AISC Night School

Thank you for joining our live webinar today.
We will begin shortly. Please standby.

Modern Methods for Learning the Basics of Structural Stability: From Behavior to Practice

Session 5: Behavior of Beam-Columns – The Fundamentals

November 10, 2020

Smarter. Stronger. Steel.

AISC Live Webinars

Today's live webinar will begin shortly. Please stand by.

Today's audio will be broadcast through the internet. Please be sure to turn up the volume on your speakers.

Please type any questions or comments through the chat feature in the left portion of your screen.

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Course Description

Behavior of Beam-Columns – The Fundamentals
November 10, 2020

This lecture will begin with a review of basic concepts related to the stability of beam-columns. Using thoughts from the lectures on compression members and flexural members, an understanding of the basics of beam-columns will be gained. The speakers will conclude by introducing the third learning module.

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Learning Objectives

- Explain the development of the AISC interaction equation for beam-columns.
- Describe how the theoretical minor axis interaction curve and the theoretical major axis interaction curve compare to the AISC interaction curve.
- List the five effects that must be considered for stability design per the AISC *Specification*.
- Explain how to derive the P-δ moment amplification factor, B_1. Explain how it changes based on the moment gradient of a member.

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Modern Methods for Learning The Basics of Structural Stability: From Behavior to Practice

Session 5: Behavior of Beam-Columns – The Fundamentals
November 10, 2020

Ronald D. Ziemian, PE, PhD
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Bucknell University

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Senior Associate
Wiss, Janney, Elstner Associates, Inc.

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Course Overview

- Topics
  - Compression Members (Weeks 1 & 2)
  - Flexural Members (Weeks 3 & 4)
  - Beam-Columns (Weeks 5 & 6)
  - Systems (Weeks 7 & 8)
- “Active” learning! Weekly virtual lab experiences…
- Case studies from the real world…

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Course Overview (2)

Course Overview (3)

- Focus of the course is on fundamentals!
- Better understanding of behavior will result in improved design
- Key Definitions
  - **Stability:** Under load, component returns to current state after applying a small disturbance such as a deflection
  - **Bifurcation (critical load)**: Theoretical point at which loading a component results in an instantaneous change from current state to significant deflection – two options: not buckled or buckled
  - **Instability:** Loading a component results in a realistic transition from small deflection to significant deflection – buckling preceded by deflection

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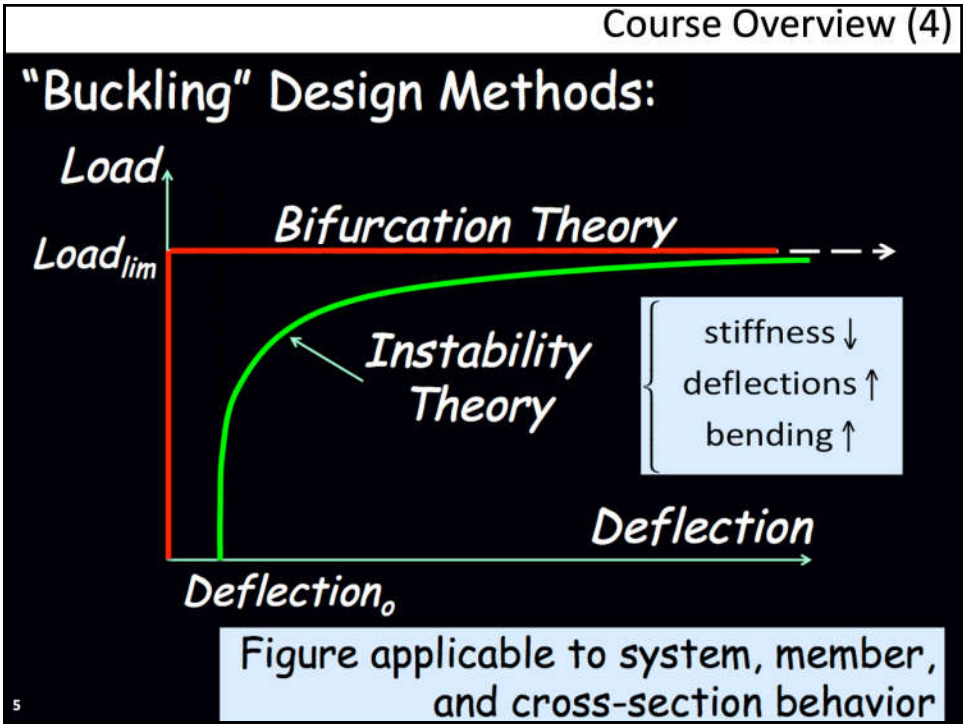
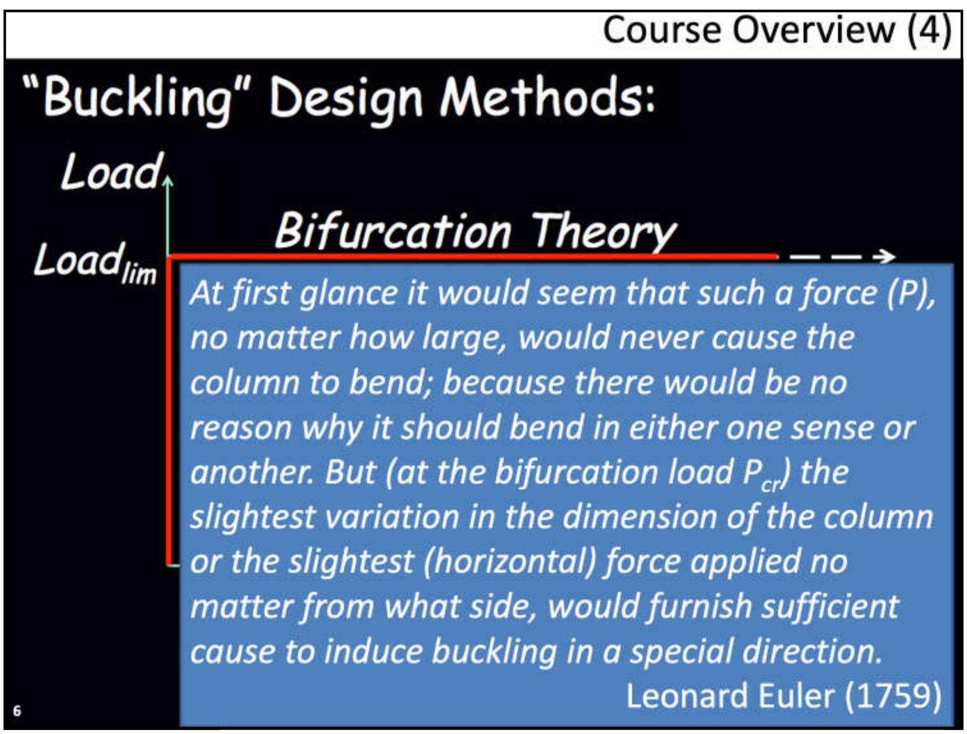
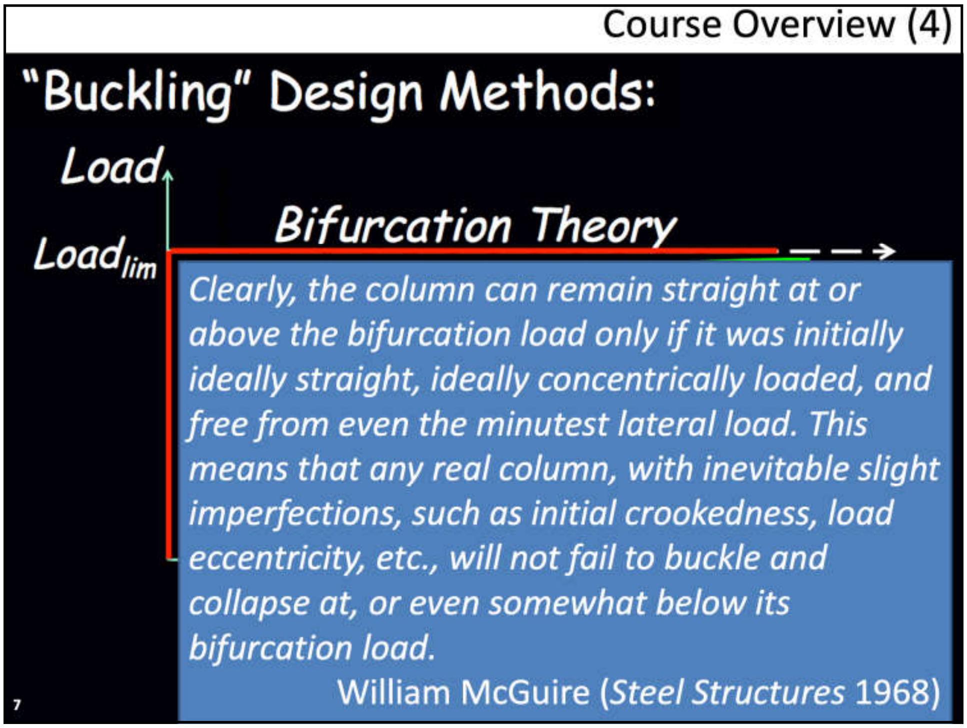
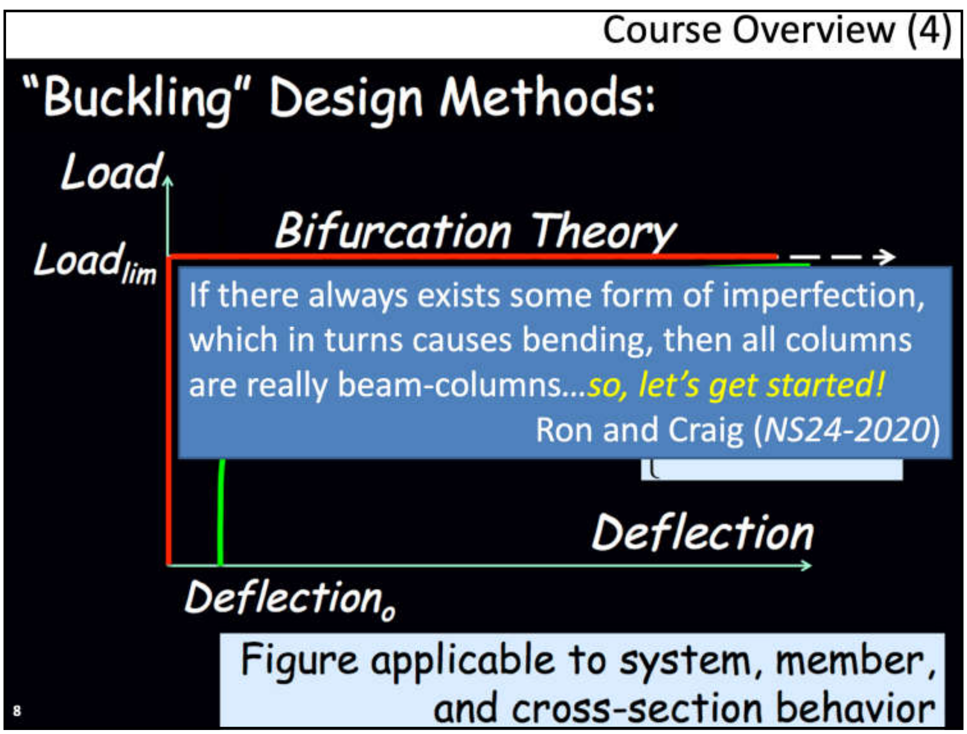

Course Overview (4)
"Buckling" Design Methods:
Load
Bifurcation Theory
Load$_{lim}$
Instability Theory
stiffness ↓
deflections ↑
bending ↑
Deflection
Deflection$_o$
Figure applicable to system, member, and cross-section behavior
5


Course Overview (4)
"Buckling" Design Methods:
Load
Bifurcation Theory
Load$_{lim}$
At first glance it would seem that such a force (P), no matter how large, would never cause the column to bend; because there would be no reason why it should bend in either one sense or another. But (at the bifurcation load P_{cr}) the slightest variation in the dimension of the column or the slightest (horizontal) force applied no matter from what side, would furnish sufficient cause to induce buckling in a special direction.
Leonard Euler (1759)
6


Course Overview (4)
"Buckling" Design Methods:
Load
Load$_{lim}$
Bifurcation Theory
Clearly, the column can remain straight at or above the bifurcation load only if it was initially ideally straight, ideally concentrically loaded, and free from even the minutest lateral load. This means that any real column, with inevitable slight imperfections, such as initial crookedness, load eccentricity, etc., will not fail to buckle and collapse at, or even somewhat below its bifurcation load.
William McGuire (*Steel Structures* 1968)
7


Course Overview (4)
"Buckling" Design Methods:
Load
Load$_{lim}$
Bifurcation Theory
If there always exists some form of imperfection, which in turns causes bending, then all columns are really beam-columns…*so, let's get started!*
Ron and Craig (*NS24-2020*)
Deflection
Deflection$_o$
Figure applicable to system, member, and cross-section behavior
8

Course Overview (5)

Analysis acronyms:

LBA: linear buckling analysis; elastic critical load analysis; elastic eigenvalue analysis; assumes bifurcation theory

GNA: geometric nonlinear analysis; 2nd-order elastic analysis; assumes equilibrium on the deformed shape and linear elastic material, with no initial imperfections

GNIA: same as GNA, but includes initial imperfections

MNA: material nonlinear analysis; 1st-order inelastic analysis; assumes equilibrium on the undeformed shape and accounts for yielding, with no initial imperfections

GMNIA: geometric and material nonlinear analysis; 2nd-order inelastic analysis; assumes equilibrium on the deformed shape, accounts for yielding, and includes initial imperfections

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Modern Methods for Learning The Basics of Structural Stability: From Behavior to Practice

Course Introduction

Compression Members

Flexural Members

Beam-Columns

Systems

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Limit States of Beam-Columns

- Full yielding (today!)
- Instability
  - Along the member length
    - Lateral-torsional buckling (slides on flexure)
    - Flexural buckling (slides on compression)
    - Torsional-flexural buckling (today!)
  - At the cross section
    - local buckling

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From Section Strength to Member Strength:

Development of AISC Interaction Equation for beam-columns (sort of!)

Limit the *P-M* strength to initial yield:

$$\left|\sigma_{res} + \frac{P}{A} + \frac{M}{S}\right| \le \sigma_y \quad \Rightarrow \quad \left|\frac{\sigma_{res}}{\sigma_y} + \frac{P}{A\sigma_y} + \frac{M}{S\sigma_y}\right| \le 1.0$$

Important Notes!

- For simplicity, not showing both *M*'s
- No factors of safety (ϕ's or Ω's) are included in today's lecture…learn behavior based on nominal strengths

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From Section Strength to Member Strength:

Development of AISC Interaction Equation for beam-columns (sort of!)

Limit the strength to initial yield:

$$\left|\sigma_{res} + \frac{P}{A} + \frac{M}{S}\right| \le \sigma_y \Rightarrow \left|\frac{\sigma_{res}}{\sigma_y} + \frac{P}{A\sigma_y} + \frac{M}{S\sigma_y}\right| \le 1.0$$

1

Cross section strength (full yield):

$$\frac{P}{P_y} \ge 0.2 \qquad \frac{P}{P_y} + \frac{8}{9}\frac{M}{M_p} \le 1.0$$

$$\frac{P}{P_y} < 0.2 \qquad \frac{1}{2}\frac{P}{P_y} + \frac{M}{M_p} \le 1.0$$

13

From Section Strength to Member Strength:

Development of AISC Interaction Equation for beam-columns (sort of!)

Limit the strength to initial yield:

$$\left|\sigma_{res} + \frac{P}{A} + \frac{M}{S}\right| \le \sigma_y \Rightarrow \left|\frac{\sigma_{res}}{\sigma_y} + \frac{P}{A\sigma_y} + \frac{M}{S\sigma_y}\right| \le 1.0$$

1

Cross section strength (full yield):

$$\frac{P}{P_y} \ge 0.2 \qquad \frac{P}{P_y} + \frac{8}{9}\frac{M}{M_p} \le 1.0$$

$$\frac{P}{P_y} < 0.2 \qquad \frac{1}{2}\frac{P}{P_y} + \frac{M}{M_p} \le 1.0$$

2

Member strength:

