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## Course Description

### **Modern Methods of Structural Analysis, from Linear to Nonlinear – Part II**

February 2, 2015

Time to go nonlinear! Using the approach provided in Lecture 1, the participants will receive a basic introduction to nonlinear methods of structural analysis. This session will give an overview of how material nonlinear behavior and/or second-order effects can be included in modern structural analysis software.



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## Learning Objectives

- Become familiar with the basic principles of structural stability.
- Gain an understanding of nonlinear analysis methods.
- Gain an understanding of material nonlinear behavior and second order effects.
- Become familiar with how material nonlinear behavior and/or second-order effects can be included in modern structural analysis computer software.



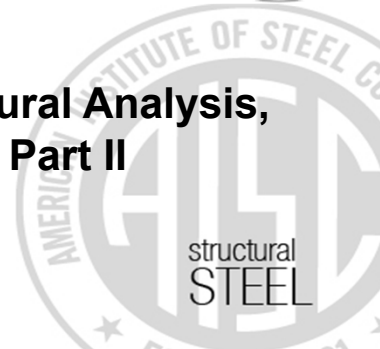
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## Stability Design of Steel Structures – Applying Modern Methods of Structural Analysis

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### Session 2 Course Introduction and Modern Methods of Structural Analysis, from Linear to Nonlinear – Part II

Ronald D. Ziemian, P.E., Ph.D.



## Course Overview

- Session Topics
  - Course Intro. and **Modern Analysis (1 & 2)**
  - Resources for Learning Stability by Analysis (3)
  - Second-Order Analysis (4)
  - Direct Analysis Method (5)
  - Low- and Medium-Rise Steel Buildings (6)
  - Advanced Application of Stability Design (7)
  - Design by Inelastic Analysis (8)
- Lectures by members of the Structural Stability Research Council (SSRC)
  - Don White and Ron Ziemian
  - Great to join AISC in this effort!

## Stability Design of Steel Structures - Applying Modern Methods of Structural Analysis

### Session 2

### Modern Methods of Structural Analysis, from Linear to Nonlinear - Part II

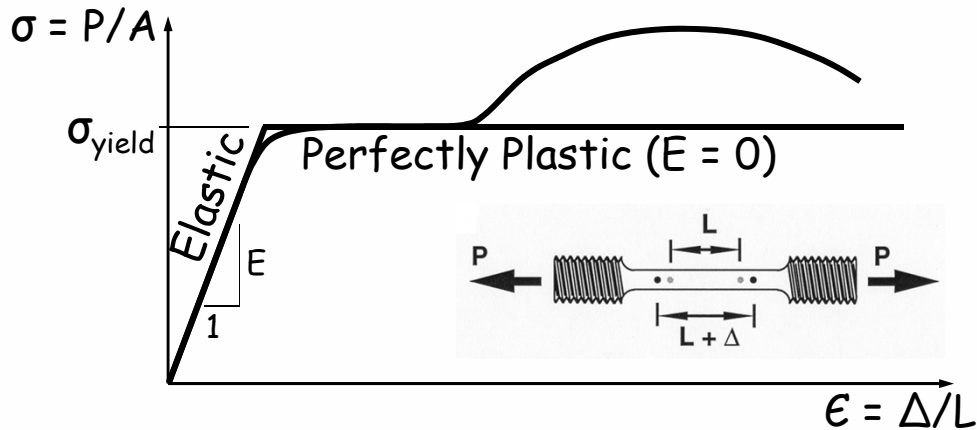


## Key Points from Session 1

- ❖ Reviewed the "Direct Stiffness Method"
  - Equilibrium  $\rightarrow$  Translator  $F(\Delta) \rightarrow$  Compatibility
- ❖ Response of structure controlled by stiffness of members (a.k.a. springs)
- ❖ First-order elastic stiffness of member function of:
  - Material Property ( $E$ )
  - Geometric Properties ( $A$ ,  $I$ ,  $L$ , and orientation)
- ❖ Time to go nonlinear...  
let's begin with material nonlinear

## Material Nonlinear (Inelastic)

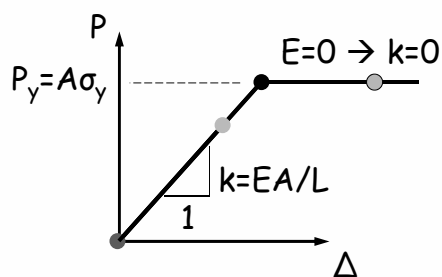
- ❖ Best place to start is with a tensile test



## Normal Stress: Structural Members 13

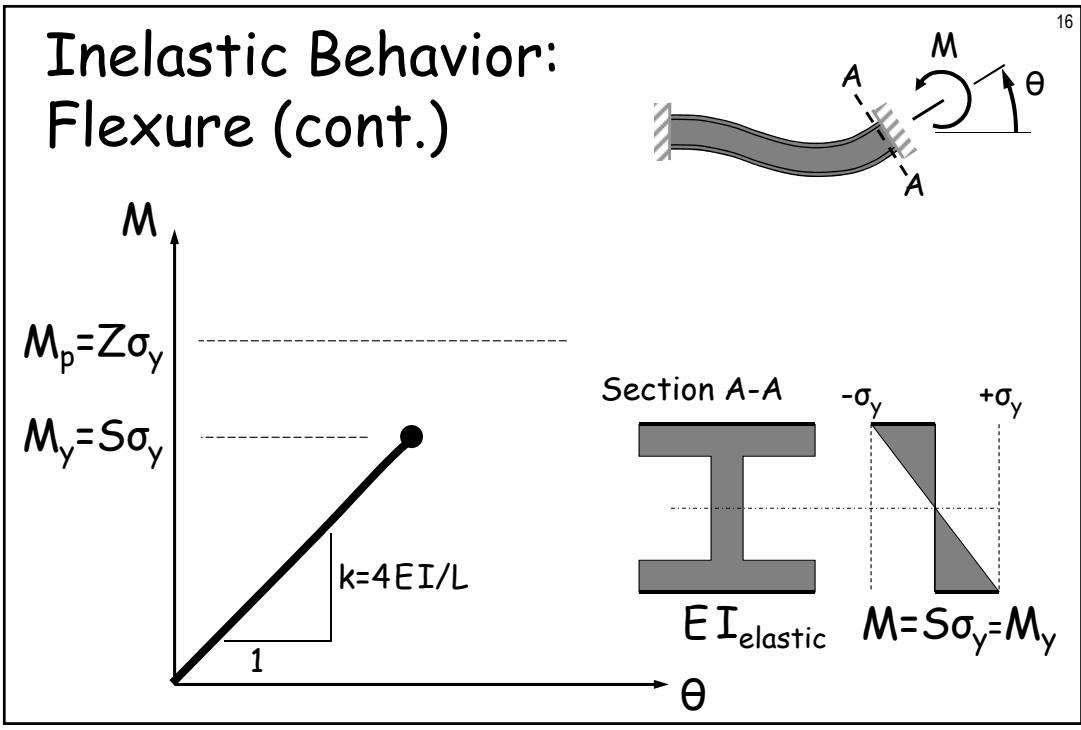
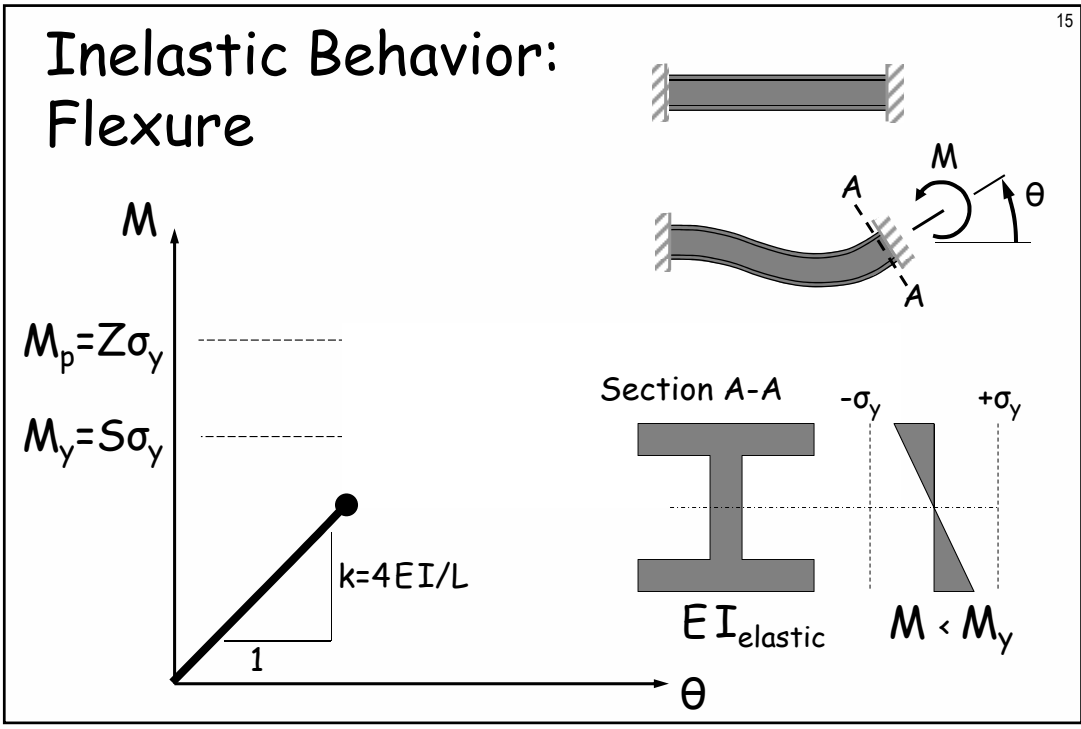
- ❖ For typical structural steel members ( $L/d > 10$ ), elastic/inelastic behavior controlled by normal stresses  $\sigma$ 's acting along the length axis of the member.
- ❖ Normal stress produced by:
  - Axial force ( $P/A$ )
  - Major and/or minor axis flexure ( $Mc/I$ )
  - Combination of above effects (i.e.  $P/A + Mc/I$ )
  - Warping (not today!)
- ❖ We will assume elastic-perfectly-plastic material (often done for steel)

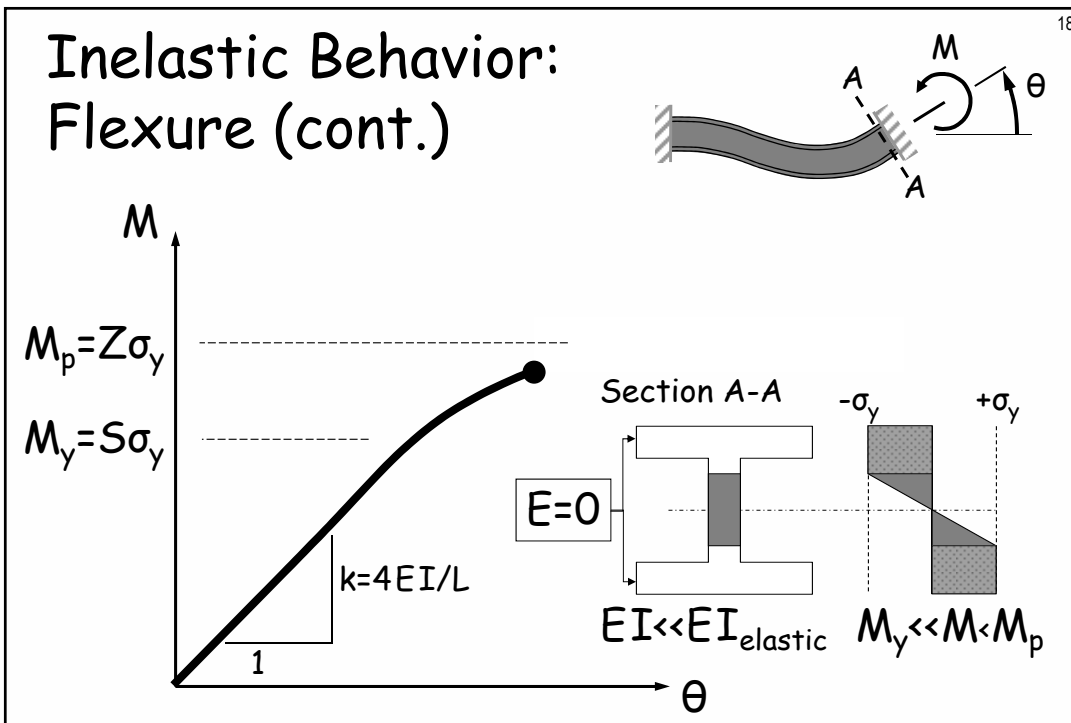
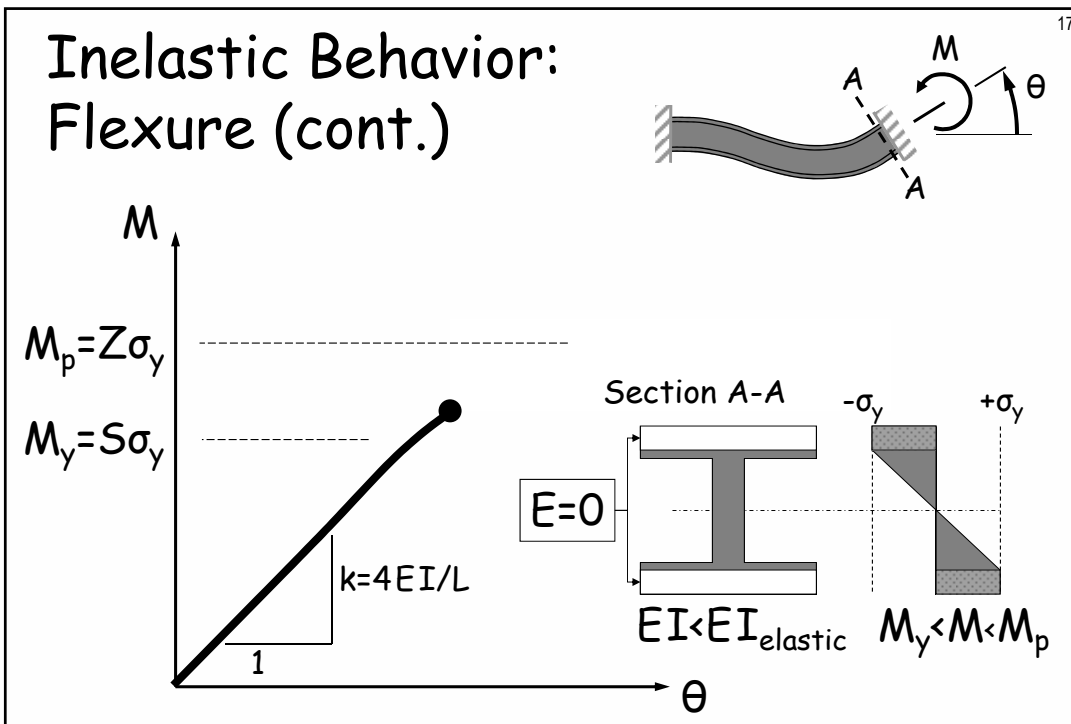
## Inelastic Behavior: Axial Force 14

