AISC Night School

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We will begin shortly. Please standby.

Modern Methods for Learning the Basics of Structural Stability: From Behavior to Practice

Session 3: Behavior of Flexural Members – The Fundamentals

October 20, 2020

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Course Description

Behavior of Flexural Members – The Fundamentals
October 20, 2020

Using an approach similar to that employed in Session 1, this lecture will provide an overview of the strengths and limitations of the solution to the differential equation that defines the elastic lateral-torsional buckling (LTB) strength of beams. Related flexural and torsional concepts, including the benefits of warping resistance, will be briefly reviewed. The assumption of elastic behavior will then be relaxed to define the inelastic LTB and plastic moment capacities of flexural members. The strength of beams without slender elements will be covered and ultimately presented in the form of beam resistance curves. The speakers will conclude by introducing the second learning module.

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Learning Objectives

- Explain the limit state of full yielding for flexural members.
- Describe lateral-torsional buckling behavior of beams.
- Explain how the length between brace points of the compression flange of a member in flexure affects lateral-torsional buckling.
- Explain the application of the lateral-torsional buckling moment gradient factor, C_b.

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Modern Methods for Learning The Basics of Structural Stability: From Behavior to Practice

Session 3: Behavior of Flexural Members – The Fundamentals
October 20, 2020

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Course Overview

- Topics
  - Compression Members (Weeks 1 & 2)
  - Flexural Members (Weeks 3 & 4)
  - Beam-Columns (Weeks 5 & 6)
  - Systems (Weeks 7 & 8)
- "Active" learning! Weekly virtual lab experiences…
- Case studies from the real world…

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Course Overview (2)

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Course Overview (3)

- Focus of the course is on fundamentals!
- Better understanding of behavior will result in improved design
- Key Definitions
  - **Stability:** Under load, component returns to current state after applying a small disturbance such as a deflection
  - **Bifurcation (critical load):** Theoretical point at which loading a component results in an instantaneous change from current state to significant deflection – two options: not buckled or buckled
  - **Instability:** Loading a component results in a realistic transition from small deflection to significant deflection – buckling preceded by deflection

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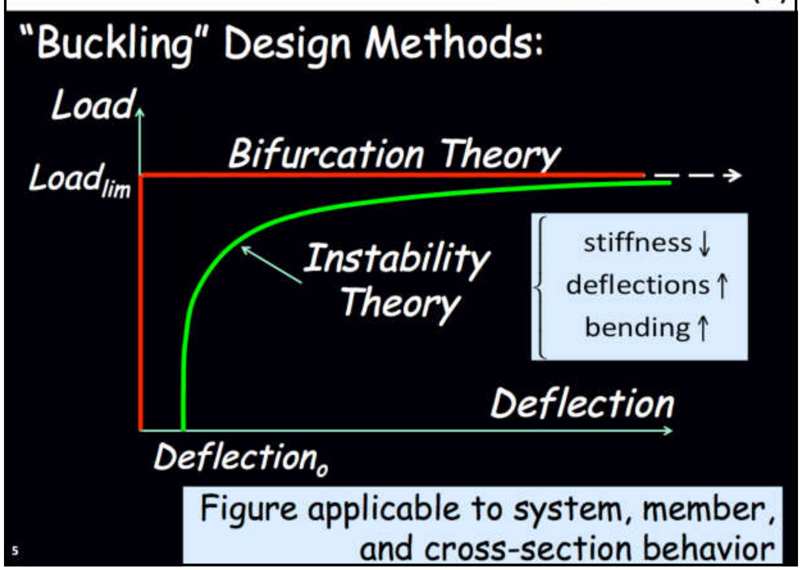
Course Overview (4)



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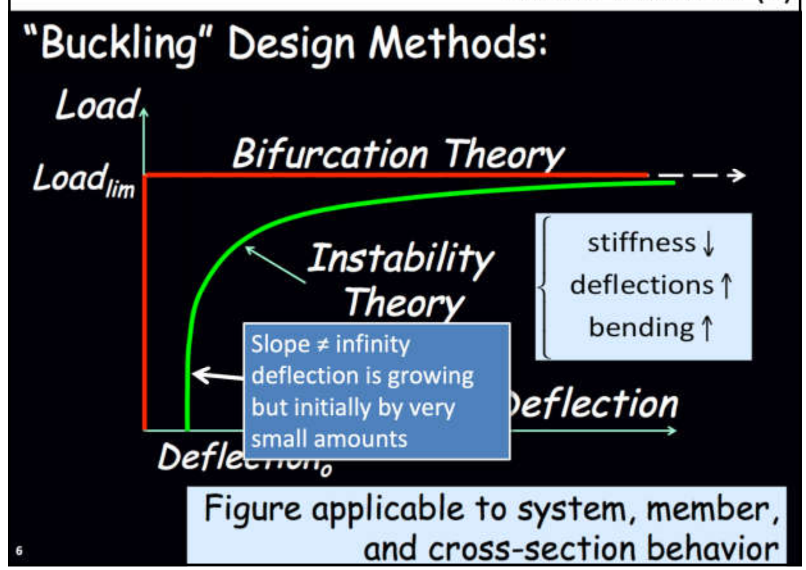
Course Overview (4)



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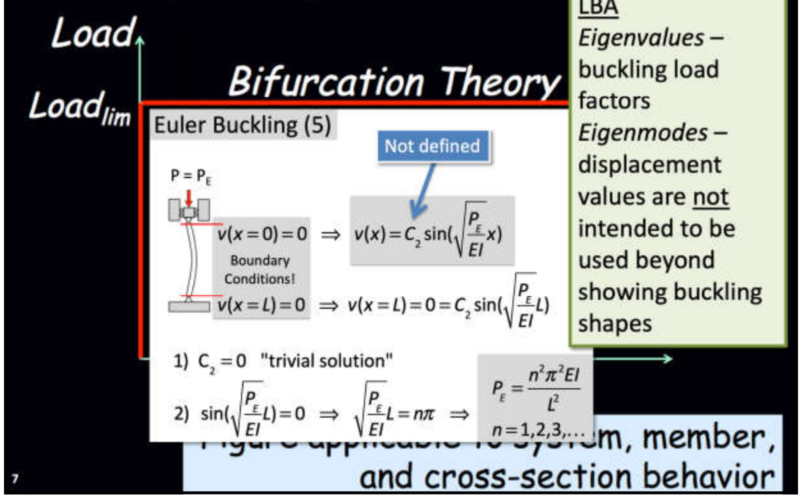
Course Overview (4a)

"Buckling" Design Methods:



Course Overview (5)

Analysis acronyms:

LBA: linear buckling analysis; elastic critical load analysis; elastic eigenvalue analysis; assumes bifurcation theory

GNA: geometric nonlinear analysis; 2nd-order elastic analysis; assumes equilibrium on the deformed shape and linear elastic material, with no initial imperfections

GNIA: same as GNA, but includes initial imperfections

MNA: material nonlinear analysis; 1st-order inelastic analysis; assumes equilibrium on the undeformed shape and accounts for yielding, with no initial imperfections

GMNIA: geometric and material nonlinear analysis; 2nd-order inelastic analysis; assumes equilibrium on the deformed shape, accounts for yielding, and includes initial imperfections

hmmm....one resource that may be of use is the textbook available at www.mastan2.com

Modern Methods for Learning The Basics of Structural Stability: From Behavior to Practice

- Course Introduction
- Compression Members
- Flexural Members
- Beam-Columns
- Systems

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Limit States of Flexural Members

- Full yielding (today!)
- Instability
  - Along the member length (today!)
    - Lateral torsional buckling
      - elastic
      - inelastic
  - At the cross section
    - local buckling

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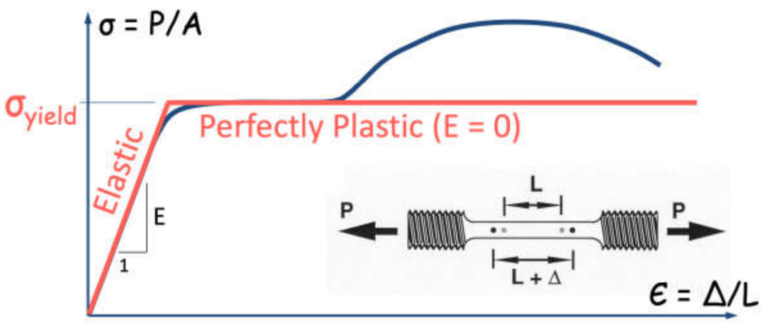
Full Yielding

- Tensile test



- Assume same response for compression
  - $\sigma_{y,compression} = \sigma_{y,tension} = \sigma_{yield}$
  - Neglect strain hardening (assume elastic-plastic)

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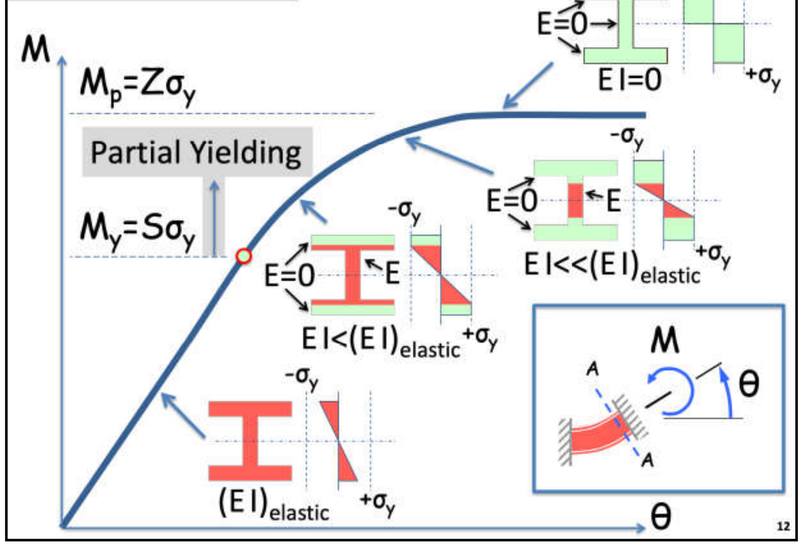
Full Yielding (2)



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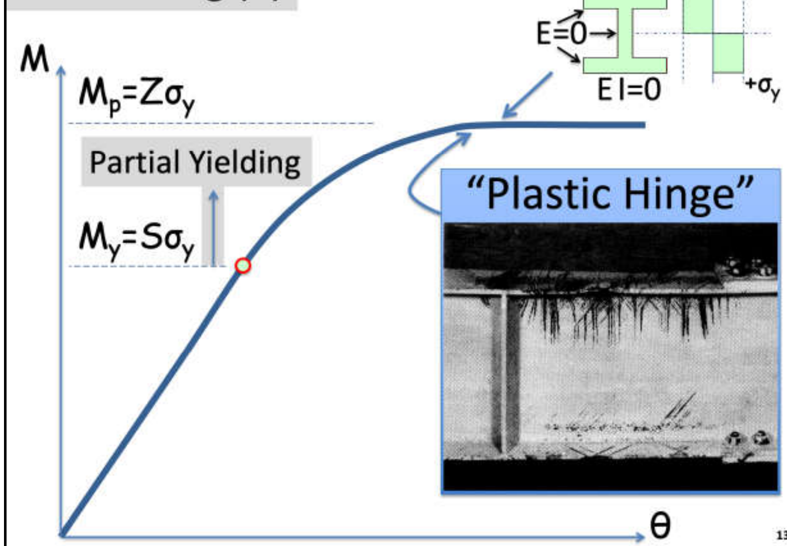
Full Yielding (3)



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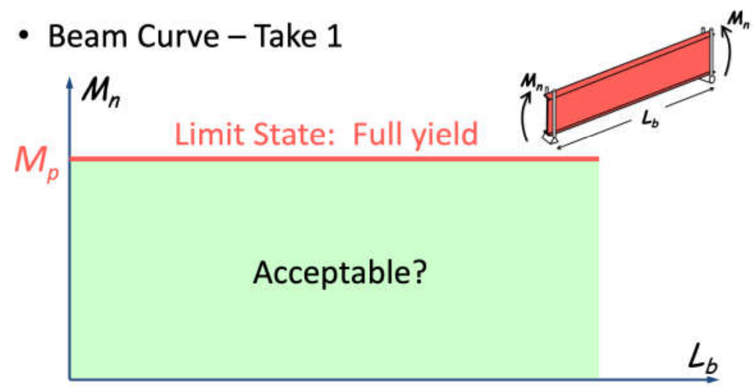
Full Yielding (4)

- Beam Curve – Take 1



- What about:
  - member instability ??? (today!)
  - cross section instability (local buckling) ???

