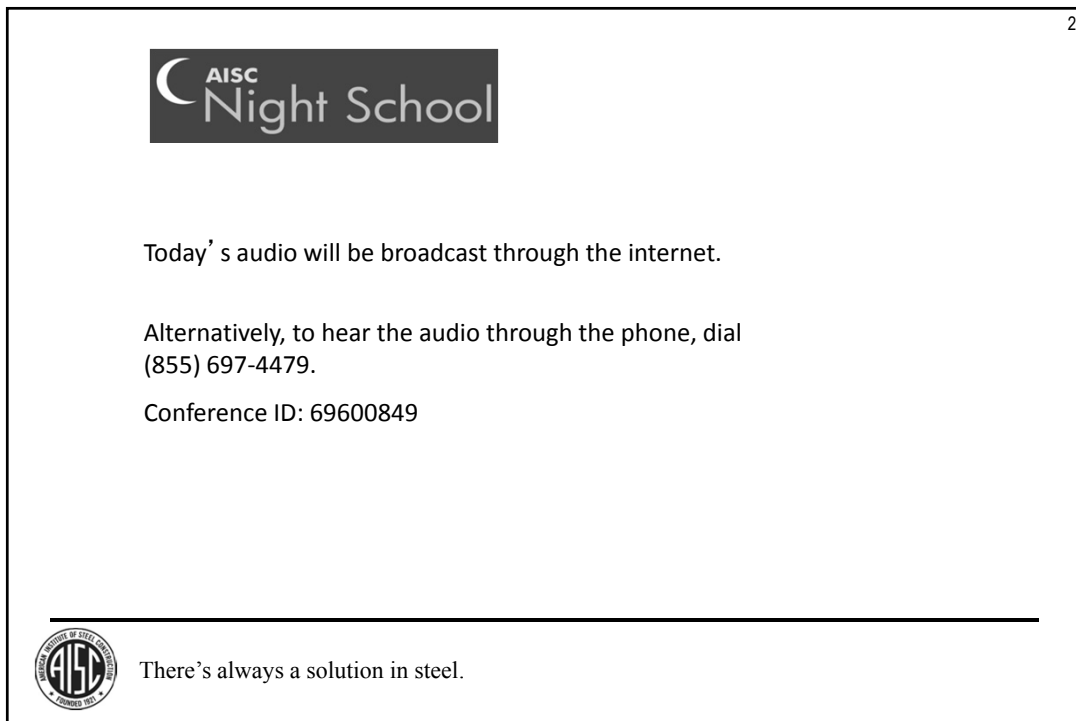
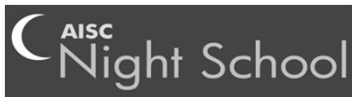
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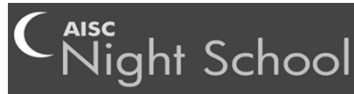
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## Course Description

### **Modern Methods of Structural Analysis, from Linear to Nonlinear – Part I**

January 26, 2015

AISC's Direct Analysis Method is based on the principle that simplifications in the design process can be granted as more and more factors known to impact structural stability are included in the analysis. After providing an overview of this logic, this two-part lecture will focus on describing how modern structural engineering software includes such factors within the analysis. Part I will cover first-order elastic analysis by the stiffness method, which is the core for almost all of today's structural engineering analysis software.



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## Learning Objectives

- Become familiar with the basic principles of structural stability.
- Become familiar with the principle that simplifications in the design process can be granted as more and more known factors are included in the analysis which is the basis of the Direct Analysis Method.
- Gain an understanding of linear and nonlinear analysis by the stiffness method and how it is applied.
- Become familiar with the methods of computer structural analysis.



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## Stability Design of Steel Structures – Applying Modern Methods of Structural Analysis

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### Session 1 Course Introduction and Modern Methods of Structural Analysis, from Linear to Nonlinear – Part I

Ronald D. Ziemian, P.E., Ph.D.



## Course Overview

- **Session Topics**
  - Course Intro. and Modern Analysis (1 & 2)
  - Resources for Learning Stability by Analysis (3)
  - Second-Order Analysis (4)
  - Direct Analysis Method (5)
  - Low- and Medium-Rise Steel Buildings (6)
  - Advanced Application of Stability Design (7)
  - Design by Inelastic Analysis (8)
- **Lectures by members of the Structural Stability Research Council (SSRC)**
  - Don White and Ron Ziemian
  - Great to join AISC in this effort!