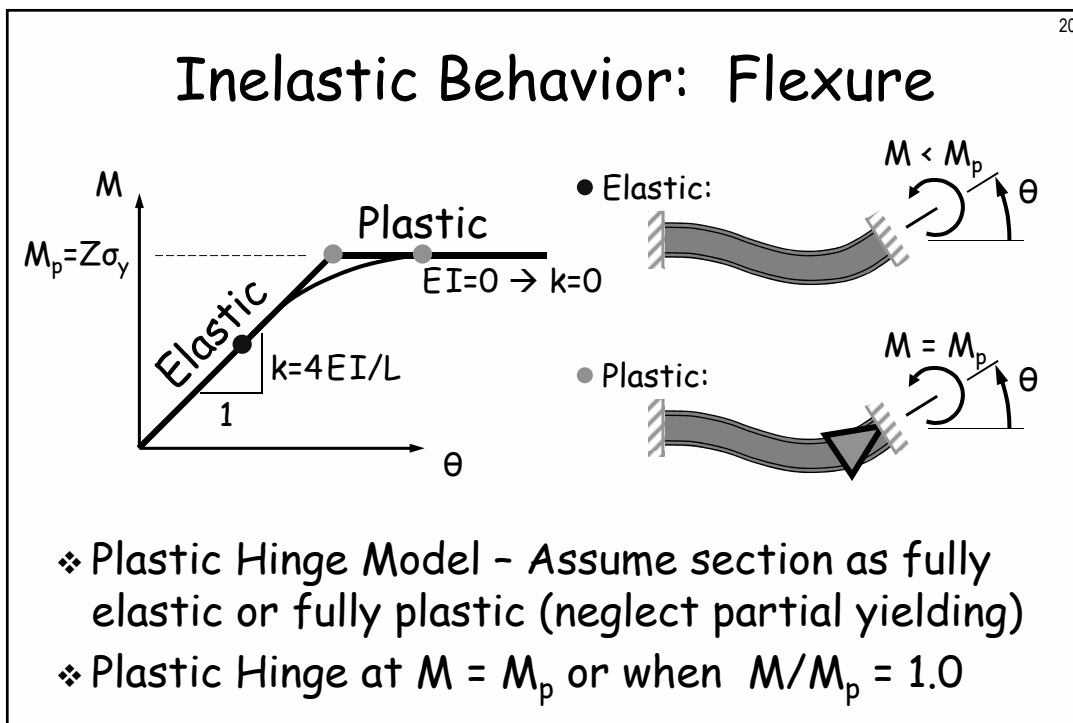
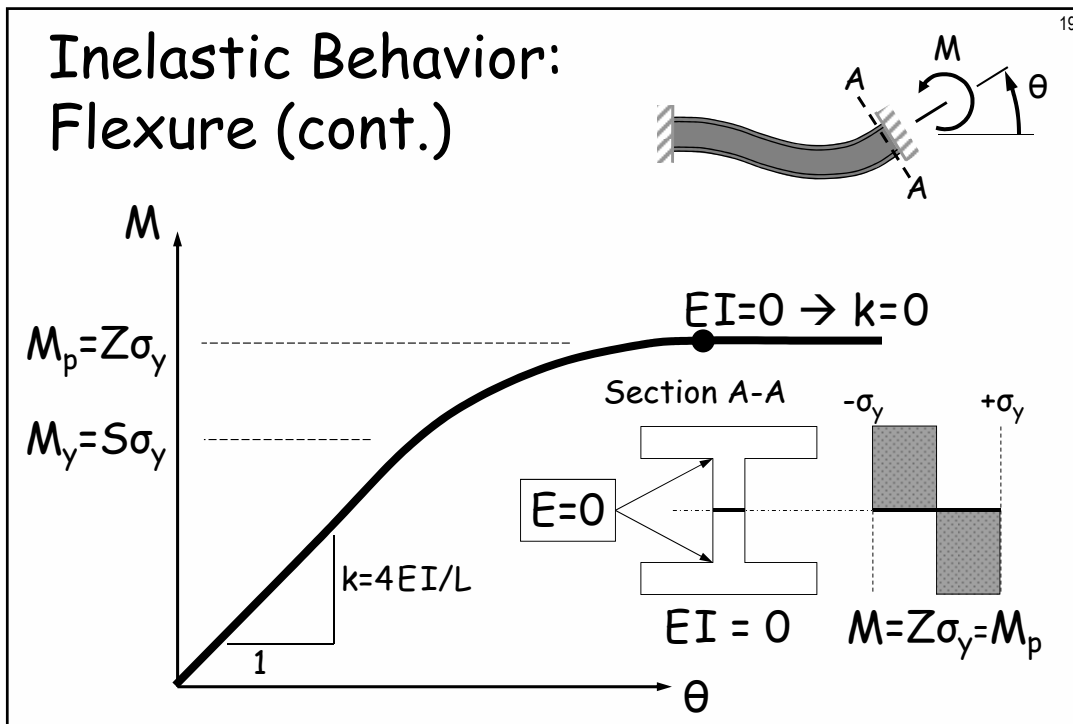


Plastic Hinge  $\triangleleft$   
at  $P = P_y$  or  
when  $P/P_y = 1.0$

- Originally:  $\sigma = P/A = 0$
- Elastic:  $\sigma < \sigma_y$
- Yield:  $\sigma = \sigma_y$
- Post-Yield:  $\sigma = \sigma_y$







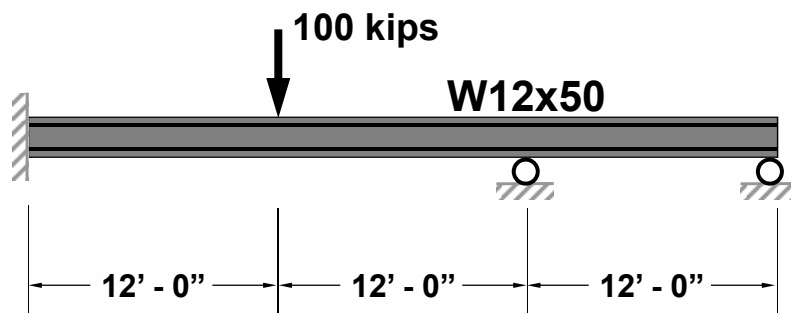
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## Types of inelastic models

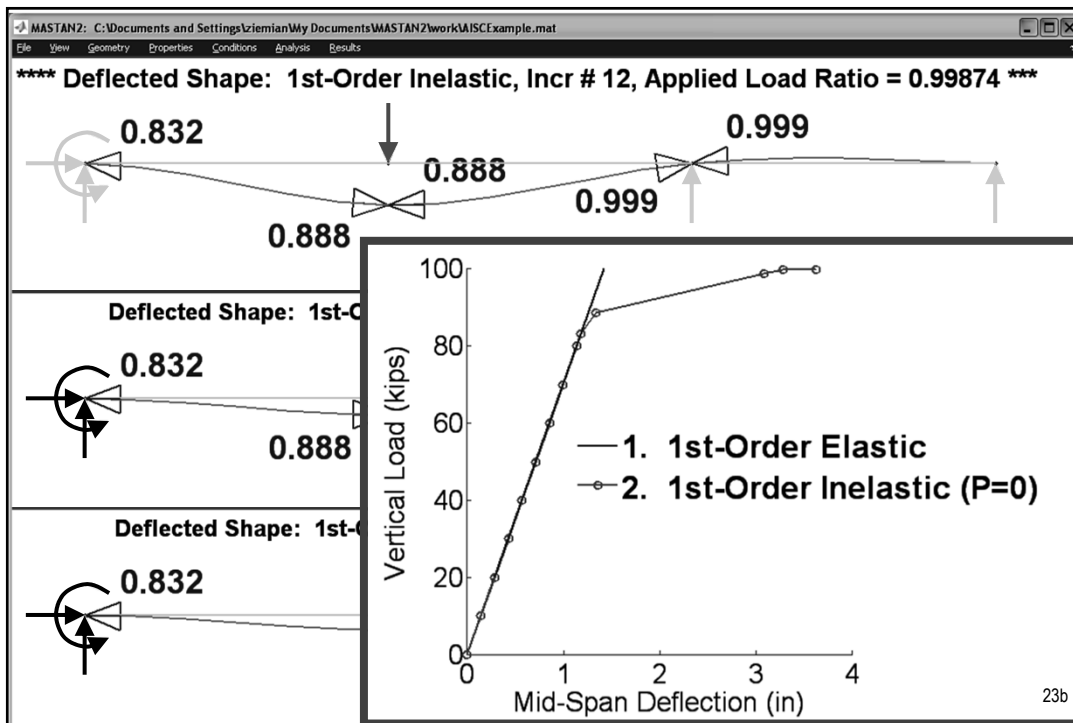
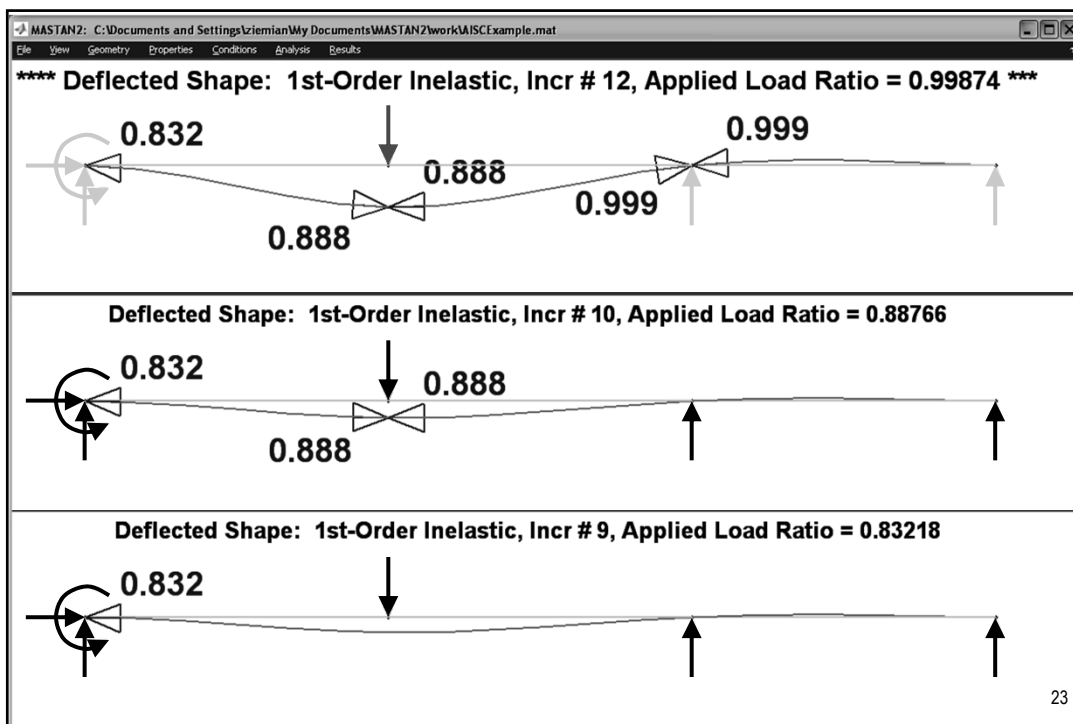
- ❖ We will employ a plastic hinge model
  - A.K.A. "Concentrated Plasticity"
  - Section is fully elastic or fully yielded
  - Plastic hinges only at element ends
  - [www.mastan2.com](http://www.mastan2.com) (software/textbook at no cost)
- ❖ Distributed plasticity (still line elements)
  - A.K.A. "Plastic Zone"
  - Captures gradual yielding through depth and along length
  - More accurate, but computationally more \$\$
- ❖ Finite element with continuum elements (\$\$\$\$)