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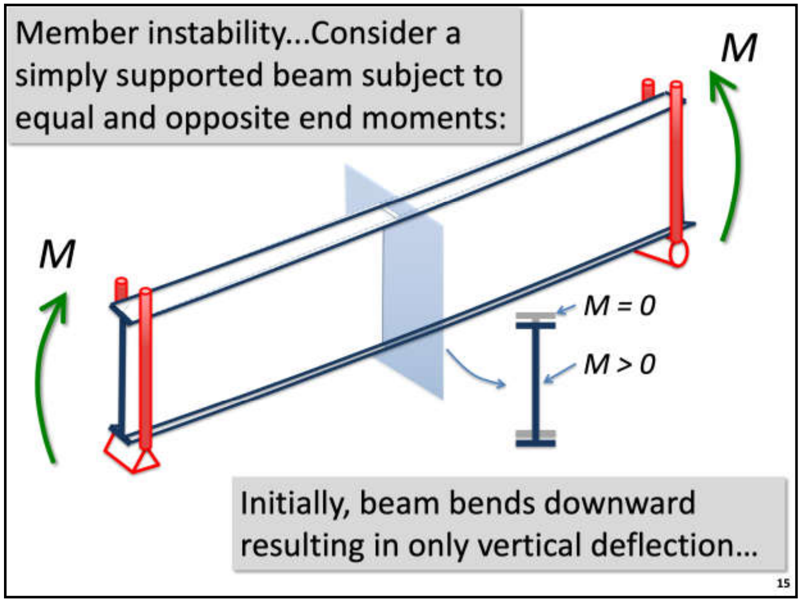
Member instability...Consider a simply supported beam subject to equal and opposite end moments:
M
M
M = 0
M > 0
Initially, beam bends downward resulting in only vertical deflection...
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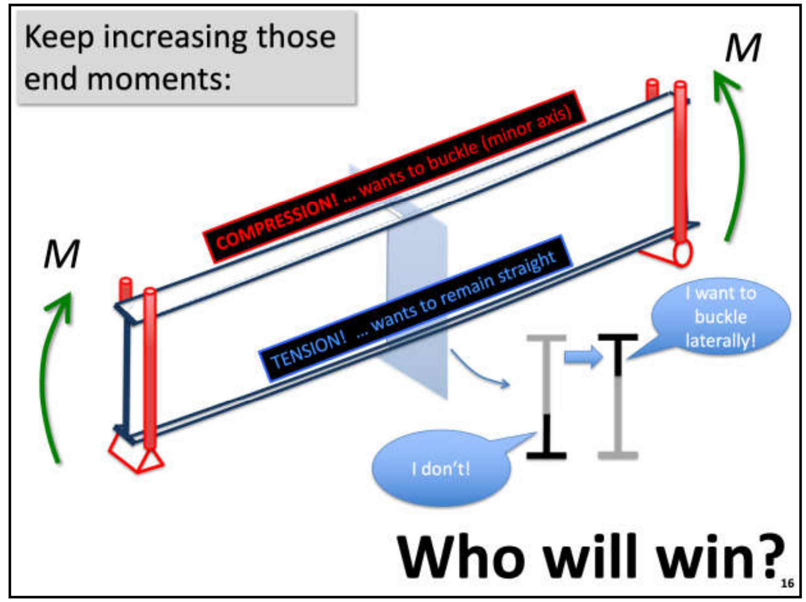
Keep increasing those end moments:
M
M
COMPRESSION! ... wants to buckle (minor axis)
TENSION! ... wants to remain straight
I want to buckle laterally!
I don't!
Who will win?
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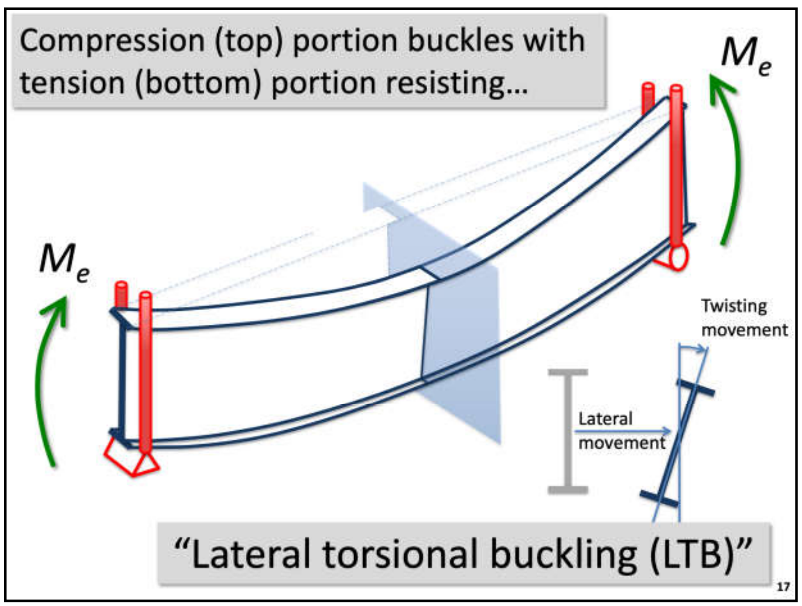
Compression (top) portion buckles with tension (bottom) portion resisting...
Me
Me
Twisting movement
Lateral movement
"Lateral torsional buckling (LTB)"
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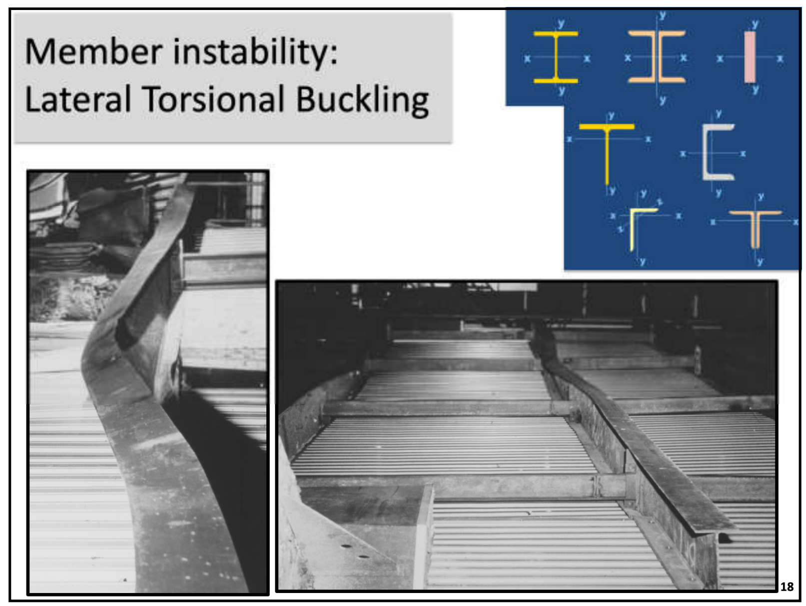
Member instability:
Lateral Torsional Buckling
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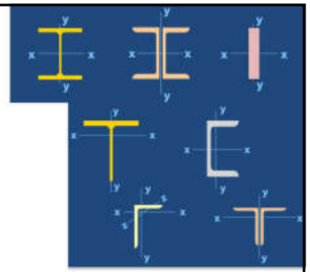
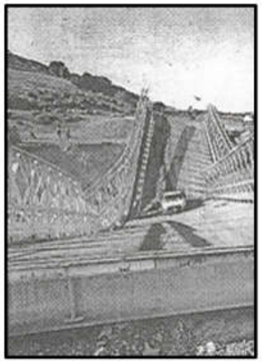
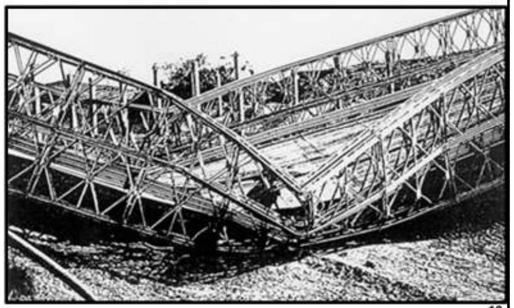
Member instability: Lateral Torsional Buckling







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Lateral Torsional Buckling

- Theoretical bifurcation
  - solution
  - assumptions
- Undoing those assumptions (approaching reality)
  - not fully elastic, partial yielding
  - alternative loading and support conditions
- Beam curves
  - AISC
  - others

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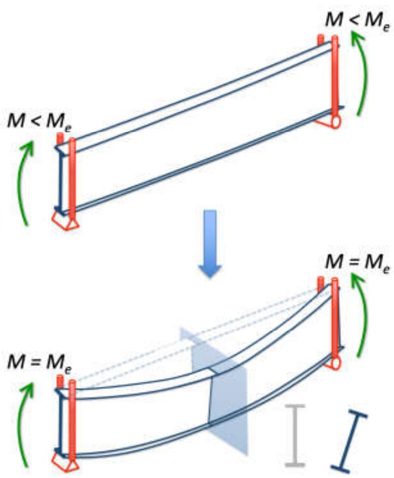
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



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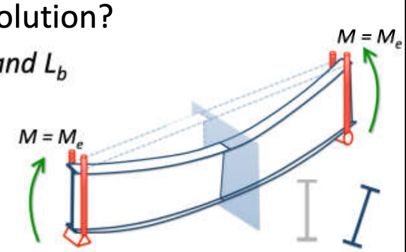
LTB (2)

- Before obtaining the theoretical solution for M_e, let's do a "parametric" analysis…
- Terms expected in the solution?
  - Minor axis buckling: EI_y *and* L_b
  - Torsion
    - St. Venant: GJ and L_b
    - Warping: EC_w and L_b
  - Others? π (of course!)



- What's their impact?

Material: $E\uparrow, G\uparrow \Rightarrow M_e\uparrow$
Terms in numerator → Section: $I_y\uparrow, J\uparrow, C_w\uparrow \Rightarrow M_e\uparrow$
Term in denominator → Unbraced length: $L_b\uparrow \Rightarrow M_e\downarrow$

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Wait…

- Minor axis buckling, I recall from earlier slides

$$P_E = \frac{\pi^2 EI}{L^2}$$

- But, I need a quick refresher on torsion!

St. Venant ?
Warping ????

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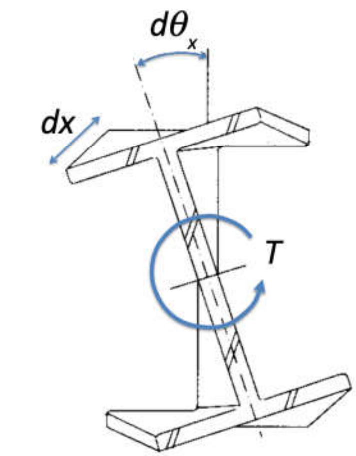
St. Venant Torsion

Consider a portion of the member of length dx subject to a torque T.
If we consider only St. Venant (uniform) torsion, the rotation per unit length is:

$$\frac{d\theta_x}{dx} = \frac{T}{GJ}$$

$\theta = TL/GJ$



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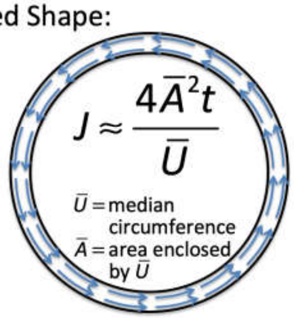
St. Venant Torsion (2)

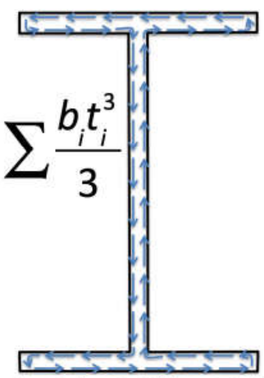
Closed Shape:



$$J \approx \frac{4\bar{A}^2 t}{\bar{U}}$$

$\bar{U}$ = median circumference
$\bar{A}$ = area enclosed by $\bar{U}$

Open Shape:



$$J \approx \sum \frac{b_i t_i^3}{3}$$

Circular Hollow Shape:
t = 0.25”, A = 3.84 in^2
$D = A/(\pi t)$ = 4.90”

$$J = \frac{4(\pi \bar{D}^2/4)^2 t}{\pi \bar{D}} = 22.95 \text{ in}^4$$

W8x13 (t_w=0.23”, t_f=0.26”):
A = 3.84 in^2
J = 0.0871 in^4

Factor of 264…closed sections rule in torsion!

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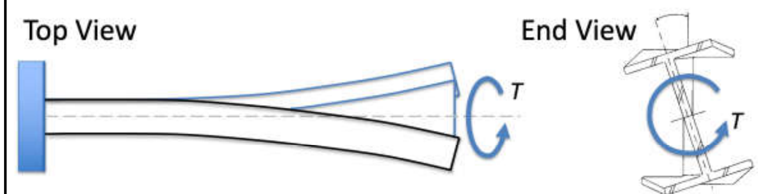
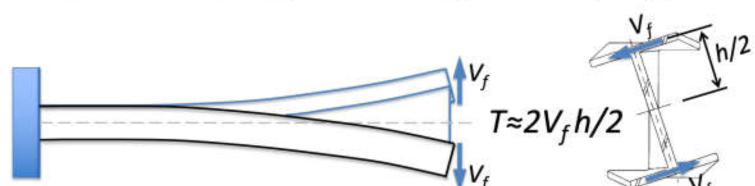
Warping Torsion (your new best friend!)



Notice that this torque T causes the flanges to bend in opposite directions. The resistance to this “cross flange” bending opposes the applied torque (yeah!)



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The twist on torsion

St. Venant (uniform):

$$T_{SV} = GJ\frac{d\theta_x}{dx}$$

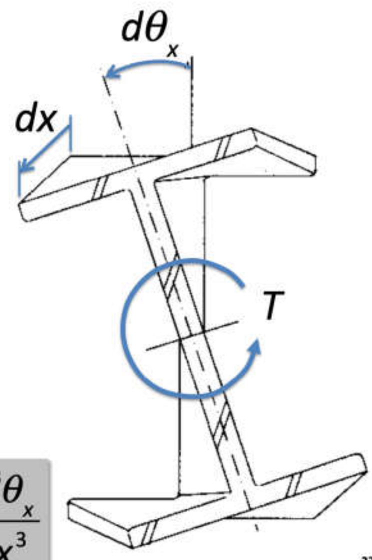
Warping(non-uniform):

$$T_w = -EC_w\frac{d^3\theta_x}{dx^3}$$

Total resisting torque:

$$T = T_{SV} + T_w$$

$$T = GJ\frac{d\theta_x}{dx} - EC_w\frac{d^3\theta_x}{dx^3}$$



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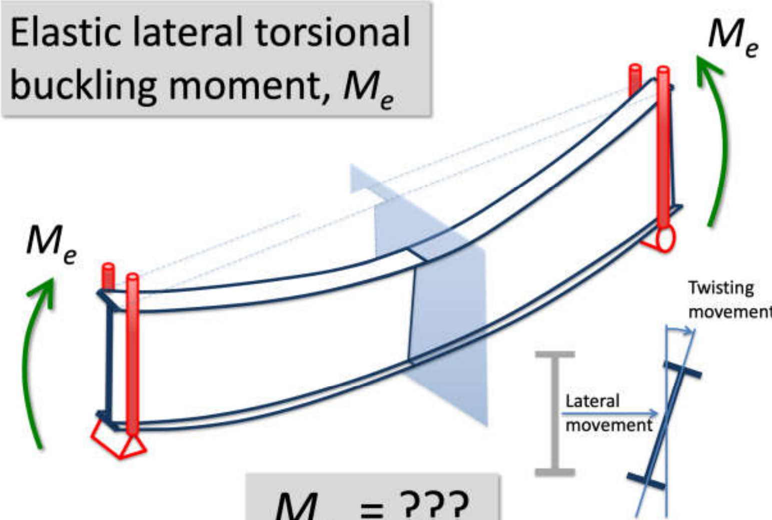
Elastic lateral torsional buckling moment, M_e



M_e = ???

