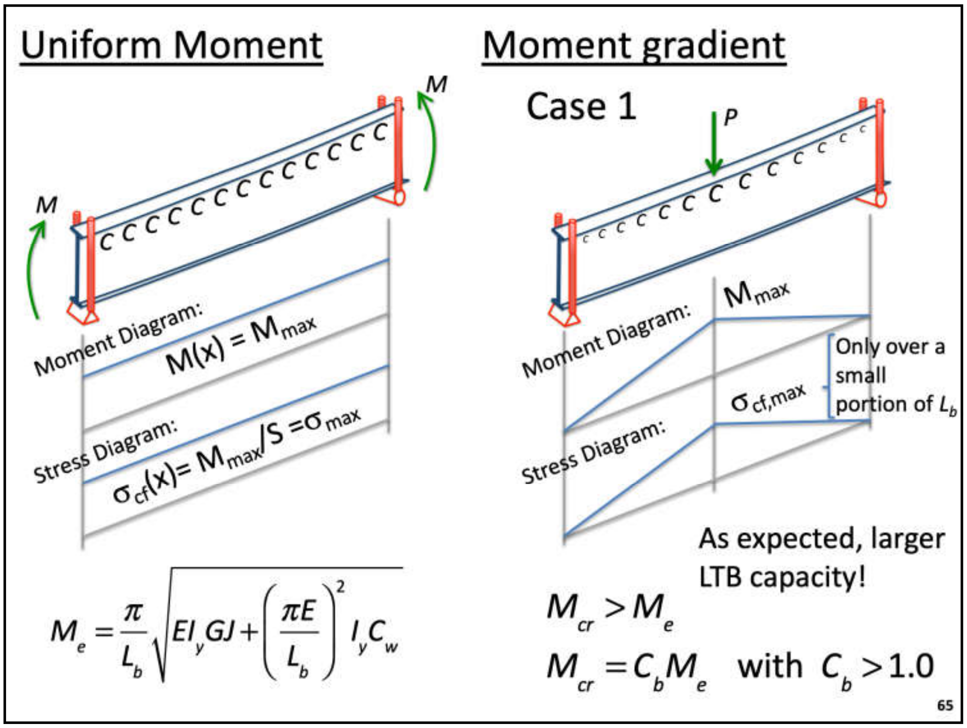
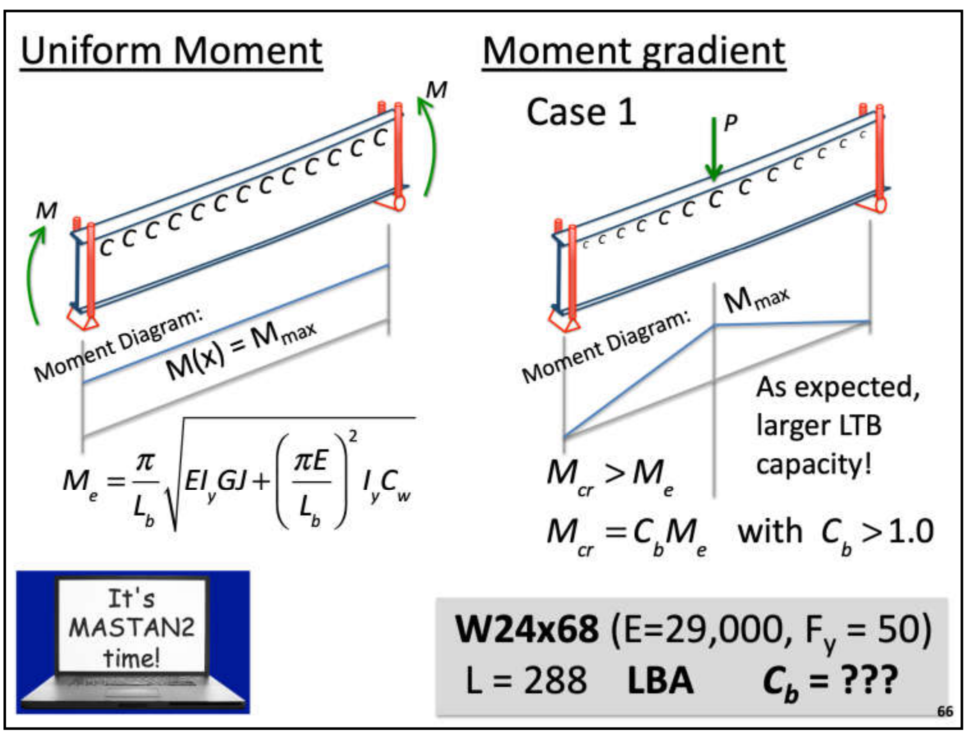
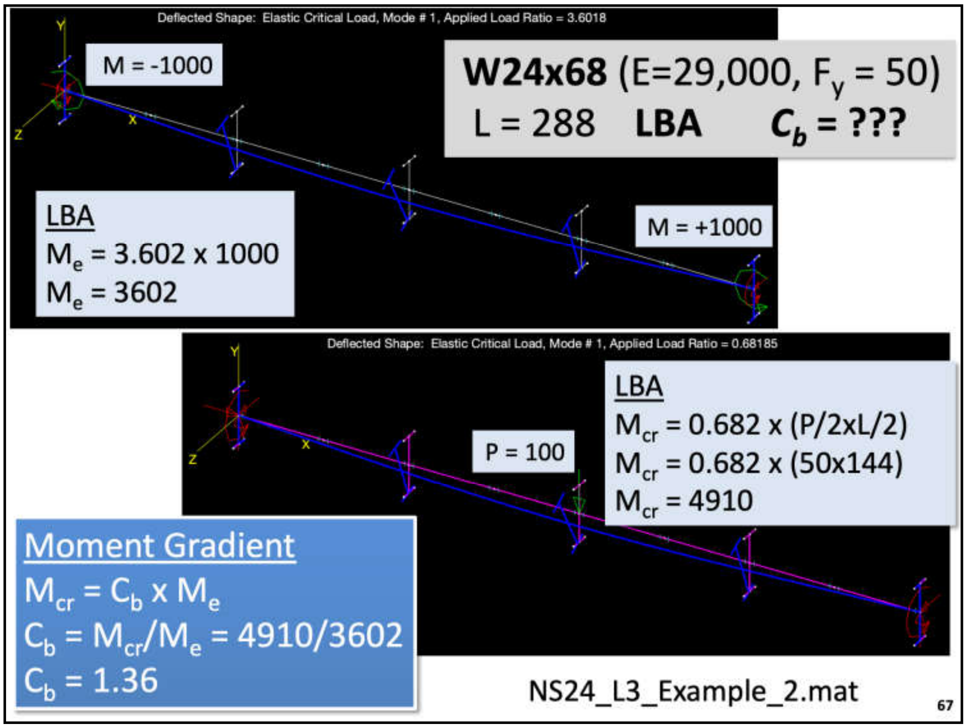
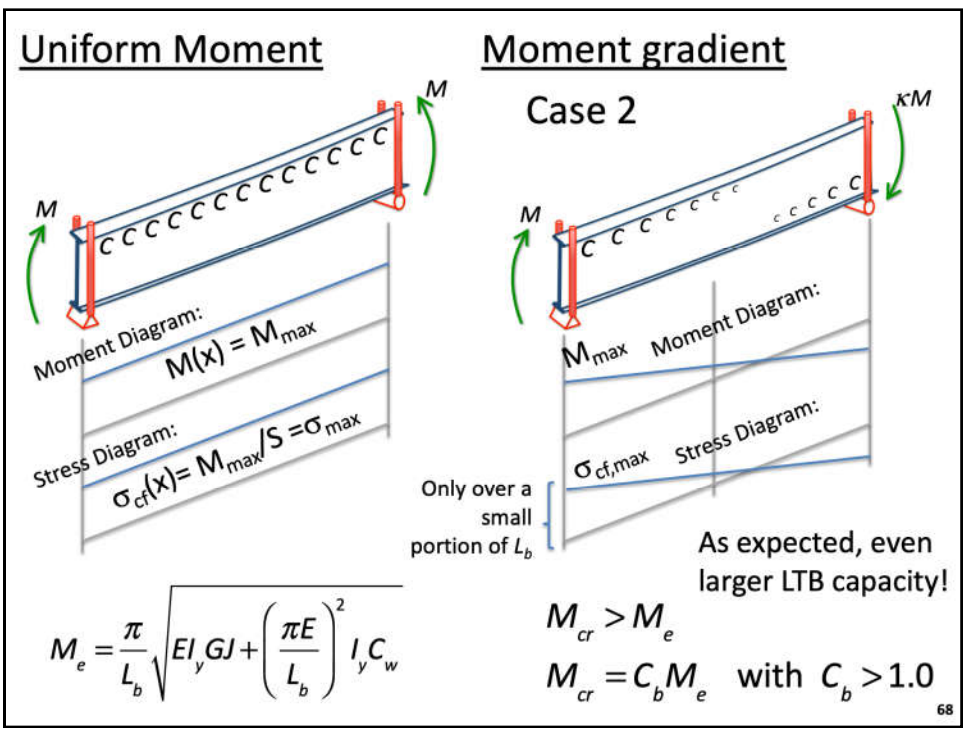
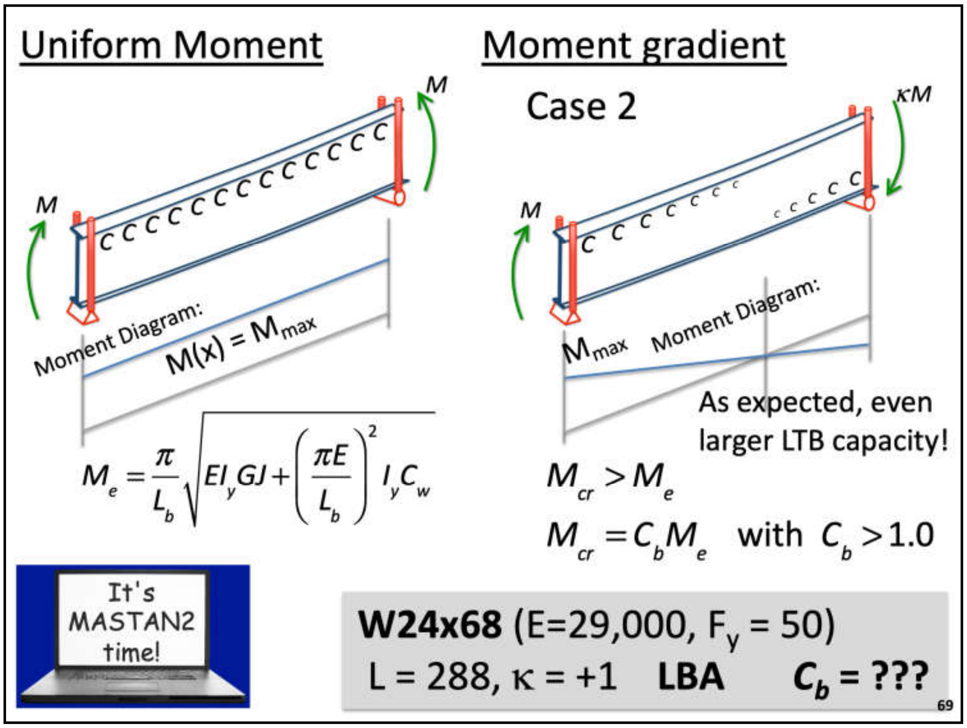
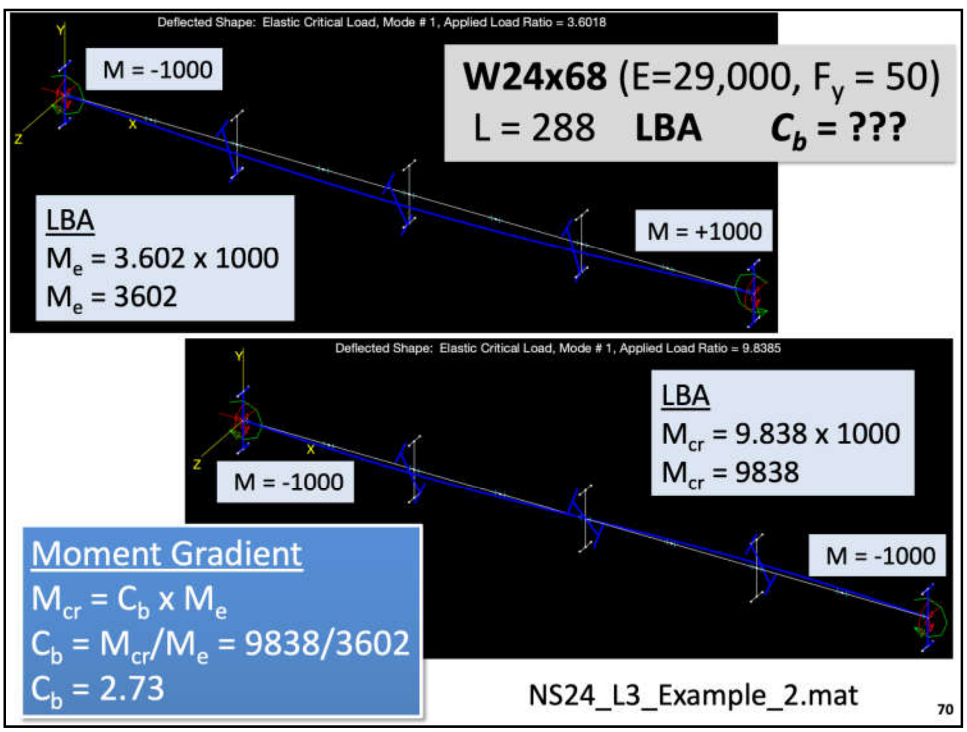