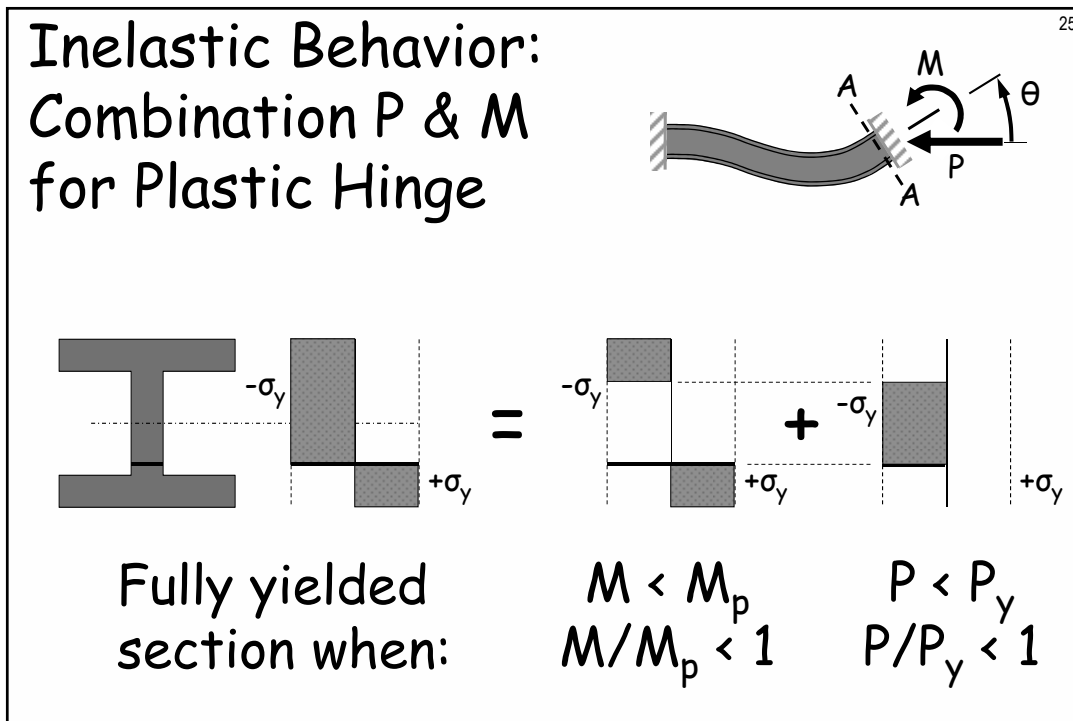
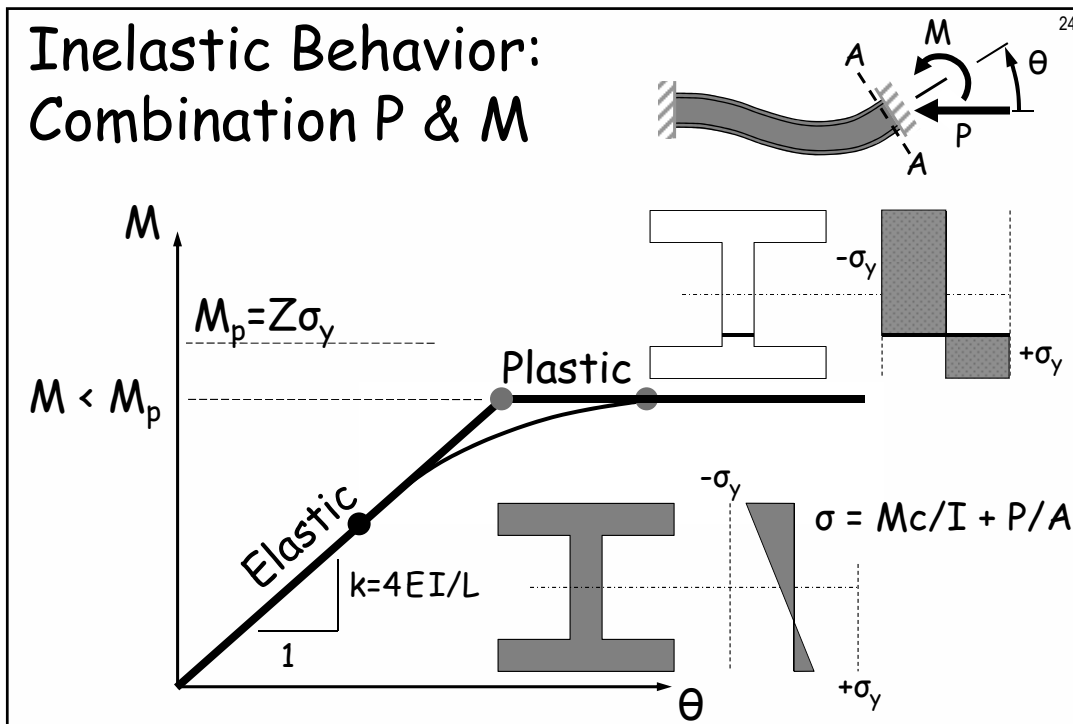
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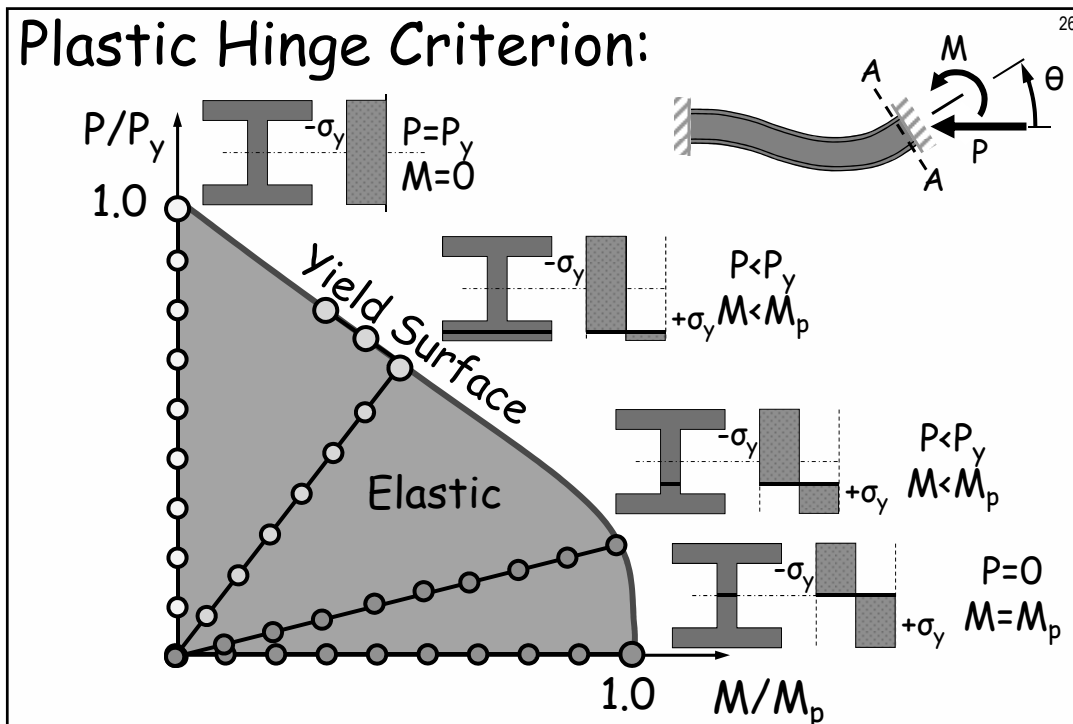
### Simple Example:



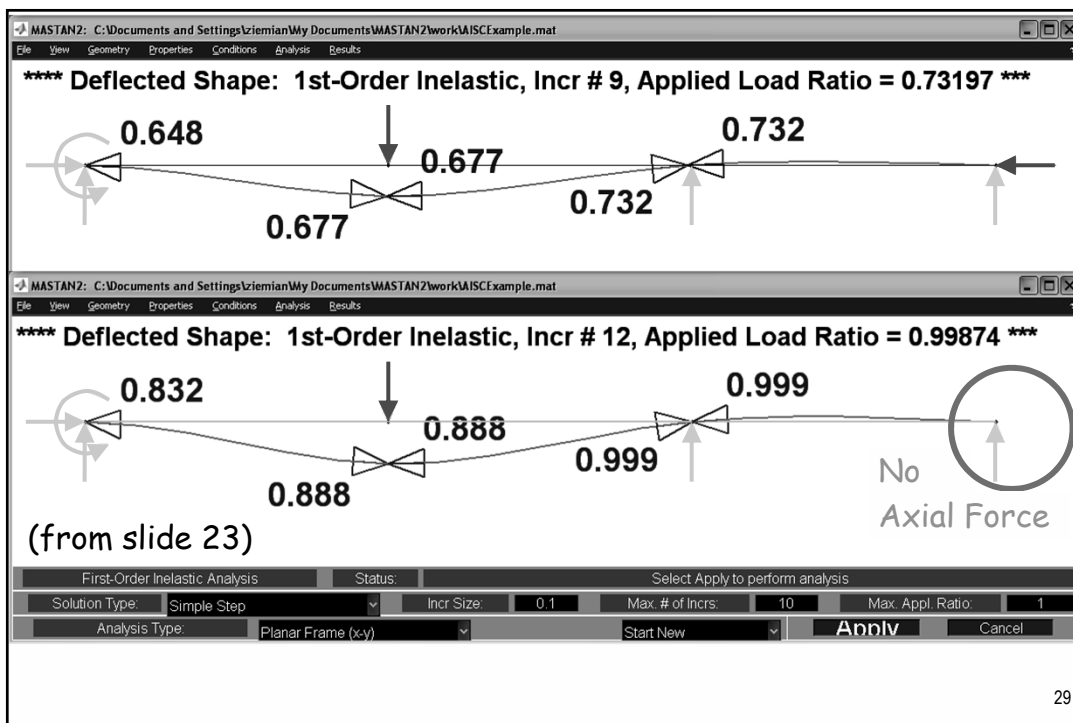
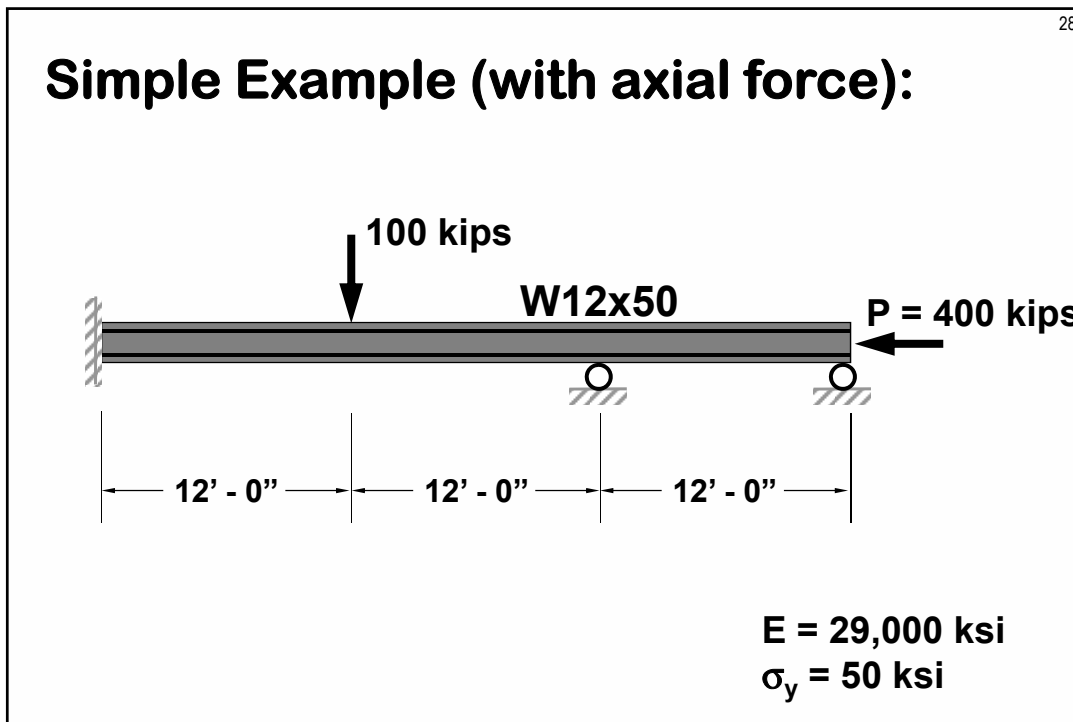
$$E = 29,000 \text{ ksi}$$
$$\sigma_y = 50 \text{ ksi}$$