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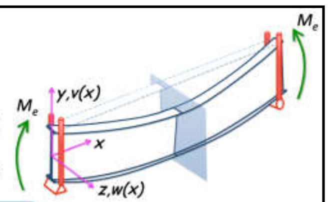
Summary

Equilibrium on deformed shape:



"applied" torque — "resisting" torque

$$T = M_e\frac{dw}{dx} = GJ\frac{d\theta_x}{dx} - EC_w\frac{d^3\theta_x}{dx^3}$$

$$\frac{d}{dx}\left(M_e\frac{dw}{dx}\right) = \frac{d}{dx}\left(GJ\frac{d\theta_x}{dx} - EC_w\frac{d^3\theta_x}{dx^3}\right)$$

$$M_e\frac{d^2w}{dx^2} = GJ\frac{d^2\theta_x}{dx^2} - EC_w\frac{d^4\theta_x}{dx^4}$$

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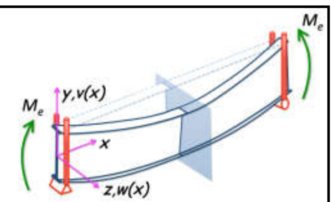
Summary (2)



$$M_e\frac{d^2w}{dx^2} = GJ\frac{d^2\theta_x}{dx^2} - EC_w\frac{d^4\theta_x}{dx^4}$$

$$M_{y'} = M_e\theta_x = -EI_{y'}\frac{d^2w}{dx^2} \quad\Rightarrow\quad \frac{d^2w}{dx^2} = -\frac{M_e}{EI_{y'}}\theta_x$$

$$-\frac{M_e^2}{EI_{y'}}\theta_x = GJ\frac{d^2\theta_x}{dx^2} - EC_w\frac{d^4\theta_x}{dx^4}$$

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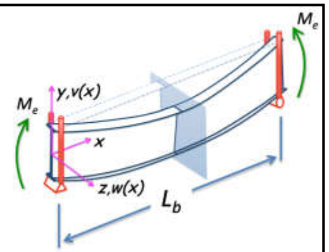
Summary (3)

Solve differential equation

$$EC_w \frac{d^4\theta_x}{dx^4} - GJ\frac{d^2\theta_x}{dx^2} - \frac{M_e^2}{EI_{y'}}\theta_x = 0$$



and apply boundary conditions

$$\theta_x(x=0)=0,\ \theta_x(x=L_b)=0$$

$$\theta_x''(x=0)=0,\ \theta_x''(x=L_b)=0$$

Results in

$$M_e^2 = \left(\frac{\pi^2 EI_y}{L_b^2}\right)\left(GJ + \frac{\pi^2}{L_b^2}EC_w\right)$$

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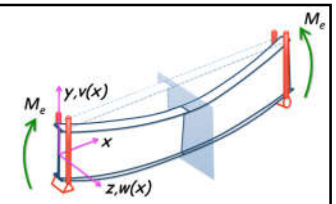
Summary (4)

This sort of makes sense!



$$M_e^2 = \left(\frac{\pi^2 EI_y}{L_b^2}\right)\left(GJ + \frac{\pi^2}{L_b^2}EC_w\right)$$

Top flange in compression trying to produce minor axis buckling

Bottom flange in tension resisting this minor axis buckling by creating a resisting torque, which includes both St. Venant and Warping components

which simplifies to:

$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

Also note that our earlier parametric study was spot on!

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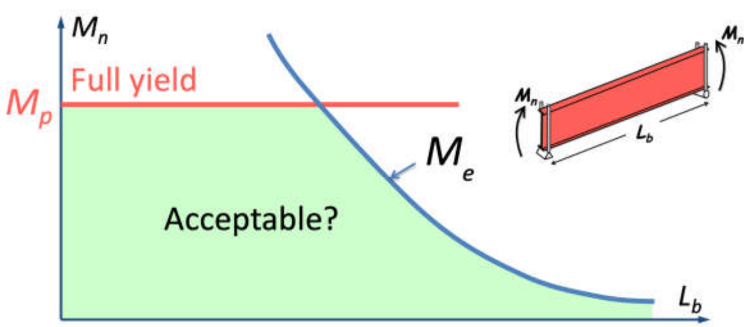
- Elastic lateral-torsional buckling

$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

- Beam Curve – Take 2



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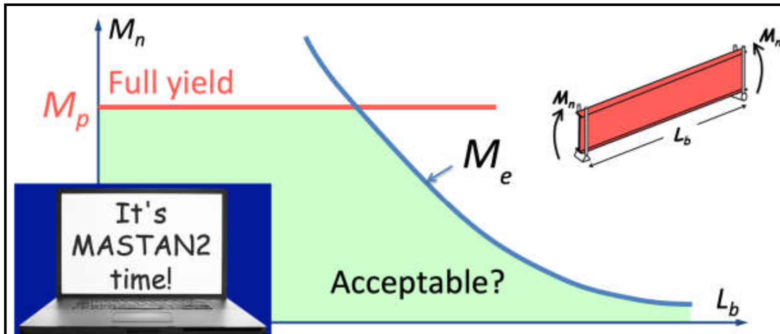

W24x68 (E=29,000, F_y = 50, Z=177, include warping)

L_b	$M_{n,LBA}$	$M_{n,GNA}$	$M_{n,MNA}$	$M_{n,min}$
100				
300				

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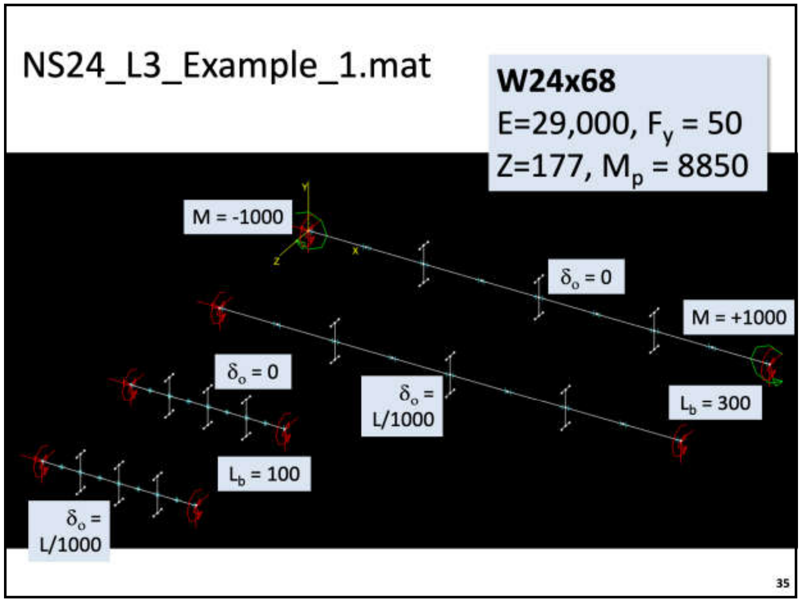
NS24_L3_Example_1.mat
W24x68
E=29,000, F_y = 50
Z=177, M_p = 8850
M = -1000
δ_o = 0
M = +1000
δ_o = 0
δ_o = L/1000
L_b = 300
L_b = 100
δ_o = L/1000
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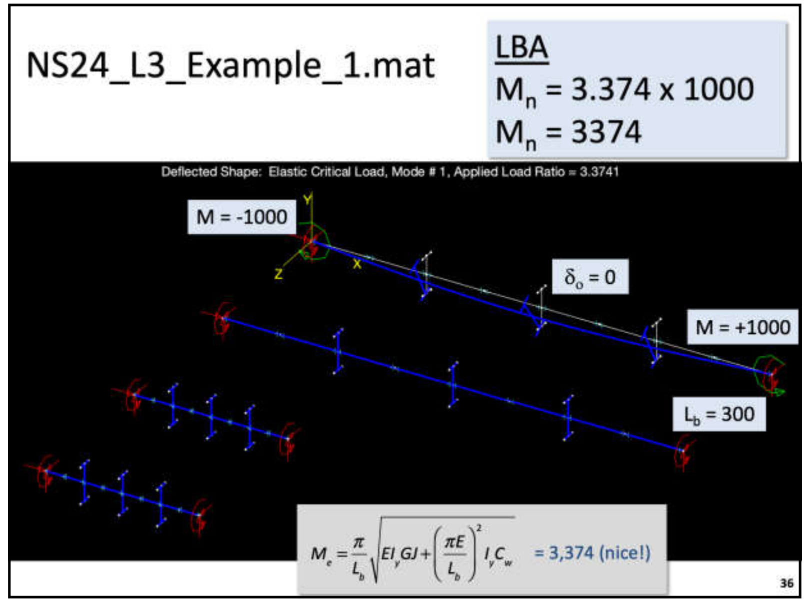
NS24_L3_Example_1.mat
LBA
M_n = 3.374 x 1000
M_n = 3374
Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 3.3741
M = -1000
δ_o = 0
M = +1000
L_b = 300
$M_e = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$ = 3,374 (nice!)
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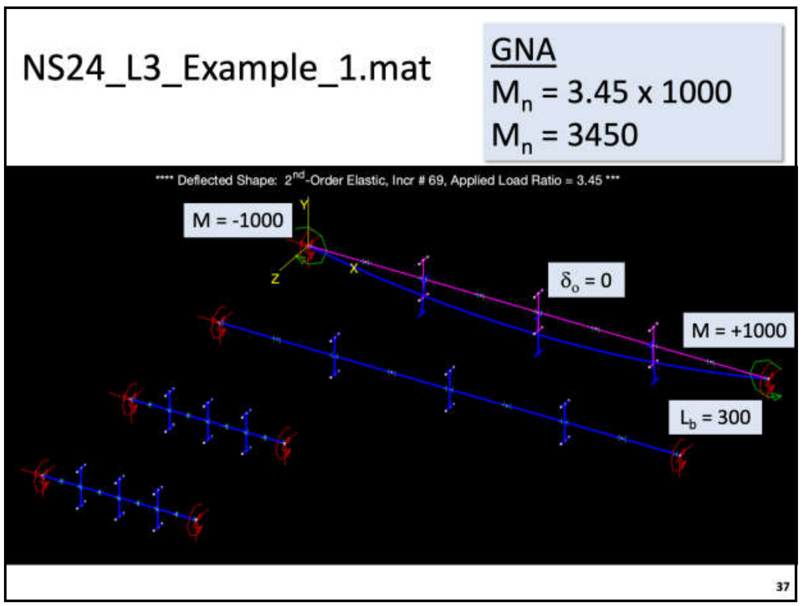
NS24_L3_Example_1.mat
GNA
M_n = 3.45 x 1000
M_n = 3450
**** Deflected Shape: 2nd-Order Elastic, Incr # 69, Applied Load Ratio = 3.45 ***
M = -1000
δ_o = 0
M = +1000
L_b = 300
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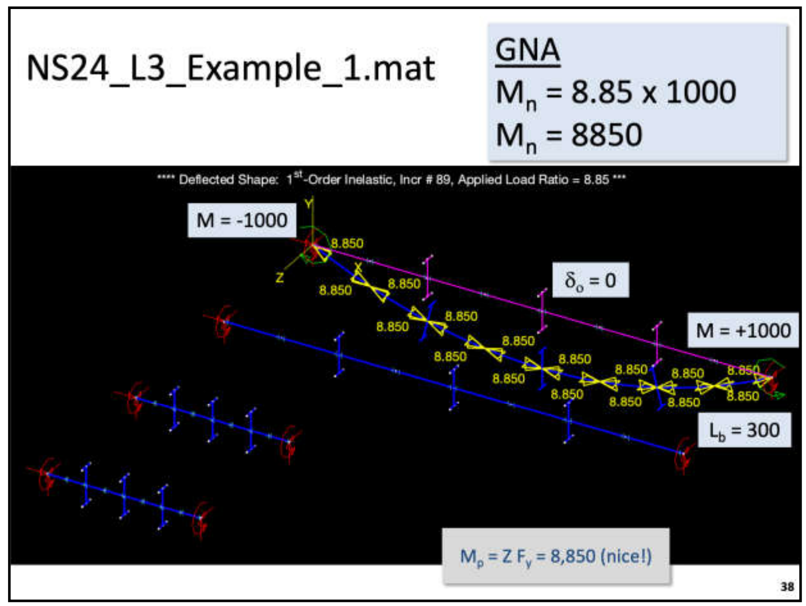
NS24_L3_Example_1.mat
GNA
M_n = 8.85 x 1000
M_n = 8850
**** Deflected Shape: 1st-Order Inelastic, Incr # 89, Applied Load Ratio = 8.85 ***
M = -1000
δ_o = 0
M = +1000
L_b = 300
$M_p = Z\,F_y$ = 8,850 (nice!)
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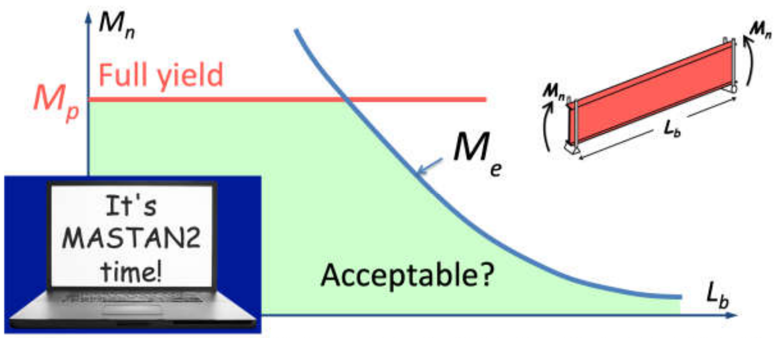

W24x68 (E=29,000, F_y = 50, Z=177, include warping)

L_b	$M_{n,LBA}$	$M_{n,GNA}$	$M_{n,MNA}$	$M_{n,min}$
100	24206	24600	8850	8850
300	3374	3450	8850	3374

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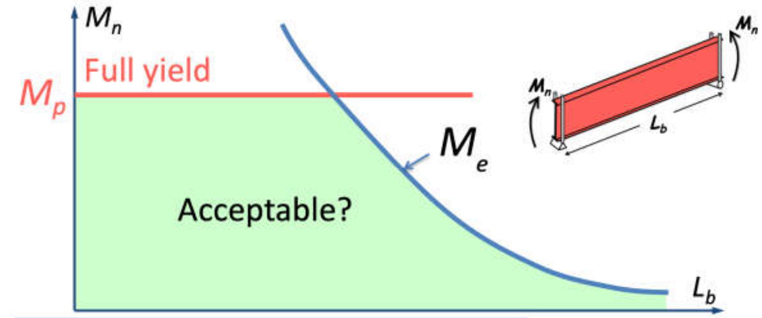
- Elastic lateral-torsional buckling

$$M_e = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$$

- Beam Curve – Take 2



- What about those assumptions?