## Stability Design of Steel Structures - Applying Modern Methods of Structural Analysis

### Course Introduction



### Fun of designing with steel:

Strength/  
Weight

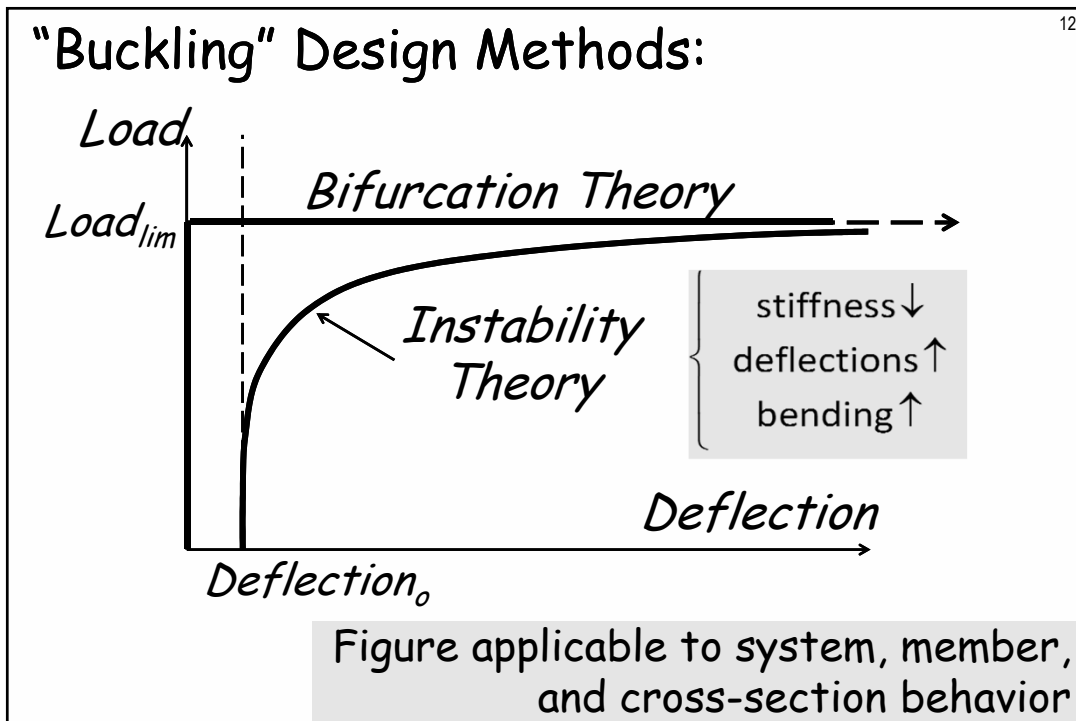
Stiffness/  
Weight

Competitive  
\$

Slender Systems, Members, and Cross-sections

Design for Stability!

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## General Stability Design Methods:

Analysis for load effects:

- Reality: Equilibrium on deformed shape


$$R_u \leq \phi R_n$$

**Effective length method**  
- bifurcation theory

**Direct analysis method**  
- instability theory

NOTE! Both methods require consideration of 2<sup>nd</sup>-order effects

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## "General" Effective Length Method

Analysis for load effects:

- Equilibrium on deformed shape

$$R_u \leq \phi R_n$$

Based on bifurcation theory (binary buckling)

- Effective lengths account for all effects known to impact system and member instability
- Given the correct effective length of all compression members, this method has been proven acceptable (50+ years of use!)



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## "General" Direct Analysis Method

Analysis for load effects:

- Equilibrium on deformed shape

Based on instability theory (approaching buckling)

$$R_u \leq \phi R_n$$

stiffness ↓  
deflections ↑  
bending ↑

- By directly modeling effects known to impact system, member, and cross-section instability, simplifications are granted in computing design resistance
- Design process will not permit system, member, and cross-section instabilities

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## General Direct Analysis Method

Analysis

**Primary effects:**

- Applied load and relative stiffness distribution
- Initial imperfections (system and component)
- Yielding accentuated by residual stresses
- All pertinent resulting deformations
- Redistribution of stresses (F/M's) resulting from changes in relative stiffness distribution

- By directly modeling effects known to impact system, member, and cross-section instability, simplifications are granted in computing design resistance
- Design process will not permit system, member, and cross-section instabilities



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## AISC's Direct Analysis Method

Analysis for load effects:

- Equilibrium on deformed shape

$$R_u \leq \phi R_n$$

Based on instability theory (approaching buckling)

stiffness ↓  
 deflections ↑  
 bending ↑

- By directly modeling effects known to impact system, member, ~~and cross-section~~ instability, simplifications are granted in computing design resistance
- Design process will not permit system, member, ~~and cross-section~~ instabilities

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## AISC's Direct Analysis Method

Analysis for load effects:

- Equilibrium on deformed shape

Based on instability theory (approaching buckling)

**Design by elastic analysis:**

- Poor man's inelastic analysis
  - member stiffness reduced by  $0.8\tau$
  - include system and member imperfections
- Redistribution of stresses (F/M's) resulting from changes in relative stiffness distribution is not accounted for

**Design inelastic analysis:**

- loss in stiffness due to yielding is directly modeled
- system and member imperfections are directly modeled
- Redistribution of stresses (F/M's) resulting from changes in relative stiffness distributions is accounted for

- By directly modeling effects known to impact system, member, ~~and cross-section~~ instability, simplifications are granted in computing design resistance
- Design process will not permit system, member, ~~and cross-section~~ instabilities



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## AISC's Direct Analysis Method (2005-16)

**Elastic Analysis** for load effects: Based on instability

- Equilibrium

**Effects modeled:**

- Applied load and stiffness distribution of components
- Initial system imperfections (out-of-plumb)
- Yielding accentuated by residual stresses (approximated by reducing member stiffness by  $0.8\tau$ )
- All pertinent deformations

- By directly modeling effects known to impact system, member, and cross-section ~~instabilities~~ **simplifications are granted in cross-section resistance**
- Design process will not permit system, member ~~instabilities~~ **cross-section instabilities**

**Simplification granted:**

- Flexural compressive strength  $P_n$  is based on unbraced length ( $KL=L$ )

**DM**

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## AISC's Direct Analysis Method (2016)

**Elastic Analysis** for load effects:

- Equilibrium

**Effects modeled:**

- Applied load and initial stiffness distribution
- Initial system imperfections (out-of-plumb)
- Initial member imperfections (out-of-straightness)
- Yielding accentuated by presence of residual stresses (approximated by reducing member stiffness by  $0.8\tau$ )
- All pertinent deformations

- By directly modeling effects known to impact system, member, and cross-section ~~instabilities~~ **simplifications are granted in cross-section resistance**
- Design process will not permit system, member ~~instabilities~~ **cross-section instabilities**

**Simplification granted:**

- Compressive strength  $P_n$  taken as cross-section axial strength

**DM-DMMI**



## AISC's Direct Analysis Method (2005-16) <sup>21</sup>

**Inelastic Analysis** for load effects:

- Eq. **Effects modeled:**
  - Applied load and initial stiffness distribution
  - Initial system imperfections (out-of-plumb)
  - Initial member imperfections (out-of-straightness)
  - Yielding accentuated by presence of residual stresses (directly modeled)
  - All pertinent deformations
  - Redistribution of stresses (F/M's) resulting from changes in relative stiffness distributions
- By directly modeling effects known to impact system, member, and ~~cross-section~~ instability, simplifications are granted in computing design
- Design process will not be **DM-Inelastic**
  - Specification design equations waived
  - Go beyond first-plastic hinge!