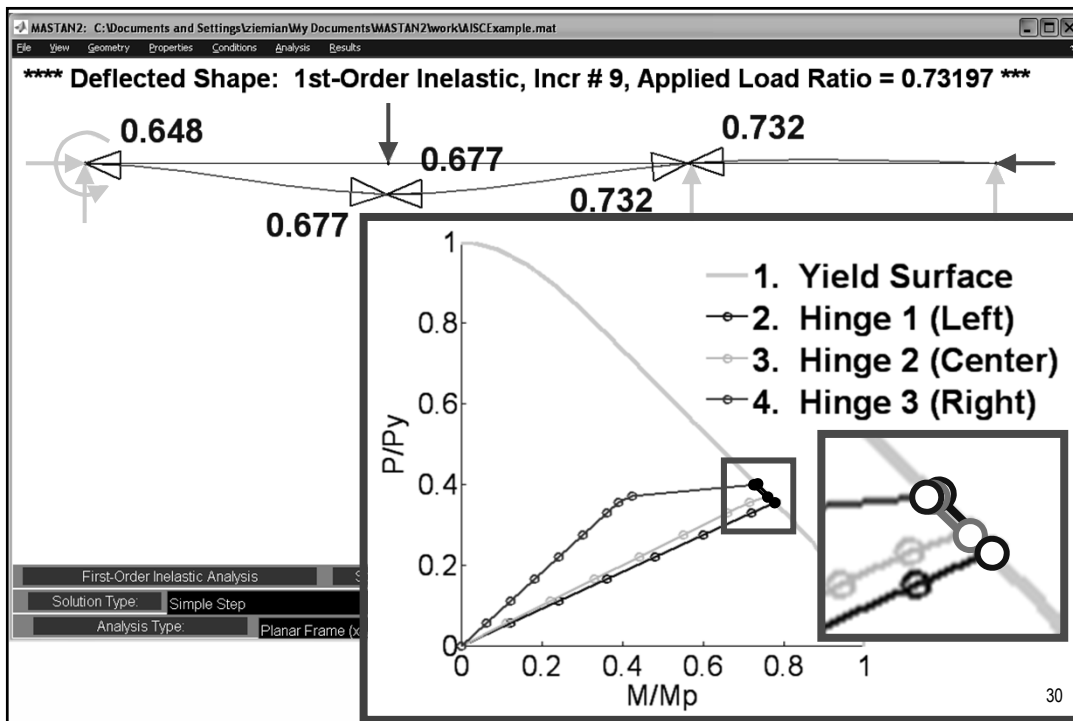
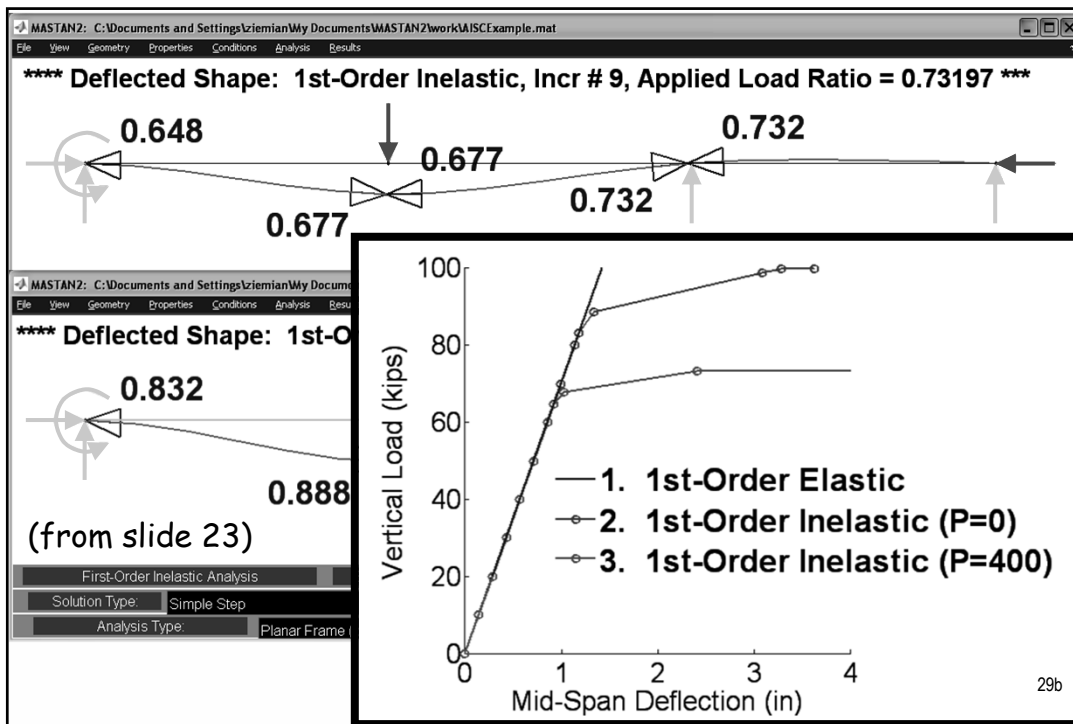






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- ## Material Nonlinear Analysis
- ❖ Employ "Direct Stiffness Method" applying loads in increments:  $[K]\{d\Delta\} = \{dF\}$
  - ❖ During the load increment, check to see if plastic hinge(s) form. If so, scale back load increment accordingly.
  - ❖ Reduce stiffness of yielded members and continue load increments
    - $k = k_{\text{elastic}} + k_{\text{plastic}}$  with  $k_{\text{plastic}} = \text{plastic reduction}$
  - ❖ Continue to accumulate results of load increments until all of load is applied or a plastic mechanism forms.





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## Second-Order Effects

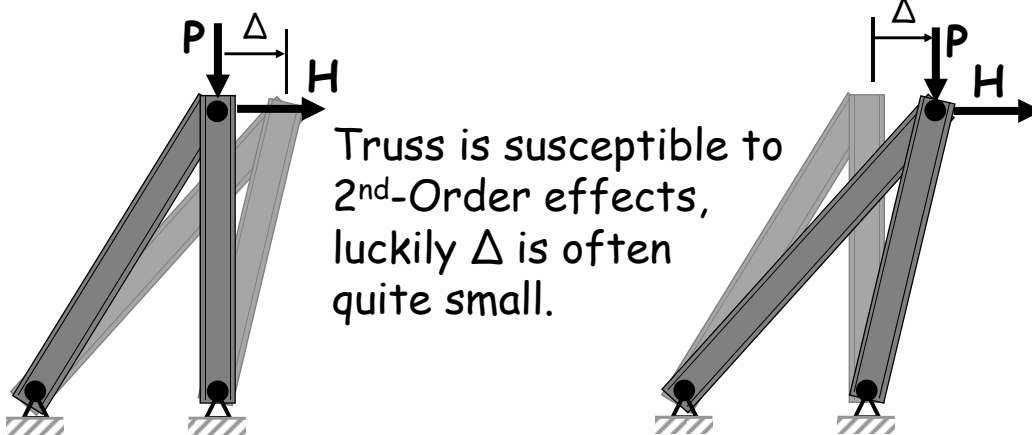
- ❖ A.K.A. "Geometric Nonlinear Behavior"
- ❖ Equilibrium Equations
  - Reality: Should be formulated on deformed shape
  - Difficulty: Deformed shape (deformations) is a function of the member forces, which are in turn a function of the deformations (Chicken 'n Egg)
  - Remedy: Perform a series of analyses with loads applied in small increments and update geometry after each load increment.

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## Equilibrium Equations

❖ Formulated on Undeformed Shape

❖ Formulated on Deformed Shape



Truss is susceptible to 2<sup>nd</sup>-Order effects, luckily  $\Delta$  is often quite small.

Different reactions and member forces.

## Equilibrium Equations

❖ Formulated on Undeformed Shape

$M = HL$

❖ Formulated on Deformed Shape

$M = HL + P\Delta$

Effective lateral stiffness is reduced!

## Focus on Lateral Stiffness

❖ Formulated on Undeformed Shape: Linear Response

• Before:

• After:

$k_{lateral} = k_{spring}$

Lateral Stiffness is slope of H-Δ response curve

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## Focus on Lateral Stiffness (cont.)

❖ Formulated on Deformed Shape: Nonlinear Response

• Before:      • After:

Effective lateral stiffness is reduced

$k_{\text{lateral}} < k_{\text{spring}}$

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## Focus on Lateral Stiffness (cont.)

❖ Equilibrium Formulated on Deformed Shape

Let's start by assuming  $L' \approx L$ ,

$$\Sigma M_o = 0 \quad RL = HL + P\Delta$$

$$R = H + P\Delta/L$$

$$k_{\text{spring}}\Delta = H + P\Delta/L$$

$$H = k_{\text{spring}}\Delta - P\Delta/L$$

$$H = (k_{\text{spring}} - P/L)\Delta$$

❖ Lateral Stiffness (slope of response curve)

$H = k_{\text{lateral}}\Delta$  with  $k_{\text{lateral}} = k_{\text{spring}} - P/L$

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## Some thoughts here...

- ❖ This simple analysis becomes less "accurate" as  $\Delta/L$  becomes large (i.e.  $\Delta/L \gg 1/5$ )
  - Remedy: Perform an incremental analysis and update geometry after each load increment...hence, limit  $\Delta/L$  in each step to some small amount
  - Keep in mind serviceability limits are often something like  $\Delta/L < 1/400$
- ❖ Most importantly,  $k_{\text{lateral}} = k_{\text{spring}} - P/L$  takes on the form:

$$k_{\text{2nd-Order El.}} = k_{\text{1st-Order El.}} + k_g$$

Geometric Stiffness ←

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## Geometric Stiffness

- ❖ Effective lateral stiffness of a member:
  - decreases as a member is compressed
    - $k_g$  is negative for compressive P
    - backpacker example
  - increases when subjected to tension
    - $k_g$  is positive for tensile P
    - guitar string example
- ❖ Employing geometric stiffness approach
  - Other methods exist (i.e. stability functions)



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## How about real members? (recall...)

❖ Flexural members subjected to axial force

❖ Stiffness k function of:

- **Geometry:** Moment of Inertia & Length ( $I \uparrow, k \uparrow$  &  $L \uparrow, k \downarrow$ )
- **Material:** Elastic Modulus ( $E \uparrow, k \uparrow$ )
- **Axial Force:** Compressive ( $P \uparrow, k \downarrow$ )

$$M = k(I, L, E, P) \theta$$

$$F = k(I, L, E, P) \Delta$$

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## Closer look at stiffness terms...

❖ Flexural members subjected to axial force

$$M = k(I, L, E, P) \theta \text{ with}$$

$$k = 4EI/L - 2PL/15$$

$$F = k(I, L, E, P) \Delta \text{ with}$$

$$k = 12EI/L^3 - 6P/5L$$

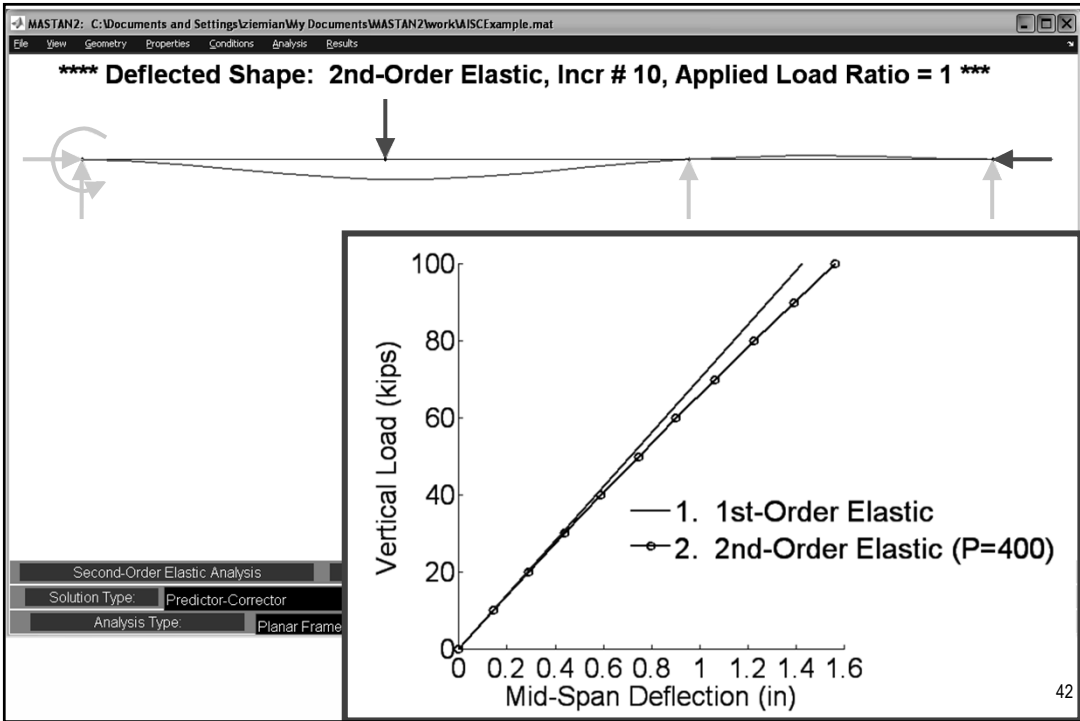
Again, basic form:

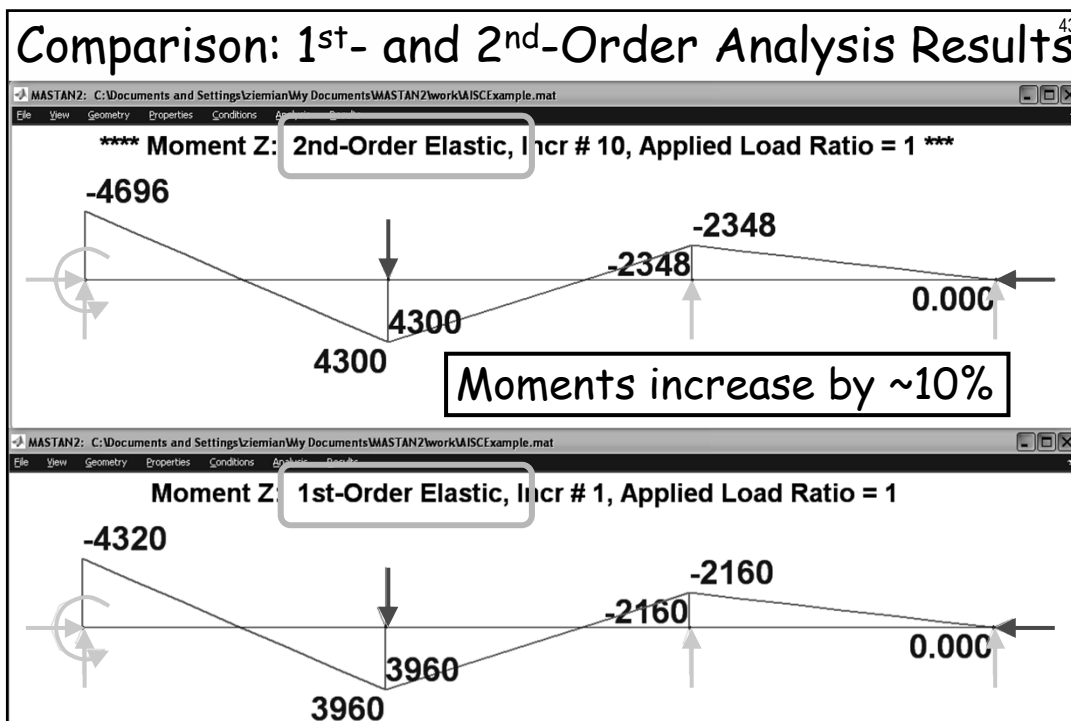
$$k_{2\text{nd-Order El.}} = k_{1\text{st-Order El.}} + k_g$$