$$\frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \le 1.0$$

$$\frac{1}{2}\frac{P}{P_n} + \frac{M}{M_n} \le 1.0$$

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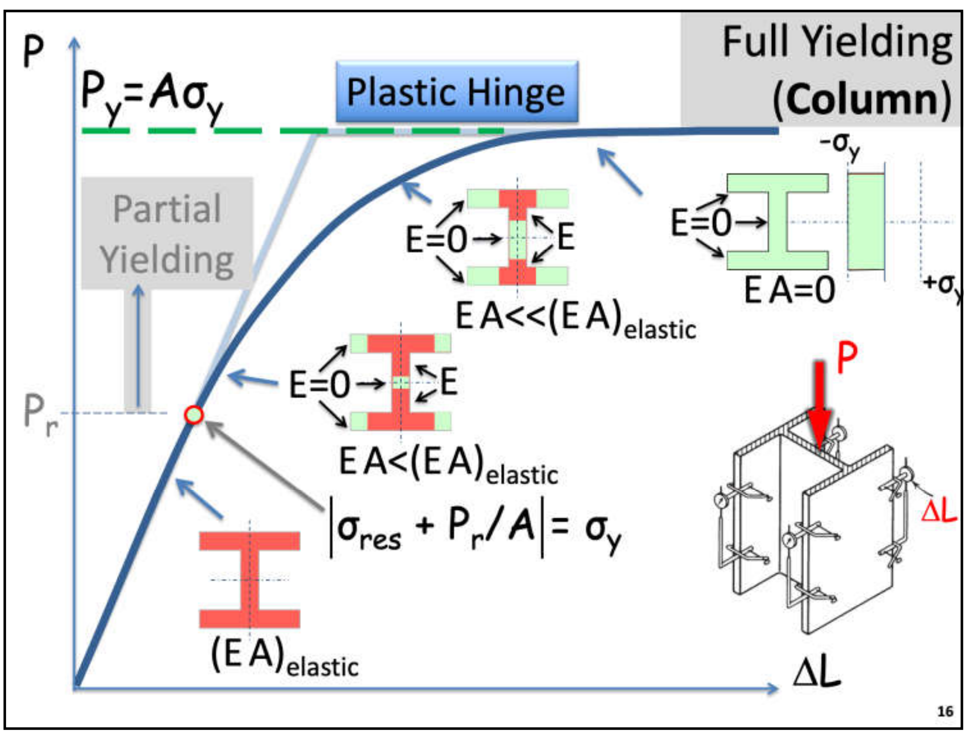
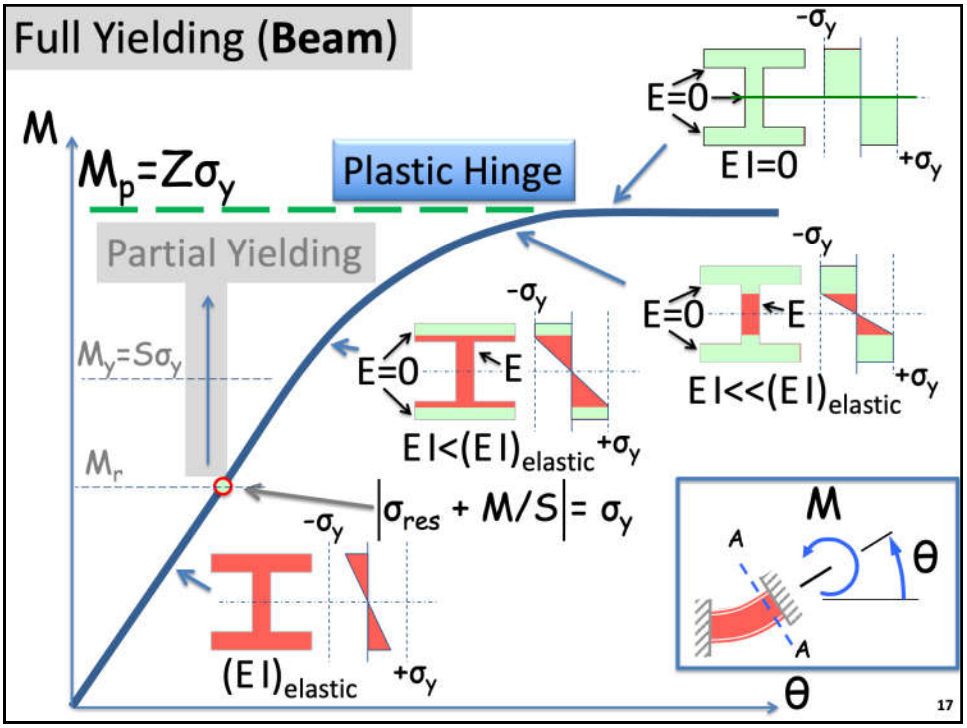
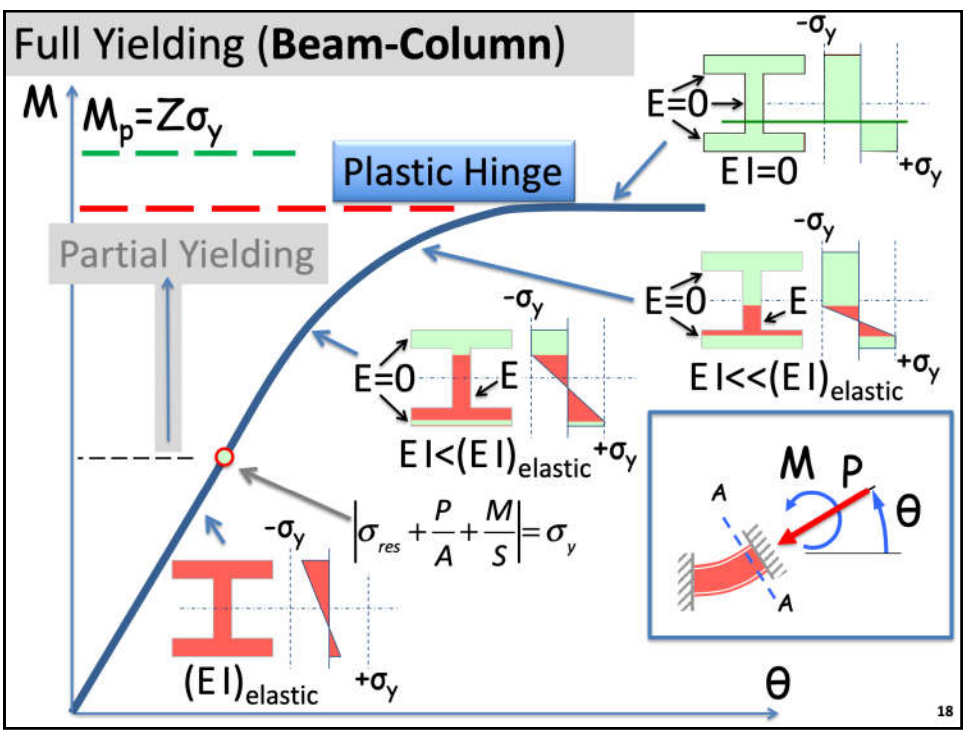
Beam-Column Strengths

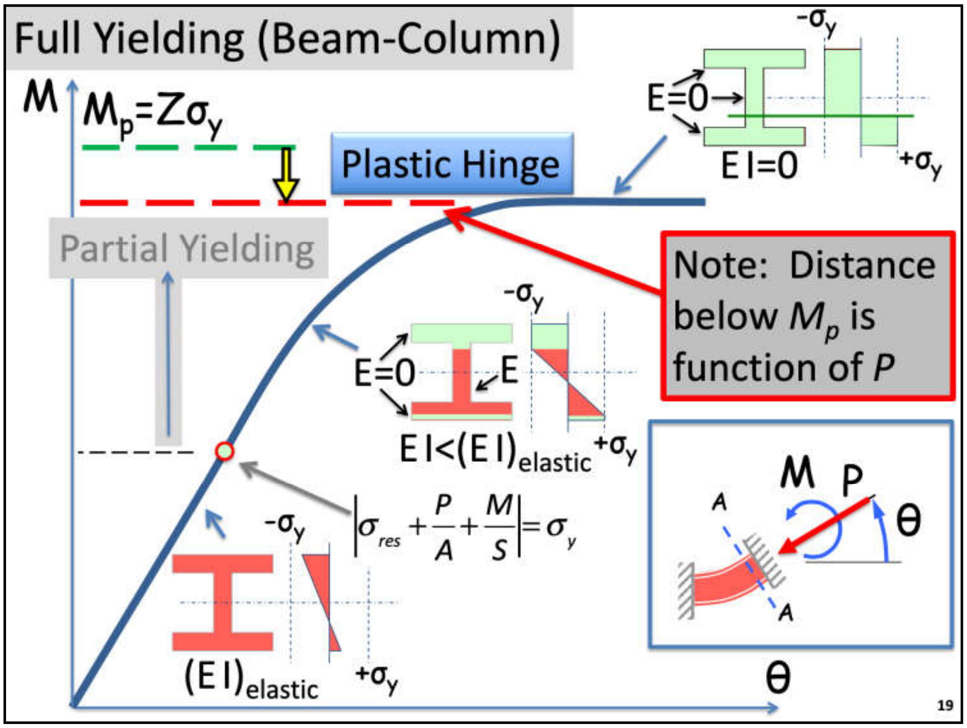
- Cross section strength (small L/r)
  - full yield
  - local buckling
- Elastic member strength (large L/r)
  - compressive (P) : flexural buckling (or torsional or flexural-torsional)
  - flexural (M_{major}): lateral torsional buckling
  - torsional-flexural buckling ($P + M_{major}$)
- Inelastic member strength (intermediate L/r)
  - same possible failure modes as elastic, except reduced due to partial yielding accentuated by presence of initial imperfections (geometric and residual stresses)

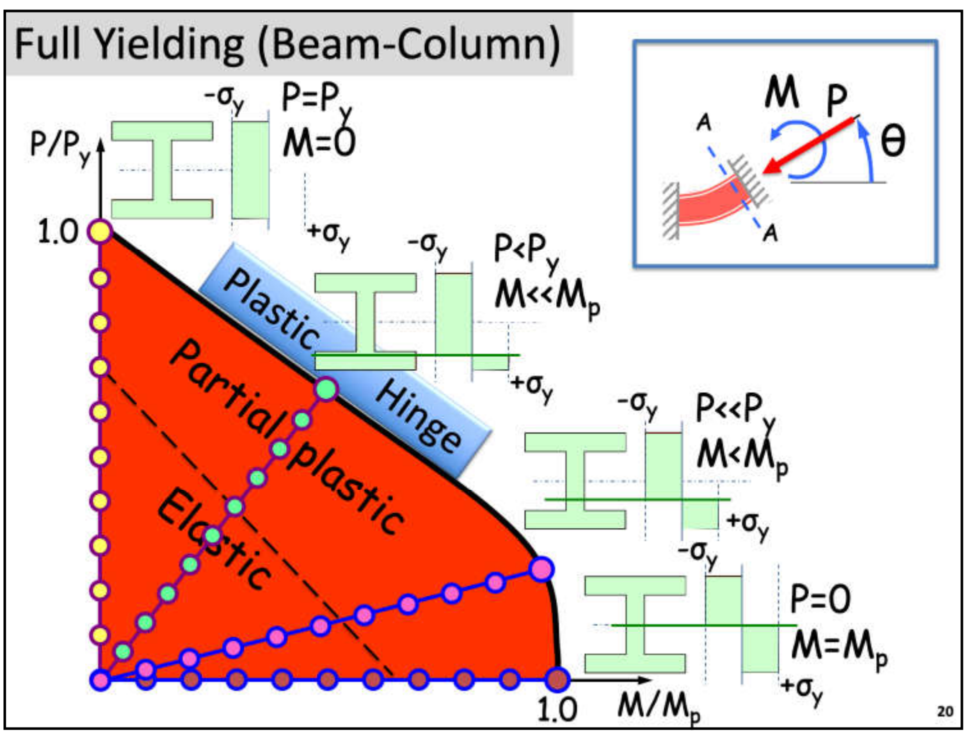
15

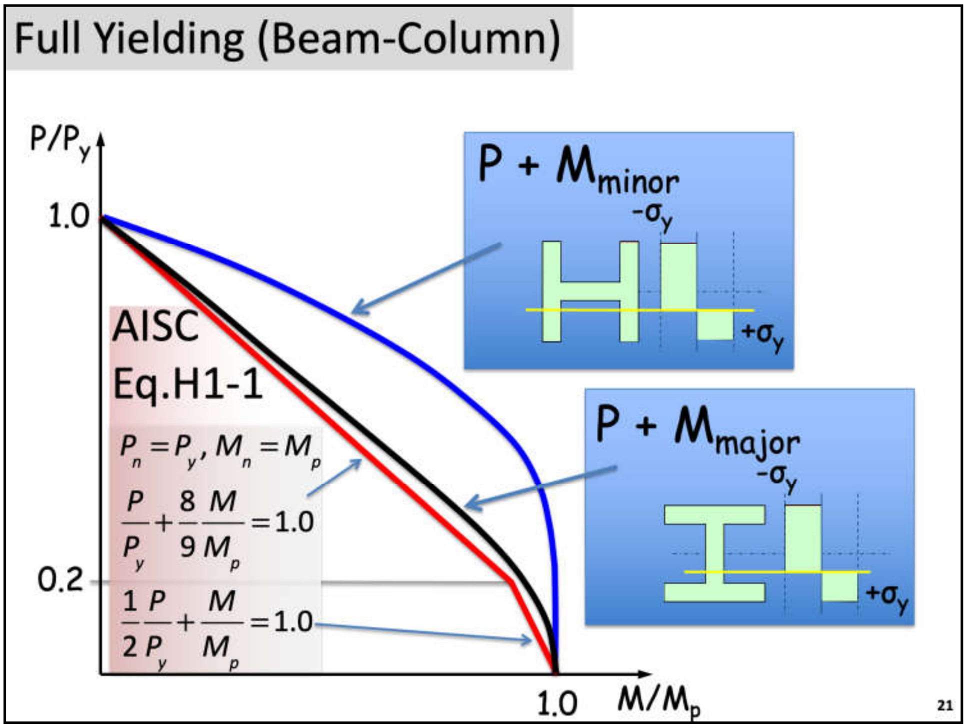


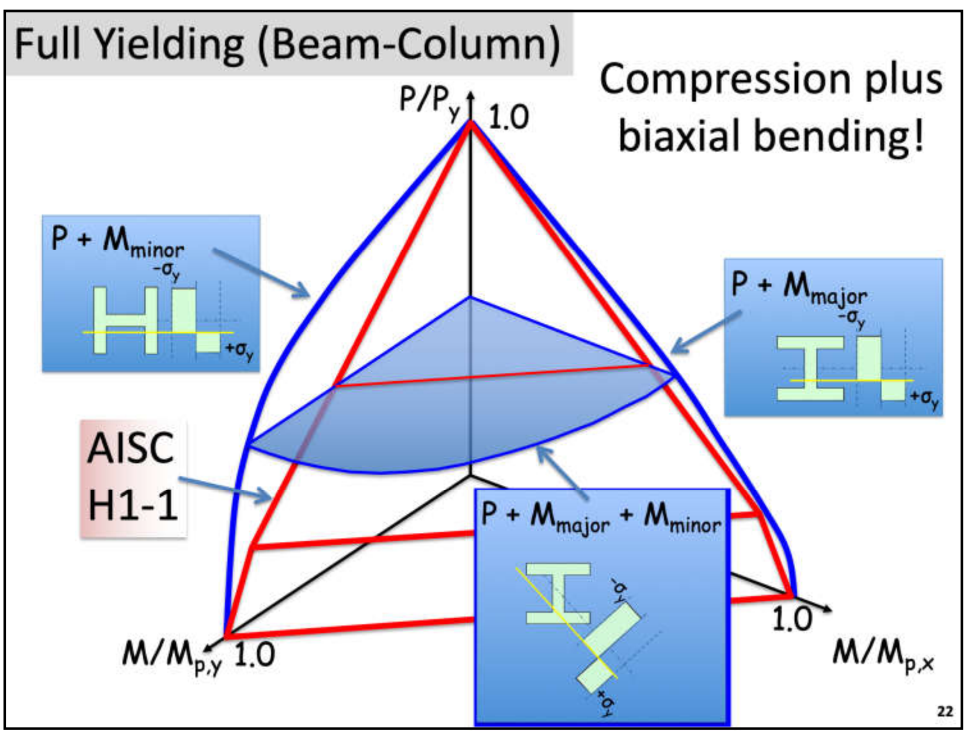

Full Yielding (Beam)
M
$M_p=Z\sigma_y$
Plastic Hinge
Partial Yielding
$M_y=S\sigma_y$
M_r
E=0
E I=0
$-\sigma_y$
$+\sigma_y$
E
E I<<(E I)elastic
E I<(E I)elastic
$|\sigma_{res} + M/S| = \sigma_y$
(E I)elastic
A
A
θ
17

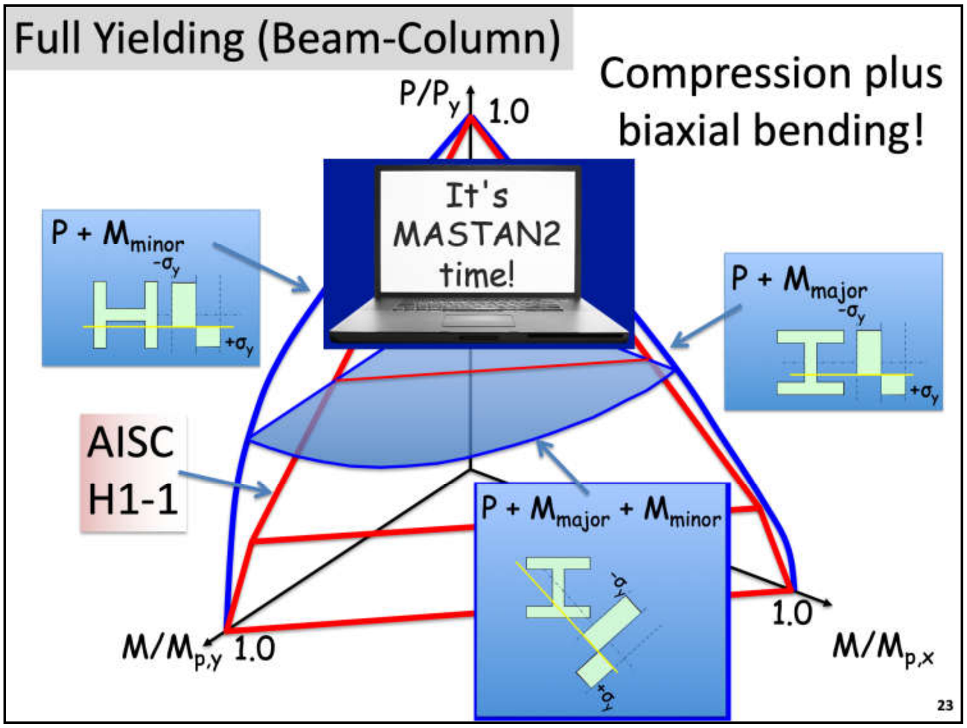

Full Yielding (Beam-Column)
M
$M_p=Z\sigma_y$
Plastic Hinge
Partial Yielding
E=0
E I=0
$-\sigma_y$
$+\sigma_y$
E
E I<<(E I)elastic
E I<(E I)elastic
$\left|\sigma_{res} + \frac{P}{A} + \frac{M}{S}\right| = \sigma_y$
(E I)elastic
M P
A
A
θ
18

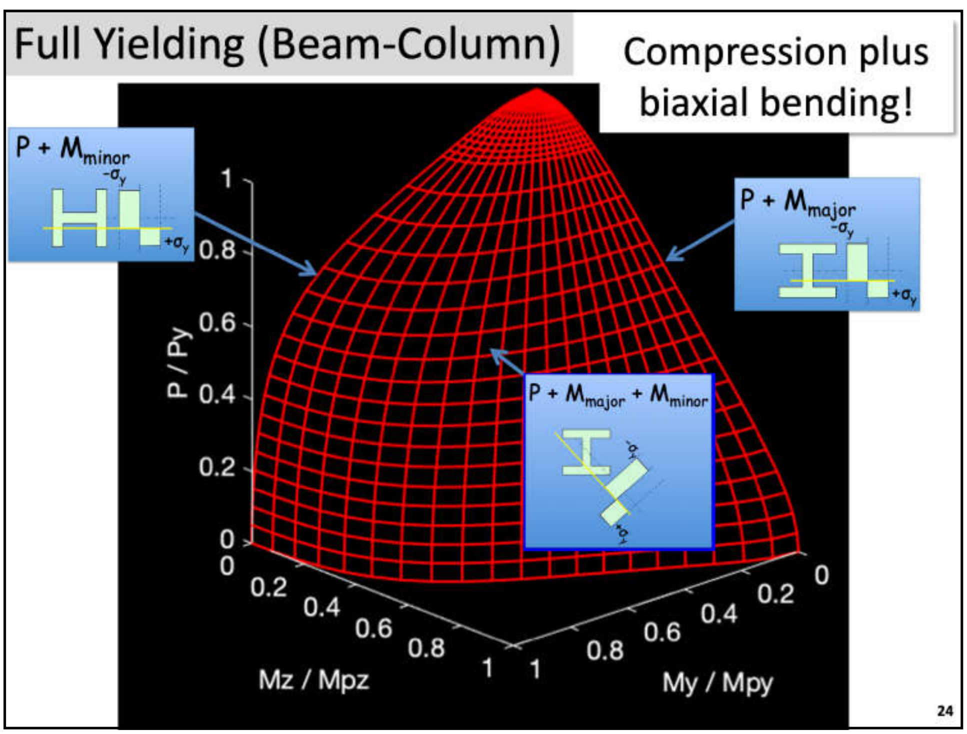

Full Yielding (Beam-Column)
M
M_p=Z$σ_y$
Plastic Hinge
Partial Yielding
Note: Distance below M_p is function of P
E=0
E I=0
-$σ_y$
+$σ_y$
E=0
E
E I<(E I)$_{elastic}$
$\left|\sigma_{res}+\frac{P}{A}+\frac{M}{S}\right|=\sigma_y$
(E I)$_{elastic}$
M
P
θ
A
A
θ
19