Uniform Moment
M
M
Moment Diagram:
M(x) = M max
Stress Diagram:
σ cf(x)= M max/S =σ max
$M_e = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$
Moment gradient
Case 1
P
M max
Moment Diagram:
Only over a small portion of L_b
σ cf,max
Stress Diagram:
As expected, larger LTB capacity!
$M_{cr} > M_e$
$M_{cr} = C_bM_e$ with $C_b > 1.0$
65


Uniform Moment
M
M
Moment Diagram:
M(x) = M max
$M_e = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$
It's MASTAN2 time!
Moment gradient
Case 1
P
M max
Moment Diagram:
As expected, larger LTB capacity!
$M_{cr} > M_e$
$M_{cr} = C_bM_e$ with $C_b > 1.0$
W24x68 (E=29,000, F_y = 50)
L = 288 LBA C_b = ???
66

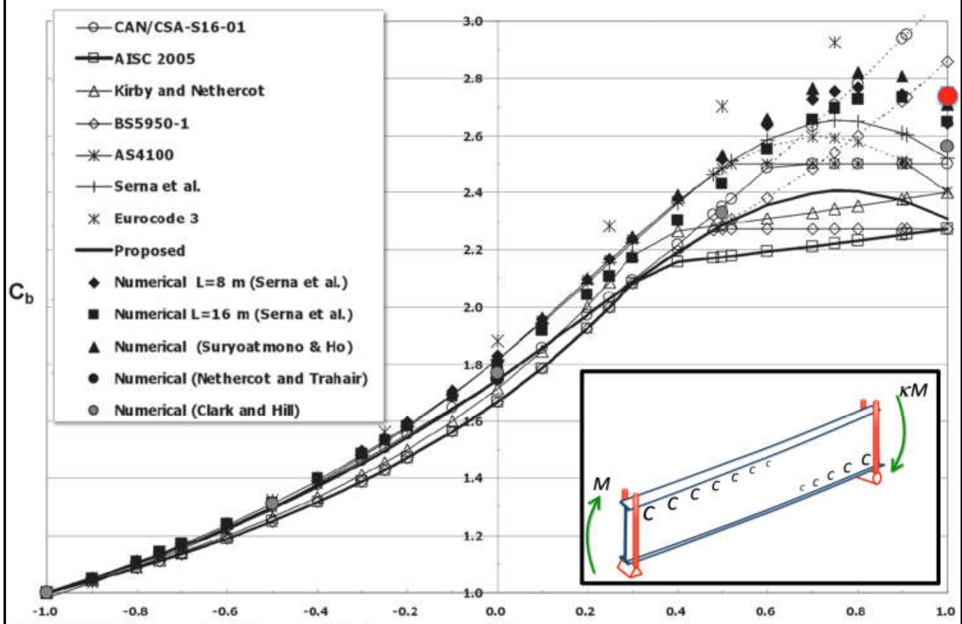

Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 3.6018
M = -1000
W24x68 (E=29,000, Fy = 50)
L = 288 LBA Cb = ???
LBA
Me = 3.602 x 1000
Me = 3602
M = +1000
Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 0.68185
LBA
Mcr = 0.682 x (P/2xL/2)
Mcr = 0.682 x (50x144)
Mcr = 4910
P = 100
Moment Gradient
Mcr = Cb x Me
Cb = Mcr/Me = 4910/3602
Cb = 1.36
NS24_L3_Example_2.mat
67