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Lateral Torsional Buckling

- Theoretical bifurcation
  - solution
  - assumptions
- Undoing those assumptions (approaching reality)
  - not fully elastic, partial yielding
  - alternative loading and support conditions
- Beam curves
  - AISC
  - others

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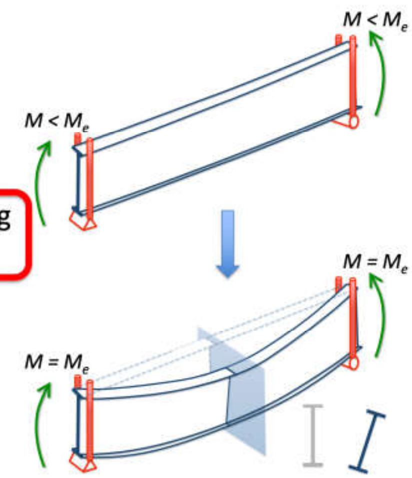
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



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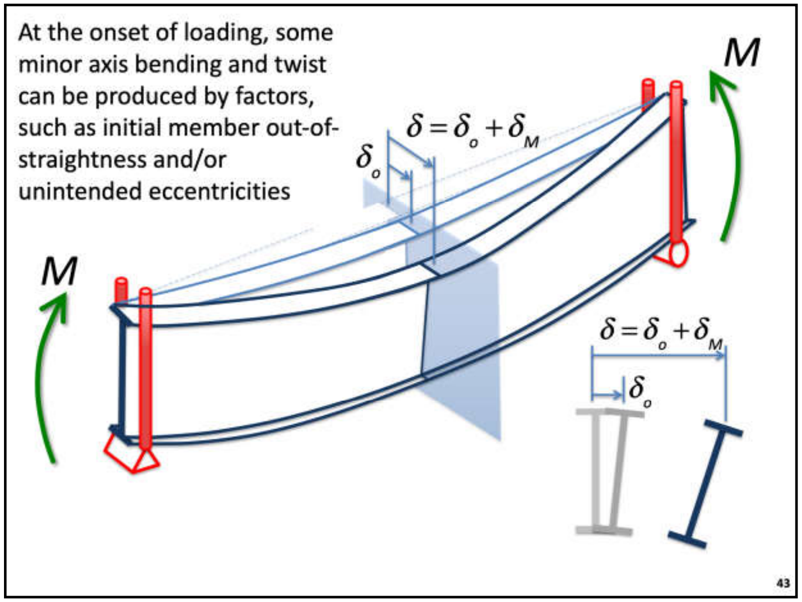
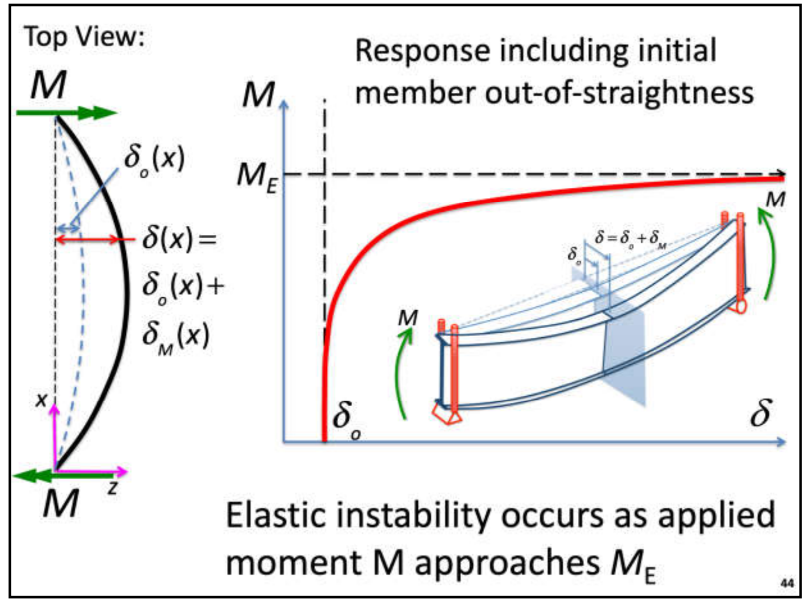
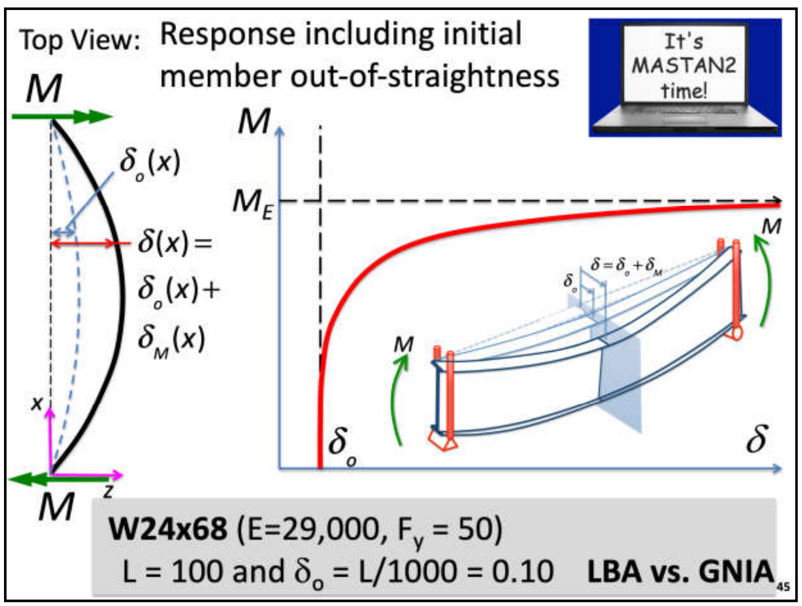
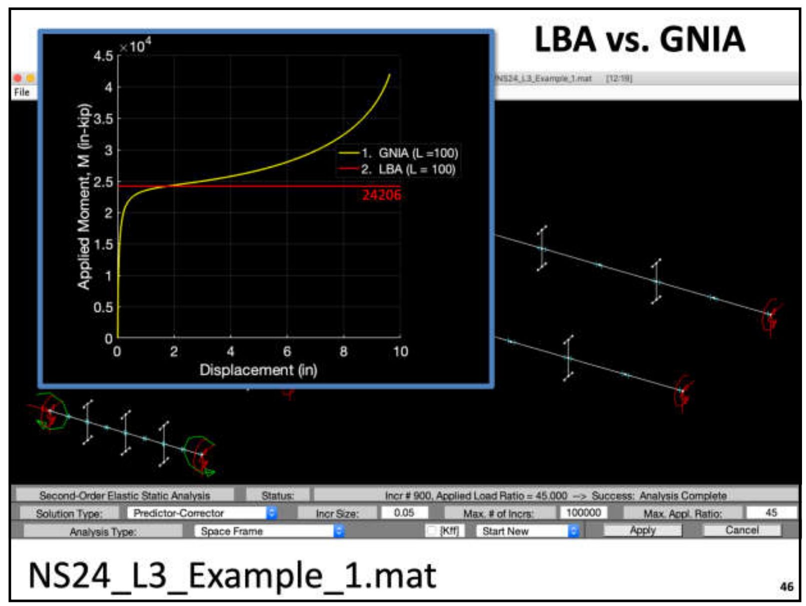
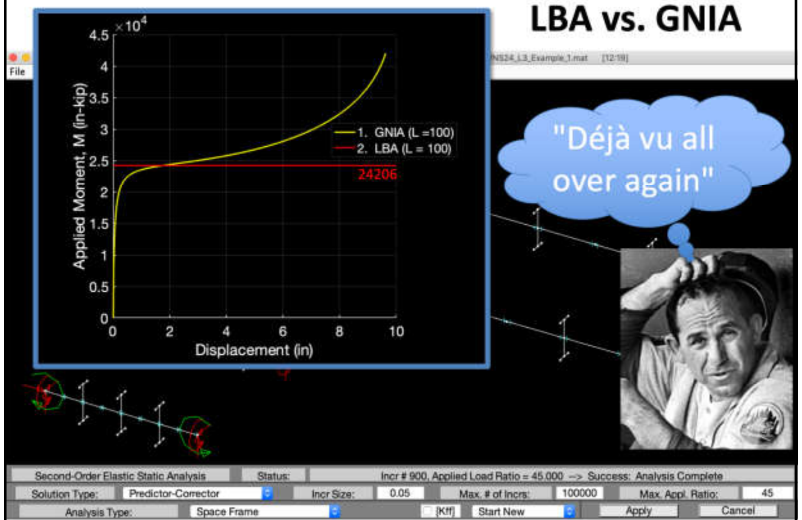

At the onset of loading, some minor axis bending and twist can be produced by factors, such as initial member out-of-straightness and/or unintended eccentricities
M
M
$\delta = \delta_o + \delta_M$
δ_o
$\delta = \delta_o + \delta_M$
δ_o
43


Top View:
M
$\delta_o(x)$
$\delta(x) = \delta_o(x) + \delta_M(x)$
x
z
M
Response including initial member out-of-straightness
M
M_E
δ_o
δ
Elastic instability occurs as applied moment M approaches M_E
44


Top View:
Response including initial member out-of-straightness
It's MASTAN2 time!
M
$\delta_o(x)$
$\delta(x) = \delta_o(x) + \delta_M(x)$
x
z
M
M
M_E
δ_o
δ
W24x68 (E=29,000, F_y = 50)
L = 100 and δ_o = L/1000 = 0.10 **LBA vs. GNIA**
45


LBA vs. GNIA
×10^4
Applied Moment, M (in-kip)
Displacement (in)
1. GNIA (L =100)
2. LBA (L = 100)
24206
Second-Order Elastic Static Analysis
Incr # 900, Applied Load Ratio = 45.000 --> Success: Analysis Complete
NS24_L3_Example_1.mat
46

LBA vs. GNIA



NS24_L3_Example_1.mat

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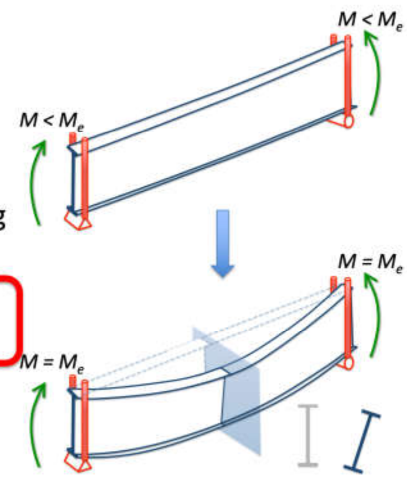
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



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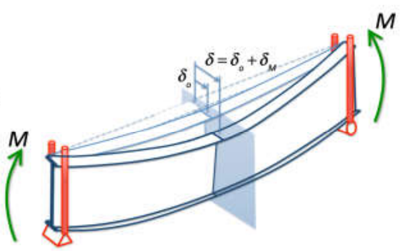
Partial Yielding

- As loading is applied, cross section may begin to yield due to
  - major axis bending
  - minor axis bending
  - torsion (warping stresses)



- Yielding is accentuated by presence of residual stresses
- Yielding results in loss of stiffness, which may result in inelastic lateral torsional buckling.

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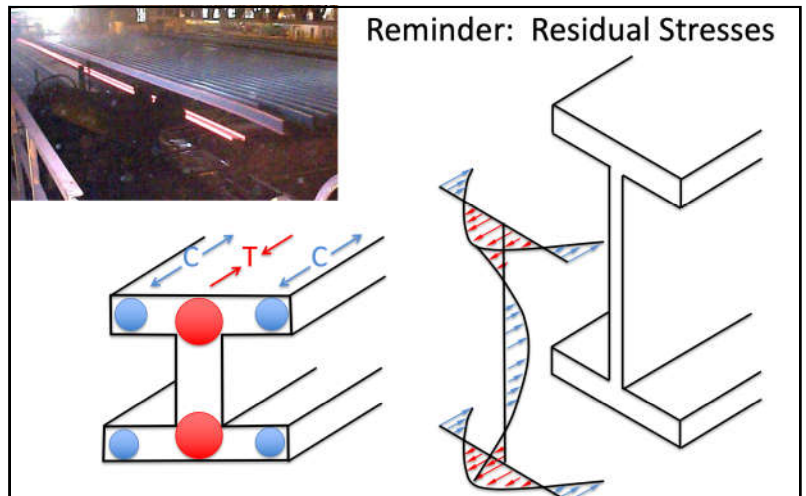
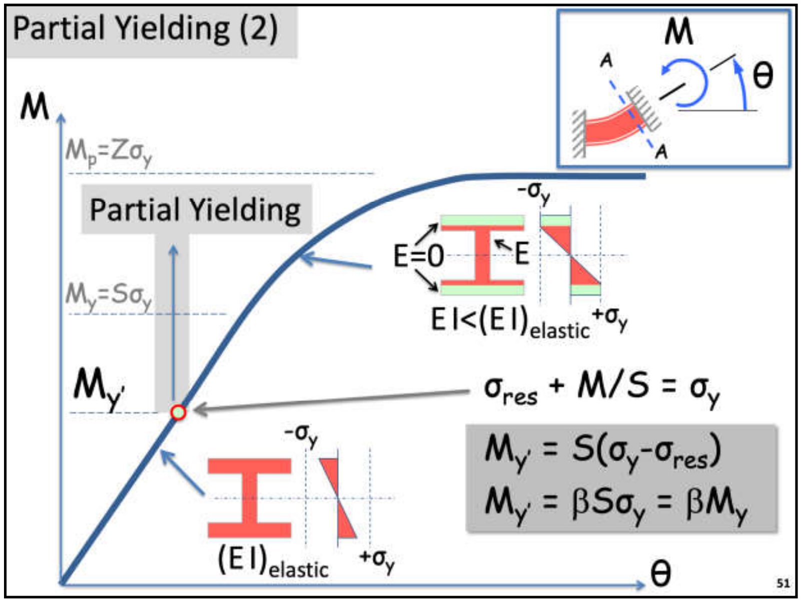
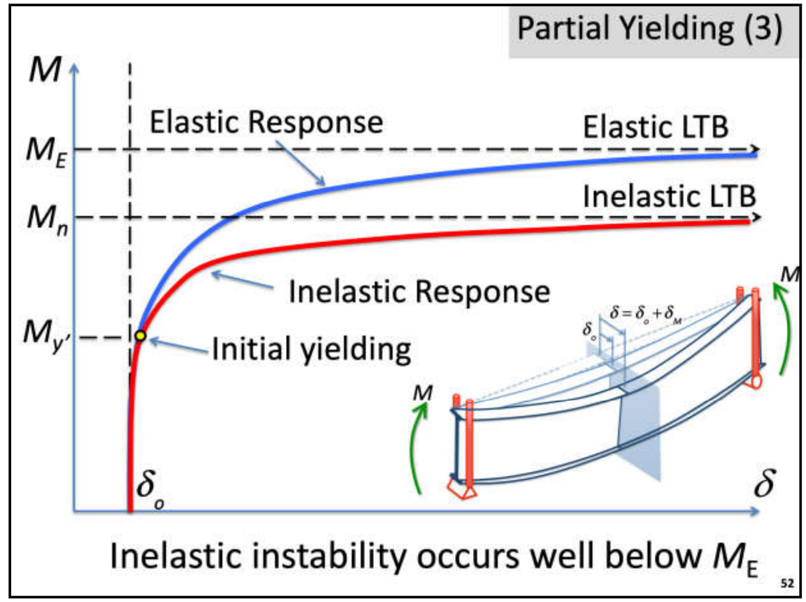
Reminder: Residual Stresses



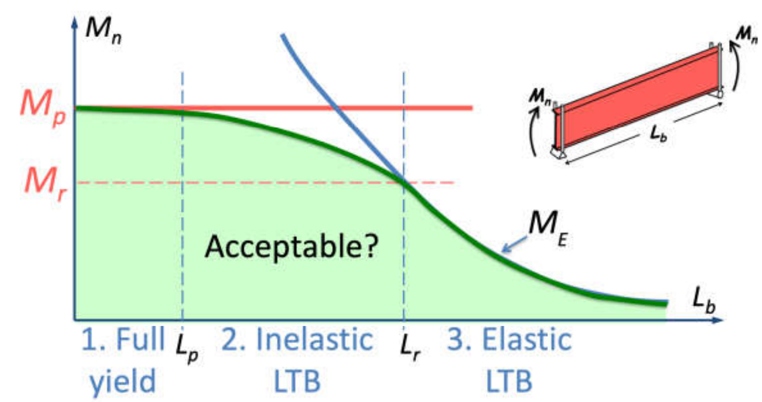
Occur in rolled wide flange shapes because locations with high surface area (e.g., flange tips) cool well before locations with smaller surface area (flange-to-web intersections)

50





- Beam Curve – Take 3



- Where did L_p and L_r come from?
- What about those assumptions?