Full Yielding (Beam-Column)
P/P_y
1.0
P=P_y
M=0
-$σ_y$
+$σ_y$
P<P_y
M<<M_p
P<<P_y
M<M_p
P=0
M=M_p
Plastic Hinge
Partial plastic
Elastic
1.0
M/M_p
M
P
θ
A
A
20


Full Yielding (Beam-Column)
P/P_y
1.0
0.2
1.0 M/M_p
P + M_minor
-σ_y
+σ_y
P + M_major
-σ_y
+σ_y
AISC
Eq.H1-1
P_n = P_y, M_n = M_p
P/P_y + 8/9 M/M_p = 1.0
1/2 P/P_y + M/M_p = 1.0
21


Full Yielding (Beam-Column)
Compression plus
biaxial bending!
P/P_y 1.0
P + M_minor
-σ_y
+σ_y
P + M_major
-σ_y
+σ_y
AISC
H1-1
P + M_major + M_minor
-σ_y
+σ_y
M/M_p,y 1.0
1.0
M/M_p,x
22


Full Yielding (Beam-Column)
Compression plus biaxial bending!
P/Py 1.0
It's MASTAN2 time!
P + Mminor
P + Mmajor
AISC H1-1
P + Mmajor + Mminor
M/Mp,y 1.0
1.0 M/Mp,x
23


Full Yielding (Beam-Column)
Compression plus biaxial bending!
P + Mminor
P + Mmajor
P + Mmajor + Mminor
P / Py
Mz / Mpz
My / Mpy
24

Beam-Column Strengths

- Cross section strength (small L/r)
  - full yield
  - local buckling
- Elastic member strength (large L/r)
  - compressive (P) : flexural buckling (or torsional or flexural-torsional)
  - flexural (M_{major}): lateral torsional buckling
  - torsional-flexural buckling ($P + M_{major}$)
- Inelastic member strength (intermediate L/r)
  - same possible failure modes as elastic, except reduced due to partial yielding accentuated by presence of initial imperfections (geometric and residual stresses)

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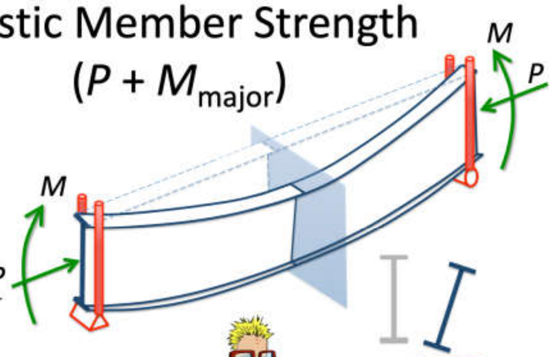
Elastic Member Strength ($P + M_{major}$)

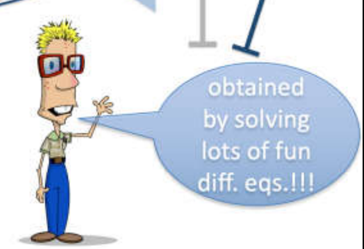
$P \neq 0$ and $M = 0$
(compression slides)

$$P_E = \pi^2 EI_y / L_b^2$$

$P = 0$ and $M \neq 0$
(flexure slides)

$$M_e = \frac{\pi}{L_b}\sqrt{EI_y GJ + \left(\frac{\pi E}{L_b}\right)^2 I_y C_w}$$





$P \neq 0$ and $M \neq 0$ (Beam-Column)
For a given M, P_{cr} is smallest root of:

$$(P_E - P_{cr})(P_t - P_{cr}) = AM^2 / I_\rho$$

with: $I_\rho = I_x + I_y$

$$P_t = \frac{A}{I_\rho}\left(GJ + \frac{\pi^2 EC_w}{L_b^2}\right)$$

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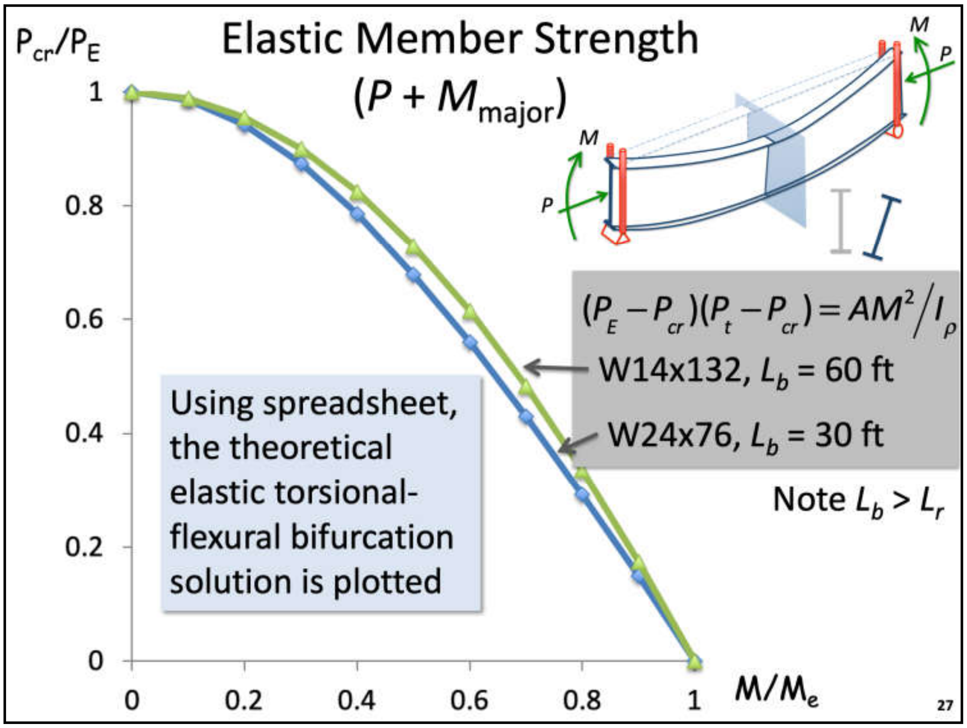
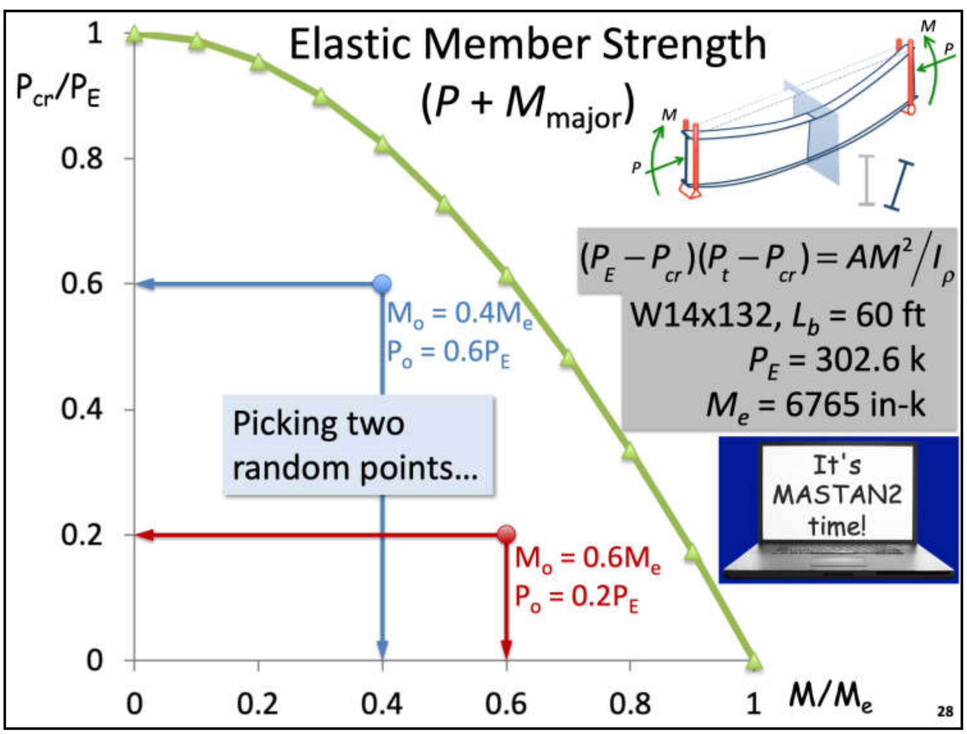
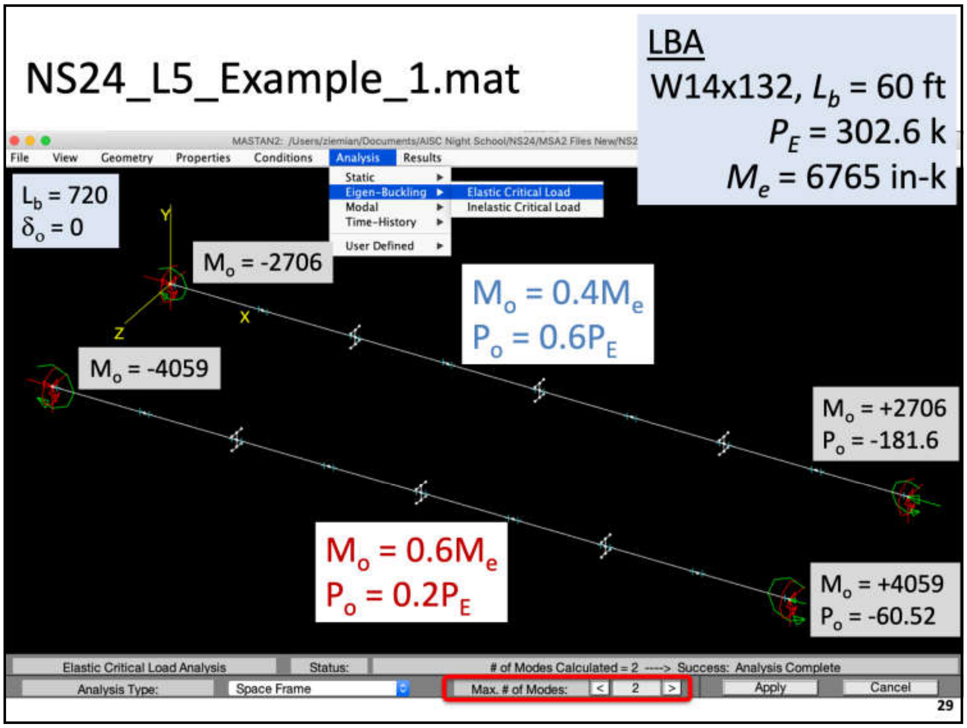
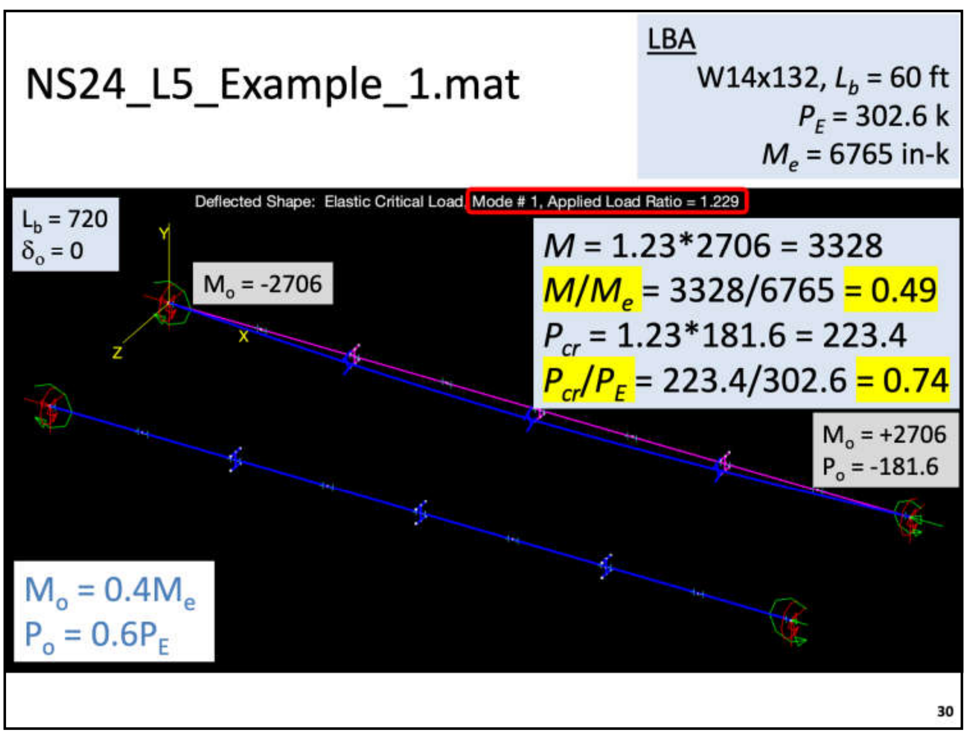
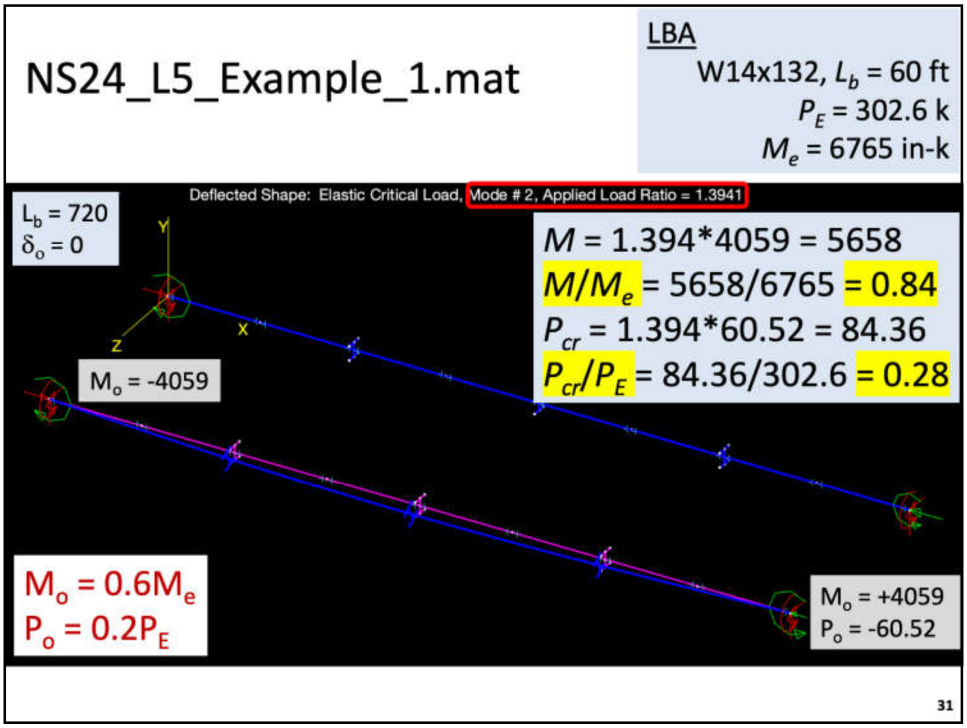
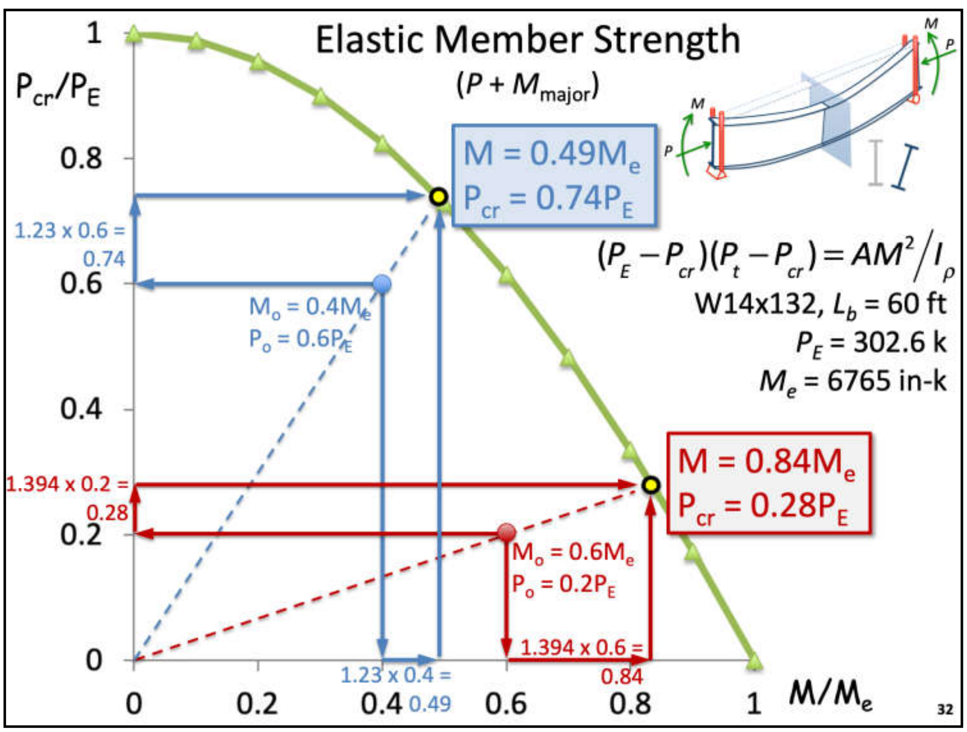

Elastic Member Strength
(P + M_major)
P_cr/P_E
M/M_e
$(P_E - P_{cr})(P_t - P_{cr}) = AM^2/I_\rho$
W14x132, L_b = 60 ft
W24x76, L_b = 30 ft
Note $L_b > L_r$
Using spreadsheet, the theoretical elastic torsional-flexural bifurcation solution is plotted
27