### Additional thoughts... <sup>22</sup>

- ❖ Automated modeling and analysis is not the objective, more opportunities for better engineering is the aim
- ❖ Goal is to improve design process by providing a more detailed, and hopefully more "realistic" understanding of structural behavior...
- ❖ Real question is *where does an engineer have the most knowledge and confidence for a given design situation?*

More ← Prescriptive specification equations → Less  
 Less ← Given Design Situation → More  
 Directly modeling effects impacting stability



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Prescriptive specification equations

← More

Given Design Situation


Less →

Directly modeling effects impacting stability

← Less

More →

**ELM  
DM**



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Prescriptive specification equations

← More

Given Design Situation


Less →

Directly modeling effects impacting stability

← Less

More →

**DM-DMMI**



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More ← Prescriptive specification equations → Less  
 ← Given Design Situation →  
 Less ← Directly modeling effects impacting stability → More

**DM-Inelastic**

Excuse me, will you please comment on those ladder columns

Sure, they were designed using AISC's Appendix 1

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More ← Prescriptive specification equations → Less  
 ← Given Design Situation →  
 Less ← Directly modeling effects impacting stability → More

**AISC's Direct Analysis Method** is intended to provide a wide range of design opportunities to the structural engineer...

- 2005** DM appears, Elastic analysis,  $P_n(KL=L)$ , App. 7
- 2010** DM upgraded and extended  
 Elastic analysis,  $P_n(KL=L)$ , upgraded to Ch. C  
 Extended to include inelasticity, fully revised App. 1
- 2016** DM, extended to include DMMI  
 to permit direct modeling of member imperfections;  
 rigorous elastic analysis,  $P_n = P_{ns}$  (cross section strength)



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## Effective Length Method (ELM)

2<sup>nd</sup>-order elastic analysis with:  
 $E = E_o$   
no initial imperfections or  
stiffness reduction

$$\frac{P_u}{\phi P_n} + \frac{8 M_u}{9 \phi M_n} \leq 1.0$$

AISC column curve  
based on KL  
with  $K \geq 1.0$

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## Direct Analysis Method (DM)

2<sup>nd</sup>-order elastic analysis with:  
 $E = 0.8 \tau E_o$   
initial frame out-of-plumbness  
 $\Delta_o = H/500$

$$\frac{P_u}{\phi P_n} + \frac{8 M_u}{9 \phi M_n} \leq 1.0$$

AISC column curve  
based on KL  
with  $K = 1.0$



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## Direct Modeling of Member Imperfections Method (DM-DMMI)

2<sup>nd</sup>-order elastic analysis with:

$$\frac{P_u}{\phi P_n} + \frac{8 M_u}{9 \phi M_n} \leq 1.0$$

$E = 0.8 \tau E_o$   
 initial frame out-of-plumbness  
 $\Delta_o = H/500$   
 initial member out-of-straightness  
 $\delta_o = L/1000$

Cross-section strength,  
 $P_n = P_{ns} = A_e F_y$

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## The big tradeoff!

$M_u$  increasing

↓

ELM  
 DM ( $\Delta_o, 0.8\tau$ )  
 DM-DMMI ( $\Delta_o, 0.8\tau, \delta_o$ )

$$\frac{P_u}{\phi P_n} + \frac{8 M_u}{9 \phi M_n} \leq 1.0$$

$P_n$  increasing

↓

ELM ( $KL > L$ )  
 DM ( $KL = L$ )  
 DMMI ( $P_n = A_e F_y$ )


Methods often give very similar designs



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# Stability Design of Steel Structures - Applying Modern Methods of Structural Analysis

**Session 1**  
**Modern Methods of  
Structural Analysis,  
from Linear to  
Nonlinear - Part I**



Welcome aboard!

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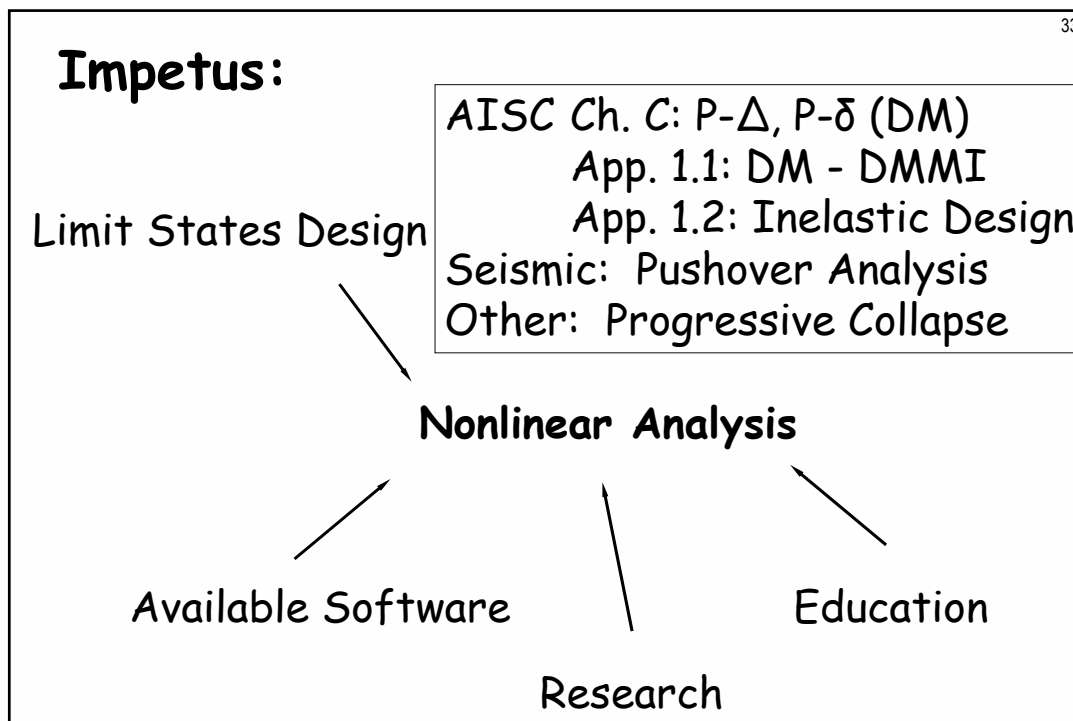
## Role of the analysis:

- forces, moments and deflections --> design equations
- insight into the behavior of a structure  
--> better the understanding, better the design

## Limit States Design:

Prior to limit of resistance, significant nonlinear response, including

- geometrical effects ( $P-\Delta$ ,  $P-\delta$ )
- material effects (yielding, cracking, crushing)
- combined effects



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## Linear to Nonlinear Analysis

- ❖ Hand methods
  - Second-order effects
    - i.e. Moment Amplification Factors (B1 and B2 factors)
  - Material nonlinear effects
    - i.e. plastic analysis (upper and lower bound theories)
- ❖ Computer Methods (focus of this lecture)
  - Lots of variations
    - all use same basic concepts (most important to today)
    - one approach will be presented (basis for MASTAN2)
- ❖ Please keep in mind
  - All methods are approximate
  - Not a substitute, but a complement to good engineering



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## Overview of Lectures

- ❖ Session 1
  - Brief Introduction (done!)
  - Computer Structural Analysis (Review?)
- ❖ Session 2
  - Basis for Material Nonlinear Models
  - Incorporating Geometric Nonlinear Behavior
  - Critical Load Analysis
  - Overview of MASTAN2 software
  - Summary and Concluding Remarks

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## How does the computer get these results?

- ❖ State-of-the-Art Crystal Ball? Not quite.
- ❖ By applying 2 requirements and 1 translator
  - Two Requirements:
    - Equilibrium (equations in terms of F's and M's, 1 per d.o.f.)
    - Compatibility (equations in terms of  $\Delta$ 's and  $\theta$ 's, 1 per d.o.f.)
  - Translator "apples to oranges"
    - Constitutive Relationship (i.e. Hooke's Law,  $\sigma = E \epsilon$ )
    - Generalized to Force-to-Displacement (i.e.  $F=k\Delta$ )
    - Re-write equilibrium eqs. in terms of unknown displacements
- ❖ # of Equil. Eqs. = # of Unknown Displacements



## Equilibrium Equations

D 40 kips

Free Body Diagram

x-d.o.f.  $u_D$ :  $\sum F_x = 0$

$$40 = f_{xD}^{AD} + f_{xD}^{BD} + f_{xD}^{CD}$$


---

y-d.o.f.  $v_D$ :  $\sum F_y = 0$

$$0 = f_{yD}^{AD} + f_{yD}^{BD} + f_{yD}^{CD}$$

## Translator: Forces $\rightarrow$ Displacements

(Undeformed)      (Deformed)

$$f_{xi} = k_{11}u_i + k_{12}v_i + k_{13}u_j + k_{14}v_j$$

$$f_{yi} = k_{21}u_i + k_{22}v_i + k_{23}u_j + k_{24}v_j$$

$$f_{xj} = k_{31}u_i + k_{32}v_i + k_{33}u_j + k_{34}v_j$$

$$f_{yj} = k_{41}u_i + k_{42}v_i + k_{43}u_j + k_{44}v_j$$

**Big Question:**  
Where do these known stiffness coefficients  $k$ 's come from?

**Little Answer:**  
Function of member's material and geometric properties, including its orientation.

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## F → Δ for all members

**Member AD:**

$$f_{xA}^{AD} = k_{11}^{AD} u_A^{AD} + k_{12}^{AD} v_A^{AD} + k_{13}^{AD} u_D^{AD} + k_{14}^{AD} v_D^{AD}$$

$$f_{yA}^{AD} = k_{21}^{AD} u_A^{AD} + k_{22}^{AD} v_A^{AD} + k_{23}^{AD} u_D^{AD} + k_{24}^{AD} v_D^{AD}$$

$$f_{xD}^{AD} = k_{31}^{AD} u_A^{AD} + k_{32}^{AD} v_A^{AD} + k_{33}^{AD} u_D^{AD} + k_{34}^{AD} v_D^{AD}$$

$$f_{yD}^{AD} = k_{41}^{AD} u_A^{AD} + k_{42}^{AD} v_A^{AD} + k_{43}^{AD} u_D^{AD} + k_{44}^{AD} v_D^{AD}$$

**Member BD:**

$$f_{xB}^{BD} = k_{11}^{BD} u_B^{BD} + k_{12}^{BD} v_B^{BD} + k_{13}^{BD} u_D^{BD} + k_{14}^{BD} v_D^{BD}$$

$$f_{yB}^{BD} = k_{21}^{BD} u_B^{BD} + k_{22}^{BD} v_B^{BD} + k_{23}^{BD} u_D^{BD} + k_{24}^{BD} v_D^{BD}$$

$$f_{xD}^{BD} = k_{31}^{BD} u_B^{BD} + k_{32}^{BD} v_B^{BD} + k_{33}^{BD} u_D^{BD} + k_{34}^{BD} v_D^{BD}$$

$$f_{yD}^{BD} = k_{41}^{BD} u_B^{BD} + k_{42}^{BD} v_B^{BD} + k_{43}^{BD} u_D^{BD} + k_{44}^{BD} v_D^{BD}$$

**Member CD:**

$$f_{xC}^{CD} = k_{11}^{CD} u_C^{CD} + k_{12}^{CD} v_C^{CD} + k_{13}^{CD} u_D^{CD} + k_{14}^{CD} v_D^{CD}$$

$$f_{yC}^{CD} = k_{21}^{CD} u_C^{CD} + k_{22}^{CD} v_C^{CD} + k_{23}^{CD} u_D^{CD} + k_{24}^{CD} v_D^{CD}$$

$$f_{xD}^{CD} = k_{31}^{CD} u_C^{CD} + k_{32}^{CD} v_C^{CD} + k_{33}^{CD} u_D^{CD} + k_{34}^{CD} v_D^{CD}$$

$$f_{yD}^{CD} = k_{41}^{CD} u_C^{CD} + k_{42}^{CD} v_C^{CD} + k_{43}^{CD} u_D^{CD} + k_{44}^{CD} v_D^{CD}$$

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## Substituting into Equil. Eqs.