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- Where did L_r come from?
  - L_r is unbraced length at transition from inelastic to elastic LTB
  - Equate M_r (moment to produce first yield including residual stresses) with M_e (elastic LTB moment)…and solve for L_b

$$S(\sigma_y - \sigma_{res}) = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

Specification for Structural Steel Buildings, July 7, 2016
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

L_r, the limiting unbraced length for the limit state of inelastic lateral-torsional buckling, in. (mm), is:

$$L_r = 1.95 r_{ts}\frac{E}{0.7F_y}\sqrt{\frac{Jc}{S_x h_o}+\sqrt{\left(\frac{Jc}{S_x h_o}\right)^2+6.76\left(\frac{0.7F_y}{E}\right)^2}} \quad \text{(F2-6)}$$

where
r_y = radius of gyration about y-axis, in. (mm)

$$r_{ts}^2 = \frac{\sqrt{I_y C_w}}{S_x} \quad \text{(F2-7)}$$

Yikes!!!

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- Where did L_p come from?
  - L_p is unbraced length at transition from full yielding to inelastic LTB
  - Game of darts in an AISC cigar filled room… …not quite!
  - Varies from code to code and is based on analytical and experimental studies
  - For a compact I-shaped member, AISC gives

$$L_p = 1.76 r_y \sqrt{E/F_y}$$

Specification for Structural Steel Buildings, July 7, 2016
AMERICAN INSTITUTE OF STEEL CONSTRUCTION

L_p, the limiting laterally unbraced length for the limit state of yielding, in. (mm), is:

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \quad \text{(F2-5)}$$

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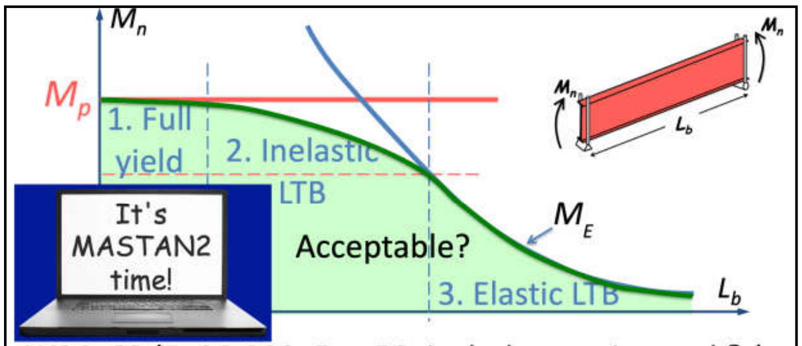

W24x68 (E=29,000, F_y = 50, include warping and δ_o)

L_b	$M_{n,LBA}$	$M_{n,MNA}$	$M_{n,GMNIA}$	$M_{n,min}$
100	24206	8850		
300	3374	8850		

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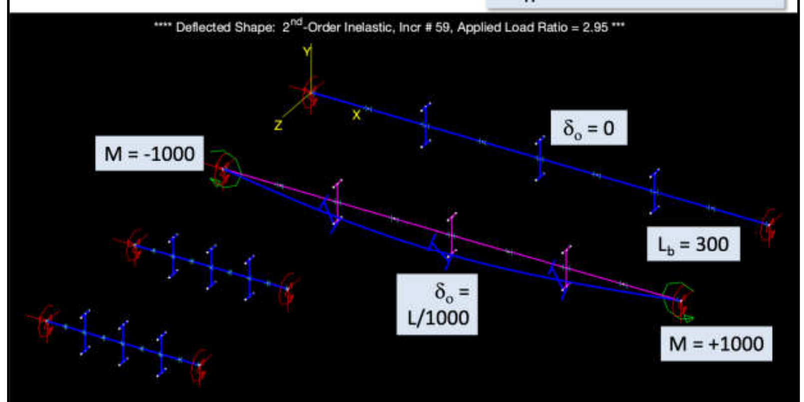
NS24_L3_Example_1.mat

GMNIA
M_n = 2.95 x 1000
M_n = 2950



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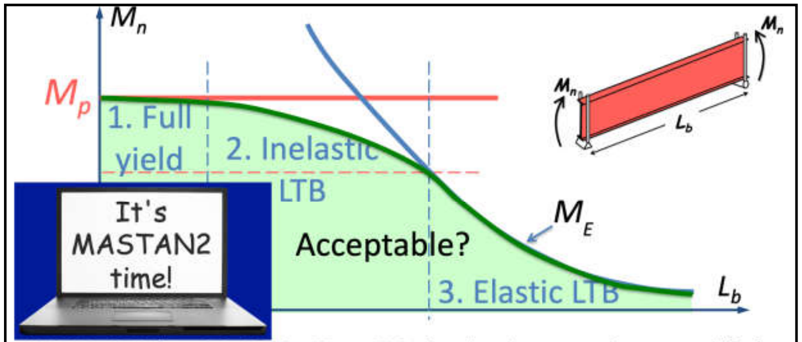

W24x68 (E=29,000, F_y = 50, include warping and δ_o)

L_b	$M_{n,LBA}$	$M_{n,MNA}$	$M_{n,GMNIA}$	$M_{n,min}$
100	24206	8850	8200	8200
300	3374	8850	2950	2950

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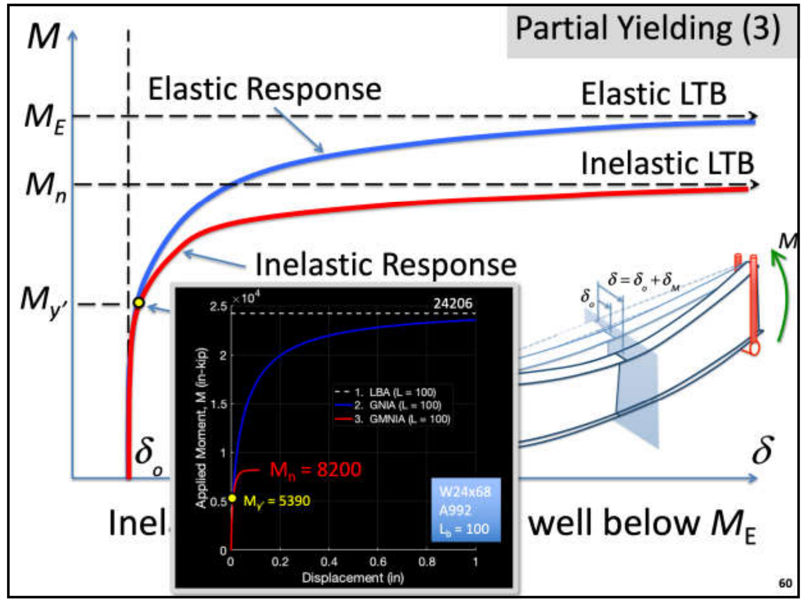

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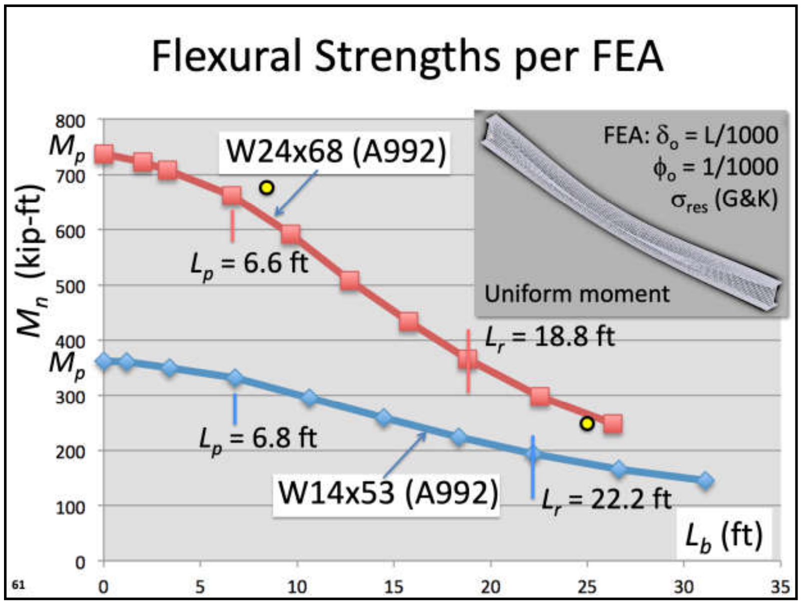

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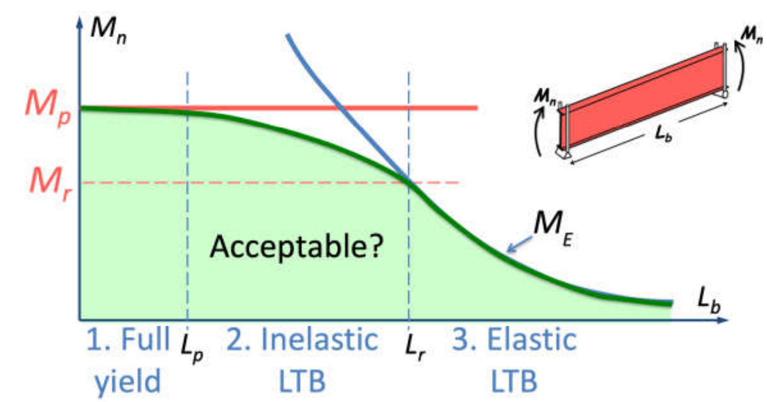
- Beam Curve – Take 3



- Where did L_p and L_r come from? (done!)
- What about those assumptions? (hmmm…)

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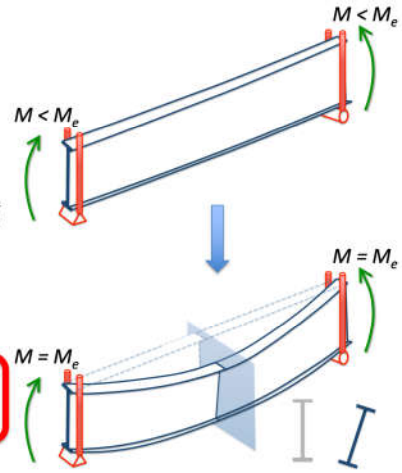
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



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Uniform Moment Distribution

- Provides for "simplest" differential equation and corresponding solution to the elastic LTB problem.
- Most conservative case
  - $M(x)$ = constant
  - maximum compressive stress occurs along entire unbraced length
- In place of formulating and solving for other moment $M(x)$ distributions, results can be adequately approximated by scaling the uniform moment in/elastic LTB solution.

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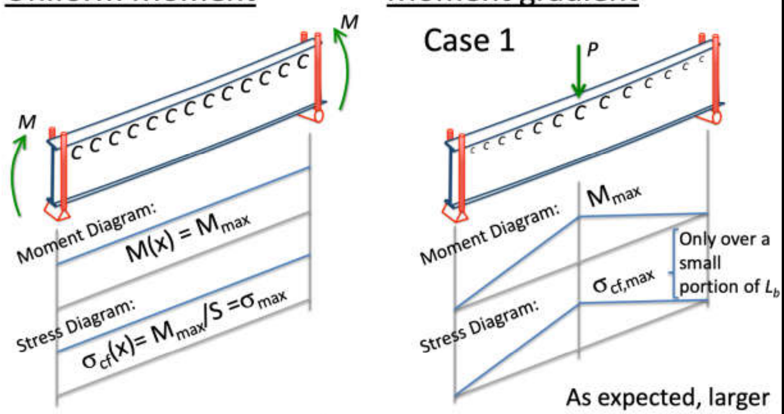
Uniform Moment | **Moment gradient**

Case 1



$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

As expected, larger LTB capacity!

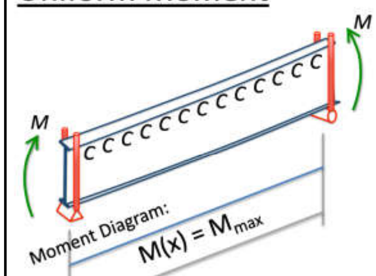
$M_{cr} > M_e$

$M_{cr} = C_b M_e$ with $C_b > 1.0$

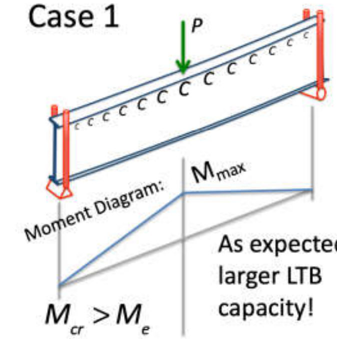
65

Uniform Moment | **Moment gradient**

Case 1



$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$



As expected, larger LTB capacity!

