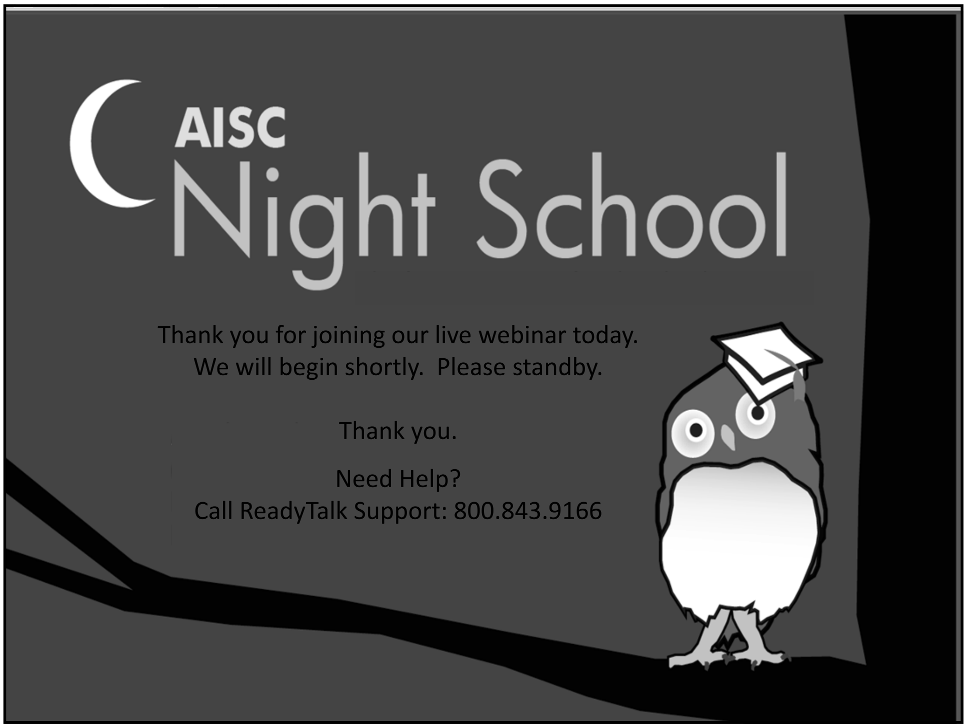

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AISC Night School

Course Description

Second-Order Elastic Analysis – Getting it Right
February 23, 2015

Various methods of second-order elastic frame analysis, ranging from intelligent application of amplification factors with first-order analysis to three-dimensional matrix structural analysis models, are reviewed. Emphasis is on the sufficiency of the methods for different problems, methods of sanity checking and ensuring that the analysis is correct, and various essential concepts important for design application.

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Learning Objectives

- Apply amplification factors in ways to obtain their best possible performance.
- Understand some of the limits of amplified 1st-order analysis.
- Calculate accurate 2nd-order elastic moments between element nodal locations.
- Understand the fundamental underpinnings of the B_2 sidesway amplifier and its terms.
- Avoid cases where simplified methods of applying B_2 can give grossly incorrect results.
- Use a proper number of elements per member in general purpose matrix structural analysis solutions.
- Avoid "gotcha's" in a general purpose matrix structural analysis due to torsional stiffness approximations.

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Stability Design of Steel Structures – Applying Modern Methods of Structural Analysis

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Session 4
Second-Order Elastic Analysis – Getting it Right

Donald W. White, Ph.D.

structural STEEL

Course Overview

- Eight Sessions
  - Course Intro. and Modern Analysis (1 & 2)
  - Resources for Learning Stability by Analysis (3)
  - Second-Order Elastic Analysis (4)
  - Direct Analysis Method (5)
  - Low- and Medium-Rise Steel Buildings (6)
  - Advanced Application of Stability Design (7)
  - Design by Inelastic Analysis (8)
- Presented by members of the Structural Stability Research Council (SSRC)
  - Don White and Ron Ziemian
  - Great to join AISC in this effort!

Second-Order Elastic Analysis – Getting it Right

Topic 4.1	P–δ Amplification, Basic Equations
Topic 4.2	P–δ Amplification, System Effects
Topic 4.3	P–δ Amplification, Calculation Of Element Internal Moments in a General Purpose 2nd–order Analysis
Topic 4.4	P–Δ Amplification, Fundamentals
Topic 4.5	Intelligent Application of B1 & B2 Amplification Factors
Topic 4.6	Accuracy Considerations for the Design Engineer when using General Purpose 2nd–Order Analysis Methods