Uniform Moment
Moment gradient
Case 2
M
κM
Moment Diagram:
M(x) = Mmax
Stress Diagram:
σcf(x)= Mmax/S =σmax
Mmax
σcf,max
Only over a small portion of Lb
As expected, even larger LTB capacity!
$M_e = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$
$M_{cr} > M_e$
$M_{cr} = C_b M_e$ with $C_b > 1.0$
68


Uniform Moment
Moment gradient
Case 2
M
M
κM
M
Moment Diagram:
M(x) = Mmax
Mmax
Moment Diagram:
As expected, even
larger LTB capacity!
$M_e = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$
$M_{cr} > M_e$
$M_{cr} = C_bM_e$ with $C_b > 1.0$
It's
MASTAN2
time!
W24x68 (E=29,000, Fy = 50)
L = 288, κ = +1 LBA Cb = ???
69


Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 3.6018
M = -1000
W24x68 (E=29,000, Fy = 50)
L = 288 LBA Cb = ???
LBA
Me = 3.602 x 1000
Me = 3602
M = +1000
Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 9.8385
LBA
Mcr = 9.838 x 1000
Mcr = 9838
M = -1000
M = -1000
Moment Gradient
Mcr = Cb x Me
Cb = Mcr/Me = 9838/3602
Cb = 2.73
NS24_L3_Example_2.mat
70

Moment Gradient Factor, C_b



2010 AISC EJ paper by Wong and Driver

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LTB Moment Gradient Factor, C_b

- In/elastic LTB M_n can be adequately approximated by scaling the uniform moment in/elastic LTB solution

$$M_n = C_b M_n^{C_b=1} \le M_p$$

- Under no conditions can M_n exceed M_p, regardless of moment gradient
- Many possibilities for C_b, AISC uses

$$C_b = \frac{12.5\left|M_{max}\right|}{2.5\left|M_{max}\right| + 3\left|M_{L_b/4}\right| + 4\left|M_{L_b/2}\right| + 3\left|M_{3L_b/4}\right|}$$

- See 2010 AISC EJ paper by Wong and Driver!

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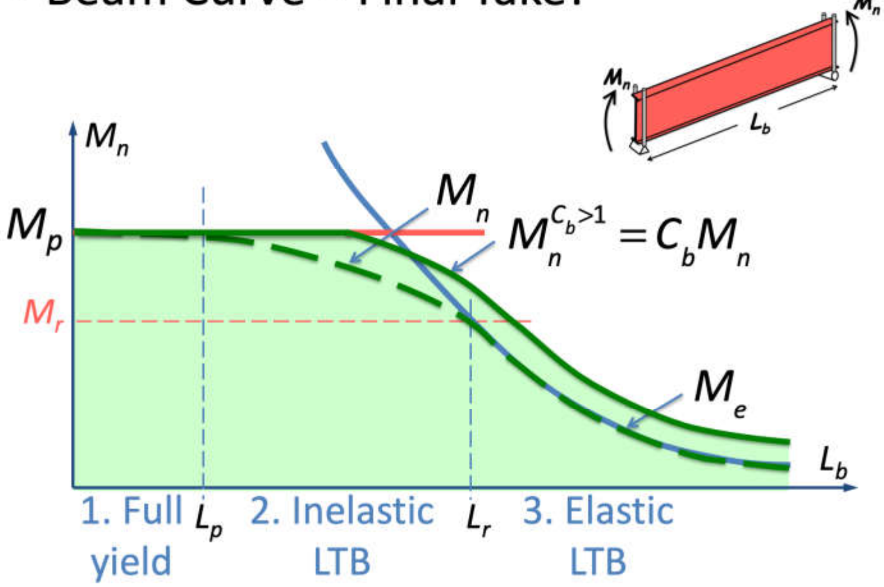
- Beam Curve – Final Take!



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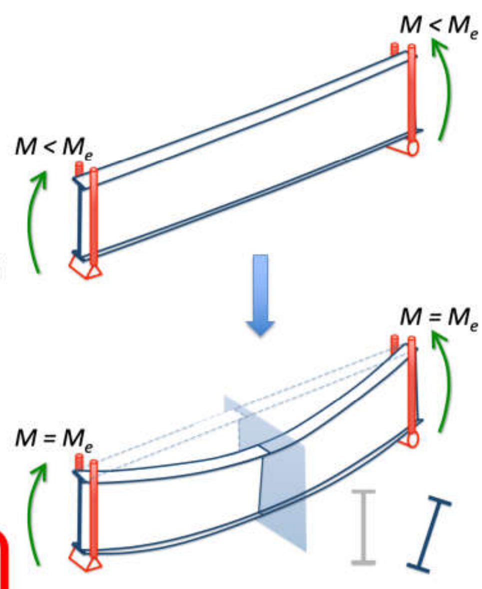
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



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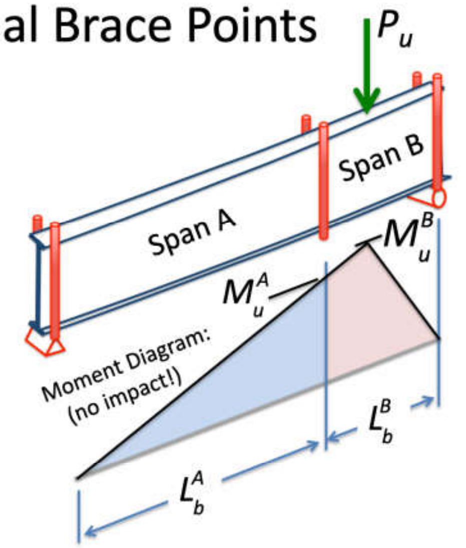
Providing Additional Brace Points

- Not vertical supports!
- Braces should restrain
  - twist
  - lateral movement
- All rules apply with L_b reduced to distance between brace points
- Must confirm strength within each unbraced span
- Design of braces!



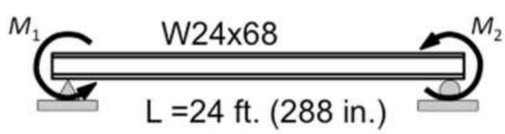
$L_b^A, C_b^A \Rightarrow M_n^A$

$M_u^A \leq \phi M_n^A$

$L_b^B, C_b^B \Rightarrow M_n^B$

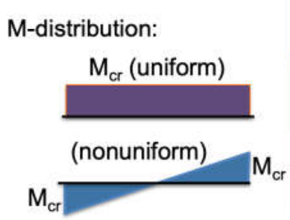
$M_u^B \leq \phi M_n^B$

75



Case 1: Braces at ends

Case 2: Braces at ends and mid-span



M_{cr} (kip-in)	Case 1	Case 2
$M_1/M_2 = -1$		
$M_1/M_2 = +1$		

Observations:

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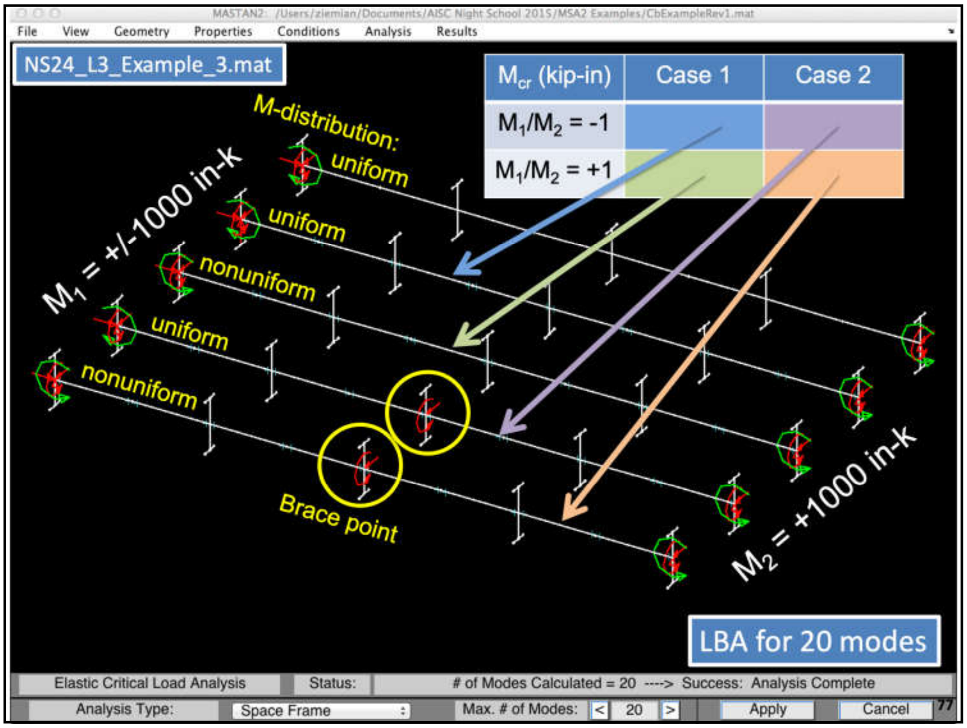
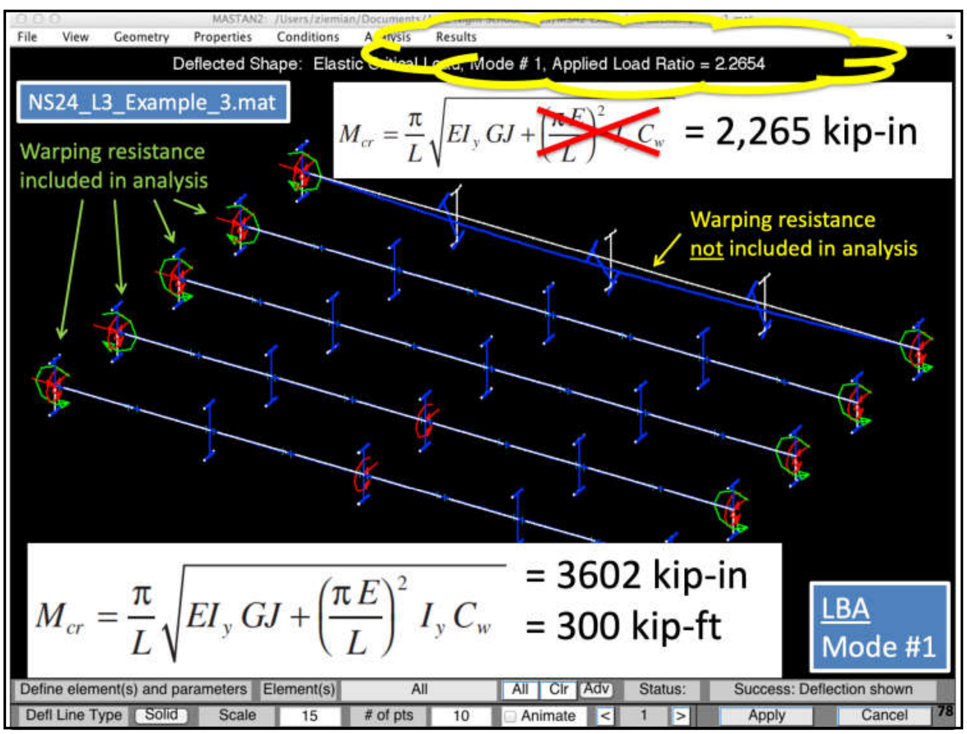
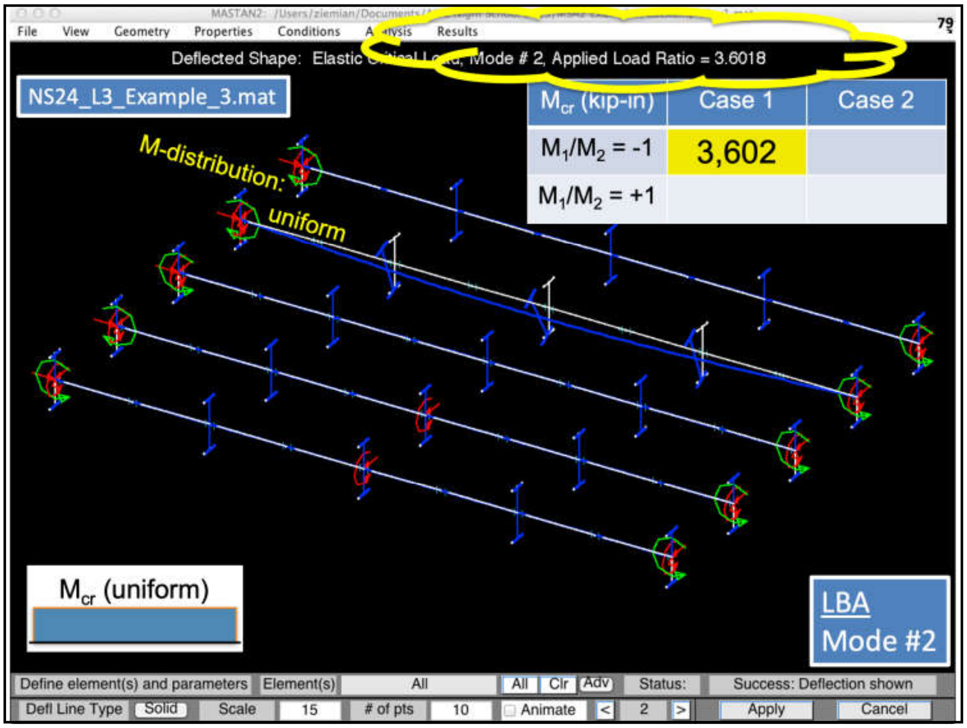
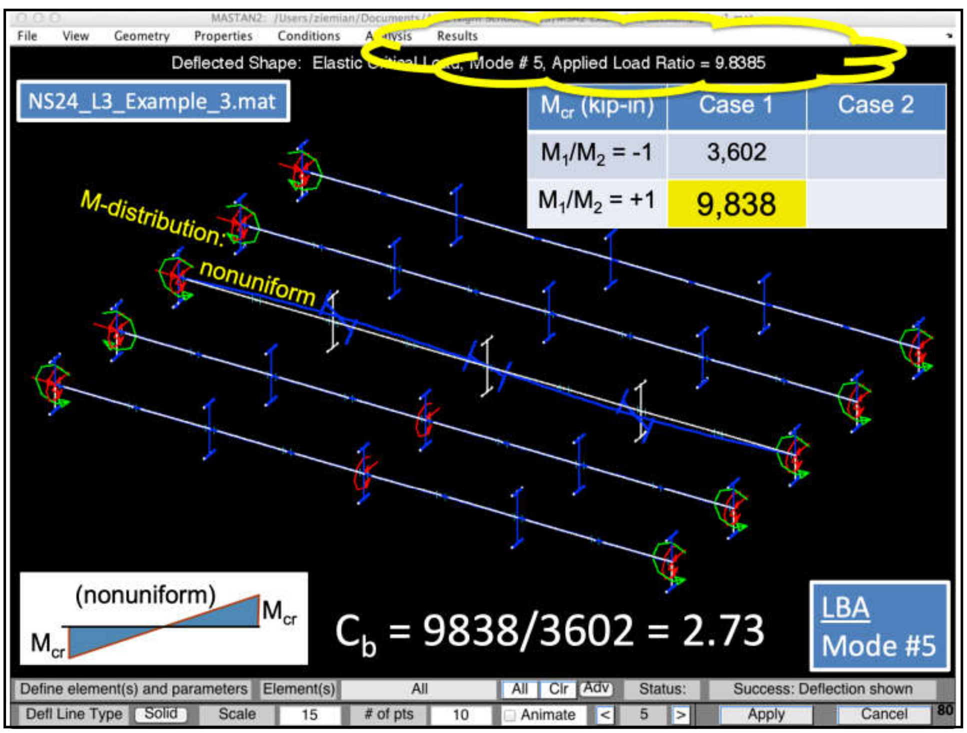

NS24_L3_Example_3.mat
M-distribution:
uniform
uniform
nonuniform
uniform
nonuniform
M1 = +/-1000 in-k
M2 = +1000 in-k
Brace point
Mcr (kip-in) | Case 1 | Case 2
M1/M2 = -1
M1/M2 = +1
LBA for 20 modes
Elastic Critical Load Analysis
Status:
# of Modes Calculated = 20 ----> Success: Analysis Complete
Analysis Type:
Space Frame
Max. # of Modes:
20
Apply
Cancel
77