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## Geometric Nonlinear Analysis

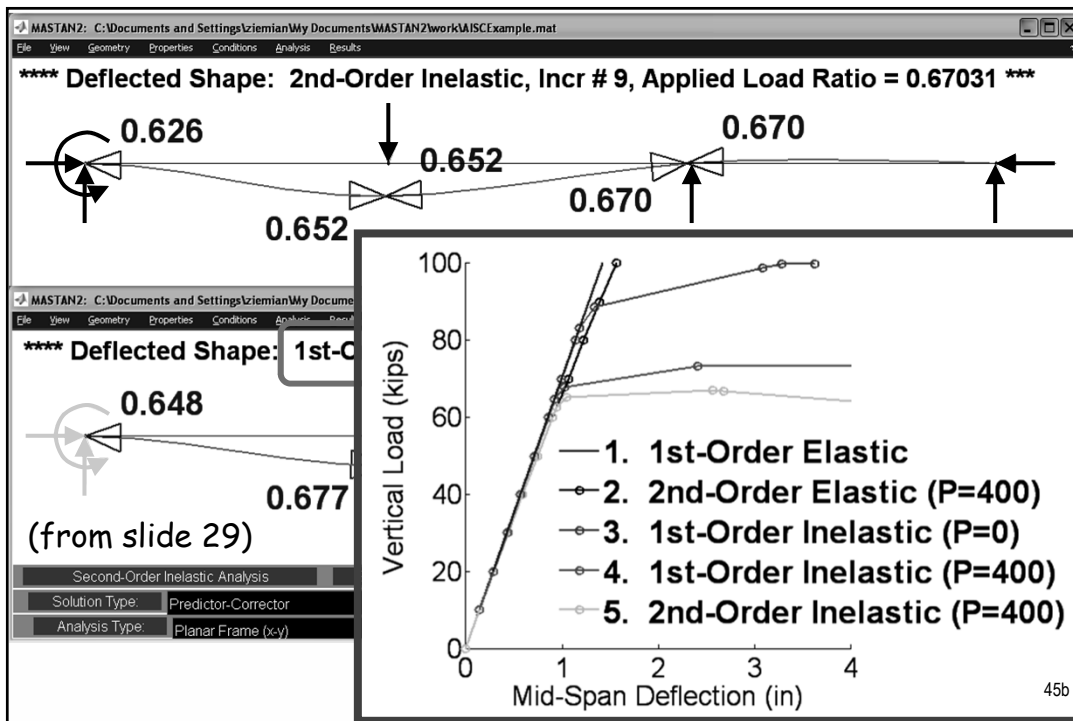
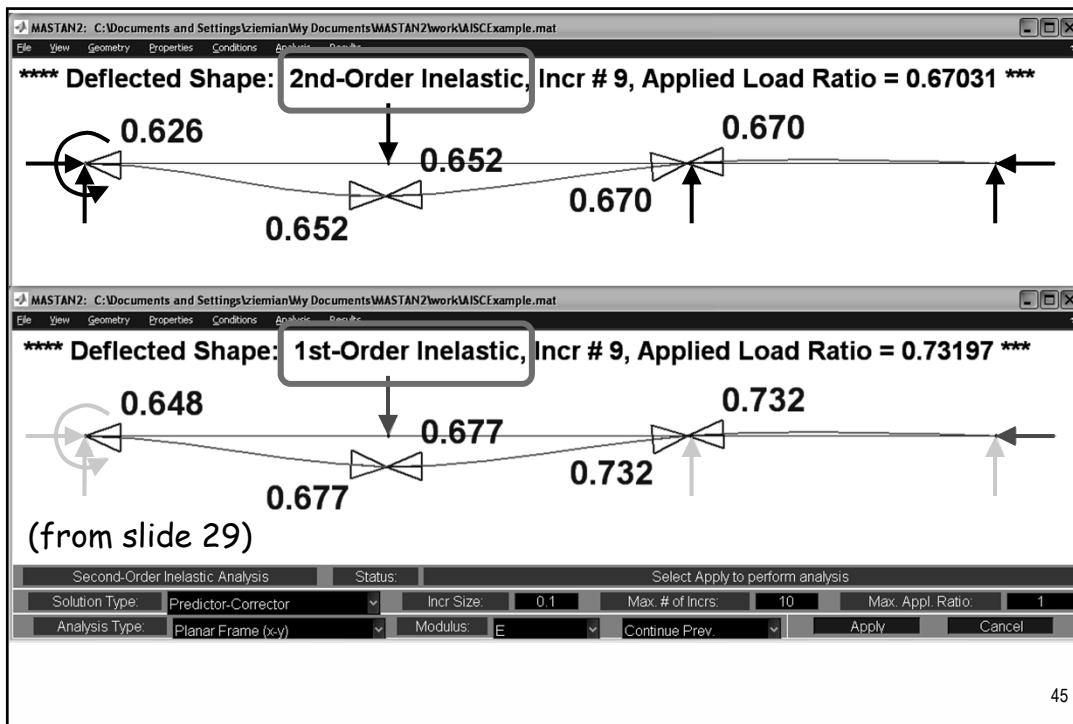
- ❖ Employ "Direct Stiffness Method" applying loads in increments: Solve Equil. Eqs.  $\{dF\} = [K]\{d\Delta\}$
- ❖ At start of increment, modify member stiffness to account for presence of member forces (such as axial force):
  - $k = k_{\text{elastic}} + k_g$  with  $k_g =$  geometric stiffness
- ❖ At end of increment, update model of structural geometry to include displacements
- ❖ Continue to accumulate results of load increments ( $\Delta_i = \Delta_{i-1} + d\Delta$  and  $f_i = f_{i-1} + df$ ) until all of load is applied or elastic instability is detected.





## 2<sup>nd</sup>-Order Inelastic Analysis 44

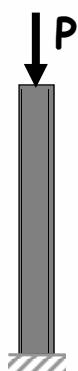
- ❖ Employ "Direct Stiffness Method" applying loads in increments: Solve Equil. Eqs.  $\{dF\} = [K]\{d\Delta\}$
- ❖ At start of increment, modify member stiffness to account for presence of member forces and any yielding:
 
$$k = k_{\text{elastic}} + k_{\text{geometric}} + k_{\text{plastic}}$$
- ❖ At end of increment, update model of structural geometry to include displacements
- ❖ Continue to accumulate results of load increments ( $\Delta_i = \Delta_{i-1} + d\Delta$  and  $f_i = f_{i-1} + df$ ) until all of load is applied or inelastic instability is detected.



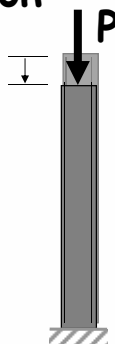
## Critical Load Analysis (Basics)

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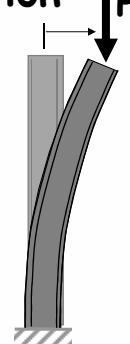
- ❖ Definition: Critical or buckling load is the load at which equilibrium may be satisfied by more than one deformed shape.



Solution #1



Solution #2



Big Q: How does computer software calculate this?

## Critical Load Analysis (Background)

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- ❖ Elastic stiffness of a member  $k = k_{el} + k_g$ 
  - $k_{el}$  is  $f(A \text{ or } I, L, \text{ and } E)$
  - $k_g$  is  $f(P, L)$ , also note directly proportional to  $P$
- ❖ Elastic stiffness of structure  $[K] = \sum k$ 
  - $[K] = [K_{el}] + [K_g]$
  - $[K_g]$  directly proportional to applied force
    - i.e. Double applied forces, hence, double internal force distribution and double  $[K_g]$
- ❖ To the computer, "buckling" will occur when our equilibrium equations  $\{F\} = [K]\{\Delta\}$  permit non-unique solutions, e.g.  $\det[K] = 0$ .

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## Example

Demonstrate computational method for calculating the elastic critical load (buckling load) for the structural system shown.

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## Example: Key Stiffness Terms

**Vertical Stiffness:**

$$P = k_{\text{vertical}} \Delta_{\text{vert}}$$

**Lateral Stiffness:**

$$H = k_{\text{lateral}} \Delta_{\text{lat}}$$

$$k_{\text{lateral}} = 12EI/L^3 - 6P/5L$$

## Example: Solution

Rigid Beam

$P$

$A, I$   
 $L, E$

1. Apply reference load, and use 1<sup>st</sup>-order elastic analysis to obtain internal force distribution.
2. Determine load factor  $\lambda$  at which system stiffness degrades to permit buckling.

$$k_{lateral} = 12EI/L^3 - 6\lambda P/5L$$

$$k_{lateral} = 0 \text{ when } \lambda P = 10EI/L^2$$

$$P_{cr} = \lambda P = 10EI/L^2 \quad (P_{theory} = 9.87EI/L^2)$$

$P_{cr}$

100 kips

W12x50

$P = 400$  kips

MASTAN2: C:\Documents and Settings\zmiemian\My Documents\MASTAN2\work\AISCExample.mat

File View Geometry Properties Conditions Analysis Results

**Deflected Shape: Elastic Critical Load, Mode # 1, Applied Load Ratio = 9.386**

$\lambda = 9.4$

MASTAN2: C:\Documents and Settings\zmiemian\My Documents\MASTAN2\work\AISCExample.mat

File View Geometry Properties Conditions Analysis Results

**Deflected Shape: Inelastic Critical Load, Mode # 1, Applied Load Ratio = 1.7367**

$\lambda = 1.7$

## Thoughts on Critical Load Analysis 52

- ❖ Computer analysis for a large system:
  - First, apply reference and perform analysis
    - Solve equilibrium eqs.  $\{F_{ref}\} = [K]\{\Delta\}$
    - With displacements solve for member forces
  - Second, assemble  $[K_{el}]$  and  $[K_g]$  based on  $\{F_{ref}\}$
  - Finally, determine load factor  $\lambda$  causing instability; computationally this means find load factor  $\lambda$  at which  $[K] = [K_{el}] + \lambda[K_g]$  becomes singular
    - Determine  $\lambda$  at which  $\det([K_{el}] + \lambda[K_g]) = 0$
    - "Eigenvalue" problem: Eigenvalues = Critical Load Factors,  $\lambda$ 's  
Eigenvectors = Buckling modes
- ❖ Accuracy increases with more elements per compression members (2 often adequate)

## Basic Introduction Complete 53

- ❖ Where do I go from here? (Learning to drive)
  - Review the slides (Read the driver's manual)
  - Acquire nonlinear software (Borrow a friend's car)
  - Work lots of examples (Go for a drive, scary at first...)
  - Apply nonlinear analysis in design (NASCAR? not quite)

### Acquire nonlinear analysis software

- Commercial programs
- Educational software (i.e. MASTAN2)



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## MASTAN2:

- Educational software
  - GUI similar to commercial programs
  - Limited # of pre- and post-processing options to reduce learning curve
  - Suite of linear and nonlinear 2D and 3D analysis routines
  - Available online with textbook at no cost
- [www.mastan2.com](http://www.mastan2.com)



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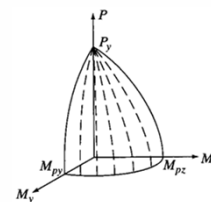
## Levels of Analysis:

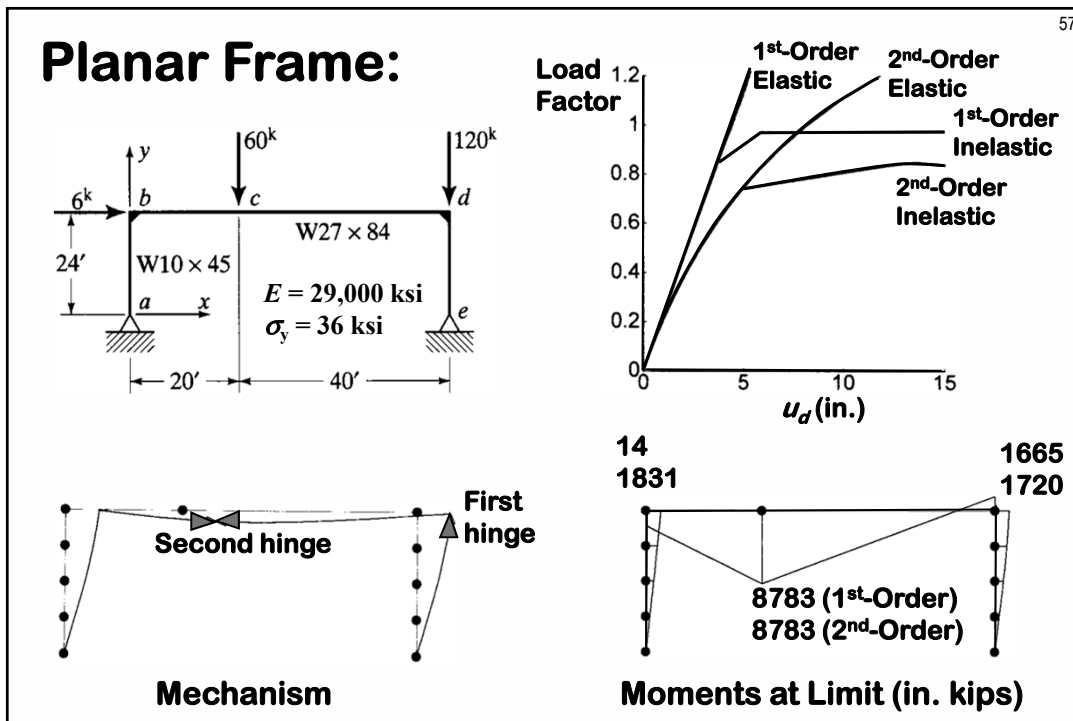
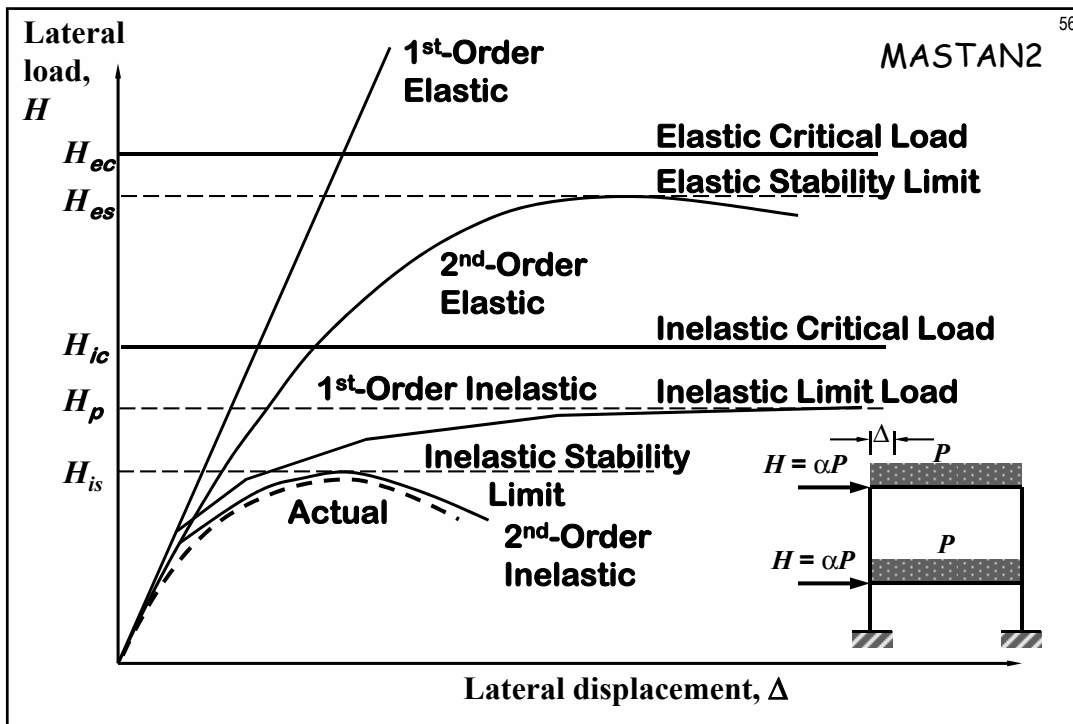
MASTAN2

- 1<sup>st</sup>-Order Elastic:  $[K_e]\{\Delta\}=\{F\}$
- 2<sup>nd</sup>-Order Elastic:  $[K_e + K_g]\{d\Delta\}=\{dF\}$
- 1<sup>st</sup>-Order Inelastic:  $[K_e + K_p]\{d\Delta\}=\{dF\}$
- 2<sup>nd</sup>-Order Inelastic:  $[K_e + K_g + K_p]\{d\Delta\}=\{dF\}$
- Critical Load:  $[K_e + \lambda K_g]\{d\Delta\}=\{0\}$

## Yield Surface:

Function of  $P$ ,  $M_{\text{major}}$ , and  $M_{\text{minor}}$





## Summary and Conclusions

- ❖ Provided an introduction to nonlinear analysis
  - Review of direct stiffness method
  - Material nonlinear analysis (Inelastic hinge)
  - Geometric nonlinear analysis (2<sup>nd</sup>-Order)
  - 2<sup>nd</sup>-Order inelastic analysis (combine above)
  - Critical load analysis ("eigenvalue analysis")
- ❖ Nonlinear...think modifying member stiffness!
- ❖ Overview and availability of MASTAN2
- ❖ Now, it's your turn to take it for a spin...

## Up Next...

- Session 3: February 9 –  
**Modules for Learning Structural Stability**  
by R.D. Ziemian, PE, PhD
- With a basic understanding of linear and nonlinear analysis now in hand, it is time to explore and gain hands-on experience in understanding factors that are known to impact structural stability. Modern computer software makes it easy to use non-linear computational analysis as a virtual lab for learning many important concepts related to designing for stability. An overview of several learning modules developed for a better understanding of topics such as elastic and inelastic flexural and lateral-torsional buckling, inelastic force redistribution, and second-order effects will be presented. Course attendees will be encouraged to download a simple analysis software package (at no charge) and work the examples and modules at the conclusion of the lecture.



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## Individual Webinar Registrants

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### CEU/PDH Certificates

Within 2 business days...

- You will receive an email on how to report attendance from: [registration@aisc.org](mailto:registration@aisc.org).
- Be on the lookout: Check your spam filter! Check your junk folder!
- Completely fill out online form. Don't forget to check the boxes next to each attendee's name!



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Within 2 business days...

- New reporting site (URL will be provided in the forthcoming email).
- Username: Same as AISC website username.
- Password: Same as AISC website password.



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## 8-Session Registrants

### CEU/PDH Certificates

One certificate will be issued at the conclusion of  
all 8 sessions.



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## 8-Session Registrants

Access to the quiz: Information for accessing the quiz will be emailed to you by Wednesday. It will contain a link to access the quiz. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG

Quiz and Attendance records: Posted Tuesday mornings. [www.aisc.org/nightschool](http://www.aisc.org/nightschool) - click on Current Course Details.

Reasons for quiz:

- EEU – must take all quizzes and final to receive EEU
- CEUs/PDHS – If you watch a recorded session you must take quiz for CEUs/PDHS.
- REINFORCEMENT – Reinforce what you learned tonight. Get more out of the course.

NOTE: If you attend the live presentation, you do not have to take the quizzes to receive CEUs/PDHS.



There's always a solution in Steel

## 8-Session Registrants

**Access to the recording:** Information for accessing the recording will be emailed to you by this Wednesday. The recording will be available for two weeks. For 8-session registrants only. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG.

**CEUs/PDHS** – If you watch a recorded session you must take AND PASS the quiz for CEUs/PDHS.



There's always a solution in Steel

There's always a solution in steel.

# Thank You

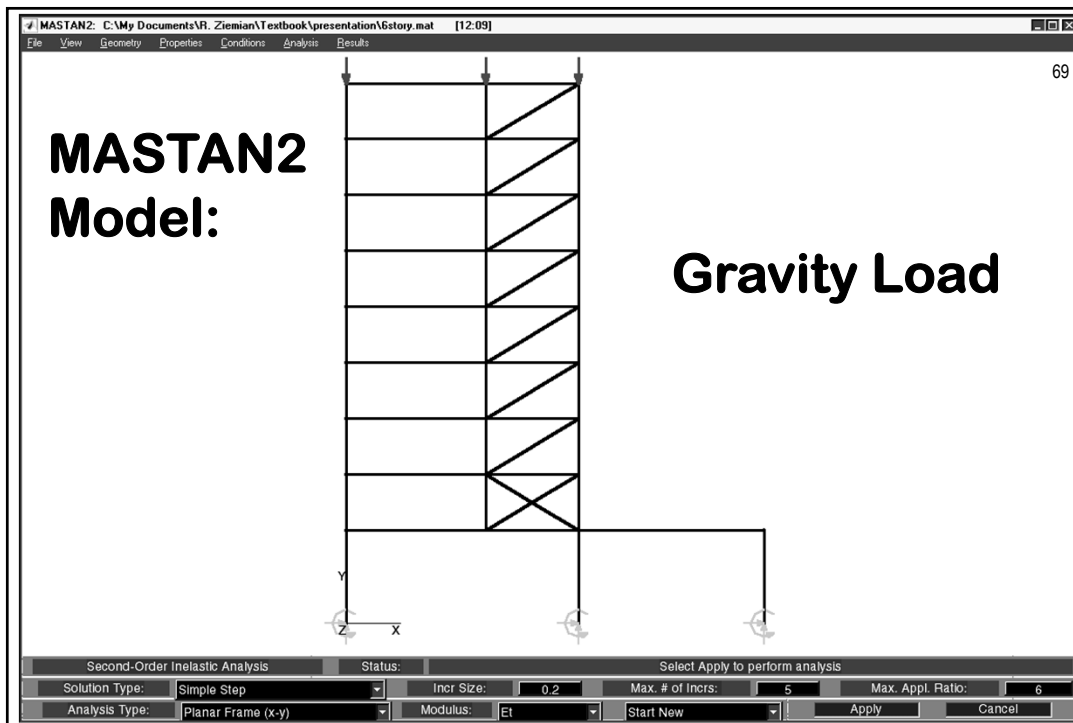
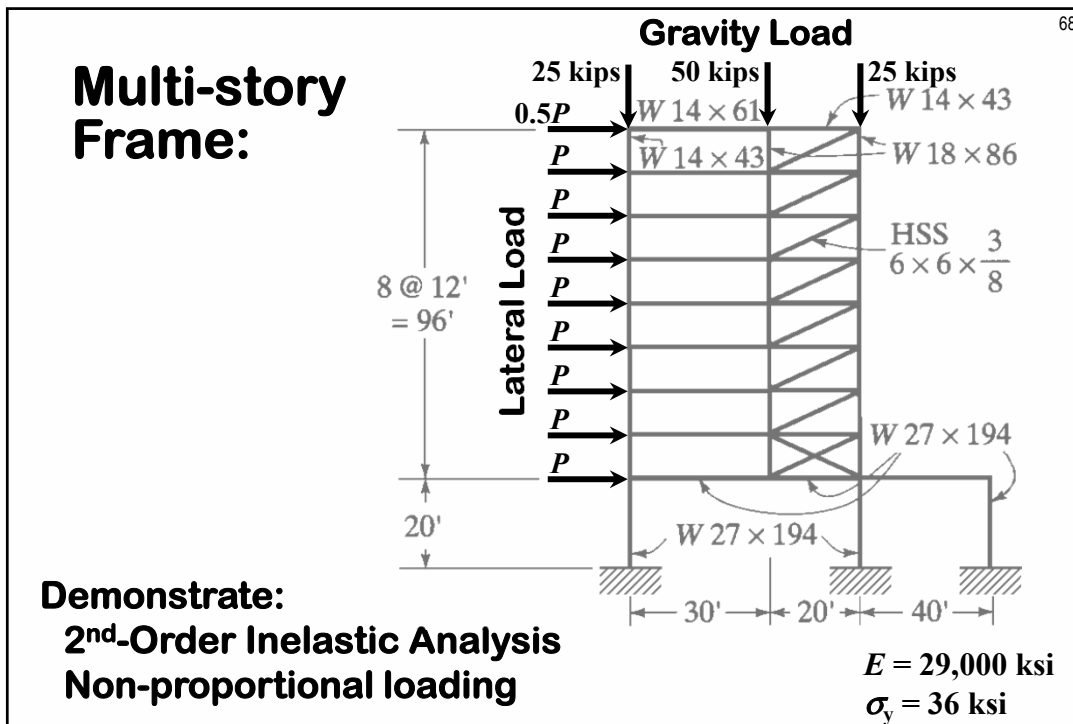
Please give us your feedback!  
*Survey at conclusion of webinar.*