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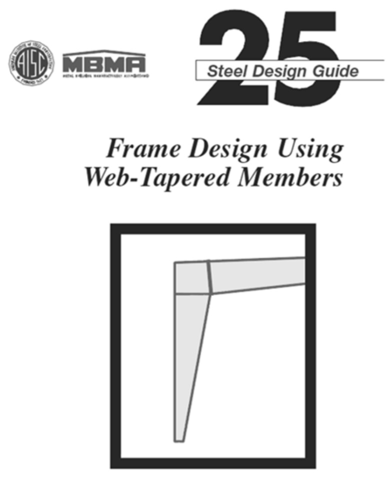
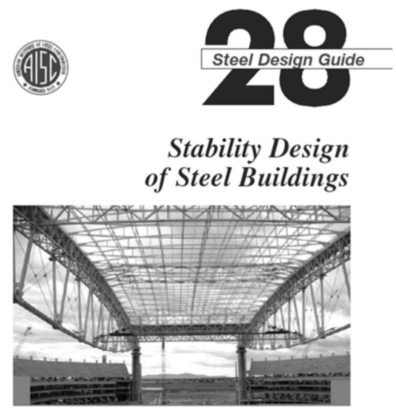
Source Materials





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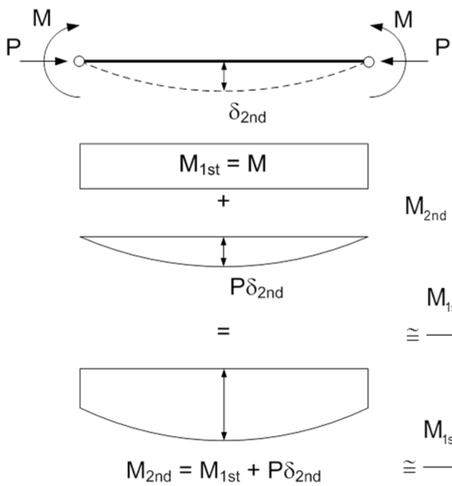
Second-Order Elastic Analysis – Getting it Right **Don White**

TOPIC 4.1

P–δ AMPLIFICATION
BASIC EQUATIONS

Simply-Supported Members

Uniform Primary Bending



$$\delta_{2nd} \cong \frac{\delta_{1st}}{1-P/P_{e1}} \qquad P_{e1} = \frac{\pi^2 EI^*}{L^2}$$

Eq. (A-8-5)

Accurate to within 3 % of analytical solution for $P/P_{e1} \leq 0.95$

$$M_{2nd} \cong M_{1st} + \frac{P\delta_{1st}}{1-P/P_{e1}} \cong \frac{M_{1st}(1-P/P_{e1}) + P\delta_{1st}}{1-P/P_{e1}}$$

$$\cong \frac{M_{1st}\left(1 - \frac{P}{P_{e1}} + \frac{P\delta_{1st}}{M_{1st}}\right)}{1-P/P_{e1}} \cong \frac{M_{1st}\left(1 + \left(\frac{P_{e1}\delta_{1st}}{M_{1st}} - 1\right)\frac{P}{P_{e1}}\right)}{1-P/P_{e1}}$$

$$\cong \frac{M_{1st}\left(1 + \psi\frac{P}{P_{e1}}\right)}{1-P/P_{e1}} \cong \frac{C_m}{1-P/P_{e1}} M_{1st} = B_1 M_{1st}$$

Eqs. (A-8-1) (A-8-3)

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Definitions

B_1 = P–δ amplification factor on the member moment

C_m = modifier in the numerator of the equation for B_1

EI^* = flexural rigidity used in the analysis ($0.8\tau_b EI$ for the Direct Analysis Method; EI for the Effective Length Method)

L = member length

M_{1st} = member maximum 1st-order moment (constant in this case)

M_{2nd} = member maximum 2nd-order moment

P = member axial load

P_{e1} = theoretical member elastic buckling strength in the plane of bending (equal to the member Euler buckling strength w/ K_1 = 1 in this case)

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Definitions (continued)

K_1 = effective length factor in the plane of bending based on the assumption of no lateral translation of the member ends

δ_{1st} = maximum 1st–order transverse displacement relative to the chord between the member ends

δ_{2nd} = maximum 2nd –order transverse displacement relative to the chord between the member ends

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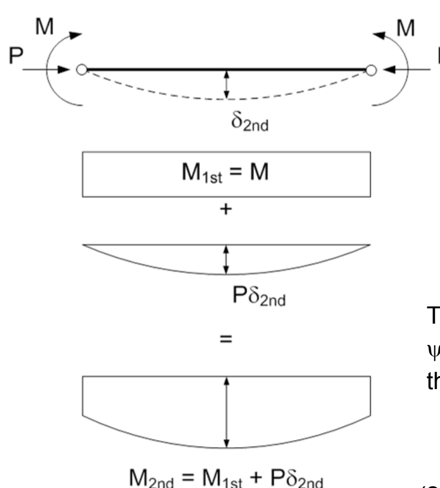
Simply–Supported Members