$M_{cr} > M_e$

$M_{cr} = C_b M_e$ with $C_b > 1.0$

W24x68 (E=29,000, F_y = 50)
L = 288 **LBA** C_b **= ???**

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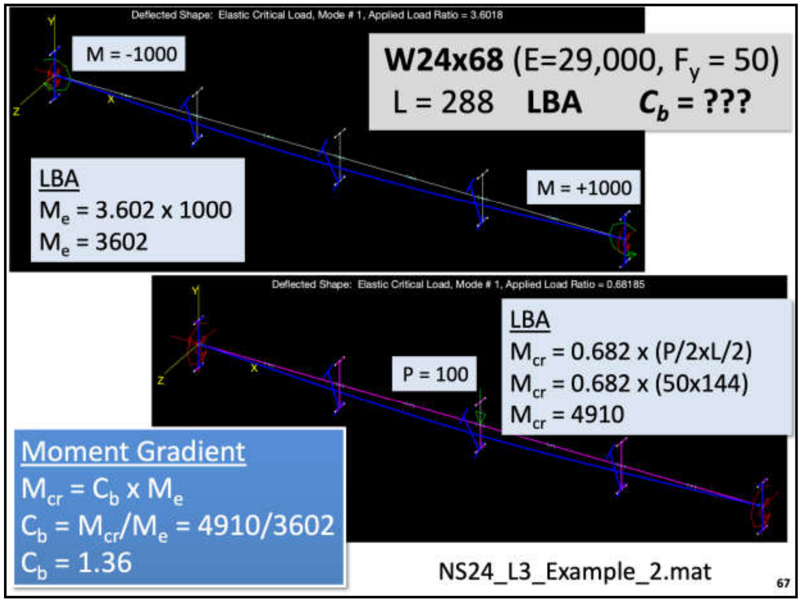
Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 3.6018
M = -1000
W24x68 (E=29,000, F_y = 50)
L = 288 LBA C_b = ???
LBA
M_e = 3.602 x 1000
M_e = 3602
M = +1000
Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 0.68185
P = 100
LBA
M_{cr} = 0.682 x (P/2xL/2)
M_{cr} = 0.682 x (50x144)
M_{cr} = 4910
Moment Gradient
M_{cr} = C_b x M_e
C_b = M_{cr}/M_e = 4910/3602
C_b = 1.36
NS24_L3_Example_2.mat
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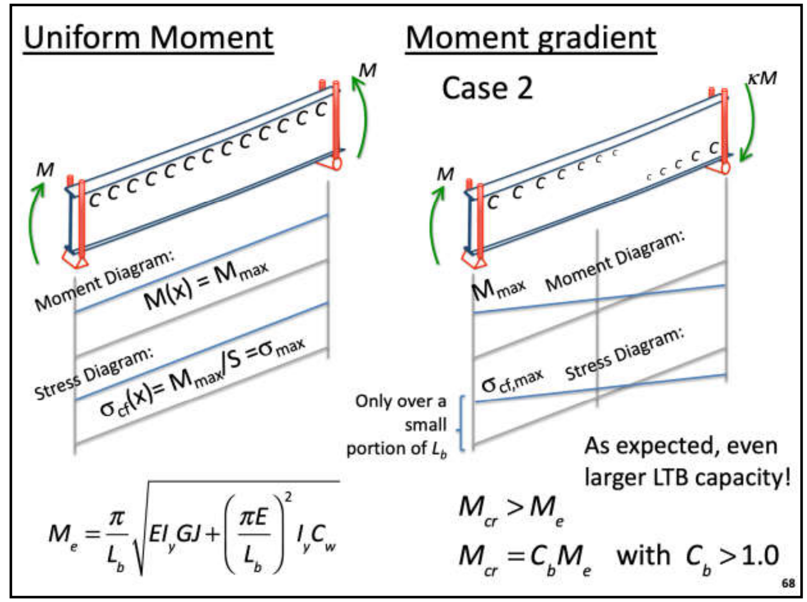
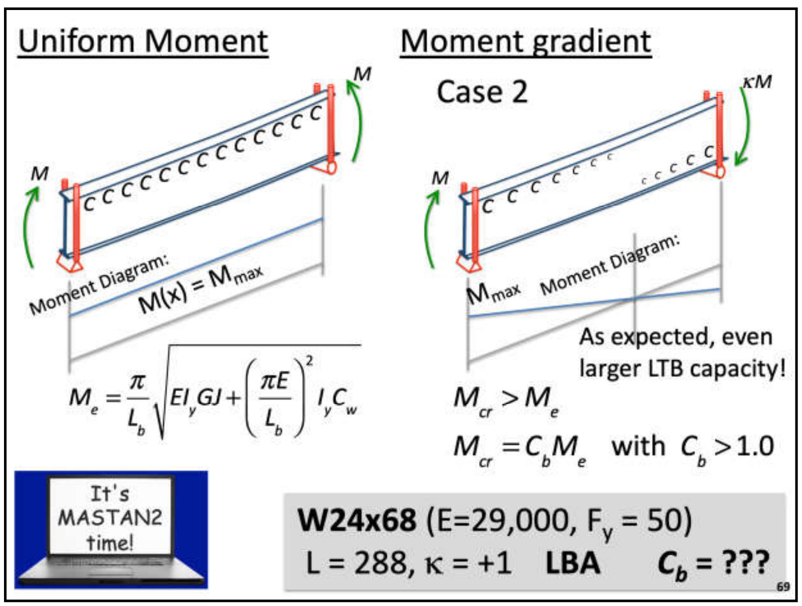
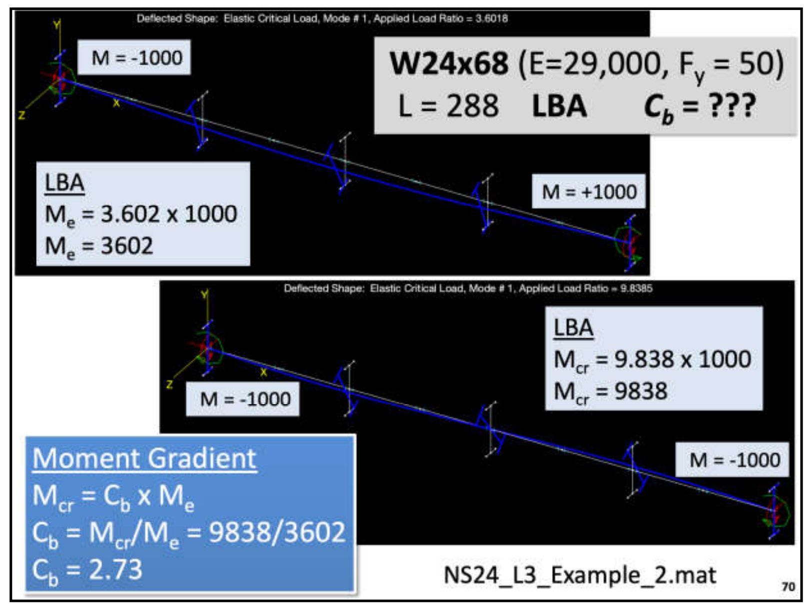

Uniform Moment
Moment gradient
Case 2
M
κM
Moment Diagram:
M(x) = M_{max}
Stress Diagram:
$\sigma_{cf}(x) = M_{max}/S = \sigma_{max}$
M_{max}
$\sigma_{cf,max}$
Only over a small portion of L_b
As expected, even larger LTB capacity!
$M_e = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$
$M_{cr} > M_e$
$M_{cr} = C_bM_e$ with $C_b > 1.0$
68


Uniform Moment
Moment gradient
Case 2
M
κM
Moment Diagram:
M(x) = M_{max}
M_{max}
As expected, even larger LTB capacity!
$M_e = \frac{\pi}{L_b}\sqrt{EI_yGJ + \left(\frac{\pi E}{L_b}\right)^2 I_yC_w}$
$M_{cr} > M_e$
$M_{cr} = C_bM_e$ with $C_b > 1.0$
It's MASTAN2 time!
W24x68 (E=29,000, F_y = 50)
L = 288, κ = +1 LBA C_b = ???
69


Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 3.6018
M = -1000
W24x68 (E=29,000, F_y = 50)
L = 288 LBA C_b = ???
LBA
M_e = 3.602 x 1000
M_e = 3602
M = +1000
Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 9.8385
LBA
M_{cr} = 9.838 x 1000
M_{cr} = 9838
M = -1000
M = -1000
Moment Gradient
M_{cr} = C_b x M_e
C_b = M_{cr}/M_e = 9838/3602
C_b = 2.73
NS24_L3_Example_2.mat
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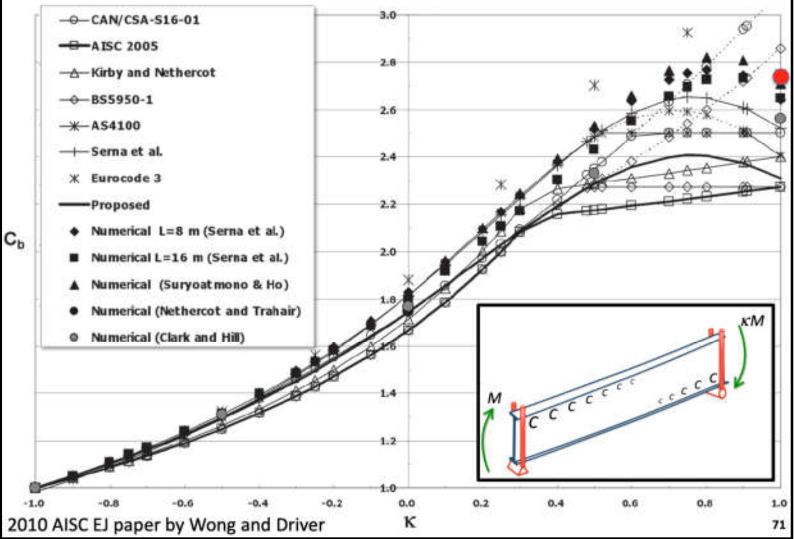
Moment Gradient Factor, C_b



2010 AISC EJ paper by Wong and Driver

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LTB Moment Gradient Factor, C_b

- In/elastic LTB M_n can be adequately approximated by scaling the uniform moment in/elastic LTB solution

$$M_n = C_b M_n^{C_b=1} \leq M_p$$

- Under no conditions can M_n exceed M_p, regardless of moment gradient
- Many possibilities for C_b, AISC uses

$$C_b = \frac{12.5|M_{\max}|}{2.5|M_{\max}| + 3|M_{L_b/4}| + 4|M_{L_b/2}| + 3|M_{3L_b/4}|}$$

- See 2010 AISC EJ paper by Wong and Driver!

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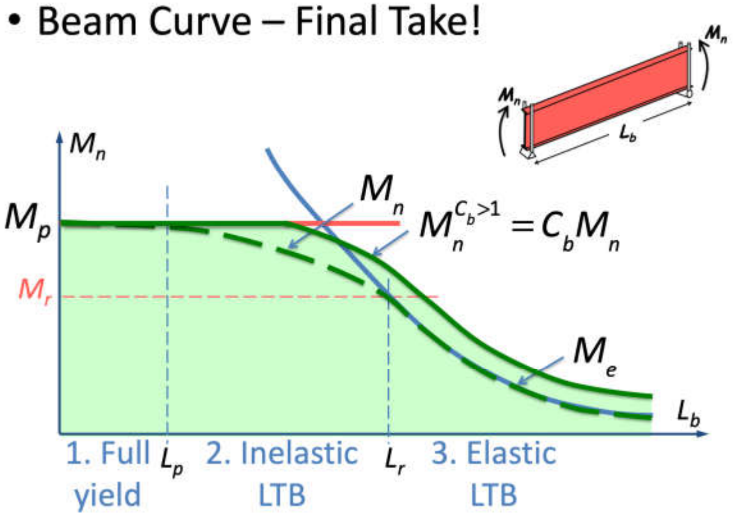
- Beam Curve – Final Take!



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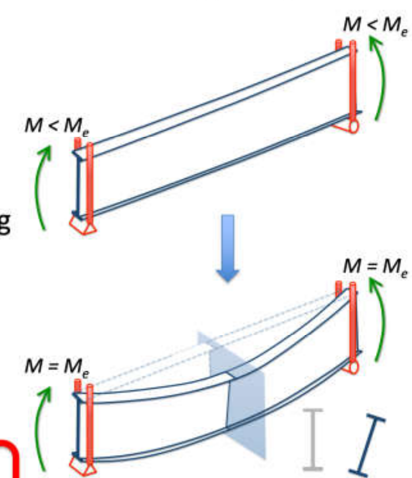
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling
  - linear elastic behavior (E = constant)
  - uniform moment distribution
  - braced at the ends (frictionless)



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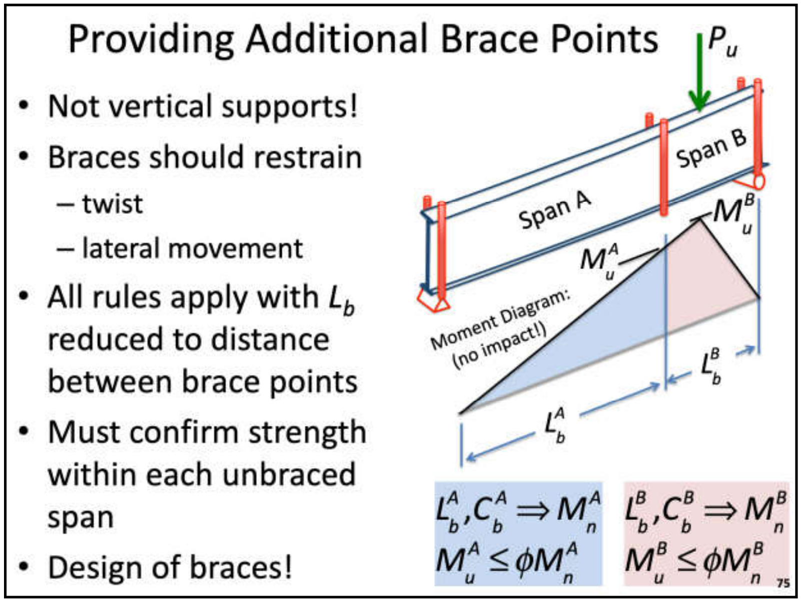
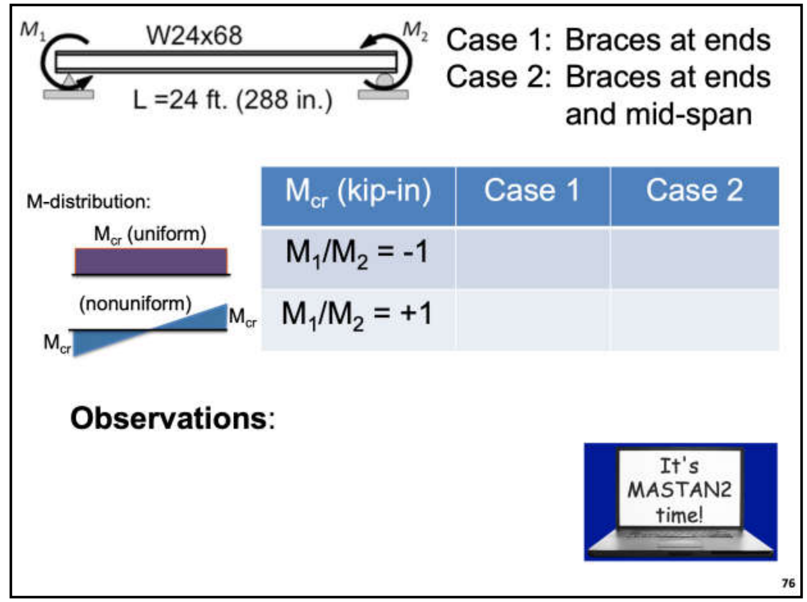

Providing Additional Brace Points
• Not vertical supports!
• Braces should restrain
– twist
– lateral movement
• All rules apply with L_b reduced to distance between brace points
• Must confirm strength within each unbraced span
• Design of braces!
P_u
Span A
Span B
M_u^A
M_u^B
Moment Diagram: (no impact!)
L_b^A
L_b^B
$L_b^A, C_b^A \Rightarrow M_n^A$
$M_u^A \le \phi M_n^A$
$L_b^B, C_b^B \Rightarrow M_n^B$
$M_u^B \le \phi M_n^B$
75