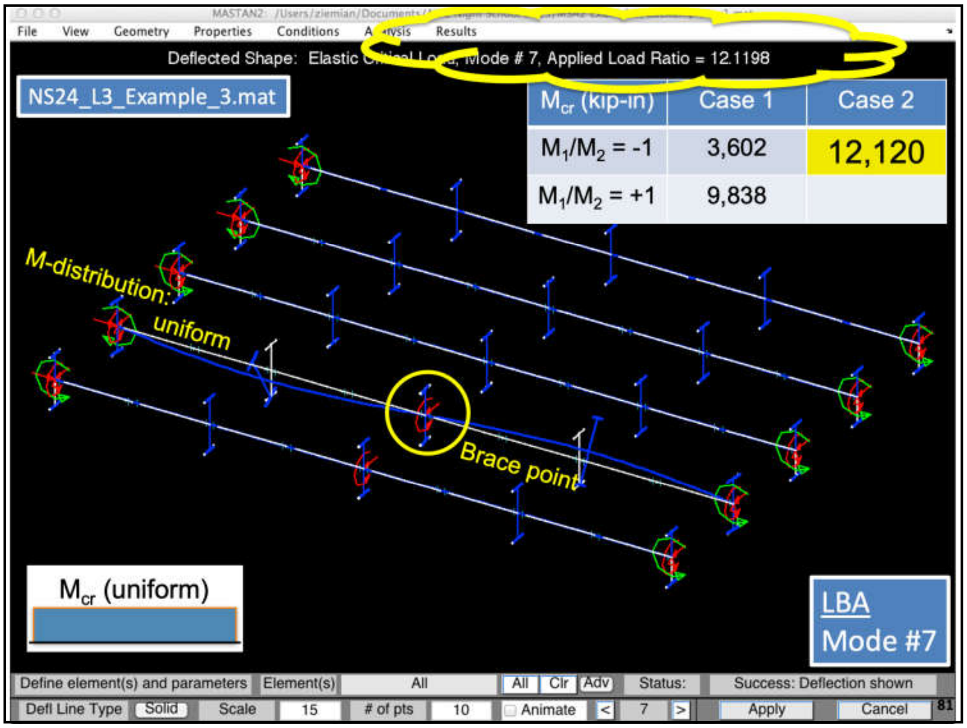

Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 2.2654
NS24_L3_Example_3.mat
Warping resistance included in analysis
Warping resistance not included in analysis
$M_{cr} = \frac{\pi}{L}\sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$ = 2,265 kip-in
$M_{cr} = \frac{\pi}{L}\sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$
= 3602 kip-in
= 300 kip-ft
LBA Mode #1
Define element(s) and parameters
Element(s)
All
Status:
Success: Deflection shown
Defl Line Type
Solid
Scale
15
# of pts
10
Animate
1
Apply
Cancel
78

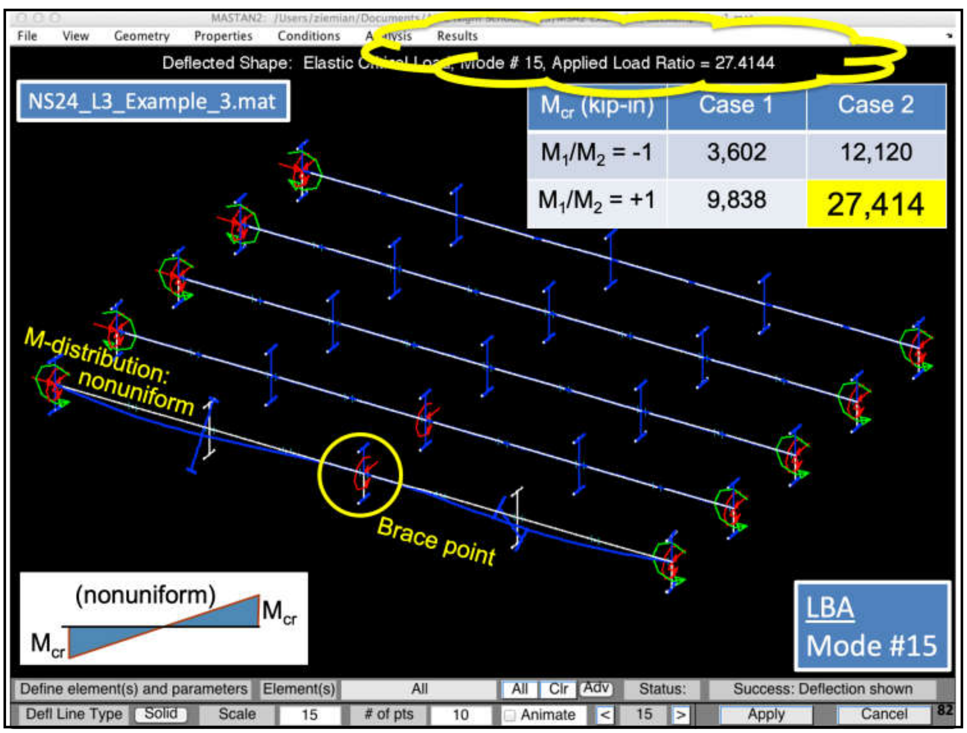


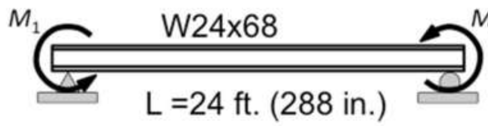
Deflected Shape: Elastic Critical Load, Mode # 2, Applied Load Ratio = 3.6018
NS24_L3_Example_3.mat
Mcr (kip-in) | Case 1 | Case 2
M1/M2 = -1 | 3,602
M1/M2 = +1
M-distribution:
uniform
Mcr (uniform)
LBA
Mode #2
79



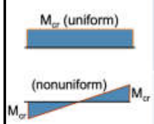
Deflected Shape: Elastic Critical Load, Mode # 5, Applied Load Ratio = 9.8385
NS24_L3_Example_3.mat
Mcr (kip-in) | Case 1 | Case 2
M1/M2 = -1 | 3,602
M1/M2 = +1 | 9,838
M-distribution:
nonuniform
(nonuniform)
Mcr
Mcr
Cb = 9838/3602 = 2.73
LBA
Mode #5
80


Deflected Shape: Elastic Critical Load, Mode # 7, Applied Load Ratio = 12.1198
NS24_L3_Example_3.mat
Mcr (kip-in) | Case 1 | Case 2
M1/M2 = -1 | 3,602 | 12,120
M1/M2 = +1 | 9,838 |
M-distribution: uniform
Brace point
Mcr (uniform)
LBA Mode #7
Success: Deflection shown
81


Deflected Shape: Elastic Critical Load, Mode # 15, Applied Load Ratio = 27.4144
NS24_L3_Example_3.mat
Mcr (kip-in) | Case 1 | Case 2
M1/M2 = -1 | 3,602 | 12,120
M1/M2 = +1 | 9,838 | 27,414
M-distribution: nonuniform
Brace point
(nonuniform)
Mcr
Mcr
LBA Mode #15
Success: Deflection shown
82



Case 1: Braces at ends
Case 2: Braces at ends and mid-span



M_{cr} (kip-in)	Case 1	Case 2
$M_1/M_2 = -1$	3,602	12,120
$M_1/M_2 = +1$	9,838	27,414