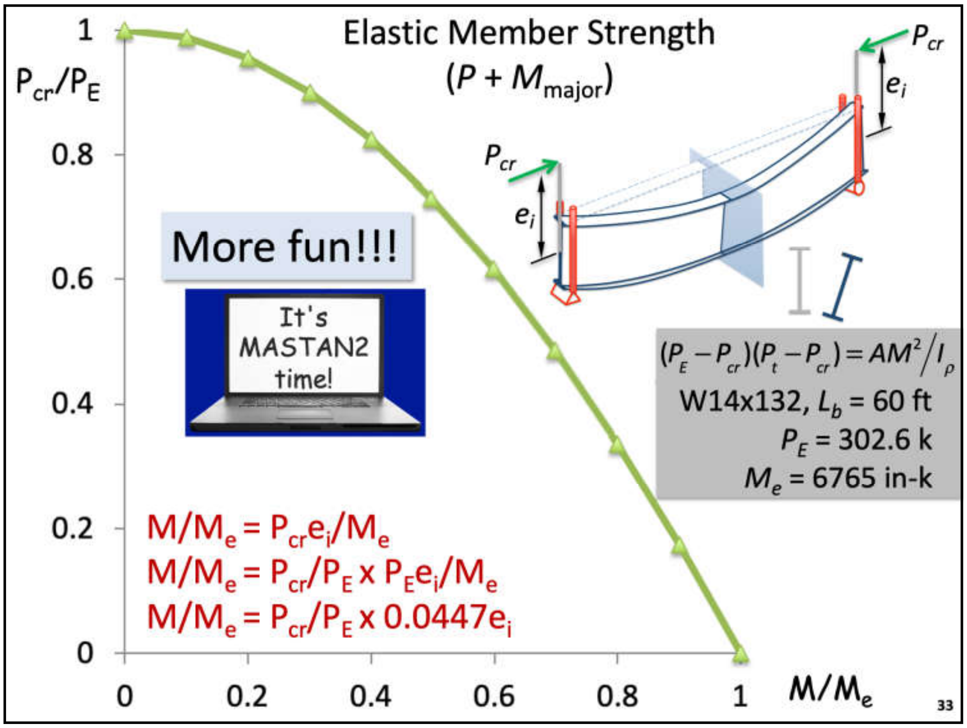

Elastic Member Strength
(P + M_major)
P_cr/P_E
M/M_e
$(P_E - P_{cr})(P_t - P_{cr}) = AM^2/I_\rho$
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k
$M_o = 0.4M_e$
$P_o = 0.6P_E$
$M_o = 0.6M_e$
$P_o = 0.2P_E$
Picking two random points...
It's MASTAN2 time!
28

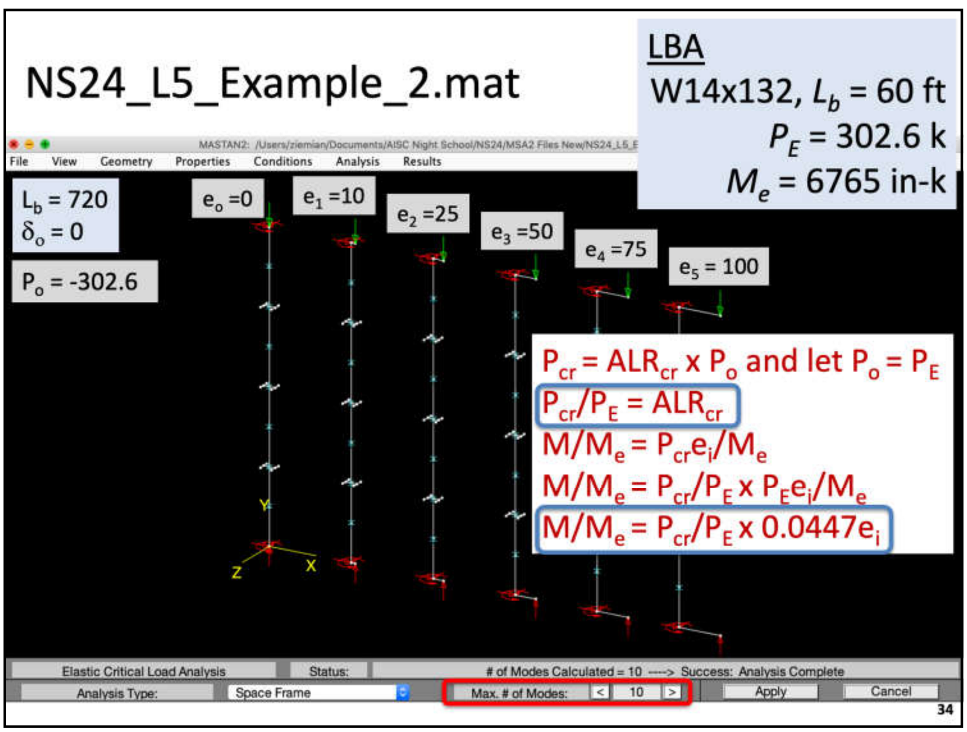

NS24_L5_Example_1.mat
LBA
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k
File View Geometry Properties Conditions Analysis Results
Static
Eigen-Buckling
Modal
Time-History
User Defined
Elastic Critical Load
Inelastic Critical Load
L_b = 720
δ_o = 0
M_o = -2706
$M_o = 0.4M_e$
$P_o = 0.6P_E$
M_o = -4059
M_o = +2706
P_o = -181.6
$M_o = 0.6M_e$
$P_o = 0.2P_E$
M_o = +4059
P_o = -60.52
Elastic Critical Load Analysis
Status:
# of Modes Calculated = 2 ----> Success: Analysis Complete
Analysis Type:
Space Frame
Max. # of Modes:
2
Apply
Cancel
29


NS24_L5_Example_1.mat
LBA
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k
Deflected Shape: Elastic Critical Load Mode # 1, Applied Load Ratio = 1.229
L_b = 720
δ_o = 0
M_o = -2706
M = 1.23*2706 = 3328
M/M_e = 3328/6765 = 0.49
P_{cr} = 1.23*181.6 = 223.4
P_{cr}/P_E = 223.4/302.6 = 0.74
M_o = +2706
P_o = -181.6
$M_o = 0.4M_e$
$P_o = 0.6P_E$
30


NS24_L5_Example_1.mat
LBA
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k
Deflected Shape: Elastic Critical Load, Mode # 2, Applied Load Ratio = 1.3941
L_b = 720
δ_o = 0
M = 1.394*4059 = 5658
M/M_e = 5658/6765 = 0.84
P_{cr} = 1.394*60.52 = 84.36
P_{cr}/P_E = 84.36/302.6 = 0.28
M_o = -4059
M_o = 0.6M_e
P_o = 0.2P_E
M_o = +4059
P_o = -60.52
31

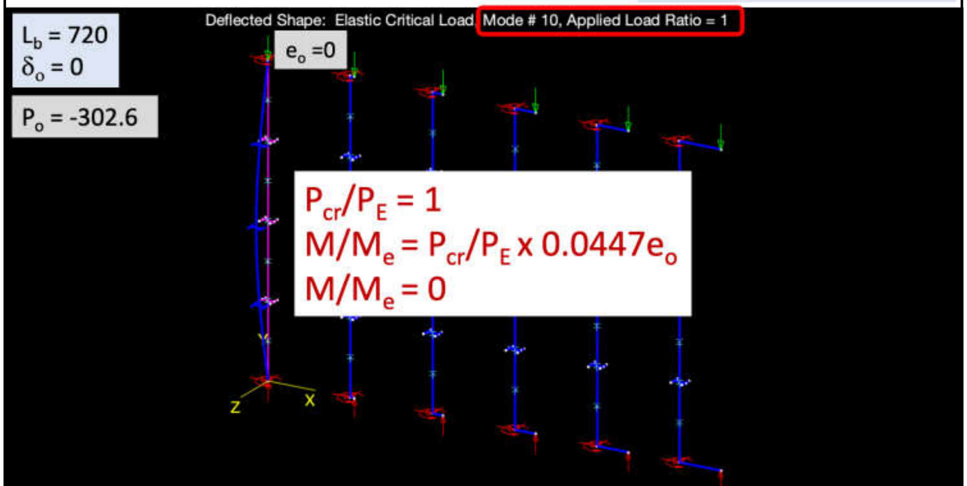

Elastic Member Strength
($P + M_{major}$)
P_{cr}/P_E
$M = 0.49M_e$
$P_{cr} = 0.74P_E$
1.23 x 0.6 = 0.74
$M_o = 0.4M_e$
$P_o = 0.6P_E$
$(P_E - P_{cr})(P_t - P_{cr}) = AM^2/I_\rho$
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k
$M = 0.84M_e$
$P_{cr} = 0.28P_E$
1.394 x 0.2 = 0.28
$M_o = 0.6M_e$
$P_o = 0.2P_E$
1.394 x 0.6 = 0.84
1.23 x 0.4 = 0.49
0
0.2
0.4
0.6
0.8
1
M/M_e
32


Elastic Member Strength
(P + M_major)
Pcr/PE
M/Me
More fun!!!
It's MASTAN2 time!
(PE – Pcr)(Pt – Pcr) = AM²/Iρ
W14x132, Lb = 60 ft
PE = 302.6 k
Me = 6765 in-k
M/Me = Pcr ei/Me
M/Me = Pcr/PE x PE ei/Me
M/Me = Pcr/PE x 0.0447ei
33


NS24_L5_Example_2.mat
LBA
W14x132, Lb = 60 ft
PE = 302.6 k
Me = 6765 in-k
Lb = 720
δo = 0
Po = -302.6
eo =0
e1 =10
e2 =25
e3 =50
e4 =75
e5 = 100
Pcr = ALRcr x Po and let Po = PE
Pcr/PE = ALRcr
M/Me = Pcr ei/Me
M/Me = Pcr/PE x PE ei/Me
M/Me = Pcr/PE x 0.0447ei
Elastic Critical Load Analysis
Status:
# of Modes Calculated = 10 ---> Success: Analysis Complete
Analysis Type:
Space Frame
Max. # of Modes:
10
Apply
Cancel
34

NS24_L5_Example_2.mat

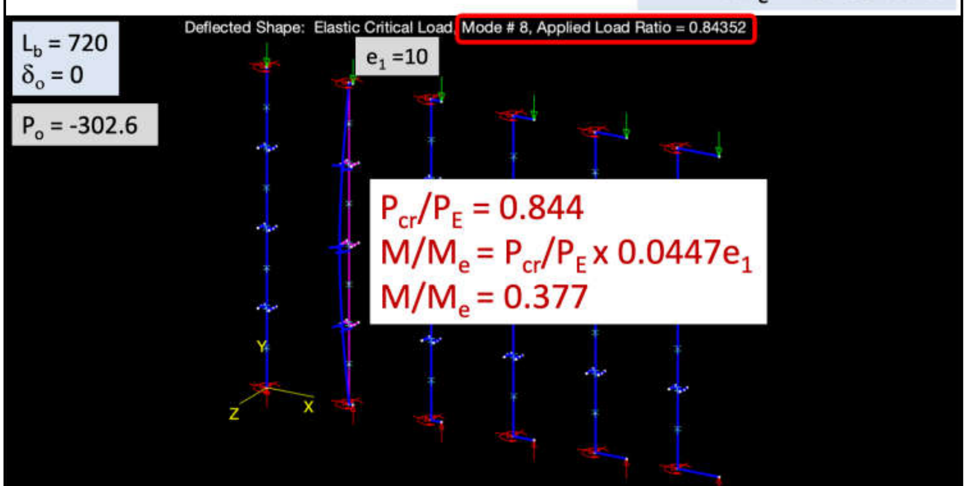
LBA
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k



35

NS24_L5_Example_2.mat

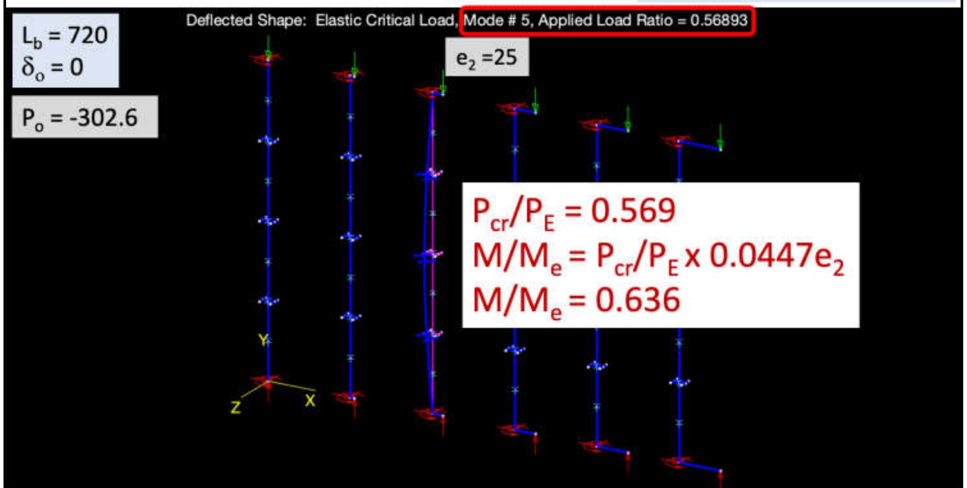
LBA
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k



36

NS24_L5_Example_2.mat

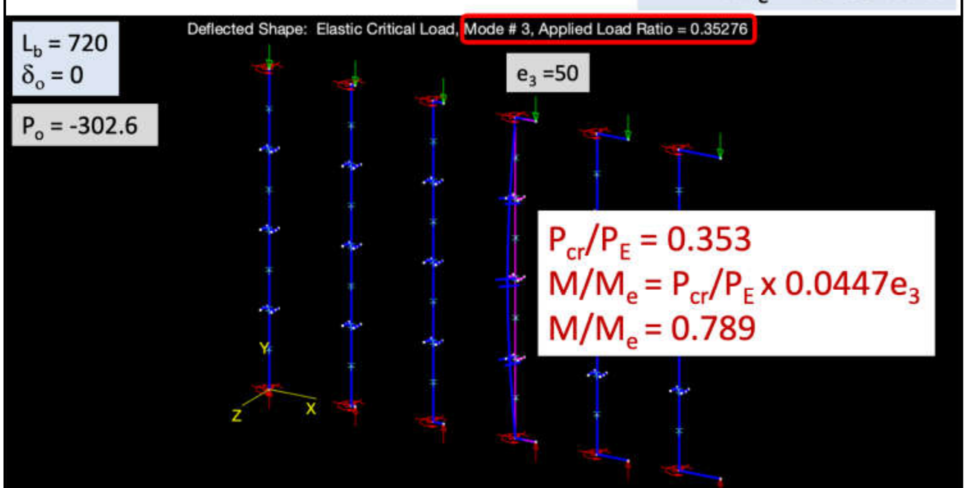
LBA
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k



37

NS24_L5_Example_2.mat

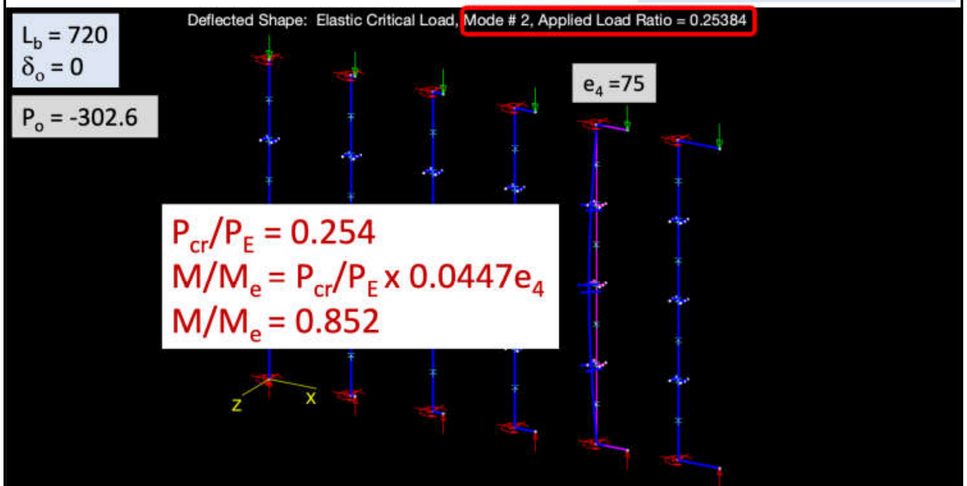
LBA
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k



38

NS24_L5_Example_2.mat

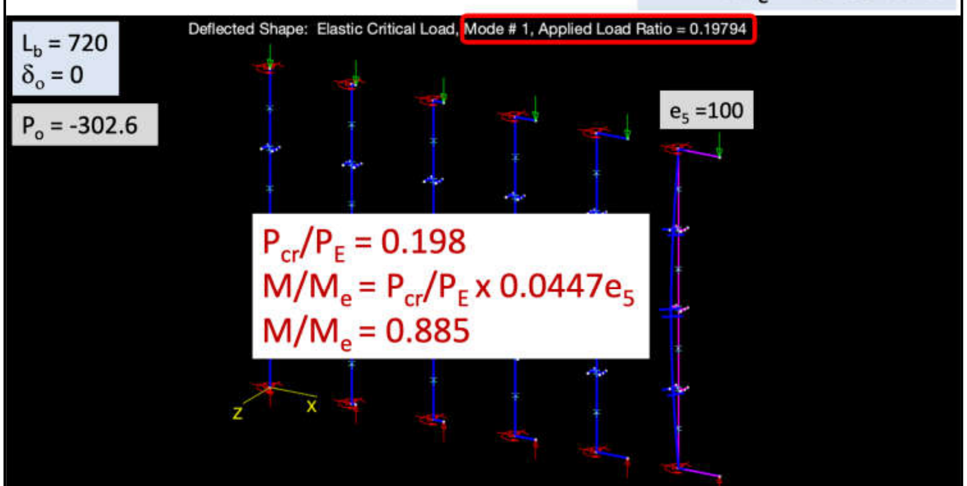
LBA
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k