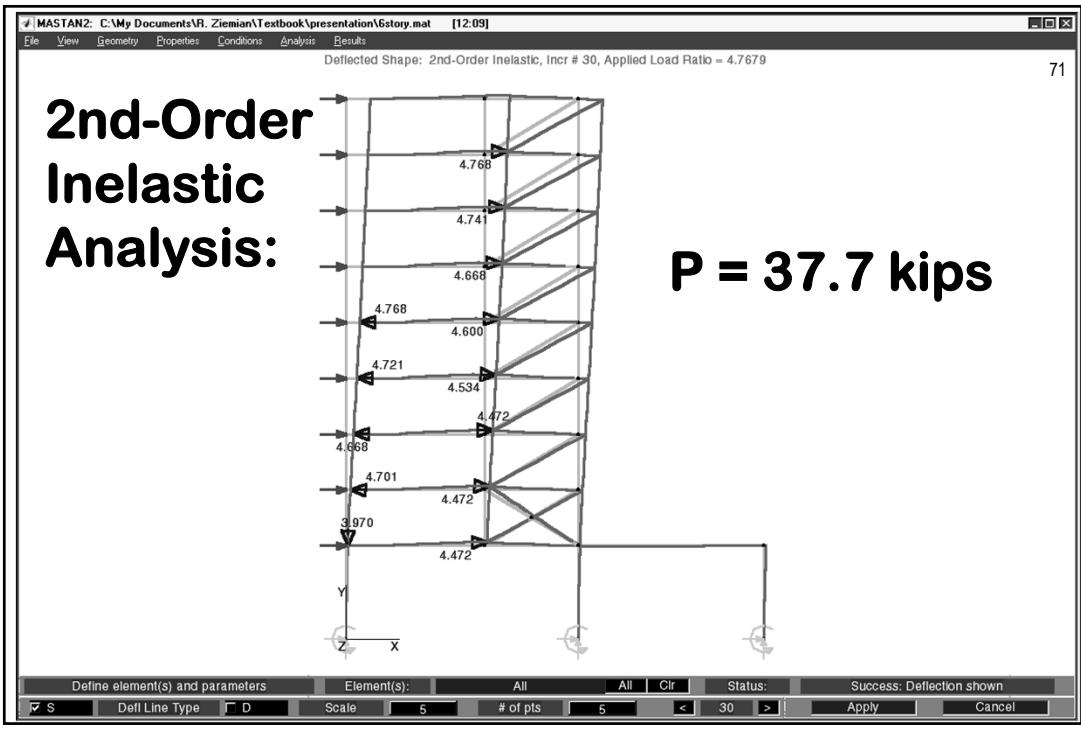
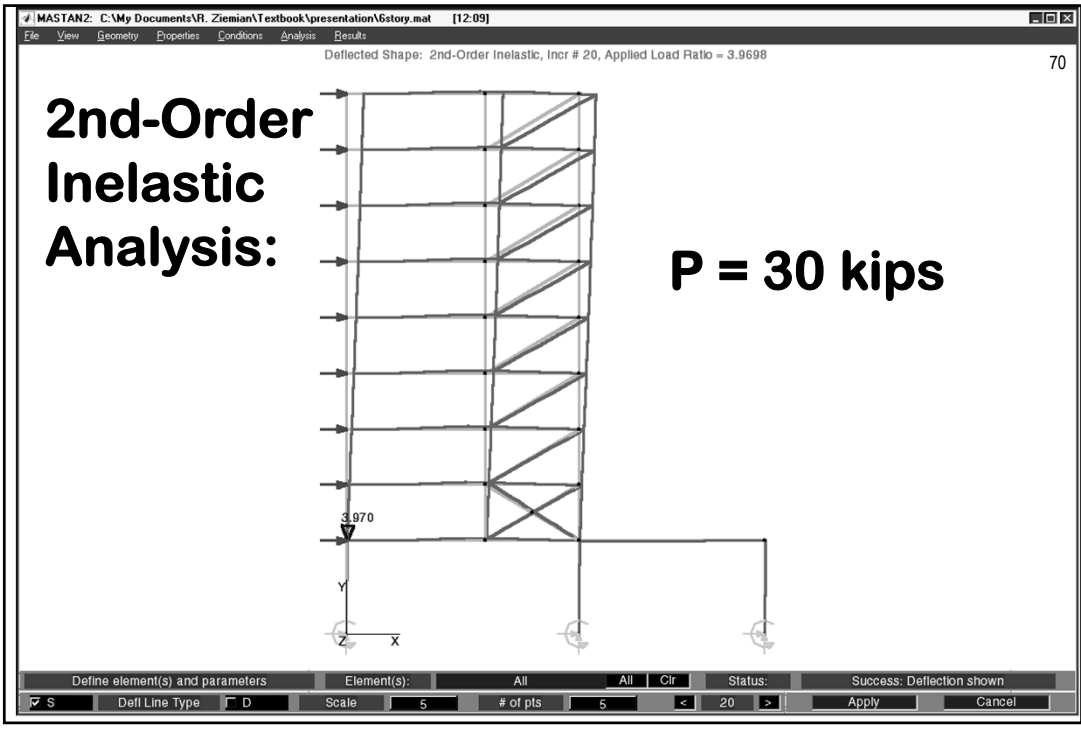


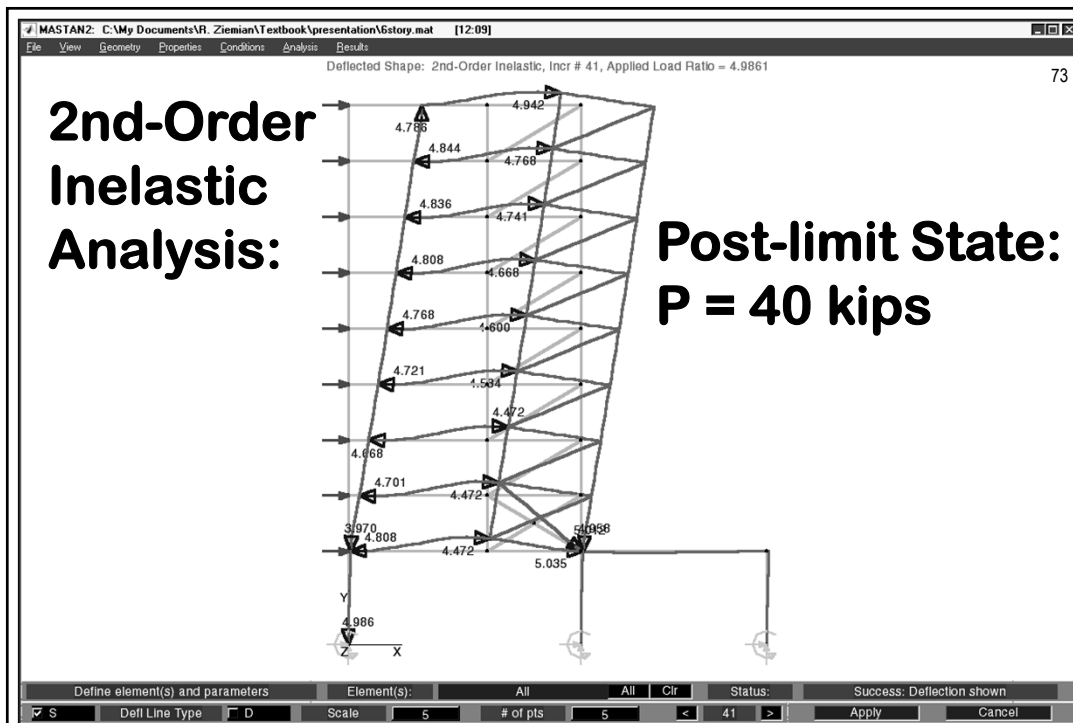
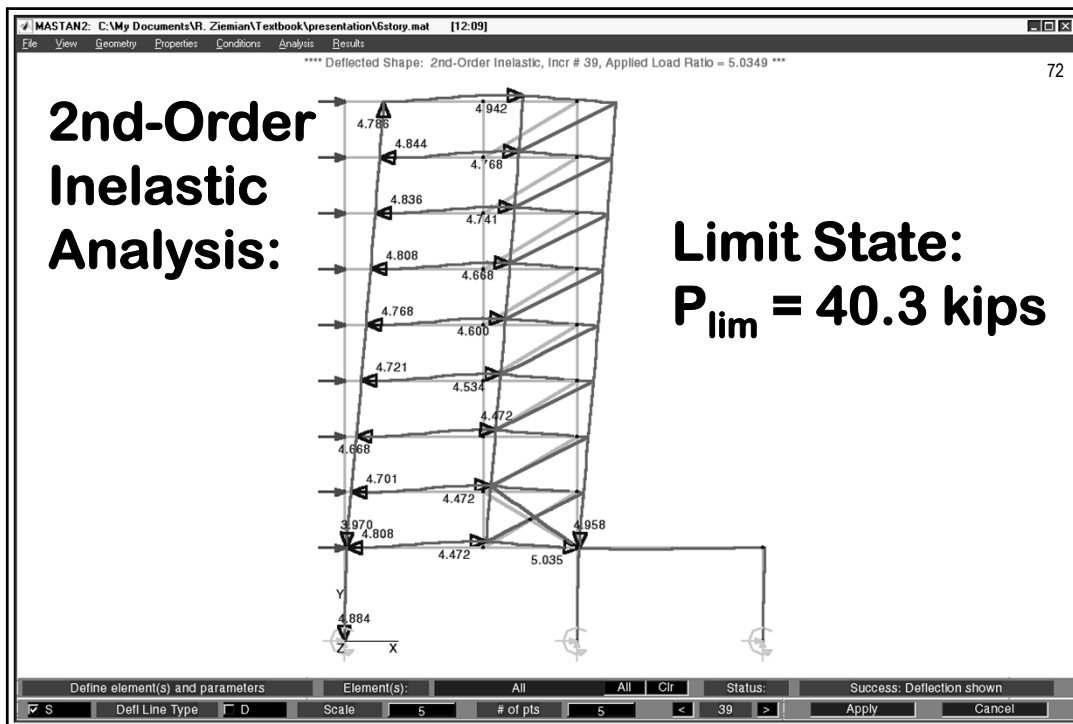
## Appendix

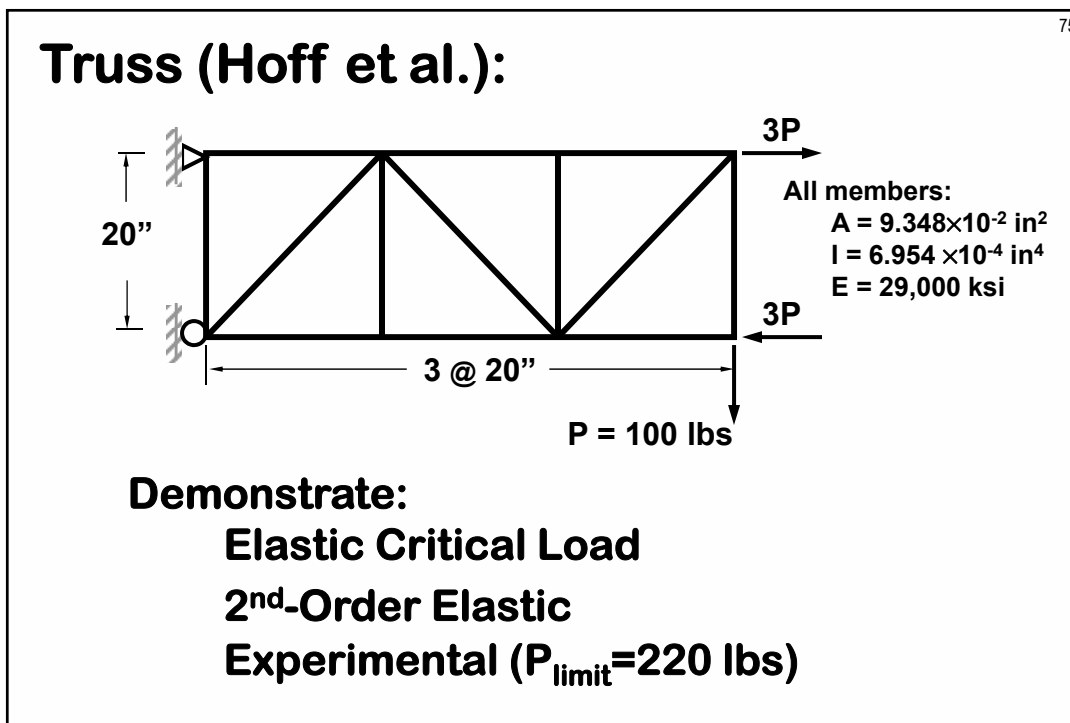
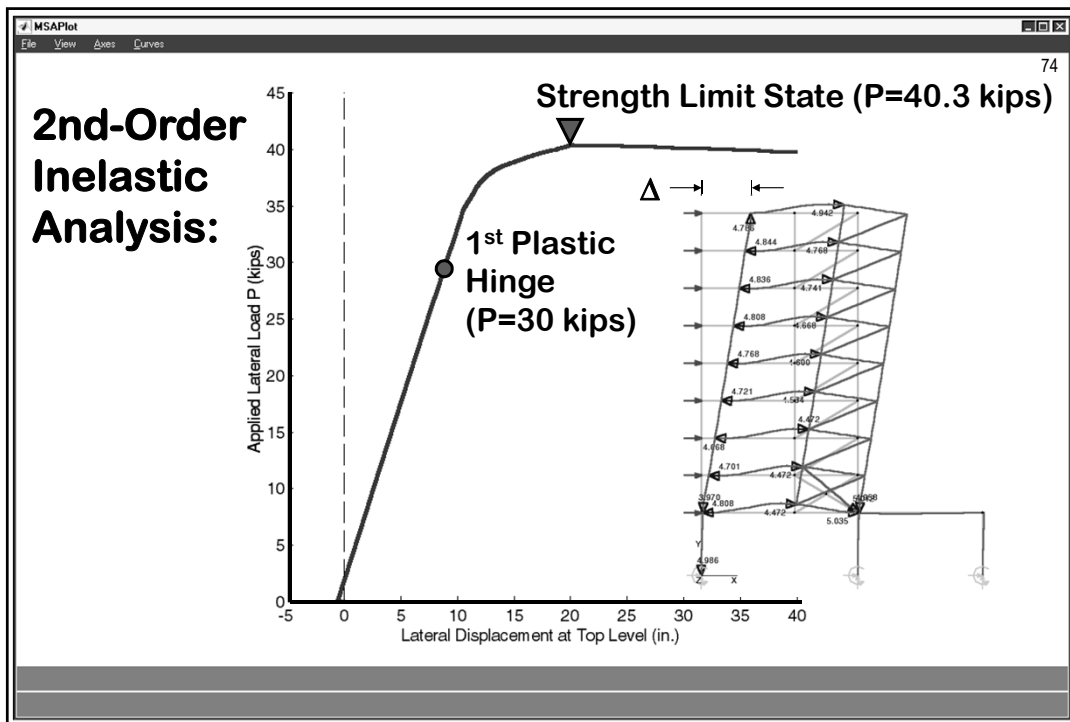
67