3.36

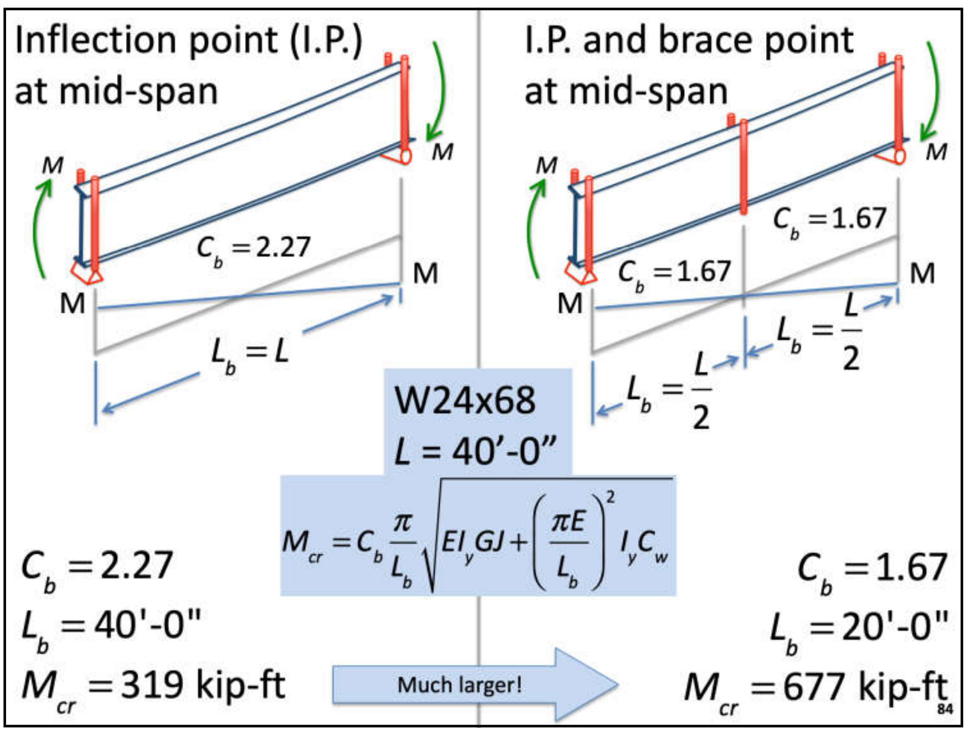
2.73

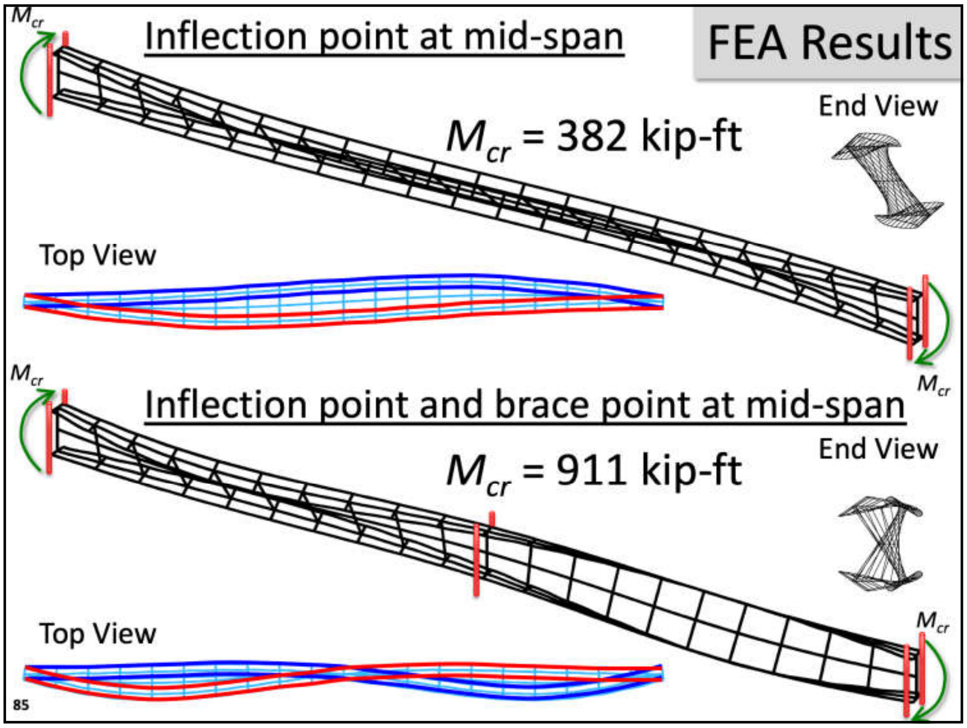
7.61

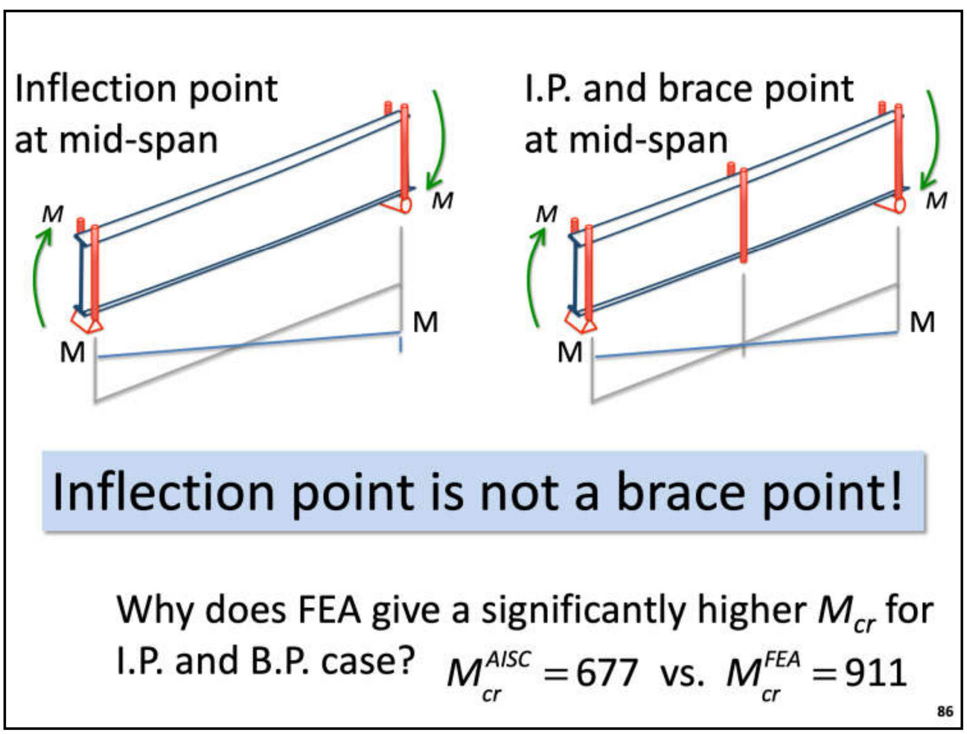
Observations:

- M_{cr} increases as L_b is reduced
- M_{cr} increases as moment gradient increases
- An inflection point is not a brace point!

83

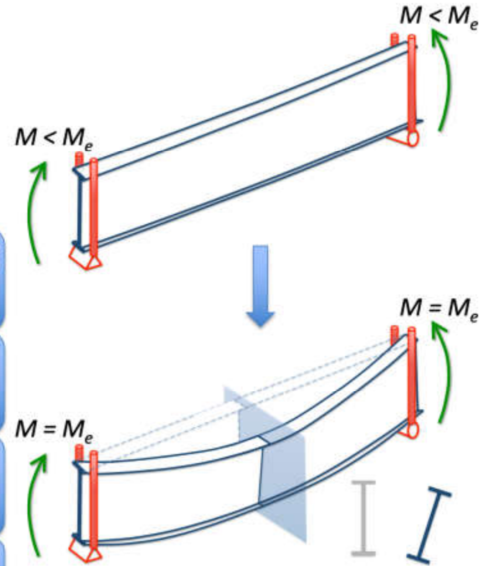



FEA Results
Inflection point at mid-span
M_{cr} = 382 kip-ft
End View
Top View
M_{cr}
Inflection point and brace point at mid-span
M_{cr} = 911 kip-ft
End View
Top View
M_{cr}
85


Inflection point
at mid-span
I.P. and brace point
at mid-span
M
M
M
M
M
M
M
M
Inflection point is not a brace point!
Why does FEA give a significantly higher M_{cr} for
I.P. and B.P. case? $M_{cr}^{AISC} = 677$ vs. $M_{cr}^{FEA} = 911$
86

Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling **Undone**
  - linear elastic behavior (E = constant) **Undone**
  - uniform moment distribution **Undone**
  - braced at the ends (frictionless) **Undone**



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Lateral Torsional Buckling

- Theoretical bifurcation
  - solution
  - assumptions
- Undoing those assumptions (approaching reality)
  - not fully elastic, partial yielding
  - alternative loading and support conditions
- Beam curves
  - AISC
  - others

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AISC Flexural Strength
(compact I-shapes)

- Initial yield
  - Moment, $M_r = S(\sigma_y - \sigma_{res}) = 0.7S\sigma_y = 0.7M_y$
  - setting $M_r = M_e$, back solve for unbraced length L_r
- For shorter unbraced lengths (full yielding)

$$L_b \le L_p,\ M_n = Z\sigma_y = M_p$$

- For longer unbraced lengths (elastic LTB)

$$L_b \ge L_r,\ M_n = C_b M_e = C_b \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\pi E/L_b\right)^2 I_y C_w} \quad \le M_p$$

- For intermediate unbraced lengths (inelastic LTB)

$$L_p < L_b < L_r,\ M_n = C_b\left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \quad \le M_p$$

89

AISC Flexural Strength
(compact I-shapes)