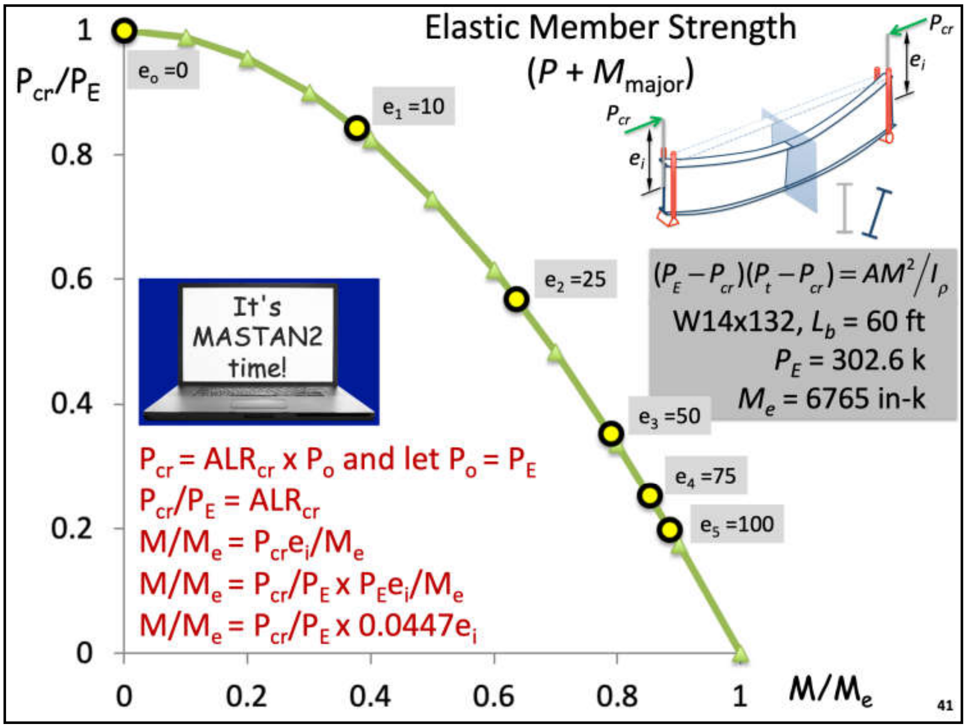
39

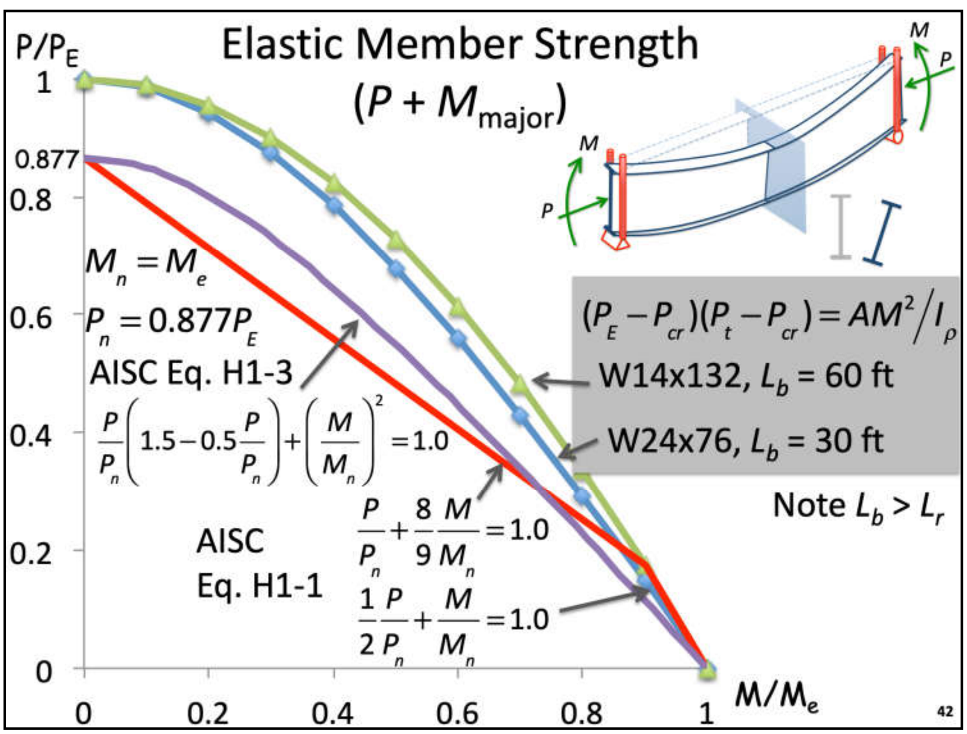
NS24_L5_Example_2.mat

LBA
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k



40


Elastic Member Strength
(P + M_major)
P_cr/P_E
M/M_e
e_o =0
e_1 =10
e_2 =25
e_3 =50
e_4 =75
e_5 =100
It's MASTAN2 time!
P_cr = ALR_cr x P_o and let P_o = P_E
P_cr/P_E = ALR_cr
M/M_e = P_cr e_i/M_e
M/M_e = P_cr/P_E x P_E e_i/M_e
M/M_e = P_cr/P_E x 0.0447e_i
(P_E − P_cr)(P_t − P_cr) = AM²/I_ρ
W14x132, L_b = 60 ft
P_E = 302.6 k
M_e = 6765 in-k
41


Elastic Member Strength
(P + M_major)
P/P_E
M/M_e
0.877
M_n = M_e
P_n = 0.877P_E
AISC Eq. H1-3
P/P_n (1.5 − 0.5 P/P_n) + (M/M_n)² = 1.0
AISC Eq. H1-1
P/P_n + 8/9 M/M_n = 1.0
1/2 P/P_n + M/M_n = 1.0
(P_E − P_cr)(P_t − P_cr) = AM²/I_ρ
W14x132, L_b = 60 ft
W24x76, L_b = 30 ft
Note L_b > L_r
42

Beam-Column Strengths

- Cross section strength (small *L/r*)
  - full yield
  - local buckling
- Elastic member strength (large *L/r*)
  - compressive (*P*) : flexural buckling (or torsional or flexural-torsional)
  - flexural (M_{major}): lateral torsional buckling
  - torsional-flexural buckling ($P + M_{major}$)
- Inelastic member strength (intermediate *L/r*)
  - same possible failure modes as elastic, except reduced due to partial yielding accentuated by presence of initial imperfections (geometric and residual stresses)

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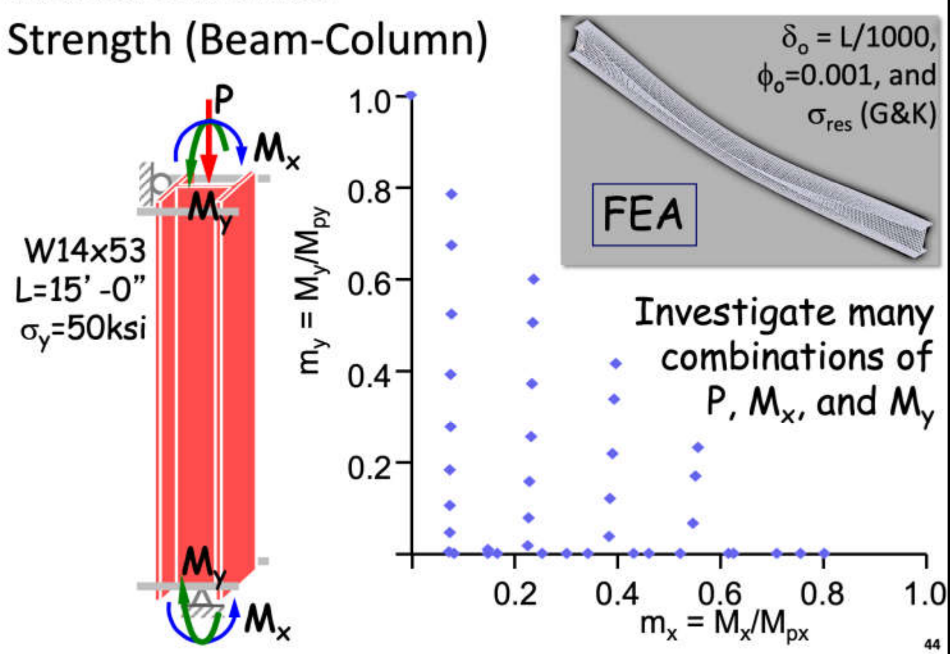
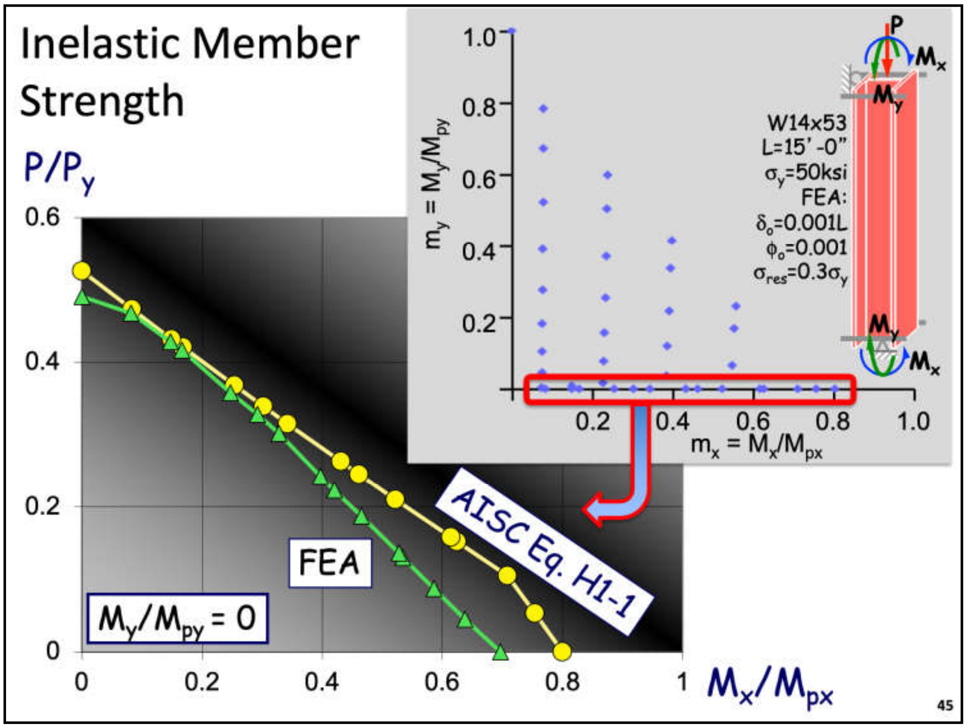
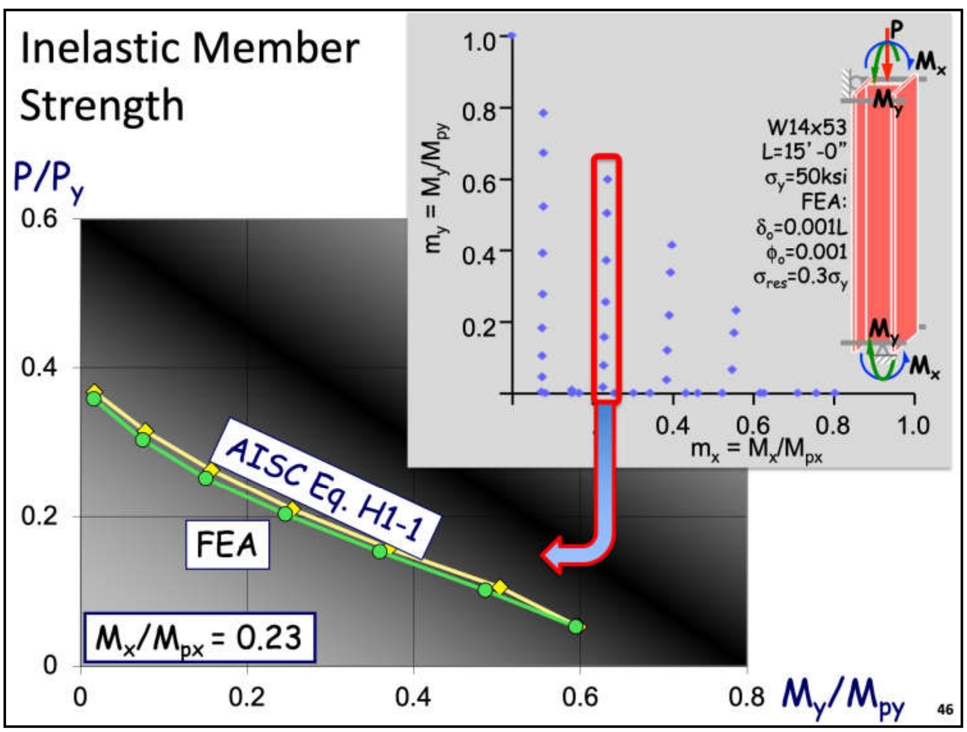
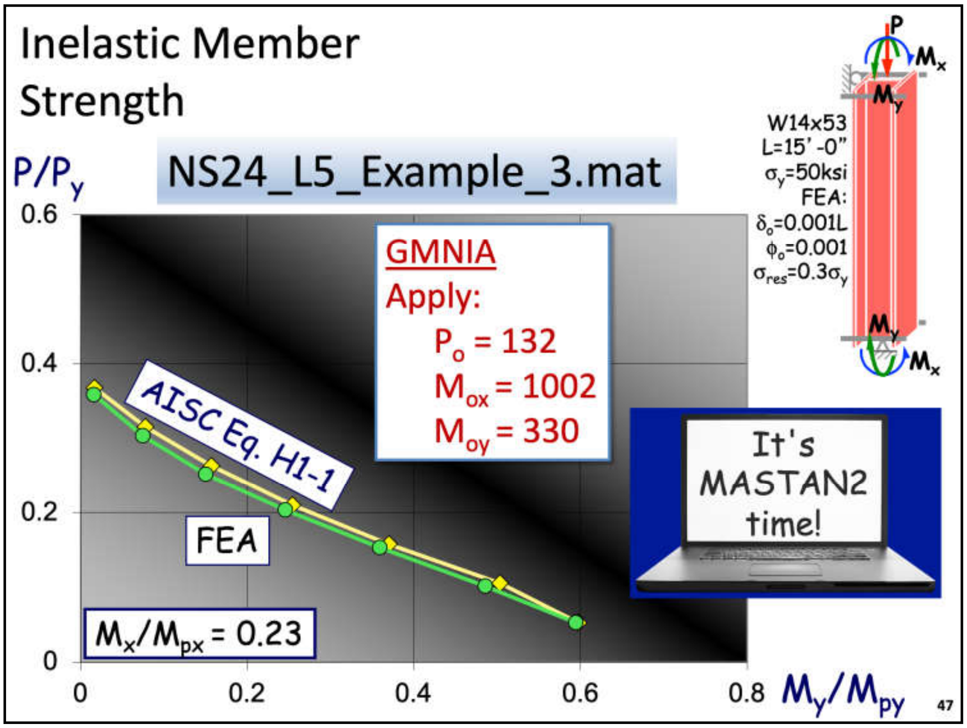
Inelastic Member Strength (Beam-Column)

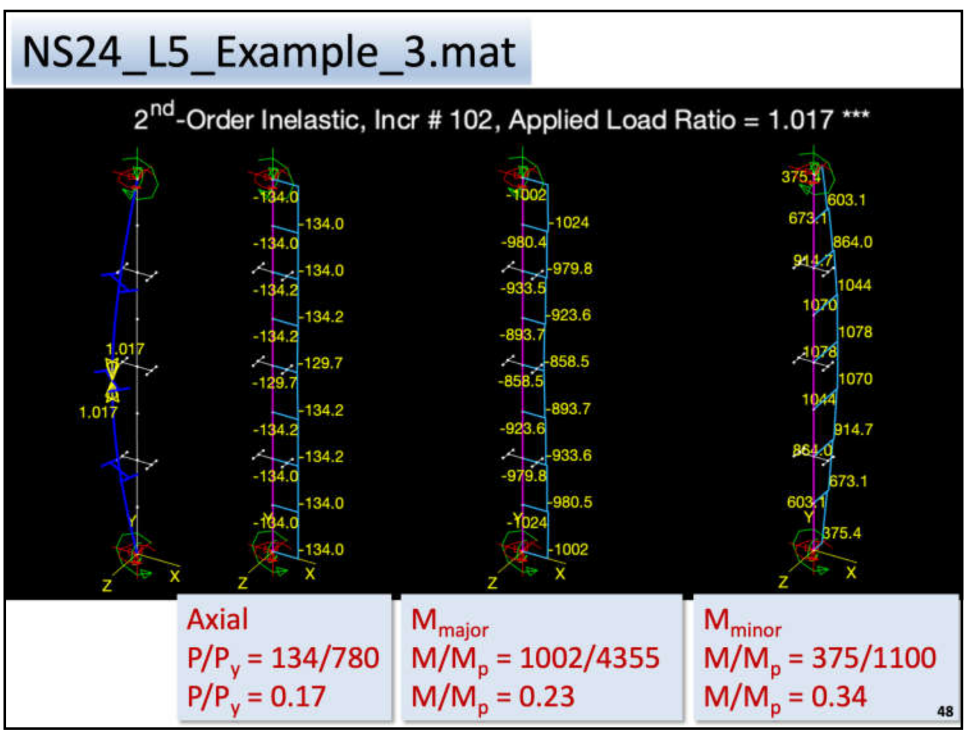


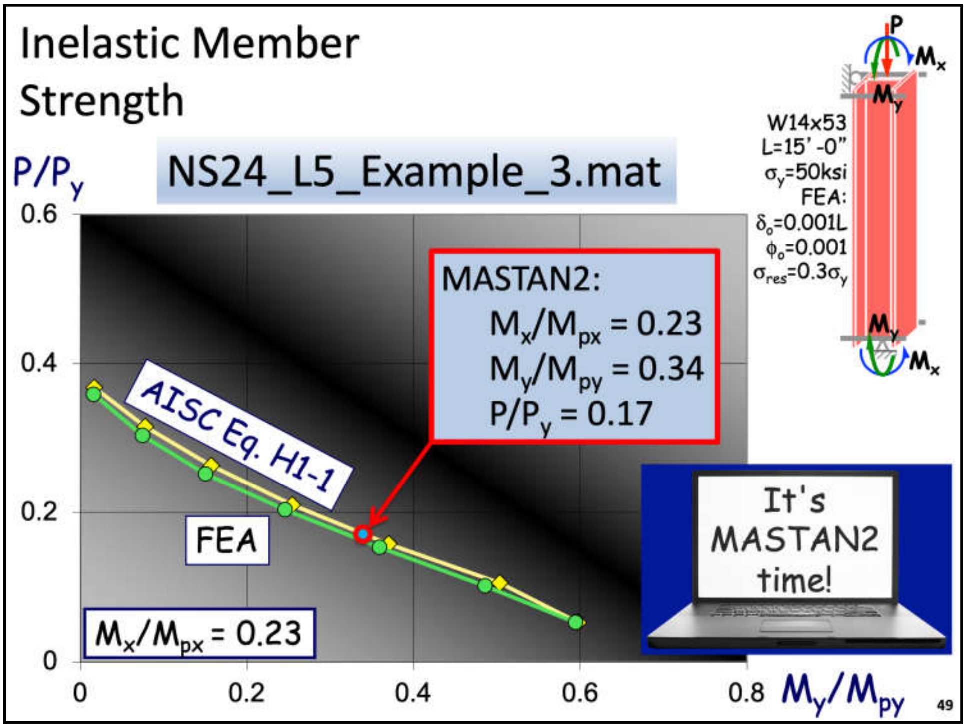
44


Inelastic Member Strength
P/Py
Mx/Mpx
My/Mpy = 0
FEA
AISC Eq. H1-1
my = My/Mpy
mx = Mx/Mpx
W14x53
L=15'-0"
σy=50ksi
FEA:
δo=0.001L
φo=0.001
σres=0.3σy
P
Mx
My
45