**Member AD:**

$$f_{xD}^{AD} = k_{31}^{AD} u_A^{AD} + k_{32}^{AD} v_A^{AD} + k_{33}^{AD} u_D^{AD} + k_{34}^{AD} v_D^{AD}$$

**Member BD:**

$$f_{xD}^{BD} = k_{31}^{BD} u_B^{BD} + k_{32}^{BD} v_B^{BD} + k_{33}^{BD} u_D^{BD} + k_{34}^{BD} v_D^{BD}$$

**Member CD:**

$$f_{xD}^{CD} = k_{31}^{CD} u_C^{CD} + k_{32}^{CD} v_C^{CD} + k_{33}^{CD} u_D^{CD} + k_{34}^{CD} v_D^{CD}$$

**x-d.o.f.  $u_D$  :**  $\sum F_x = 0$

$$40 = f_{xD}^{AD} + f_{xD}^{BD} + f_{xD}^{CD}$$

$u_D$  :

$$\sum F_x = 0$$

$$40 = \left( k_{31}^{AD} u_A^{AD} + k_{32}^{AD} v_A^{AD} + k_{33}^{AD} u_D^{AD} + k_{34}^{AD} v_D^{AD} \right) +$$

$$\left( k_{31}^{BD} u_B^{BD} + k_{32}^{BD} v_B^{BD} + k_{33}^{BD} u_D^{BD} + k_{34}^{BD} v_D^{BD} \right) +$$

$$\left( k_{31}^{CD} u_C^{CD} + k_{32}^{CD} v_C^{CD} + k_{33}^{CD} u_D^{CD} + k_{34}^{CD} v_D^{CD} \right)$$



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## Substituting into Equil. Eqs. (cont.)

**Member AD:**  
 $f_{yD}^{AD} = k_{41}^{AD} u_A^{AD} + k_{42}^{AD} v_A^{AD} + k_{43}^{AD} u_D^{AD} + k_{44}^{AD} v_D^{AD}$

**Member BD:**  
 $f_{yD}^{BD} = k_{41}^{BD} u_B^{BD} + k_{42}^{BD} v_B^{BD} + k_{43}^{BD} u_D^{BD} + k_{44}^{BD} v_D^{BD}$

**Member CD:**  
 $f_{yD}^{CD} = k_{41}^{CD} u_C^{CD} + k_{42}^{CD} v_C^{CD} + k_{43}^{CD} u_D^{CD} + k_{44}^{CD} v_D^{CD}$

**y-d.o.f.  $v_D$ :**  $\sum F_y = 0$

$0 = f_{yD}^{AD} + f_{yD}^{BD} + f_{yD}^{CD}$

$$v_D: \sum F_y = 0$$

$$0 = \left( k_{41}^{AD} u_A^{AD} + k_{42}^{AD} v_A^{AD} + k_{43}^{AD} u_D^{AD} + k_{44}^{AD} v_D^{AD} \right) +$$

$$\left( k_{41}^{BD} u_B^{BD} + k_{42}^{BD} v_B^{BD} + k_{43}^{BD} u_D^{BD} + k_{44}^{BD} v_D^{BD} \right) +$$

$$\left( k_{41}^{CD} u_C^{CD} + k_{42}^{CD} v_C^{CD} + k_{43}^{CD} u_D^{CD} + k_{44}^{CD} v_D^{CD} \right)$$

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## So, where are we at?

- ❖ We have two equilibrium equations (1 per d.o.f.) in terms of a lot of displacements:

$$u_D: 40 = \left( k_{31}^{AD} u_A^{AD} + k_{32}^{AD} v_A^{AD} + k_{33}^{AD} u_D^{AD} + k_{34}^{AD} v_D^{AD} \right) +$$

$$\left( k_{31}^{BD} u_B^{BD} + k_{32}^{BD} v_B^{BD} + k_{33}^{BD} u_D^{BD} + k_{34}^{BD} v_D^{BD} \right) +$$

$$\left( k_{31}^{CD} u_C^{CD} + k_{32}^{CD} v_C^{CD} + k_{33}^{CD} u_D^{CD} + k_{34}^{CD} v_D^{CD} \right)$$

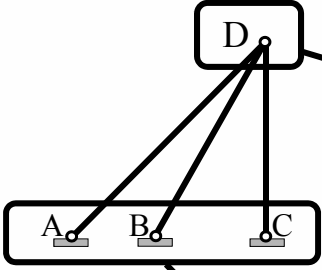
$$v_D: 0 = \left( k_{41}^{AD} u_A^{AD} + k_{42}^{AD} v_A^{AD} + k_{43}^{AD} u_D^{AD} + k_{44}^{AD} v_D^{AD} \right) +$$

$$\left( k_{41}^{BD} u_B^{BD} + k_{42}^{BD} v_B^{BD} + k_{43}^{BD} u_D^{BD} + k_{44}^{BD} v_D^{BD} \right) +$$

$$\left( k_{41}^{CD} u_C^{CD} + k_{42}^{CD} v_C^{CD} + k_{43}^{CD} u_D^{CD} + k_{44}^{CD} v_D^{CD} \right)$$