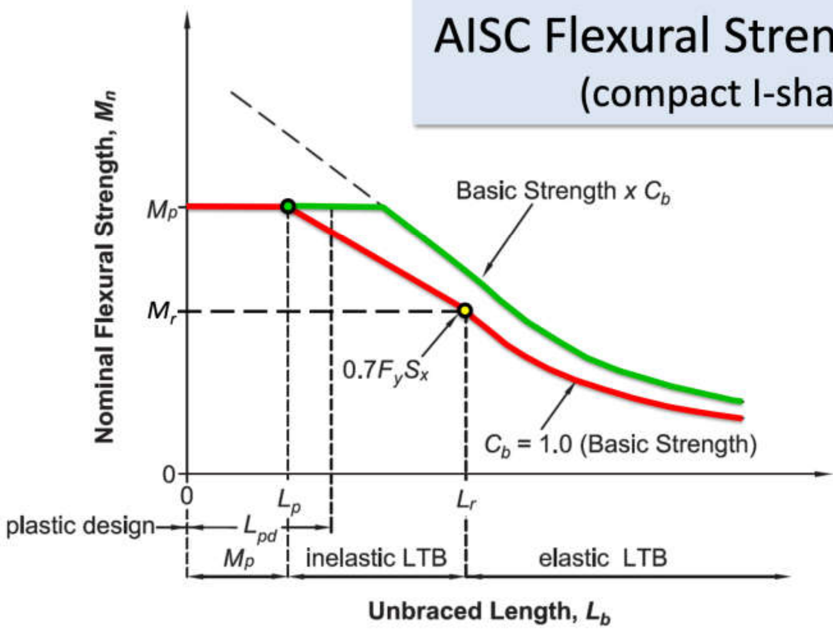
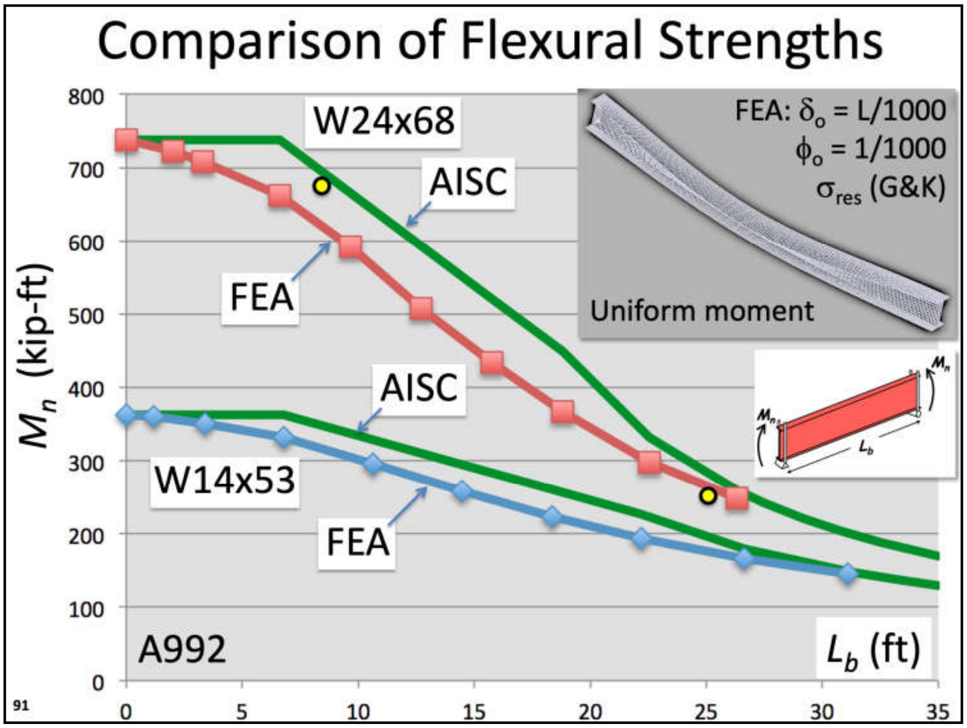
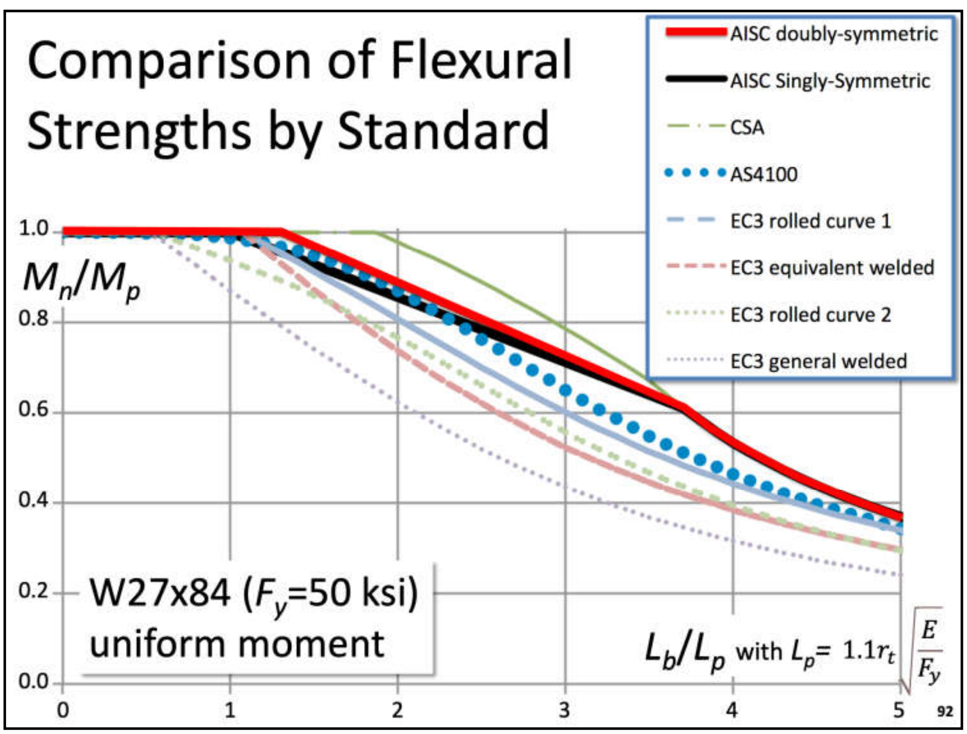


Fig. C-F1.2. Nominal flexural strength as a function of unbraced length and moment gradient.

90


Comparison of Flexural Strengths
W24x68
AISC
FEA
AISC
W14x53
FEA
FEA: δo = L/1000
φo = 1/1000
σres (G&K)
Uniform moment
Mn (kip-ft)
A992
Lb (ft)
91


Comparison of Flexural Strengths by Standard
AISC doubly-symmetric
AISC Singly-Symmetric
CSA
AS4100
EC3 rolled curve 1
EC3 equivalent welded
EC3 rolled curve 2
EC3 general welded
Mn/Mp
W27x84 (Fy=50 ksi)
uniform moment
Lb/Lp with Lp= 1.1rt √(E/Fy)
92

Summary – Flexure

- Limit states of flexural members with focus on full yielding and lateral torsional buckling
- LTB Theory -to- Flexural Strength Beam Curve
- Beam curve accounts for:
  - full yielding
  - bending due to initial imperfection (out-of-straightness)
  - partial yielding accentuated by presence of residual stresses
  - moment gradient and brace points

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Summary – Flexure (cont.)