M_1
W24x68
M_2
L =24 ft. (288 in.)
Case 1: Braces at ends
Case 2: Braces at ends and mid-span
M-distribution:
M_{cr} (uniform)
(nonuniform)
M_{cr}
M_{cr}
M_{cr} (kip-in)	Case 1	Case 2
M_1/M_2 = -1		
M_1/M_2 = +1		
Observations:
It's MASTAN2 time!
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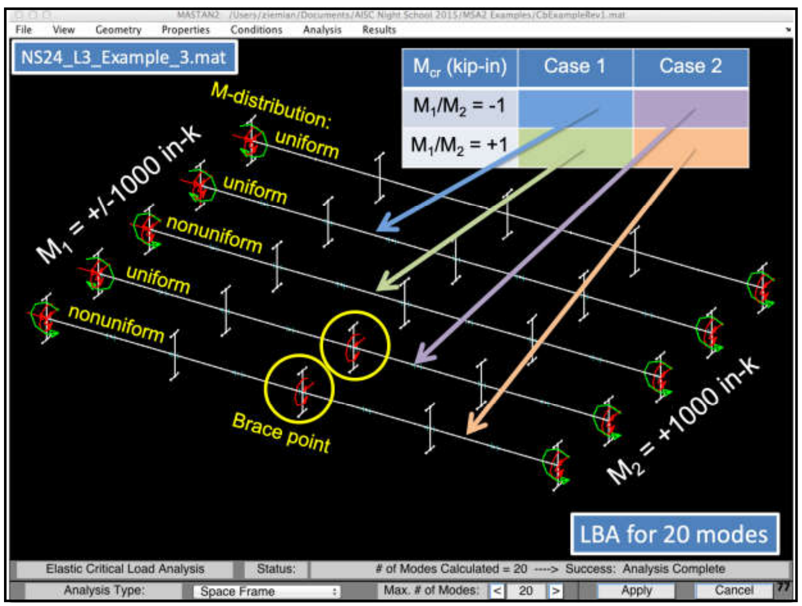
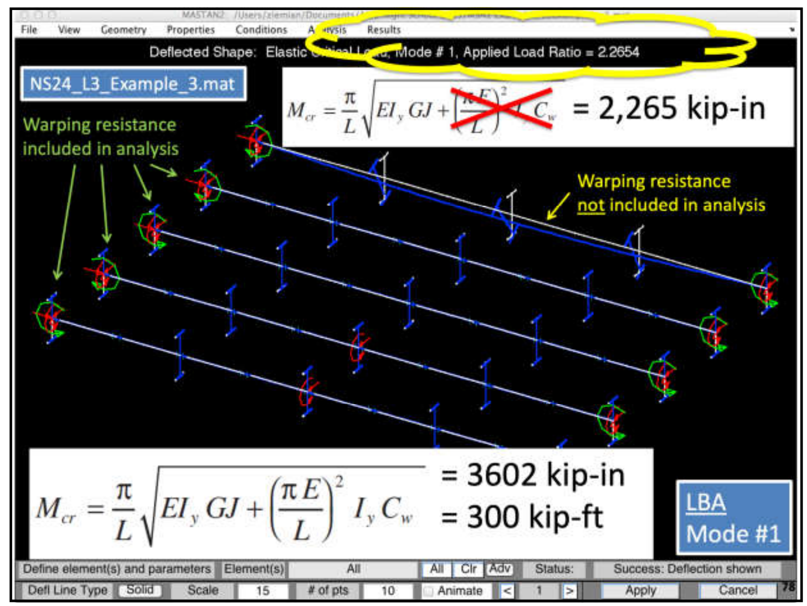
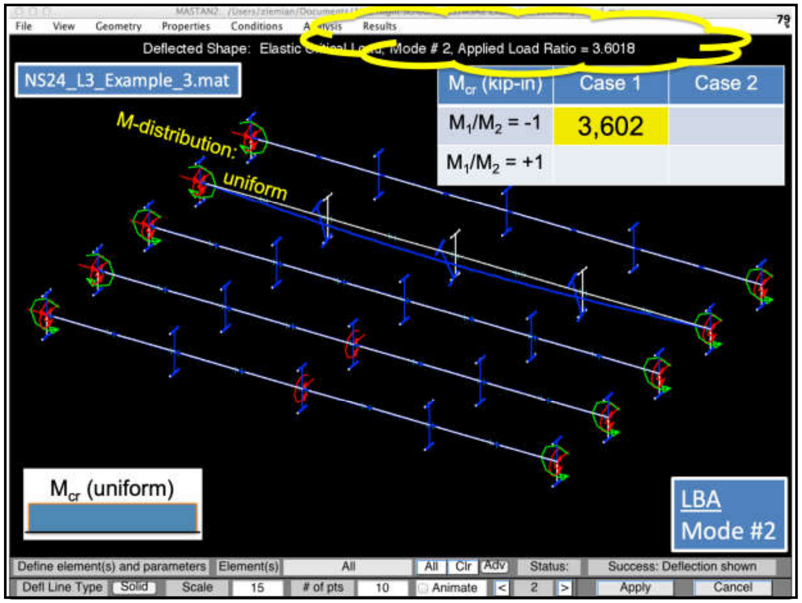
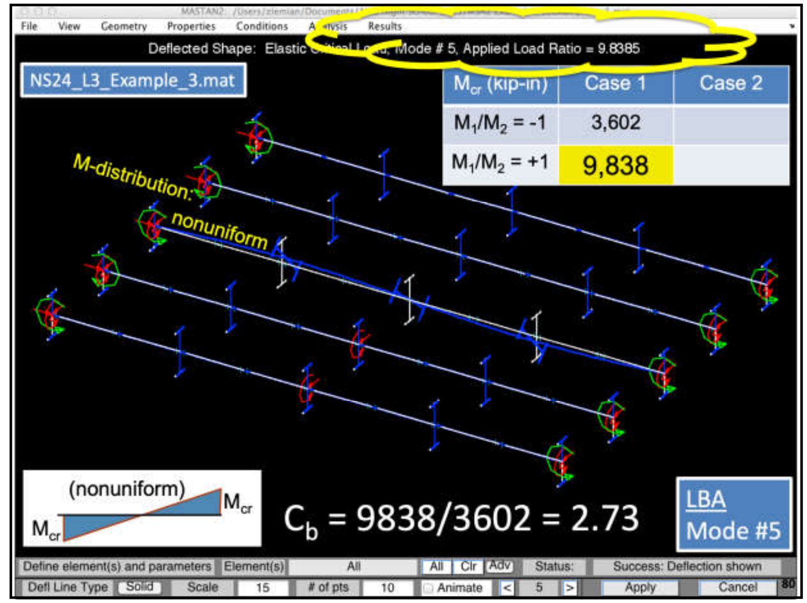
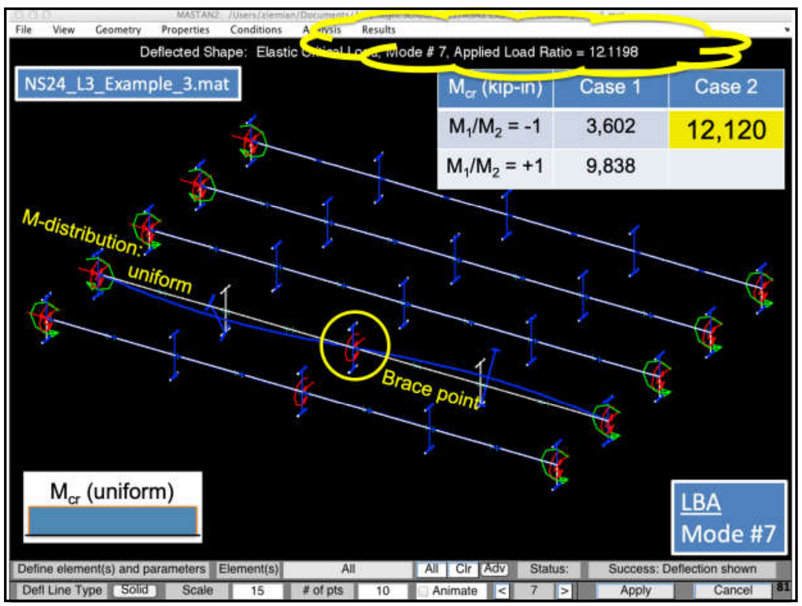
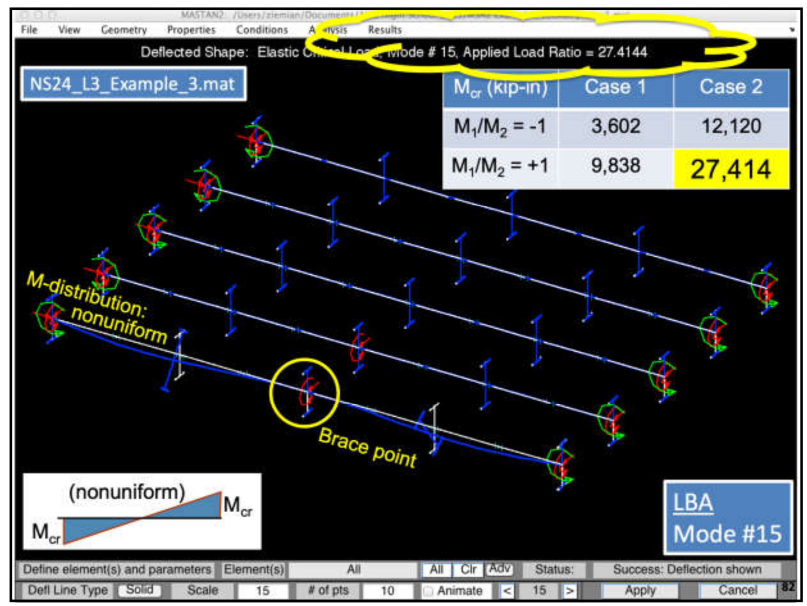

NS24_L3_Example_3.mat
M-distribution:
uniform
uniform
nonuniform
uniform
nonuniform
M_1 = +/-1000 in-k
M_2 = +1000 in-k
Brace point
M_{cr} (kip-in)	Case 1	Case 2
M_1/M_2 = -1		
M_1/M_2 = +1		
LBA for 20 modes
Elastic Critical Load Analysis
# of Modes Calculated = 20 ---> Success: Analysis Complete
Analysis Type: Space Frame
Max. # of Modes: 20
Apply
Cancel
77

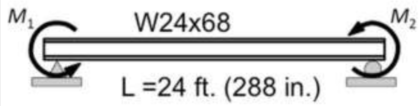

Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 2.2654
NS24_L3_Example_3.mat
$M_{cr} = \frac{\pi}{L}\sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$ = 2,265 kip-in
Warping resistance included in analysis
Warping resistance not included in analysis
$M_{cr} = \frac{\pi}{L}\sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$ = 3602 kip-in = 300 kip-ft
LBA Mode #1
Success: Deflection shown
Apply
Cancel
78


Deflected Shape: Elastic Critical Load, Mode # 2, Applied Load Ratio = 3.6018
NS24_L3_Example_3.mat
Mcr (kip-in) | Case 1 | Case 2
M1/M2 = -1 | 3,602
M1/M2 = +1
M-distribution: uniform
Mcr (uniform)
LBA Mode #2


Deflected Shape: Elastic Critical Load, Mode # 5, Applied Load Ratio = 9.8385
NS24_L3_Example_3.mat
Mcr (kip-in) | Case 1 | Case 2
M1/M2 = -1 | 3,602
M1/M2 = +1 | 9,838
M-distribution: nonuniform
(nonuniform) Mcr Mcr
Cb = 9838/3602 = 2.73
LBA Mode #5


Deflected Shape: Elastic Critical Load, Mode # 7, Applied Load Ratio = 12.1198
NS24_L3_Example_3.mat
Mcr (kip-in) | Case 1 | Case 2
M1/M2 = -1 | 3,602 | 12,120
M1/M2 = +1 | 9,838
M-distribution: uniform
Brace point
Mcr (uniform)
LBA Mode #7


Deflected Shape: Elastic Critical Load, Mode # 15, Applied Load Ratio = 27.4144
NS24_L3_Example_3.mat
Mcr (kip-in) | Case 1 | Case 2
M1/M2 = -1 | 3,602 | 12,120
M1/M2 = +1 | 9,838 | 27,414
M-distribution: nonuniform
Brace point
(nonuniform) Mcr Mcr
LBA Mode #15



Case 1: Braces at ends
Case 2: Braces at ends and mid-span

M_{cr} (kip-in)	Case 1	Case 2	
M_1/M_2 = -1	3,602	12,120	3.36
M_1/M_2 = +1	9,838	27,414	7.61
	2.73		

Observations:

- M_{cr} increases as L_b is reduced
- M_{cr} increases as moment gradient increases
- An inflection point is not a brace point!

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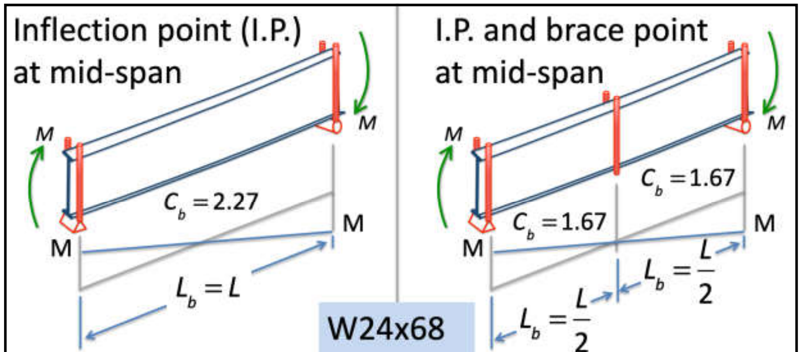

W24x68
L = 40'-0"

$$M_{cr} = C_b \frac{\pi}{L_b} \sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$

$C_b = 2.27$
$L_b = 40'\text{-}0"$
$M_{cr} = 319$ kip-ft

Much larger!

$C_b = 1.67$
$L_b = 20'\text{-}0"$
$M_{cr} = 677$ kip-ft

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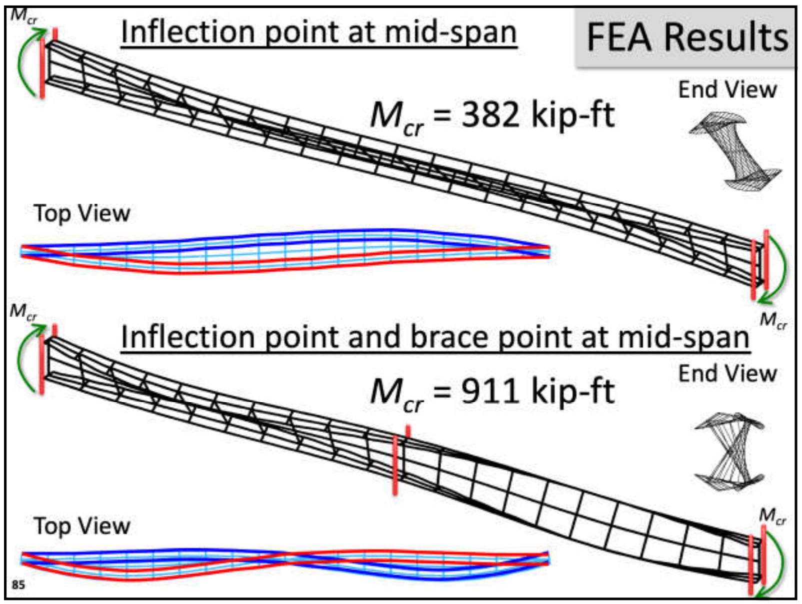

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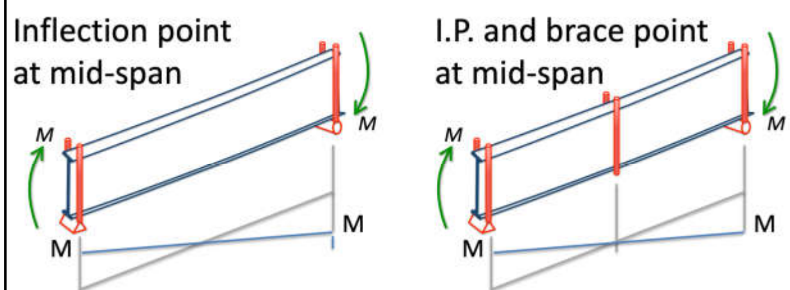

Inflection point is not a brace point!