**What card haven't we played yet?**

### Compatibility Eqs. (consistent deflections) 43



**Member-to-Support**

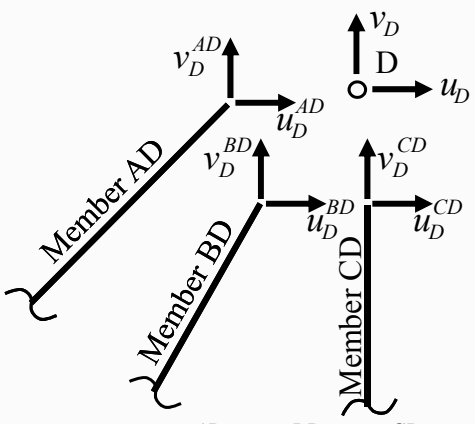
$$u_A = u_A^{AD} = 0 \quad u_C = u_C^{CD} = 0$$

$$v_A = v_A^{AD} = 0 \quad v_C = v_C^{CD} = 0$$

$$u_B = u_B^{BD} = 0$$

$$v_B = v_B^{BD} = 0$$

**Member-to-Member**



$$u_D = u_D^{AD} = u_D^{BD} = u_D^{CD}$$

$$v_D = v_D^{AD} = v_D^{BD} = v_D^{CD}$$

### Time for some serious simplifying 44

❖ Applying Compatibility to Equil. Eqs.:

All = 0

$$u_D : 40 = (k_{31}^{AD} u_A^{AD} + k_{32}^{AD} v_A^{AD} + k_{33}^{AD} u_D^{AD} + k_{34}^{AD} v_D^{AD}) +$$

$$(k_{31}^{BD} u_B^{BD} + k_{32}^{BD} v_B^{BD} + k_{33}^{BD} u_D^{BD} + k_{34}^{BD} v_D^{BD}) +$$

$$(k_{31}^{CD} u_C^{CD} + k_{32}^{CD} v_C^{CD} + k_{33}^{CD} u_D^{CD} + k_{34}^{CD} v_D^{CD})$$

$$v_D : 0 = (k_{41}^{AD} u_A^{AD} + k_{42}^{AD} v_A^{AD} + k_{43}^{AD} u_D^{AD} + k_{44}^{AD} v_D^{AD}) +$$

$$(k_{41}^{BD} u_B^{BD} + k_{42}^{BD} v_B^{BD} + k_{43}^{BD} u_D^{BD} + k_{44}^{BD} v_D^{BD}) +$$

$$(k_{41}^{CD} u_C^{CD} + k_{42}^{CD} v_C^{CD} + k_{43}^{CD} u_D^{CD} + k_{44}^{CD} v_D^{CD})$$

All =  $u_D$

All = 0

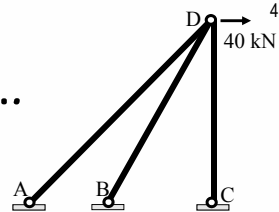
All =  $v_D$

Which simplifies to...




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## After simplifying...



$$u_D: 40 = (k_{33}^{AD} + k_{33}^{BD} + k_{33}^{CD})u_D^? + (k_{34}^{AD} + k_{34}^{BD} + k_{34}^{CD})v_D^?$$

$$v_D: 0 = (k_{43}^{AD} + k_{43}^{BD} + k_{43}^{CD})u_D^? + (k_{44}^{AD} + k_{44}^{BD} + k_{44}^{CD})v_D^?$$

Since k's are known, we have  
2 Equations and 2 Unknowns 

### Solve for Unknown Displacements

$u_D = \#$       and       $v_D = \#\#$

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## With all displacements, solve for member forces...

$u_A = u_A^{AD} = 0$   
 $v_A = v_A^{AD} = 0$

$u_B = u_B^{BD} = 0$   
 $v_B = v_B^{BD} = 0$

$u_C = u_C^{CD} = 0$   
 $v_C = v_C^{CD} = 0$

$u_D = u_D^{AD} = u_D^{BD} = u_D^{CD} = \#$   
 $v_D = v_D^{AD} = v_D^{BD} = v_D^{CD} = \#\#$

**Member AD:**

$$f_{xA}^{AD} = k_{11}^{AD}u_A^{AD} + k_{12}^{AD}v_A^{AD} + k_{13}^{AD}u_D^{AD} + k_{14}^{AD}v_D^{AD}$$

$$f_{yA}^{AD} = k_{21}^{AD}u_A^{AD} + k_{22}^{AD}v_A^{AD} + k_{23}^{AD}u_D^{AD} + k_{24}^{AD}v_D^{AD}$$

$$f_{xD}^{AD} = k_{31}^{AD}u_A^{AD} + k_{32}^{AD}v_A^{AD} + k_{33}^{AD}u_D^{AD} + k_{34}^{AD}v_D^{AD}$$

$$f_{yD}^{AD} = k_{41}^{AD}u_A^{AD} + k_{42}^{AD}v_A^{AD} + k_{43}^{AD}u_D^{AD} + k_{44}^{AD}v_D^{AD}$$

**Member BD:**

$$f_{xB}^{BD} = k_{11}^{BD}u_B^{BD} + k_{12}^{BD}v_B^{BD} + k_{13}^{BD}u_D^{BD} + k_{14}^{BD}v_D^{BD}$$

$$f_{yB}^{BD} = k_{21}^{BD}u_B^{BD} + k_{22}^{BD}v_B^{BD} + k_{23}^{BD}u_D^{BD} + k_{24}^{BD}v_D^{BD}$$

$$f_{xD}^{BD} = k_{31}^{BD}u_B^{BD} + k_{32}^{BD}v_B^{BD} + k_{33}^{BD}u_D^{BD} + k_{34}^{BD}v_D^{BD}$$

$$f_{yD}^{BD} = k_{41}^{BD}u_B^{BD} + k_{42}^{BD}v_B^{BD} + k_{43}^{BD}u_D^{BD} + k_{44}^{BD}v_D^{BD}$$