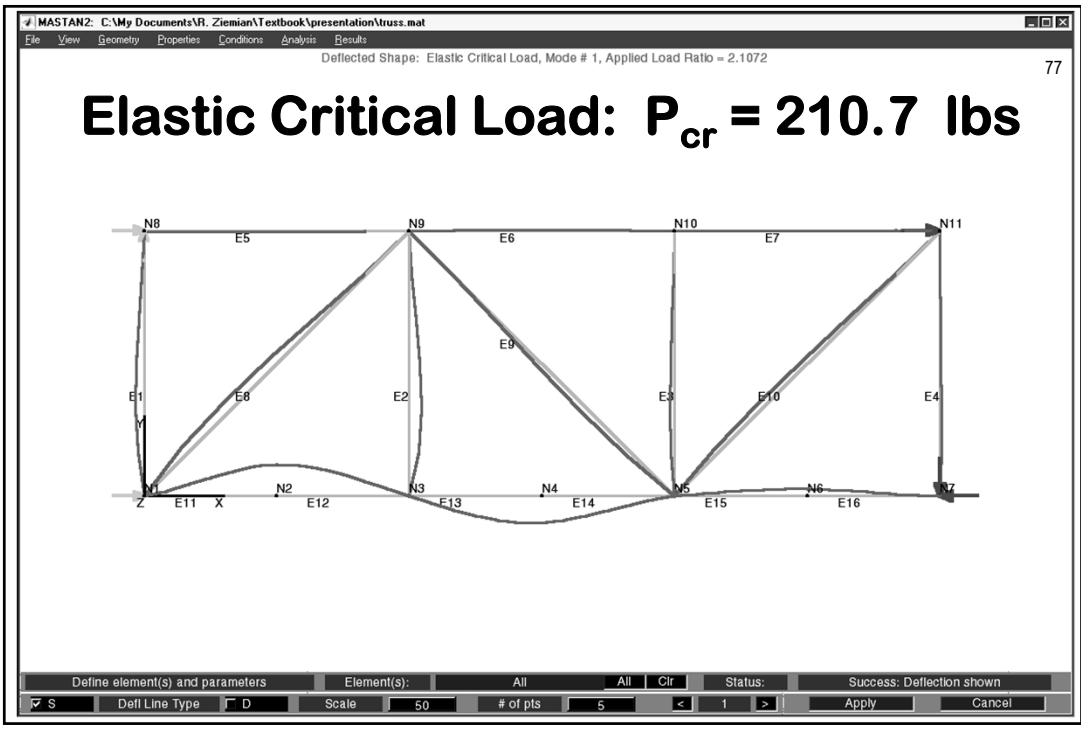
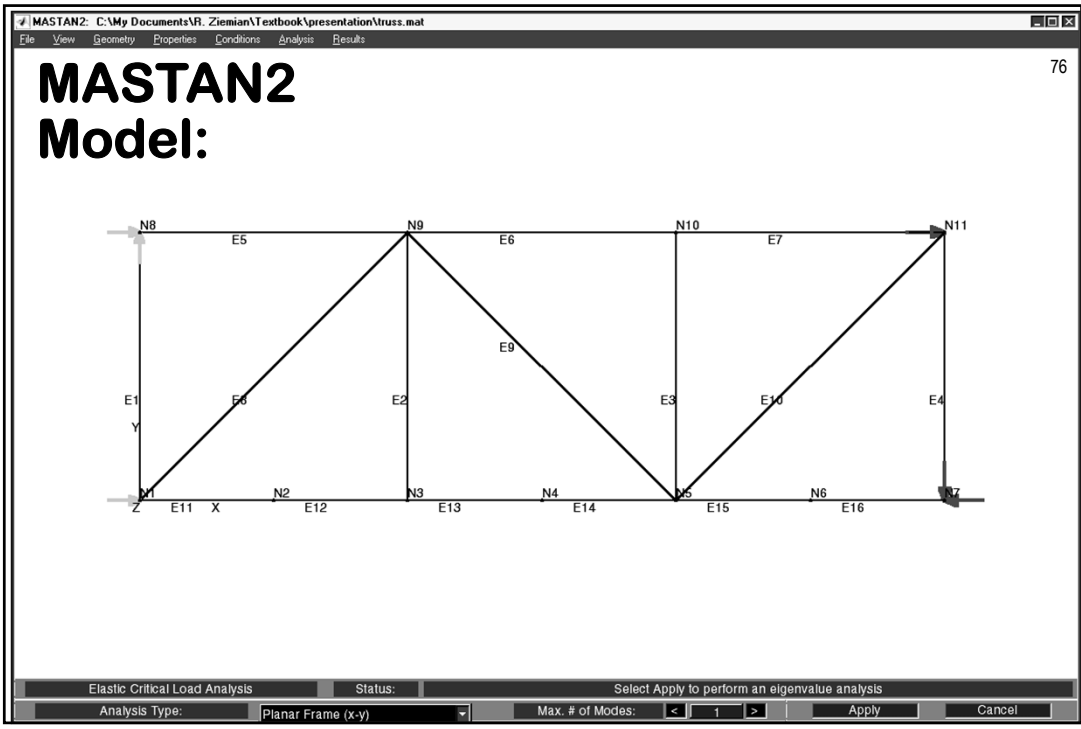
- ❖ Several examples to try out
- ❖ Solutions by MASTAN2
- ❖ Need a reference text with many examples? see Matrix Structural Analysis, 2<sup>nd</sup> Ed., by McGuire, Gallagher, and Ziemian (Wiley, 2000)
- ❖ See tutorial that comes with MASTAN2
- ❖ OK, time to jump in and start driving...

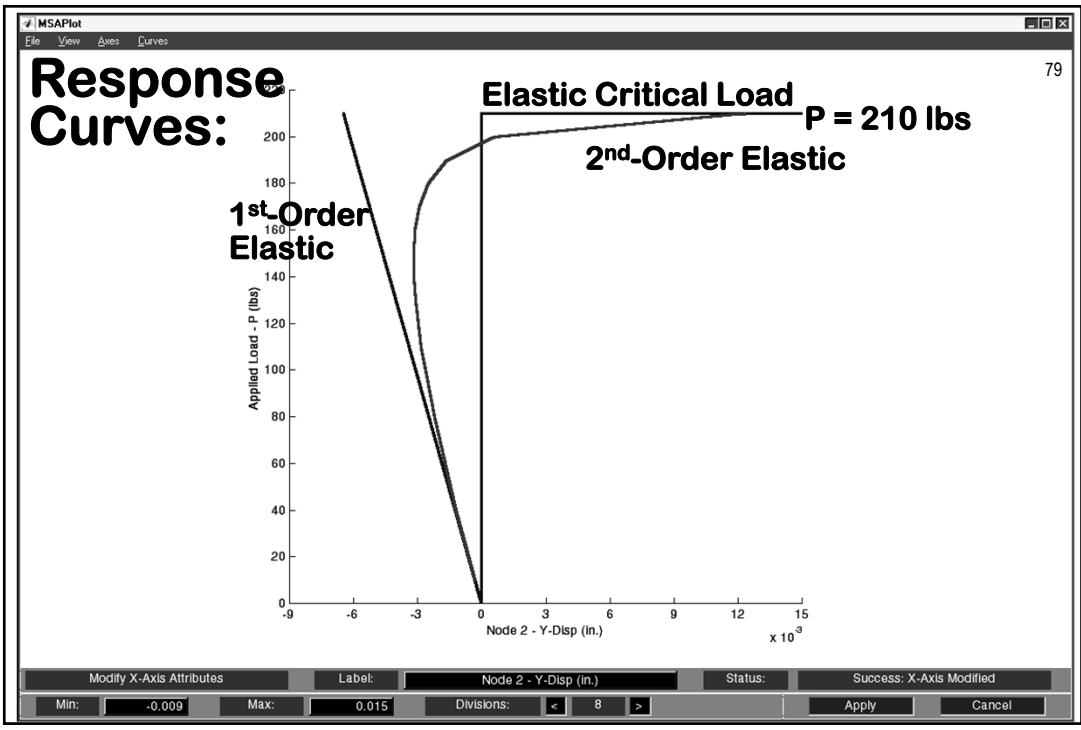
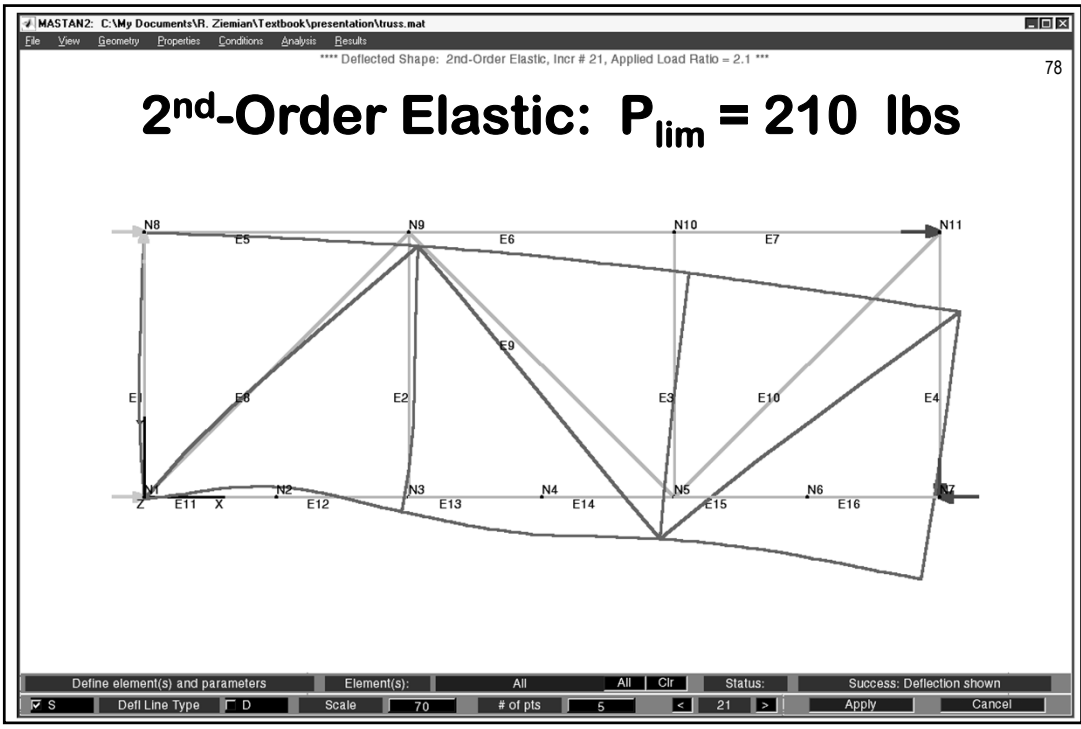








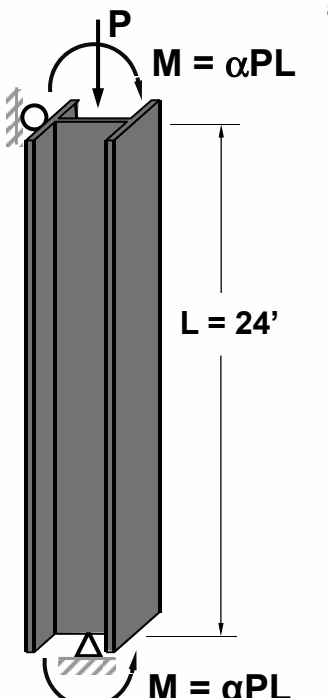




**Beam-Column:**

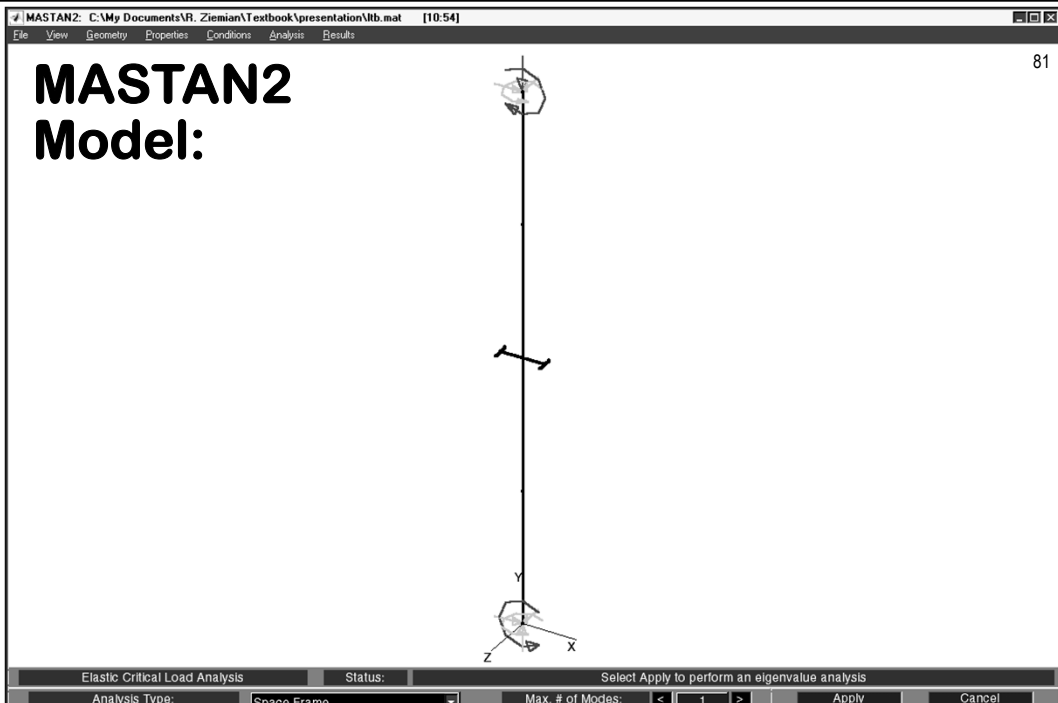
**W24x76**  
 $E = 29,000 \text{ ksi}$

**Demonstrate:**  
**Elastic Critical Load Analysis**  
1. Flexural Buckling ( $\alpha=0.0$ )  
2. Torsional Flexural Buckling ( $\alpha=0.04$ )



80

**MASTAN2 Model:**



81

Elastic Critical Load Analysis      Status:      Select Apply to perform an eigenvalue analysis

Analysis Type: Space Frame      Max. # of Modes: < 1 >      Apply      Cancel