Inelastic Member Strength
P/Py
My/Mpy
Mx/Mpx = 0.23
FEA
AISC Eq. H1-1
my = My/Mpy
mx = Mx/Mpx
W14x53
L=15'-0"
σy=50ksi
FEA:
δo=0.001L
φo=0.001
σres=0.3σy
P
Mx
My
46


Inelastic Member Strength
NS24_L5_Example_3.mat
P/Py
GMNIA
Apply:
Po = 132
Mox = 1002
Moy = 330
AISC Eq. H1-1
FEA
Mx/Mpx = 0.23
It's MASTAN2 time!
W14x53
L=15'-0"
σy=50ksi
FEA:
δo=0.001L
φo=0.001
σres=0.3σy
P
Mx
My
0.6
0.4
0.2
0
0
0.2
0.4
0.6
0.8
My/Mpy
47


NS24_L5_Example_3.mat
2nd-Order Inelastic, Incr # 102, Applied Load Ratio = 1.017 ***
1.017
1.017
Axial
P/Py = 134/780
P/Py = 0.17
Mmajor
M/Mp = 1002/4355
M/Mp = 0.23
Mminor
M/Mp = 375/1100
M/Mp = 0.34
48



AISC Interaction Equation for Member Strength

P and *M* are required strengths from the structural analysis and must account for effects that may impact stability of member

For $\dfrac{P}{P_n} \geq 0.2$, $\quad \dfrac{P}{P_n} + \dfrac{8}{9}\dfrac{M}{M_n} \leq 1.0$

For $\dfrac{P}{P_n} < 0.2$, $\quad \dfrac{1}{2}\dfrac{P}{P_n} + \dfrac{M}{M_n} \leq 1.0$

M_n = flexural strength (earlier slides)

P_n = Compressive strength (earlier slides)

50

Analysis Essentials to obtain Required Strengths

$$\text{For } \frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$$

Effects that may impact stability of member:

- axial, flexural, shear, torsional, connection deformations, …
- second-order effects
- geometric imperfections
- stiffness reductions due to inelasticity

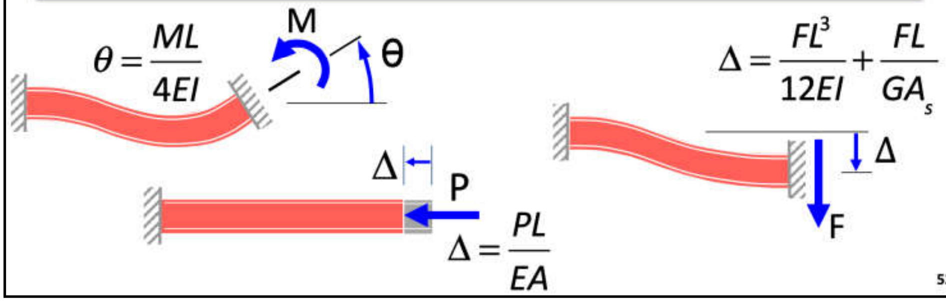
51

Analysis Essentials to obtain Required Strengths

$$\text{For } \frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$$

Effects that may impact stability of member:

- axial, flexural, shear, torsional, connection deformations, …

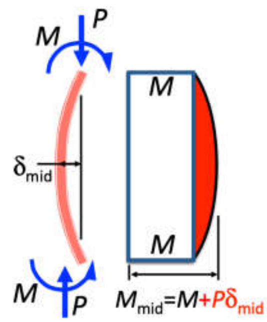


52

Analysis Essentials to obtain Required Strengths

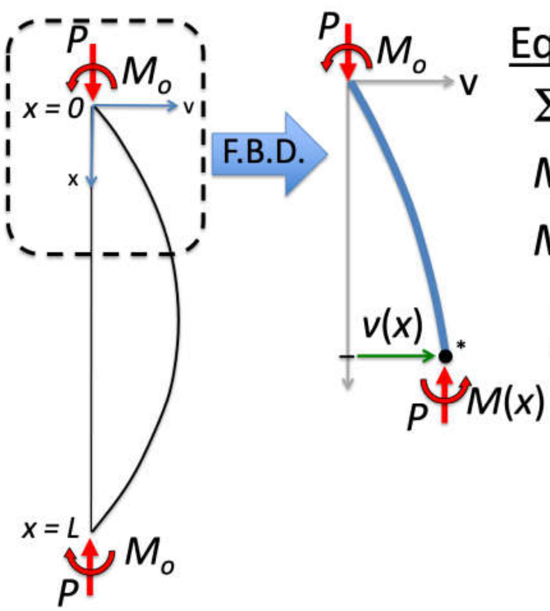
For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact stability of member:
- second-order effects



53

Axial plus uniform bending: P-δ (1)



<u>Equilibrium</u>:

$$\Sigma M_* = 0$$

$$M(x) + M_o + Pv(x) = 0$$

$$M(x) + Pv(x) = -M_o$$

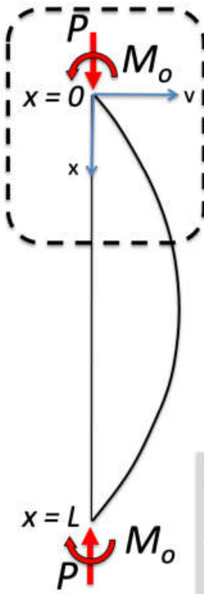
<u>Moment-curvature</u>:

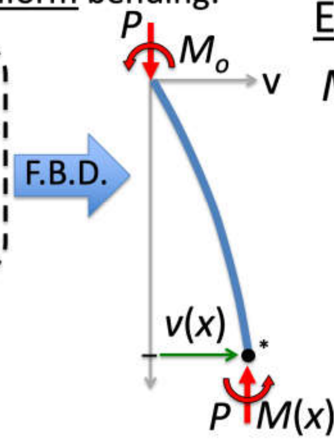
$$M(x) = EI\frac{d^2v(x)}{dx^2}$$

54

Axial plus uniform bending:

P-δ (2)





Equilibrium:

$$M(x)+Pv(x)=-M_o$$

Moment-curvature:

$$M(x)=EI\frac{d^2v(x)}{dx^2}$$

$$EI\frac{d^2v}{dx^2}+Pv=-M_o$$

Solution:

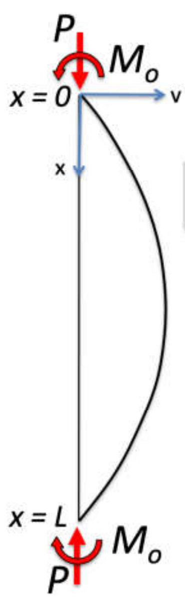
$$v(x)=C_1\cos\left(\sqrt{\frac{P}{EI}}x\right)+C_2\sin\left(\sqrt{\frac{P}{EI}}x\right)-\frac{M_o}{P}$$

wolframalpha.com: a2*y"(x)+a1*y(x)=a3

55

Axial plus uniform bending:

P-δ (3)



$$v(x)=C_1\cos\left(\sqrt{\frac{P}{EI}}x\right)+C_2\sin\left(\sqrt{\frac{P}{EI}}x\right)-\frac{M_o}{P}$$

Boundary Conditions!

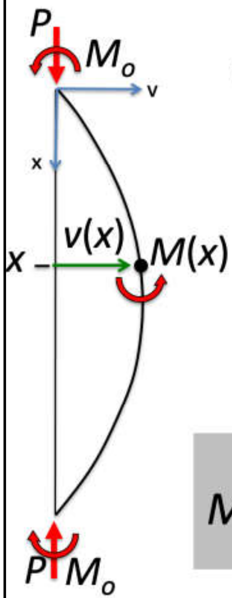
$$v(x=0)=0 \Rightarrow C_1=M_o/P$$

$$v(x=L)=0 \Rightarrow C_2=\frac{M_o}{P}\frac{1-\cos\left(\sqrt{\frac{P}{EI}}L\right)}{\sin\left(\sqrt{\frac{P}{EI}}L\right)}$$

56

Axial plus uniform bending:

P-δ (4)



$$v(x) = C_1 \cos\left(\sqrt{\frac{P}{EI}}x\right) + C_2 \sin\left(\sqrt{\frac{P}{EI}}x\right) - \frac{M_o}{P}$$

Moment-curvature:

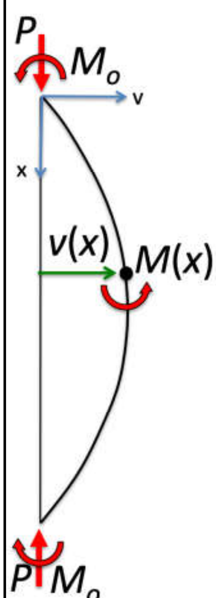
$$M(x) = EI\frac{d^2v(x)}{dx^2}$$

$$M(x) = -C_1 P \cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2 P \sin\left(\sqrt{\frac{P}{EI}}x\right)$$

57

Axial plus uniform bending:

P-δ (5)



$$M(x) = -C_1 P \cos\left(\sqrt{\frac{P}{EI}}x\right) - C_2 P \sin\left(\sqrt{\frac{P}{EI}}x\right)$$

Substituting C_1 and C_2 from previous slide and letting

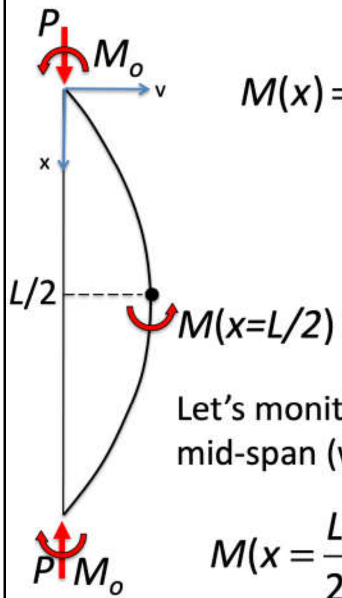
$$\beta = \left(1 - \cos\left(\sqrt{\frac{P}{EI}}L\right)\right) \Big/ \sin\left(\sqrt{\frac{P}{EI}}L\right)$$

$$M(x) = -M_o\left(\cos\left(\sqrt{\frac{P}{EI}}x\right) + \beta \sin\left(\sqrt{\frac{P}{EI}}x\right)\right)$$

58

Axial plus uniform bending:

P-δ (6)



$$M(x) = -M_o\left(\cos\left(\sqrt{\frac{P}{EI}}x\right) + \beta\sin\left(\sqrt{\frac{P}{EI}}x\right)\right)$$

with $\beta = \left(1 - \cos\left(\sqrt{\frac{P}{EI}}L\right)\right) \Big/ \sin\left(\sqrt{\frac{P}{EI}}L\right)$

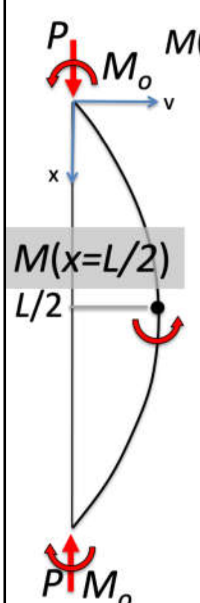
Let's monitor the moment at some point, say the mid-span (which should be the largest)

$$M\left(x = \frac{L}{2}\right) = -M_o\left(\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \beta\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\right)$$

59

Axial plus uniform bending:

P-δ (7)



$$M\left(x = \frac{L}{2}\right) = -M_o\left(\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right) + \beta\sin\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\right)$$

with $\beta = \left(1 - \cos\left(\sqrt{\frac{P}{EI}}L\right)\right) \Big/ \sin\left(\sqrt{\frac{P}{EI}}L\right)$

Letting:

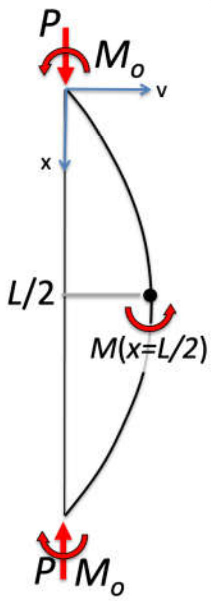
$$u = \sqrt{\frac{P}{EI}}\frac{L}{2}$$

$$M\left(x = \frac{L}{2}\right) = -M_o\left(\cos(u) + \beta\sin(u)\right)$$

with $\beta = \left(1 - \cos(2u)\right) / \sin(2u)$

60

Axial plus uniform bending: P-δ (8)



$$M(x=\frac{L}{2})=-M_o\left(\cos(u)+\beta\sin(u)\right)$$

with $\beta=\left(1-\cos(2u)\right)/\sin(2u)$

Employing trig. identities

$$\cos(2u)=1-2\sin^2(u)$$
$$\sin(2u)=2\sin(u)\cos(u)$$

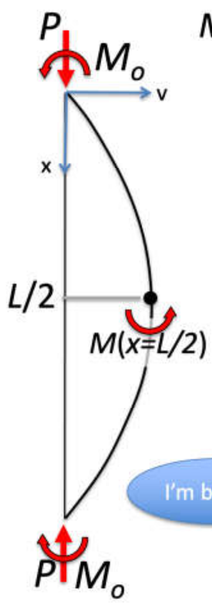
after simplifying

$$M(x=\frac{L}{2})=-M_o\left(\frac{1}{\cos(u)}\right) \quad \text{with} \quad u=\sqrt{\frac{P}{EI}}\frac{L}{2}$$

$$M(x=\frac{L}{2})=-M_o\left(\frac{1}{\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)}\right)$$

61

Axial plus uniform bending: P-δ (9)



$$M(x=\frac{L}{2})=-M_o\left(\frac{1}{\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)}\right)$$

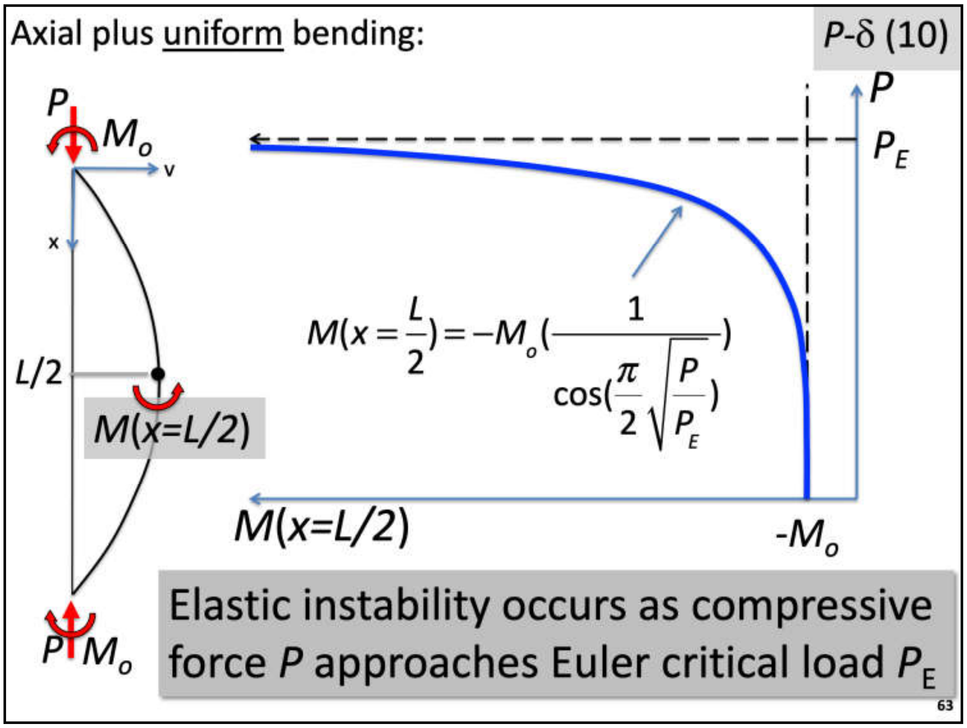
BUT!!! As $\sqrt{\frac{P}{EI}}\frac{L}{2}\rightarrow\frac{\pi}{2}$, $\frac{1}{\cos\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)}\rightarrow\infty$

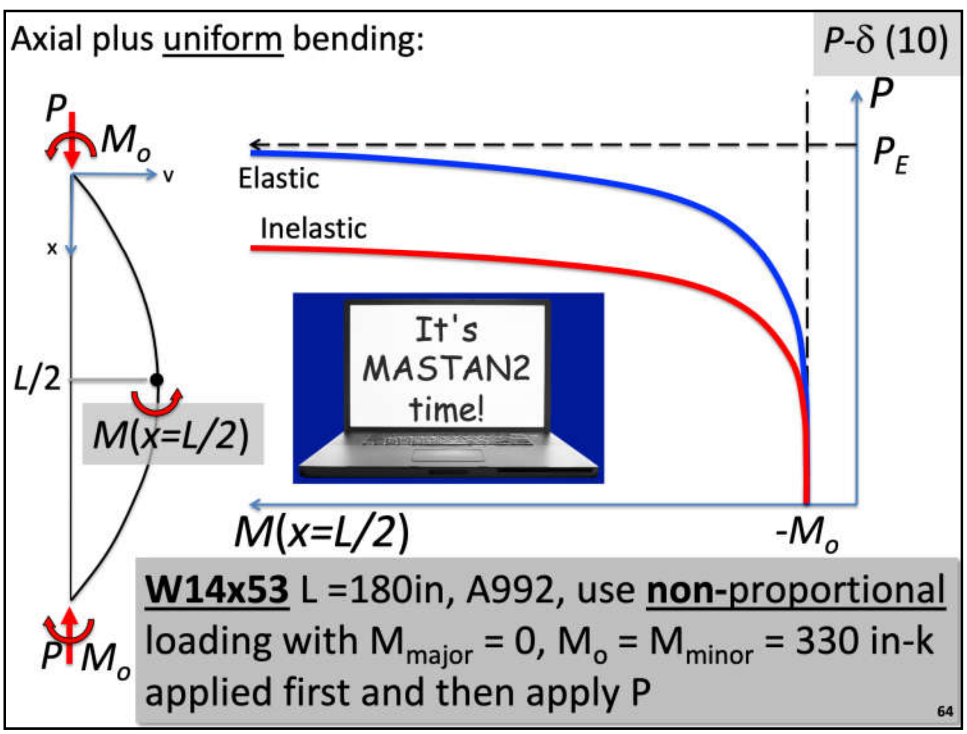
Noting:

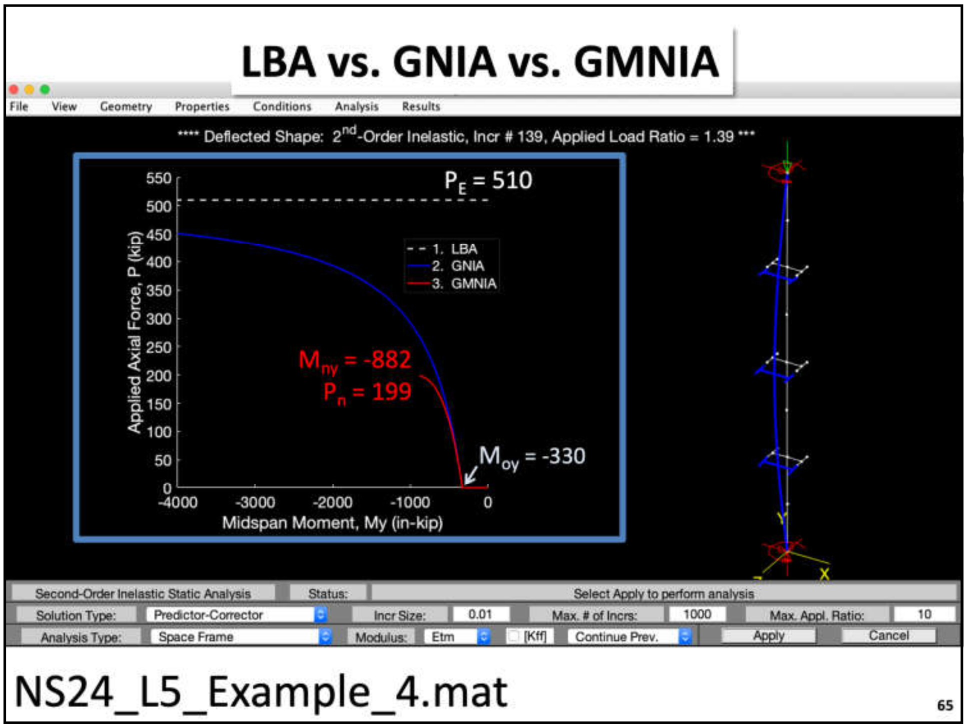
$$\sqrt{\frac{P}{EI}}\frac{L}{2}=\frac{\pi}{2} \Rightarrow P=\frac{\pi^2 EI}{L^2}=P_E \quad \text{and} \quad \sqrt{\frac{P}{EI}}\frac{L}{2}=\frac{\pi}{2}\sqrt{\frac{P}{P_E}}$$

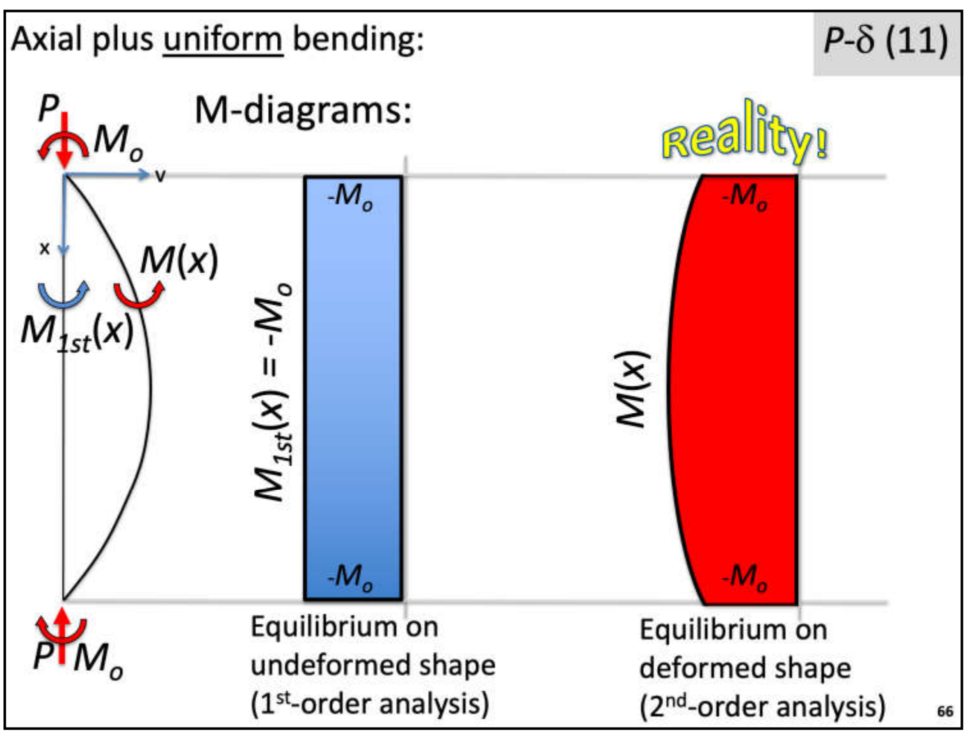
$$M(x=\frac{L}{2})=-M_o\left(\frac{1}{\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)}\right)$$

62

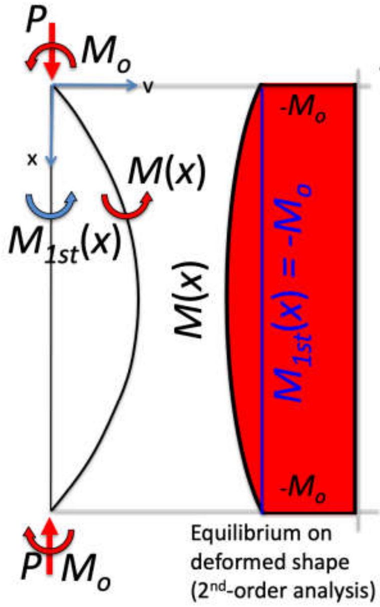

Axial plus uniform bending:
P-δ (10)
P
M_o
v
x
L/2
M(x=L/2)
P
M_o
P
P_E
$M(x=\frac{L}{2}) = -M_o(\frac{1}{\cos(\frac{\pi}{2}\sqrt{\frac{P}{P_E}})})$
M(x=L/2)
$-M_o$
Elastic instability occurs as compressive force P approaches Euler critical load P_E
63


Axial plus uniform bending:
P-δ (10)
P
M_o
v
x
L/2
M(x=L/2)
P
M_o
P
P_E
Elastic
Inelastic
It's MASTAN2 time!
M(x=L/2)
$-M_o$
W14x53 L =180in, A992, use non-proportional loading with M_{major} = 0, M_o = M_{minor} = 330 in-k applied first and then apply P
64


LBA vs. GNIA vs. GMNIA
File View Geometry Properties Conditions Analysis Results
**** Deflected Shape: 2nd-Order Inelastic, Incr # 139, Applied Load Ratio = 1.39 ***
P_E = 510
1. LBA
2. GNIA
3. GMNIA
M_ny = -882
P_n = 199
M_oy = -330
Applied Axial Force, P (kip)
Midspan Moment, My (in-kip)
Second-Order Inelastic Static Analysis
Status:
Select Apply to perform analysis
Solution Type: Predictor-Corrector
Incr Size: 0.01
Max. # of Incrs: 1000
Max. Appl. Ratio: 10
Analysis Type: Space Frame
Modulus: Etm
Continue Prev.
Apply
Cancel
NS24_L5_Example_4.mat
65


Axial plus uniform bending:
P-δ (11)
M-diagrams:
Reality!
P
M_o
v
x
M(x)
M_1st(x)
-M_o
M_1st(x) = -M_o
M(x)
Equilibrium on undeformed shape (1st-order analysis)
Equilibrium on deformed shape (2nd-order analysis)
66

Axial plus <u>uniform</u> bending: *P*-δ (12)



$$M(x) = -M_o\left(\cos\left(\sqrt{\frac{P}{EI}}x\right) + \beta\sin\left(\sqrt{\frac{P}{EI}}x\right)\right)$$

In terms of a moment amplification factor, B_1:

$$M(x) = B_1 M_{1st}(x)$$

with

$$B_1 = \cos\left(\sqrt{\frac{P}{EI}}x\right) + \beta\sin\left(\sqrt{\frac{P}{EI}}x\right)$$

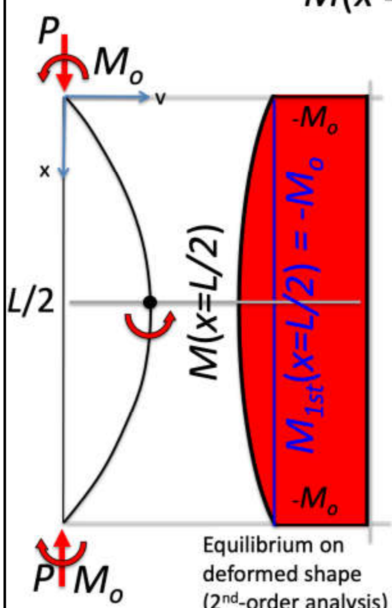
and

$$\beta = \left(1 - \cos\left(\sqrt{\frac{P}{EI}}L\right)\right) \Big/ \sin\left(\sqrt{\frac{P}{EI}}L\right)$$

67

At mid-span: *P*-δ (13)

$$M\left(x = \frac{L}{2}\right) = -M_o\left(\frac{1}{\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)}\right)$$



In terms of a moment amplification factor, B_1:

$$M\left(x = \frac{L}{2}\right) = B_1 M_{1st}\left(x = \frac{L}{2}\right)$$

with

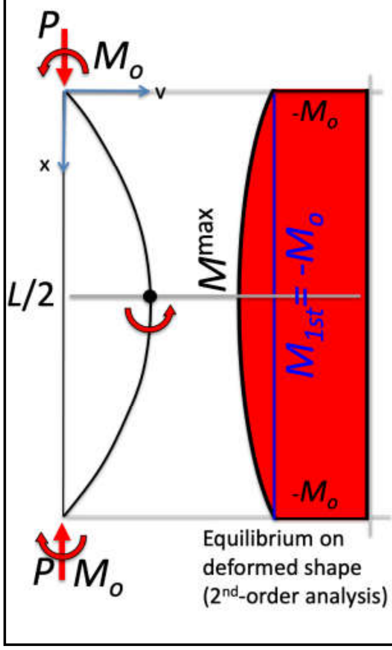
$$B_1 = \frac{1}{\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)}$$

-or-

$$M^{max} = B_1 M_{1st}^{max}$$

68

Axial plus uniform bending: **P-δ (14)**



In terms of a moment amplification factor, B_1:

$$M^{max} = B_1 M^{max}_{1st}$$

with

$$B_1 = \frac{1}{\cos\left(\frac{\pi}{2}\sqrt{\frac{P}{P_E}}\right)}$$

which is often approximated by

$$B_1 = \frac{1}{1 - \frac{P}{P_E}}$$

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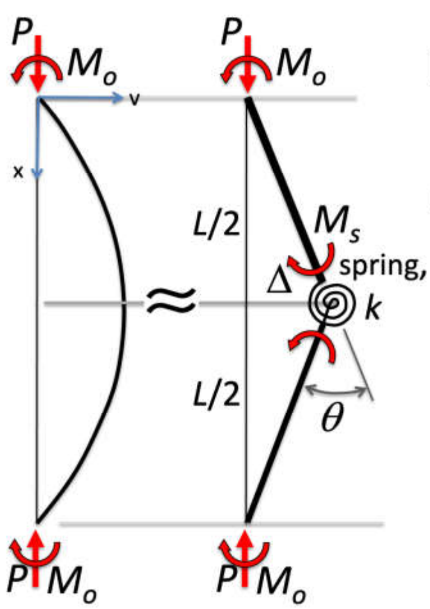
Approximate B_1: **P-δ (15)**



Spring: $M_s = k\theta = \frac{4k}{L}\Delta$

Equilibrium w/ M_o=0, provides P_{cr}

$$M_s = P_{cr}\Delta \;\Rightarrow\; P_{cr} = \frac{4k}{L}$$

Equilibrium w/ M_o≠0,

$$M_s = M_o + P\Delta$$

$$\text{w/ } \Delta = \frac{L}{4k}M_s = \frac{1}{P_{cr}}M_s$$

$$M_s = M_o + \frac{P}{P_{cr}}M_s$$

$$M_s = \left(\frac{1}{1 - \frac{P}{P_{cr}}}\right)M_o = B_1 M_o$$

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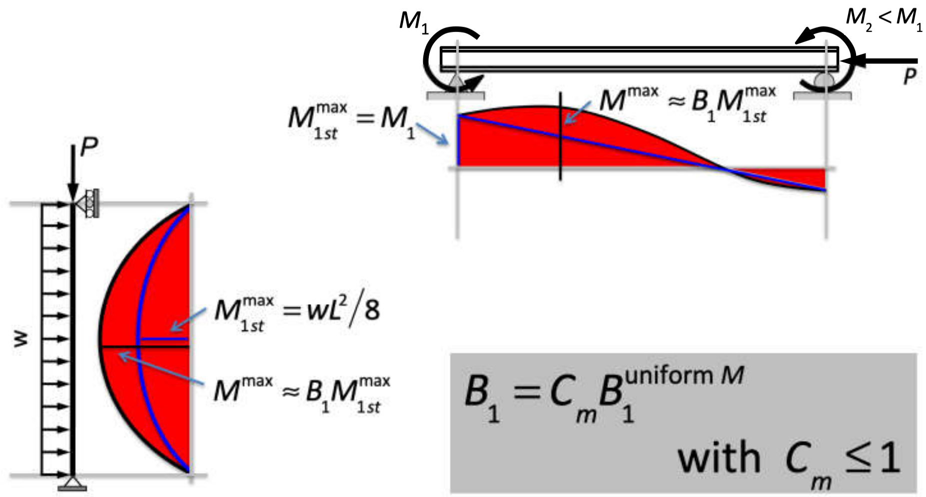
P-δ (16)

What about cases where the M(x) is <u>not</u> uniform?



$$B_1 = C_m B_1^{\text{uniform } M}$$

with $C_m \le 1$

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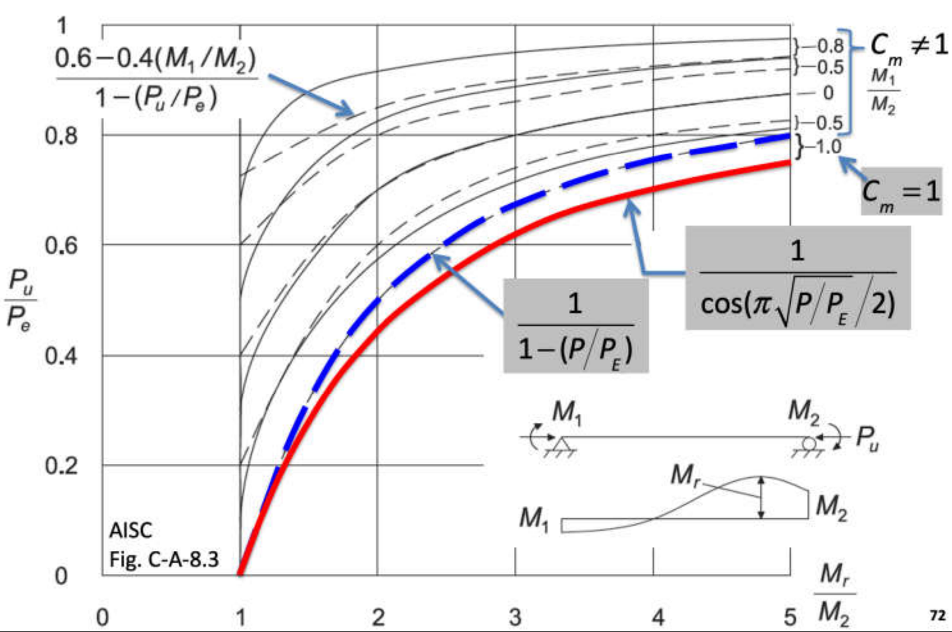
Comparison of Exact and Approximate B_1's — P-δ (17)



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SPECIFICATION FOR THE DESIGN, FABRICATION AND ERECTION OF STRUCTURAL STEEL FOR BUILDINGS (RIVETED, BOLTED AND ARC-WELDED CONSTRUCTION)

REVISED JUNE, 1949

1949

SECTION 12. COMBINED STRESSES.

(a) Axial and Bending.