**Member CD:**

$$f_{xC}^{CD} = k_{11}^{CD}u_C^{CD} + k_{12}^{CD}v_C^{CD} + k_{13}^{CD}u_D^{CD} + k_{14}^{CD}v_D^{CD}$$

$$f_{yC}^{CD} = k_{21}^{CD}u_C^{CD} + k_{22}^{CD}v_C^{CD} + k_{23}^{CD}u_D^{CD} + k_{24}^{CD}v_D^{CD}$$

$$f_{xD}^{CD} = k_{31}^{CD}u_C^{CD} + k_{32}^{CD}v_C^{CD} + k_{33}^{CD}u_D^{CD} + k_{34}^{CD}v_D^{CD}$$

$$f_{yD}^{CD} = k_{41}^{CD}u_C^{CD} + k_{42}^{CD}v_C^{CD} + k_{43}^{CD}u_D^{CD} + k_{44}^{CD}v_D^{CD}$$



## Summary of Computer Approach

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- ❖ For each d.o.f., write an equilibrium equation:

$$F_{\text{external}} = \sum f_{\text{member}} \quad (\text{Equil. Eqs.})$$

- ❖ Re-write (translate) each member force in terms of its end displacements (Stiffness Eqs.)

$$f_{\text{member}} = \sum k_{\text{member}} \Delta_{\text{member end}}$$

- ❖ Substitute Stiffness Eqs. into above Equil. Eqs.
- ❖ Simplify Equil. Eqs. by applying member-to-member and member-to-support compatibility conditions
- ❖ Solve n Equil. Eqs. for the n unknown displacements
- ❖ Use Stiffness Eqs. to calculate member forces
- ❖ Apply Equil. Eqs. to solve for reactions

## Lot's of Questions

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- ❖ So, this is how most commercial programs such as SAP2000, RISA, STAAD, Strand7, etc. get the answer?
  - Yes! Known as "Direct Stiffness Method"
- ❖ So, all such programs will give the same answer?
  - Yes, as long as it is a static 1<sup>st</sup>-order elastic analysis.
- ❖ Wait a minute...Is this the basic analysis procedure for the "finite element method"?
  - Yes! Bit more tricky to get k's,  $\sigma$ 's, and  $\epsilon$ 's



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## Two Big Questions

- ❖ Where do those stiffness coefficients come from?
  - You mean the ones that relate member end forces to member end displacements?
  - Yeah, those k's! <More to come on this>
- ❖ What happens when we go static nonlinear or even dynamic?
  - Same basic procedure, but apply loads in increments and perform a series of analyses. Then, sum incremental results.  
< Much more to come on this! >

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## Important Points

- ❖ The only opportunity for most computer analysis software to model the actual behavior of the structure is through the member stiffness terms.
- ❖ So, to include
  - first-order effects
  - second-order effects
  - material nonlinear behavior

Must modify member stiffness!!!
- ❖ Let's review member stiffness

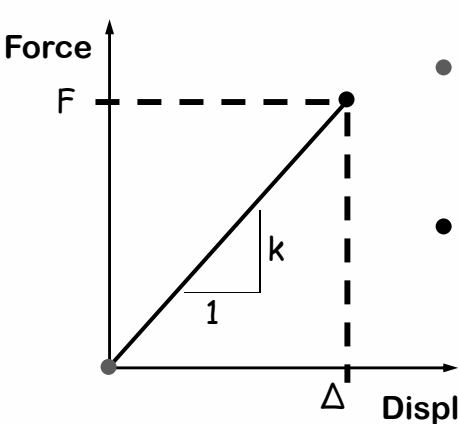



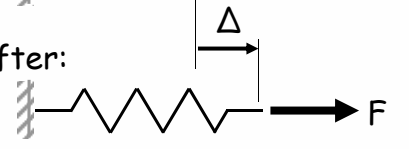
51

## Stiffness Coefficients, $k$ 's

❖ Let's start with high school physics

- Extension Spring Experiment



- Before: 
- After: 

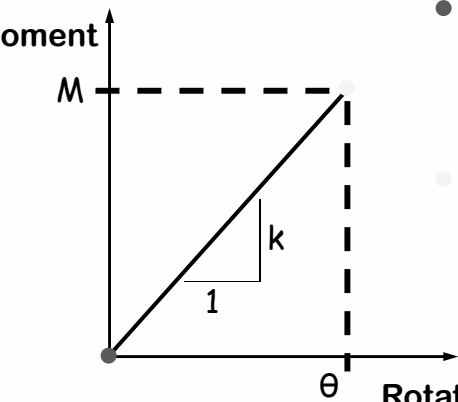
$F = k \Delta$

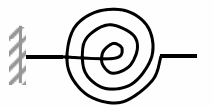
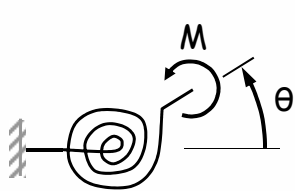
52

## Stiffness Coefficients, $k$ 's (cont.)

❖ More "advanced" high school physics lab

- Rotational Spring Experiment



- Before: 
- After: 

$M = k \theta$

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## How about real structural members?

❖ Axial force member

- Before:
- After:

❖ Stiffness k function of:

- **Geometry:** Area and Length ( $A \uparrow, k \uparrow$  &  $L \uparrow, k \downarrow$ )
- **Material:** Elastic Modulus ( $E \uparrow, k \uparrow$ )

$$F = k(A, L, E) \Delta$$

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## How about real members? (cont.)