- AISC and other standards

- Your virtual laboratory assignment…
  - Try to recreate examples from this lecture
  - Try to recreate Learning Module 5 results presented in this lecture (C_b fun!)
  - Complete a portion of Learning Module 4

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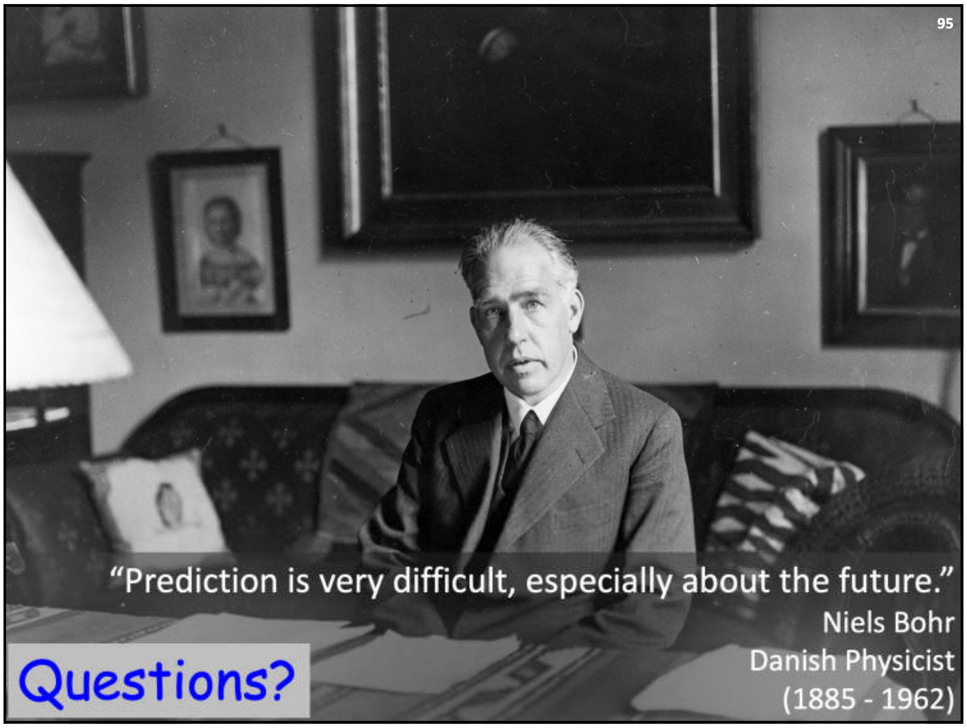

Thank You!

Hope you enjoyed this lecture!

structural STEEL

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Single-Session Registrants

CEU / PDH Certificates

- You will receive an email on how to report attendance from: registration@aisc.org.
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!

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Stronger.
Steel.

Single-Session Registrants

CEU / PDH Certificates

- Reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.

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8-Session Registrants

CEU / PDH Certificates

One certificate will be issued at the conclusion of the course.

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8-Session Registrants

Attendance and PDH Certificates

- You have two options to receive credit for a given session.
  - Option 1: Watch the live session. Credit for live attendance will be displayed on the Course Resources table within two days of the session.
  - Option 2: Watch the recording and pass the associated quiz.

Videos and Quizzes

- For each session, find access within two business days after the live air date. (An email will be sent from nightschool@aisc.org.)
- Reasons for quiz:
  - EEU – You must take all quizzes and the final exam to receive EEU.
  - PDHs – If you watch a recorded session, you must pass quiz for PDHs.
  - Reinforce what you learn in the lectures and get more out of the course!

Distribution of Certificates

All certificates will be issued after the course is completed. Only the registrant will receive a certificate for the course.

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8-Session Registrants

Course Resources

Find all your handouts, quizzes and quiz scores, recording access, and attendance information in one place!

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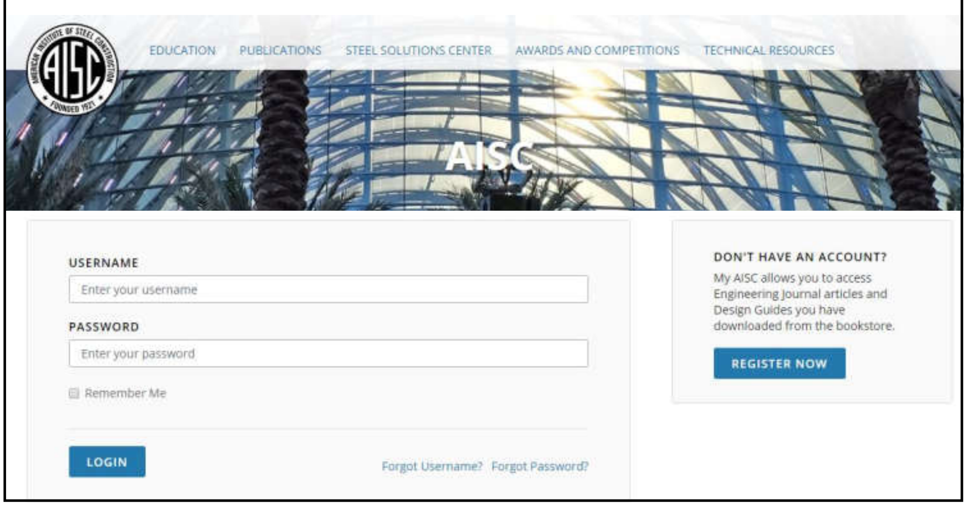
8-Session Registrants

Course Resources

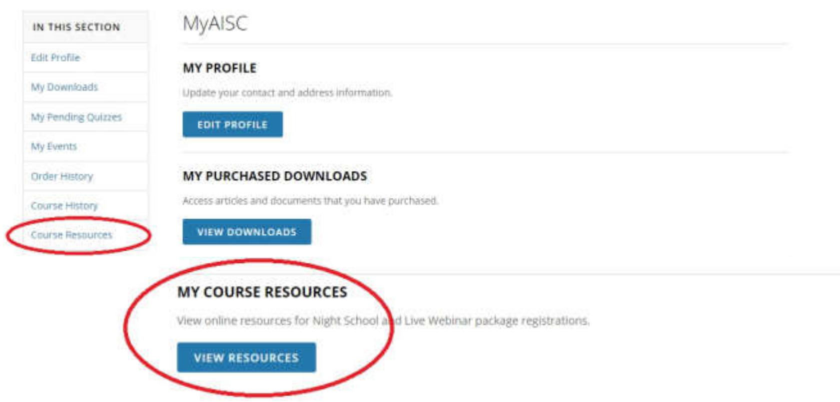
Go to www.aisc.org and sign in.



8-Session Registrants

Course Resources

Go to www.aisc.org and sign in.



8-Session Registrants

Course Resources

Course Resources

Event	Start Date
Seismic Design in Steel	1/1/1900 12:00:00 AM
4-Session Package-Design of Facade Attachments	5/9/2019 1:30:00 PM
NS 15 8-Session Package-Night School 15 - Fundamentals of Connection Design	10/3/2017 7:00:00 PM
NS 16 8-Session Package-Night School 16 - Seismic Design in Steel	2/5/2018 7:00:00 PM
NS 17 4-Session Package-Night School 17: Design of Facade Attachments	7/16/2018 7:00:00 PM
NS 18 8-Session Package-Night School 18: Steel Construction: Mill To Topping Out	10/15/2018 7:00:00 PM
NS 19 8-Session Package-Night School 19: Connection Design	2/4/2019 7:00:00 PM
NS 20 8-Session Package-Night School 20: Classical Methods of Structural Analysis	6/3/2019 7:00:00 PM
8-Session Package-Seismic Design in Steel - Concepts & Examples	7/16/2018 1:30:00 PM

8-Session Registrants

Course Resources

AISC › MY ACCOUNT › COURSE RESOURCES › NS24 8-SESSION PACKAGE RESOURCES

Night School 24: Modern Methods for Learning Structural Stability

8-SESSION PACKAGE RESOURCES

Event	Date	Handouts	Video	Quiz	Attendance
NS24.1 - Compression Members - The Fundamentals	Oct 6 2020 7:00PM EDT	Handouts	Available 10/08/2020 5:00PM EDT	Available 10/08/2020 5:00PM EDT	Pending
NS24.2 - Compression Members - Practical Considerations	Oct 13 2020 7:00PM EDT	Handouts	Available 10/15/2020 5:00PM EDT	Available 10/15/2020 5:00PM EDT	Pending
NS24.3 - Behavior of Flexural Members - The Fundamentals	Oct 20 2020 7:00PM EDT	Handouts	Available 10/22/2020 5:00PM EDT	Available 10/22/2020 5:00PM EDT	Pending
NS24.4 - Flexural Members - Practical Considerations	Oct 27 2020 7:00PM EDT	Handouts	Available 10/29/2020 5:00PM EDT	Available 10/29/2020 5:00PM EDT	Pending
NS24.5 - Stability of Beam-Columns - The Fundamentals	Nov 10 2020 7:00PM EST	Handouts	Available 11/12/2020 5:00PM EST	No longer available	Pending
NS24.6 - Stability of Beam-Columns - Practical Consideration	Nov 17 2020 7:00PM EST	Handouts	Available 11/19/2020 5:00PM EST	No longer available	Pending
NS24.7 - Behavior of Structural Systems - The Fundamentals	Dec 1 2020 7:00PM EST	Handouts	Available 12/03/2020 5:00PM EST	No longer available	Pending
NS24.8 - Structural Systems - Practical Considerations	Dec 8 2020 7:00PM EST	Handouts	Available 12/10/2020 5:00PM EST	No longer available	Pending
NS24 - Final Exam	N/A			No longer available	

AISC | Thank you.

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