Members subject to both axial and bending stresses shall be so proportioned that the quantity

$$\frac{f_a}{F_a} + \frac{f_b}{F_b}$$ shall not exceed unity, in which

SPECIFICATION FOR THE DESIGN, FABRICATION & ERECTION OF STRUCTURAL STEEL FOR BUILDINGS (ADOPTED NOVEMBER 30, 1961)

1961

SECTION 1.6 COMBINED STRESSES

1.6.1 Axial Compression and Bending

Members subject to both axial compression and bending stresses shall be proportioned to meet the requirements of both Formula (6) and Formula (7).

$$\frac{f_a}{F_a} + \frac{C_m f_b}{\left(1 - \frac{f_a}{F'_e}\right) F_b} \leq 1.0 \qquad \text{Formula (6)}$$

$B_1 = C_m B_1^{\text{uniform } M}$ with $C_m \leq 1$

2[nd]-order effects introduced before RZ and CQ were born…

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SPECIFICATION FOR THE DESIGN, FABRICATION & ERECTION OF STRUCTURAL STEEL FOR BUILDINGS (ADOPTED NOVEMBER 30, 1961)

1961

By the Committee,

William C. Alsmeyer	Theodore R. Higgins	James A. Munro
Ethan F. Ball	Sol Horwitz	William H. Munse
Lynn S. Beedle	John W. Hubler	Charles I. Orr
Walter E. Blessey	Bruce G. Johnston	Egor P. Popov
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Milton E. Eliot	Carl A. Metz	David M. Wilson
Edward R. Estes, Jr.	William A. Milek, Jr.	George Winter
Edwin H. Gaylord, Jr.		Robert J. Wood

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LM7. Second-order Effects (P-Δ and P-δ) Effects

Learning Objectives

– Employ second-order elastic analysis
– Observe the relationship between extent of moment amplification and P/P_e
– Investigate the impact of single or double(reverse) curvature bending has on the degree of moment amplification
– Compute and assess the AISC moment amplification factor B_1 defined in App. 8

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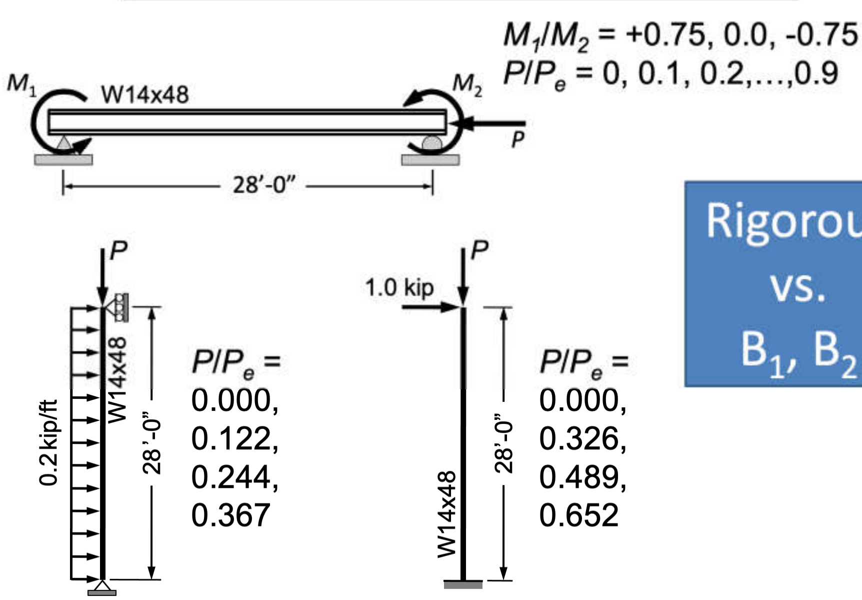
1st-order vs. 2nd-order



Rigorous vs. B_1, B_2

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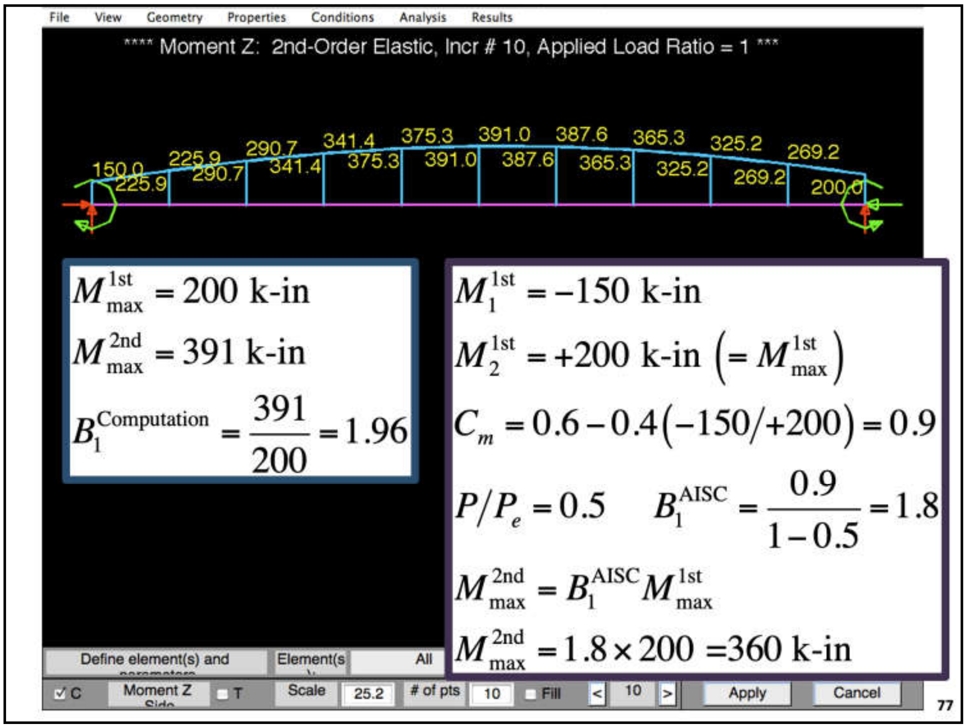
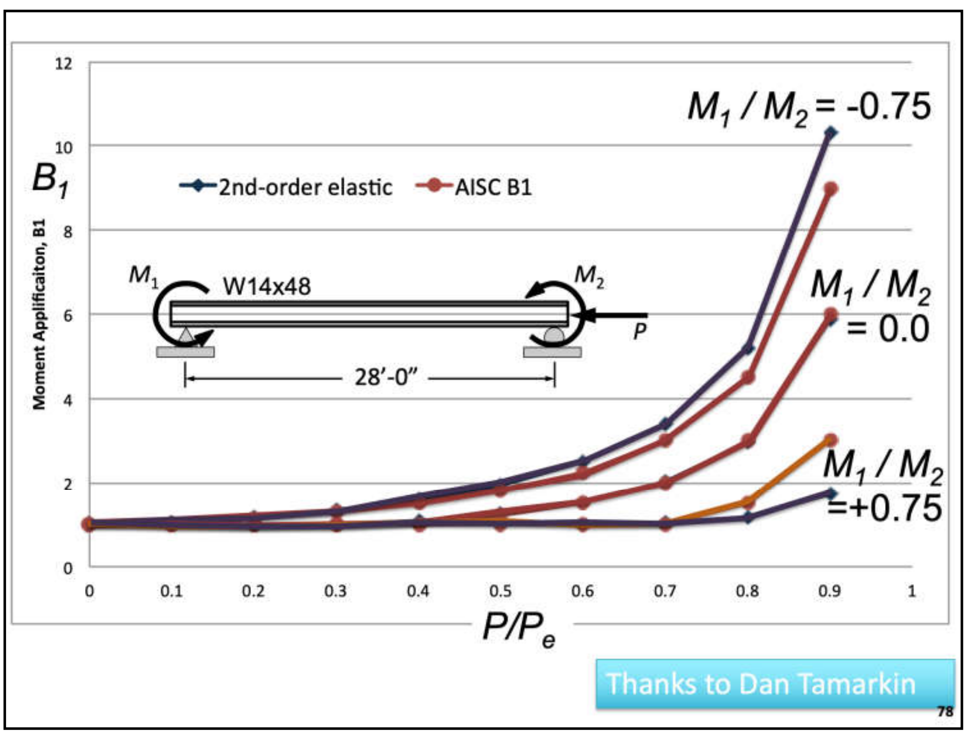

File View Geometry Properties Conditions Analysis Results
**** Moment Z: 2nd-Order Elastic, Incr # 10, Applied Load Ratio = 1 ***
150.0 225.9 290.7 341.4 375.3 391.0 387.6 365.3 325.2 269.2
225.9 290.7 341.4 375.3 391.0 387.6 365.3 325.2 269.2 200.0
$M_{max}^{1st} = 200$ k-in
$M_{max}^{2nd} = 391$ k-in
$B_1^{Computation} = \frac{391}{200} = 1.96$
$M_1^{1st} = -150$ k-in
$M_2^{1st} = +200$ k-in $\left(= M_{max}^{1st}\right)$
$C_m = 0.6 - 0.4(-150/+200) = 0.9$
$P/P_e = 0.5 \quad B_1^{AISC} = \frac{0.9}{1-0.5} = 1.8$
$M_{max}^{2nd} = B_1^{AISC} M_{max}^{1st}$
$M_{max}^{2nd} = 1.8 \times 200 = 360$ k-in
Define element(s) and
Element(s
All
C
Moment Z
T
Scale
25.2
# of pts
10
Fill
10
Apply
Cancel
77


B_1
Moment Applificaiton, B1
2nd-order elastic
AISC B1
$M_1 / M_2 = -0.75$
$M_1 / M_2 = 0.0$
$M_1 / M_2 = +0.75$
M_1
M_2
W14x48
P
28'-0"
0
2
4
6
8
10
12
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
P/P_e
Thanks to Dan Tamarkin
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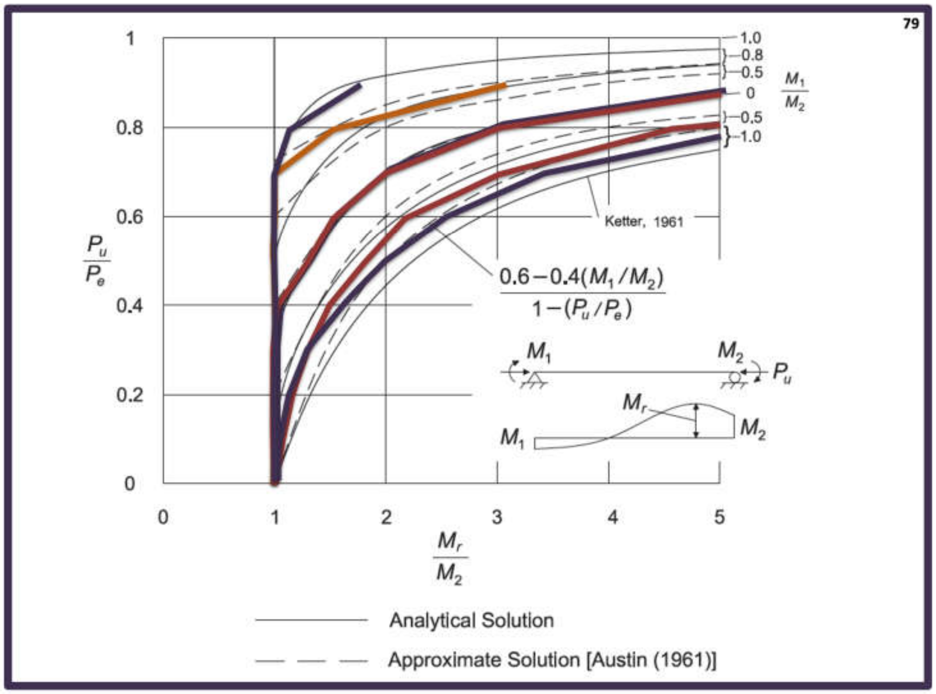


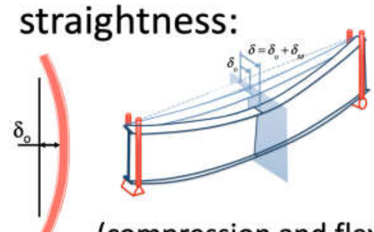
Analysis Essentials to obtain Required Strengths

For $\frac{P}{P_n} \geq 0.2$, $\quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact stability of member:

- geometric imperfections

Member out-of-straightness:



(compression and flexure slides)

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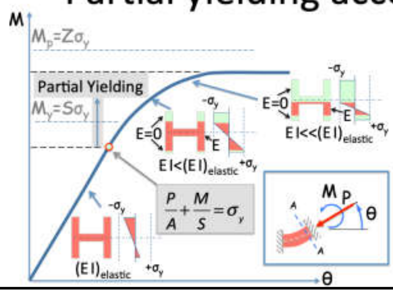
Analysis Essentials to obtain Required Strengths

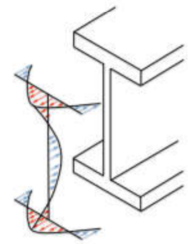
For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact stability of member:

- stiffness reductions due to inelasticity

Partial yielding accentuated by residual stresses:





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Analysis Essentials to obtain Required Strengths

For $\frac{P}{P_n} \geq 0.2, \quad \frac{P}{P_n} + \frac{8}{9}\frac{M}{M_n} \leq 1.0$

Effects that may impact stability of member:

- axial, flexural, shear, torsional, connection deformations, … ✓
- second-order effects ✓
- geometric imperfections ✓
- stiffness reductions due to inelasticity ✓

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Summary – Beam-Columns

- Stability of members
  - Compression (earlier slides)
  - Flexural (earlier slides)
  - Combined compression and flexure
    - Behavior of beam-columns (today!)
    - Design
- Behavior of Beam-Columns
  - full yield (interaction surface)
  - member instability
    - from flexural buckling -to- lateral torsional buckling, including torsional-flexural buckling
  - AISC Interaction Equations

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Summary – Beam-Columns (cont.)

- Factors impacting stability
  - axial, flexural, shear, torsional, connection deformations, …
  - second-order effects
  - geometric imperfections
  - stiffness reductions due to inelasticity

Your virtual laboratory assignment…

Rerun examples presented in this lecture

Try to recreate Learning Module 7 results

Complete a portion of Learning Module 8

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Thank You!

Hope you enjoyed this lecture!

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Stronger.
Steel.

Single-Session Registrants

CEU / PDH Certificates

- You will receive an email on how to report attendance from: registration@aisc.org.
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!

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Single-Session Registrants

CEU / PDH Certificates

- Reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.

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8-Session Registrants

CEU / PDH Certificates

One certificate will be issued at the conclusion of the course.

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Attendance and PDH Certificates

- You have two options to receive credit for a given session.
  - Option 1: Watch the live session. Credit for live attendance will be displayed on the Course Resources table within two days of the session.
  - Option 2: Watch the recording and pass the associated quiz.

Videos and Quizzes

- For each session, find access within two business days after the live air date. (An email will be sent from nightschool@aisc.org.)
- Reasons for quiz:
  - EEU – You must take all quizzes and the final exam to receive EEU.
  - PDHs – If you watch a recorded session, you must pass quiz for PDHs.
  - Reinforce what you learn in the lectures and get more out of the course!

Distribution of Certificates

All certificates will be issued after the course is completed. Only the registrant will receive a certificate for the course.

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8-Session Registrants

Course Resources

Find all your handouts, quizzes and quiz scores, recording access, and attendance information in one place!

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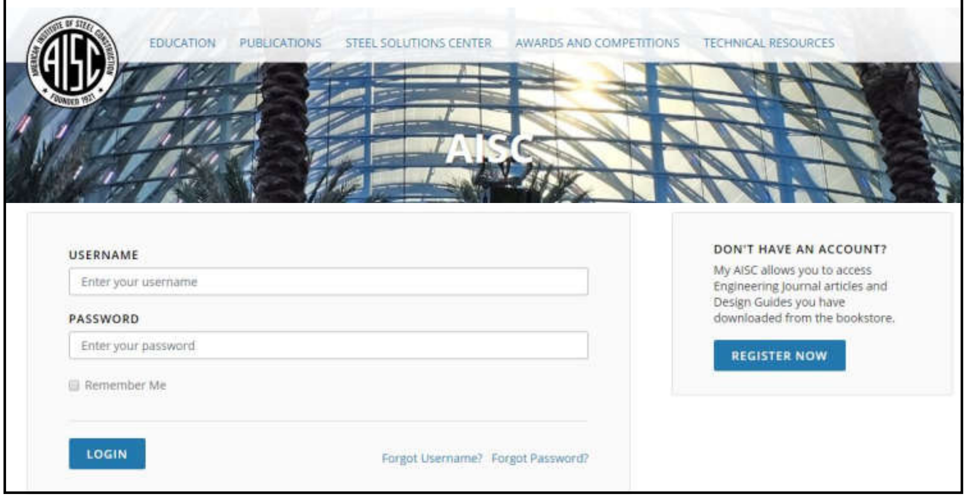
Course Resources

Go to www.aisc.org and sign in.



8-Session Registrants

Course Resources

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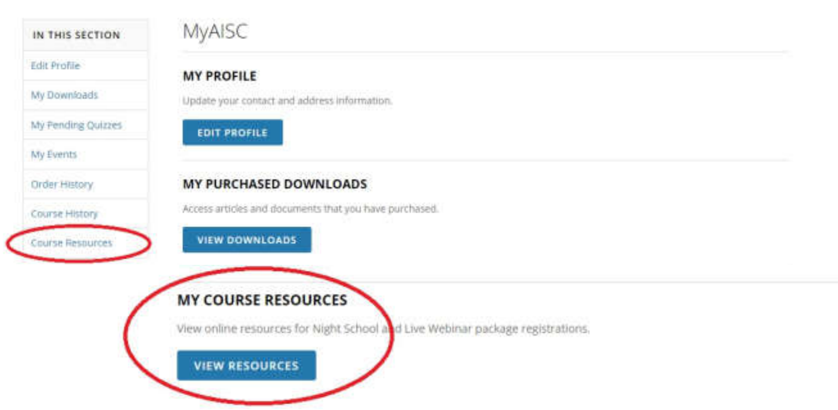

8-Session Registrants

Course Resources

AISC › MY ACCOUNT › COURSE RESOURCES

Course Resources

Event	Start Date
Seismic Design in Steel	1/1/1900 12:00:00 AM
4-Session Package-Design of Facade Attachments	5/9/2019 1:30:00 PM
NS 15 8-Session Package-Night School 15 - Fundamentals of Connection Design	10/3/2017 7:00:00 PM
NS 16 8-Session Package-Night School 16 - Seismic Design in Steel	2/5/2018 7:00:00 PM
NS 17 4-Session Package-Night School 17: Design of Facade Attachments	7/16/2018 7:00:00 PM
NS 18 8-Session Package-Night School 18: Steel Construction: Mill To Topping Out	10/15/2018 7:00:00 PM
NS 19 8-Session Package-Night School 19: Connection Design	2/4/2019 7:00:00 PM
NS 20 8-Session Package-Night School 20: Classical Methods of Structural Analysis	6/3/2019 7:00:00 PM
8-Session Package-Seismic Design in Steel - Concepts & Examples	7/16/2018 1:30:00 PM

8-Session Registrants

Course Resources

AISC › MY ACCOUNT › COURSE RESOURCES › NS24 8-SESSION PACKAGE RESOURCES

Night School 24: Modern Methods for Learning Structural Stability

8-SESSION PACKAGE RESOURCES

Event	Date	Handouts	Video	Quiz	Attendance
NS24.1 - Compression Members - The Fundamentals	Oct 6 2020 7:00PM EDT	Handouts	Available 10/08/2020 5:00PM EDT	Available 10/08/2020 5:00PM EDT	Pending
NS24.2 - Compression Members - Practical Considerations	Oct 13 2020 7:00PM EDT	Handouts	Available 10/15/2020 5:00PM EDT	Available 10/15/2020 5:00PM EDT	Pending
NS24.3 - Behavior of Flexural Members - The Fundamentals	Oct 20 2020 7:00PM EDT	Handouts	Available 10/22/2020 5:00PM EDT	Available 10/22/2020 5:00PM EDT	Pending
NS24.4 - Flexural Members - Practical Considerations	Oct 27 2020 7:00PM EDT	Handouts	Available 10/29/2020 5:00PM EDT	Available 10/29/2020 5:00PM EDT	Pending
NS24.5 - Stability of Beam-Columns - The Fundamentals	Nov 10 2020 7:00PM EST	Handouts	Available 11/12/2020 5:00PM EST	No longer available	Pending
NS24.6 - Stability of Beam-Columns - Practical Consideration	Nov 17 2020 7:00PM EST	Handouts	Available 11/19/2020 5:00PM EST	No longer available	Pending
NS24.7 - Behavior of Structural Systems - The Fundamentals	Dec 1 2020 7:00PM EST	Handouts	Available 12/03/2020 5:00PM EST	No longer available	Pending
NS24.8 - Structural Systems - Practical Considerations	Dec 8 2020 7:00PM EST	Handouts	Available 12/10/2020 5:00PM EST	No longer available	Pending
NS24 - Final Exam	N/A			No longer available	

AISC | Thank you.

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