❖ Flexural members

- Before:
- After:

- Before:
- After:

❖ Stiffness k function of:

- **Geometry:** Moment of Inertia & Length ( $I \uparrow, k \uparrow$  &  $L \uparrow, k \downarrow$ )
- **Material:** Elastic Modulus ( $E \uparrow, k \uparrow$ )

$M = k(I, L, E) \theta$

$F = k(I, L, E) \Delta$

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## Other factor impacting stiffness

- ❖ Orientation of member
  - consider axial force member:

Vertical Member                      Horizontal Member

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## Orientation of axial force member

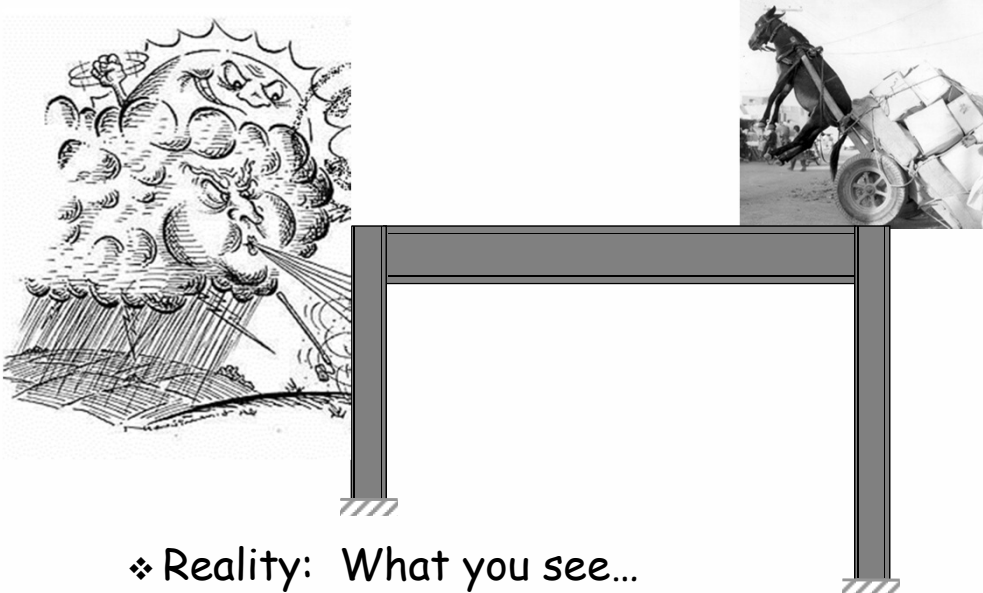
$k_h = (\cos^2 \varphi)EA/L$                        $k_v = (\sin^2 \varphi)EA/L$

$\varphi$                        $\varphi$

**Important Point:** Less vertical a member, the less stiffness to resist vertical loads.

## Summary: Three Perspectives

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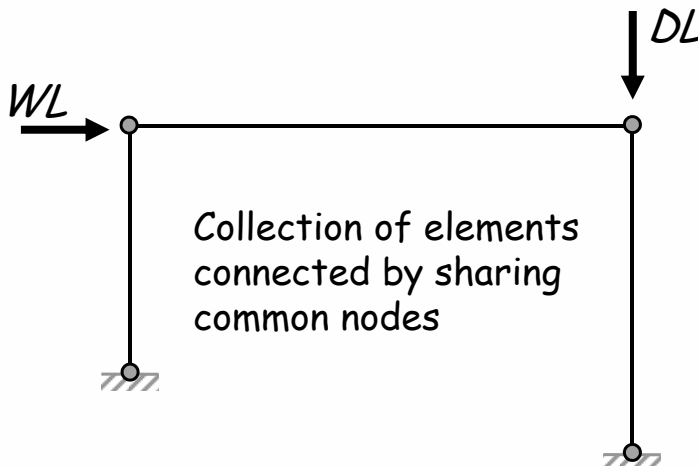


❖ Reality: What you see...

## Three Perspectives (cont.)

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❖ What you see on your computer screen:



Collection of elements  
connected by sharing  
common nodes

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## Three Perspectives

❖ What your computer actually sees:

The diagram shows a horizontal beam with a vertical column on the left and a vertical column on the right. The left column is fixed to a ground. The right column is fixed to a ground. The beam is supported by the top of both columns. A horizontal load  $WL$  is applied to the left end of the beam. A vertical load  $DL$  is applied to the top of the right column. The beam and columns are represented by a series of springs. Arrows point from these springs to a central symbol  $\Sigma k$ . Below the diagram, the text reads: "Assemblage of equivalent springs  $\{F\} = [K]\{\Delta\}$ ".

Assemblage of equivalent springs  $\{F\} = [K]\{\Delta\}$

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## Analysis Review: Key Points

- ❖ Reviewed the "Direct Stiffness Method"
  - Equilibrium  $\rightarrow$  Translator  $F(\Delta) \rightarrow$  Compatibility
- ❖ Response of structure controlled by stiffness of members (a.k.a. springs)
- ❖ First-order elastic stiffness of member function of:
  - Material Property ( $E$ )
  - Geometric Properties ( $A$ ,  $I$ ,  $L$ , and orientation)
- ❖ Time to go nonlinear...

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## Up Next...

- Session 2: February 2 –  
**Modern Methods of Structural Analysis,  
from Linear to Nonlinear – Part II**  
by R.D. Ziemian, PE, PhD
- Time to go nonlinear! Using the approach provided in Lecture 1, the participants will receive a basic introduction to nonlinear methods of structural analysis. This session will give an overview of how material nonlinear behavior and/or second-order effects can be included in modern structural analysis software.



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- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!



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Access to the quiz: Information for accessing the quiz will be emailed to you by Thursday. It will contain a link to access the quiz. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG

Quiz and Attendance records: Posted Tuesday mornings. [www.aisc.org/nightsschool](http://www.aisc.org/nightsschool) - click on Current Course Details.

Reasons for quiz:

- EEU – must take all quizzes and final to receive EEU
- CEUs/PDHS – If you watch a recorded session you must take quiz for CEUs/PDHS.
- REINFORCEMENT – Reinforce what you learned tonight. Get more out of the course.

NOTE: If you attend the live presentation, you do not have to take the quizzes to receive CEUs/PDHS.



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# Thank You

Please give us your feedback!  
*Survey at conclusion of webinar.*