Why does FEA give a significantly higher M_{cr} for I.P. and B.P. case? $M_{cr}^{AISC} = 677$ vs. $M_{cr}^{FEA} = 911$

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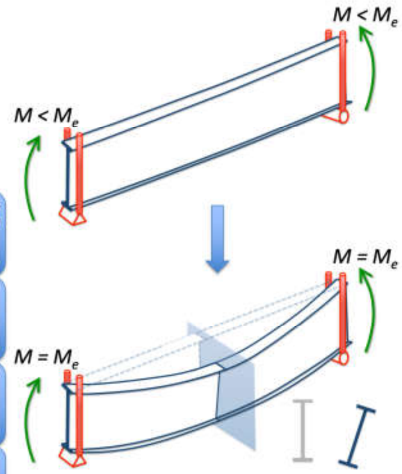
Lateral Torsional Buckling (LTB)

- Bifurcation solution
- Assumptions!
  - prismatic member (I = constant)
  - only major axis bending occurs before buckling — Undone
  - linear elastic behavior (E = constant) — Undone
  - uniform moment distribution — Undone
  - braced at the ends (frictionless) — Undone



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Lateral Torsional Buckling

- Theoretical bifurcation
  - solution
  - assumptions
- Undoing those assumptions (approaching reality)
  - not fully elastic, partial yielding
  - alternative loading and support conditions
- Beam curves
  - AISC
  - others

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AISC Flexural Strength (compact I-shapes)

- Initial yield
  - Moment, $M_r = S(\sigma_y - \sigma_{res}) = 0.7S\sigma_y = 0.7M_y$
  - setting $M_r = M_e$, back solve for unbraced length L_r
- For shorter unbraced lengths (full yielding)

$$L_b \le L_p,\ M_n = Z\sigma_y = M_p$$

- For longer unbraced lengths (elastic LTB)

$$L_b \ge L_r,\ M_n = C_b M_e = C_b \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\pi E/L_b\right)^2 I_y C_w} \quad \le M_p$$

- For intermediate unbraced lengths (inelastic LTB)

$$L_p < L_b < L_r,\ M_n = C_b\left[M_p - (M_p - M_r)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \quad \le M_p$$

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AISC Flexural Strength (compact I-shapes)

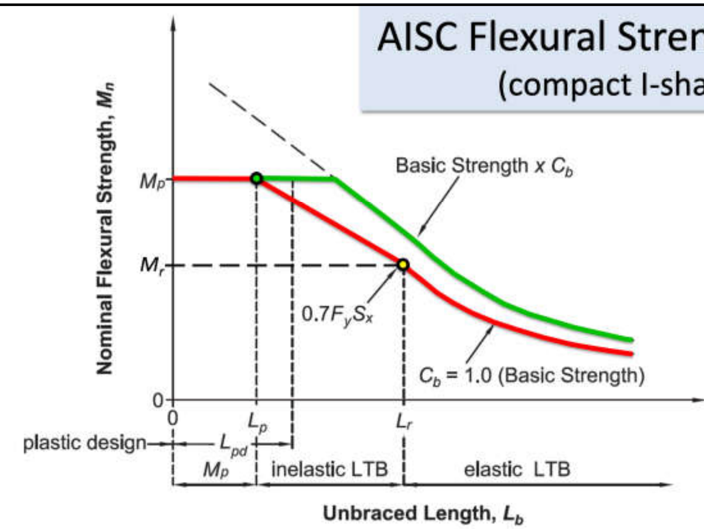

Fig. C-F1.2. Nominal flexural strength as a function of unbraced length and moment gradient.

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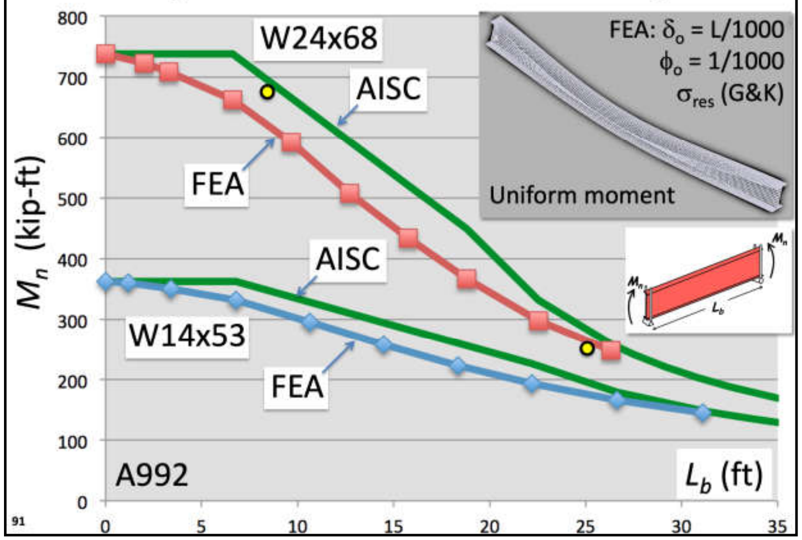
Comparison of Flexural Strengths



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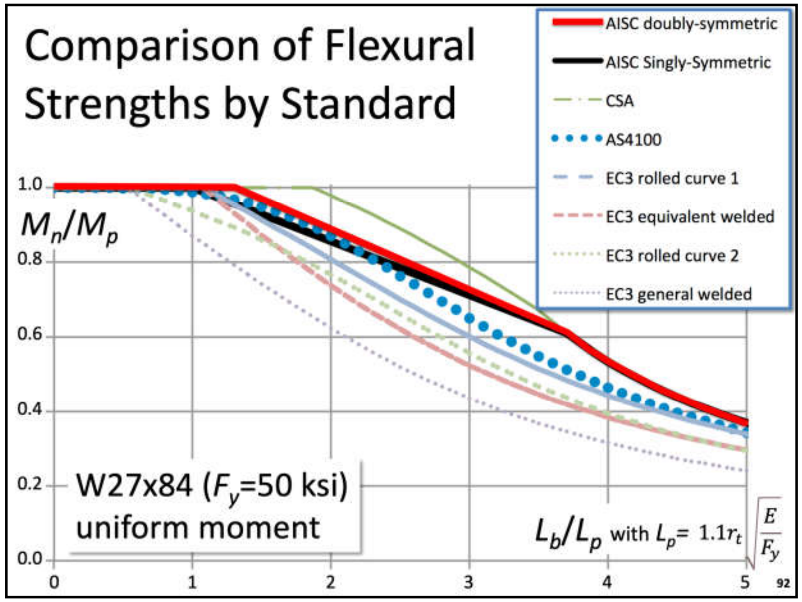
Comparison of Flexural Strengths by Standard



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Summary – Flexure

- Limit states of flexural members with focus on full yielding and lateral torsional buckling
- LTB Theory -to- Flexural Strength Beam Curve
- Beam curve accounts for:
  - full yielding
  - bending due to initial imperfection (out-of-straightness)
  - partial yielding accentuated by presence of residual stresses
  - moment gradient and brace points

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Summary – Flexure (cont.)

- AISC and other standards
- Your virtual laboratory assignment…
  - Try to recreate examples from this lecture
  - Try to recreate Learning Module 5 results presented in this lecture (C_b fun!)
  - Complete a portion of Learning Module 4

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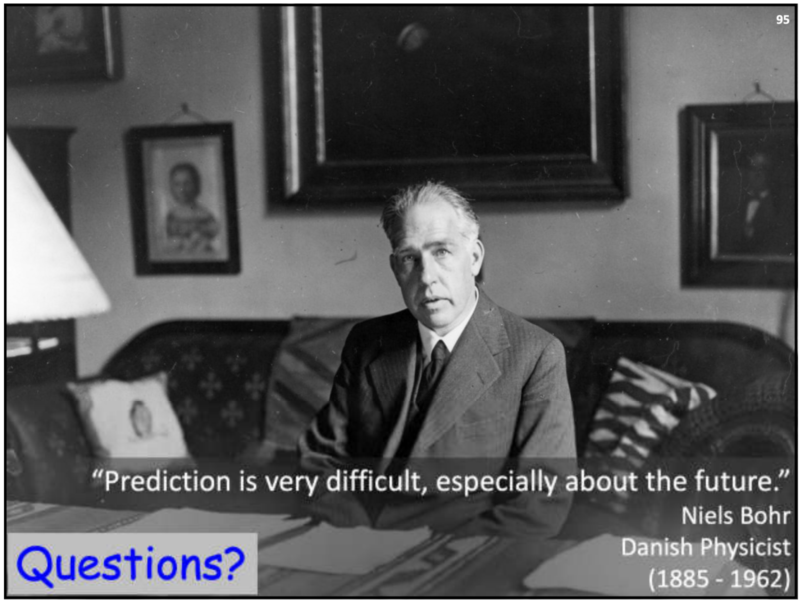

Thank You!

Hope you enjoyed this lecture!

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Single-Session Registrants

CEU / PDH Certificates

- You will receive an email on how to report attendance from: registration@aisc.org.
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!

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Single-Session Registrants

CEU / PDH Certificates

- Reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.

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8-Session Registrants

CEU / PDH Certificates

One certificate will be issued at the conclusion of the course.

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8-Session Registrants

Attendance and PDH Certificates

- You have two options to receive credit for a given session.
  - Option 1: Watch the live session. Credit for live attendance will be displayed on the Course Resources table within two days of the session.
  - Option 2: Watch the recording and pass the associated quiz.

Videos and Quizzes

- For each session, find access within two business days after the live air date. (An email will be sent from nightschool@aisc.org.)
- Reasons for quiz:
  - EEU – You must take all quizzes and the final exam to receive EEU.
  - PDHs – If you watch a recorded session, you must pass quiz for PDHs.
  - Reinforce what you learn in the lectures and get more out of the course!

Distribution of Certificates

All certificates will be issued after the course is completed. Only the registrant will receive a certificate for the course.

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8-Session Registrants

Course Resources

Find all your handouts, quizzes and quiz scores, recording access, and attendance information in one place!

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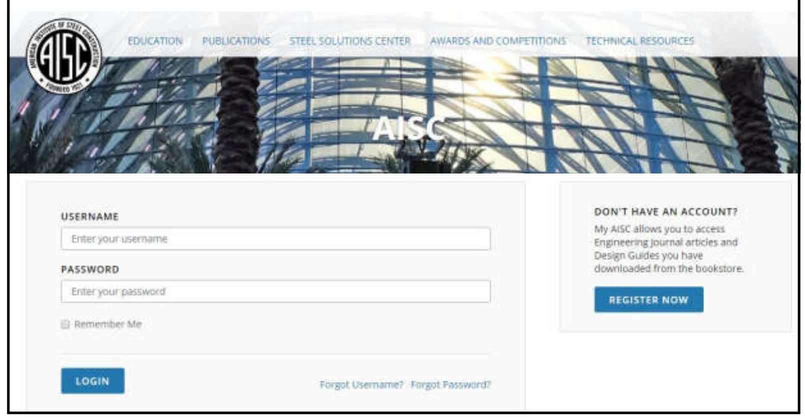
8-Session Registrants

Course Resources

Go to www.aisc.org and sign in.



8-Session Registrants

Course Resources

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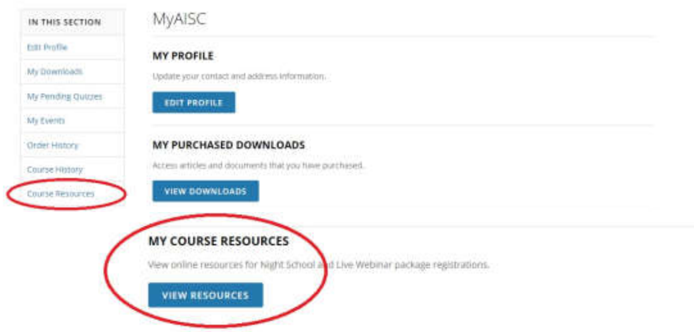
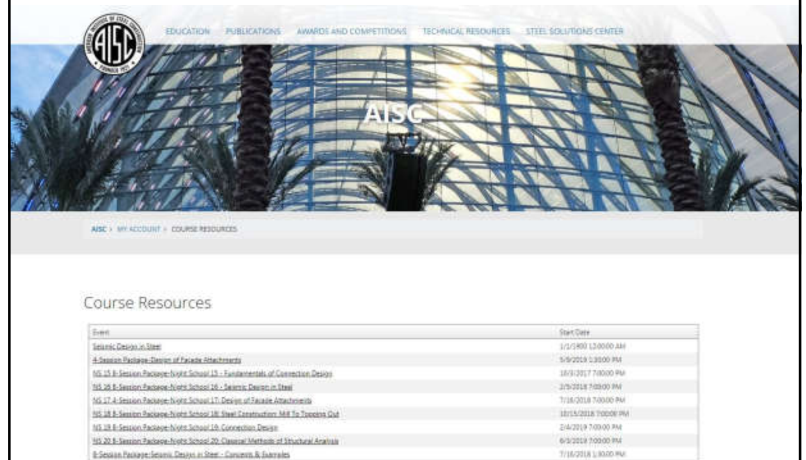


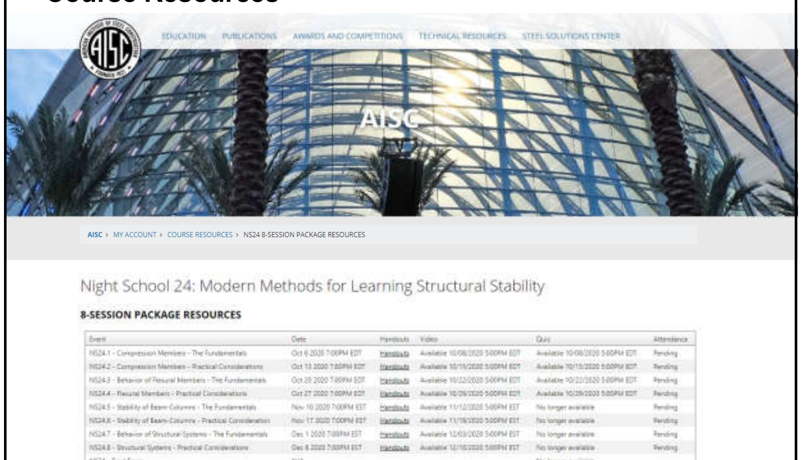
8-Session Registrants

Course Resources



8-Session Registrants

Course Resources



AISC | Thank you.

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