Uniform Primary Bending



$$M_{2nd} \cong B_1 M_{1st} \qquad B_1 = \frac{C_m}{1 - P/P_{e1}}$$

Eqs. (A-8-1) (A-8-3)

$$C_m = 1 + \psi \frac{P}{P_{e1}}$$

Eq. (C-A-8-2)

$$\psi = \frac{P_{e1}\delta_{1st}}{M_{1st}} - 1 = \frac{\frac{\pi^2 EI}{L^2}\frac{M_{1st}L^2}{8EI^*}}{M_{1st}} - 1 = +0.23$$

The AISC Spec. uses ψ = 0 for this case; ψ = +0.23 is a more accurate approx. of the analytical solution $M_{2nd} = \sec\left[\frac{\pi}{2}\sqrt{\frac{P}{P_{e1}}}\right]M_{1st}$

(2% vs 15% error at $\frac{P}{P_{e1}} = 0.7$)

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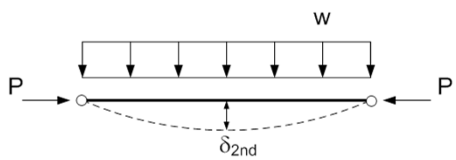
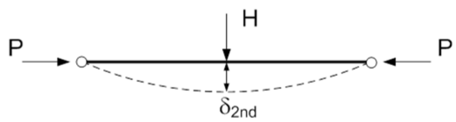
Simply-Supported Members

Transversely loaded

$\delta_{2nd} \cong \frac{\delta_{1st}}{1-P/P_{e1}}$ and $M_{2nd} = M_{1st} + P\delta_{2nd}$ … still works well…



$\psi = 0$



$\psi = -0.2$

Table (C-A-8.1)

… at least for symmetrical loading, where the maximum M & δ are at the same location

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Fixed-Fixed & Propped Cantilever Beams

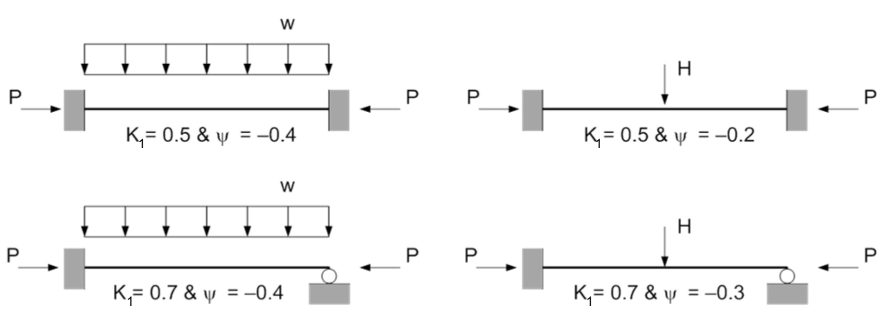
The following modified form works well…

$$M_{2nd.max} \cong \overset{B_1}{\cancel{\frac{C_m}{1-P/P_{e1}}}} M_{1st.max} \qquad C_m = \left(1+\psi\frac{P}{P_{e1}}\right) \qquad P_{e1} = \frac{\pi^2 EI^*}{(K_1L)^2}$$

Eqs. (A-8-1) (A-8-3) (C-A-8-2) (A-8-5)

where K_1 < 1) … however, ψ must be determined generally as a curve-fit

→ AISC (2010) **Table C-A-8.1**…



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General End-Restrained Members

Transversely loaded, moment-connected to other framing or flexible supports

Forget it !…

e.g., constant ψ doesn't work all that well

Generally speaking:

$C_m = 1$ is accurate to conservative

$\therefore$ Use $\quad B_1 = \dfrac{1}{1 - P/P_{e1}} \qquad P_{e1} = \dfrac{\pi^2 EI^*}{(K_1 L)^2}$

Eqs. (A-8-3) (A-8-5)

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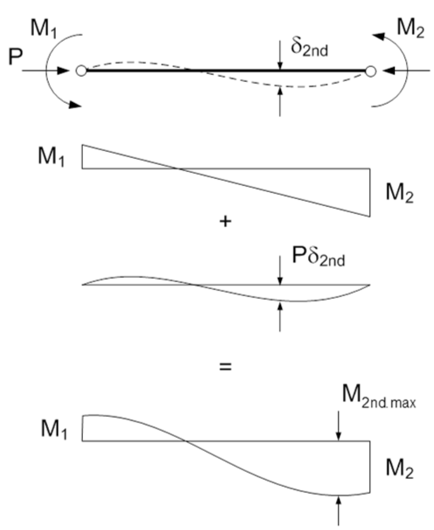
Simply-Supported Members

End-Loaded Members, Unequal End Moments



$$M_{2nd.max} \cong \underbrace{\frac{C_m}{1 - P/P_{e1}}}_{\text{< 1.0 in many practical situations}} M_2 \geq M_2 \qquad \left(\cancel{\frac{C_m}{1-P/P_{e1}}} \rightarrow B_1\right)$$

Eqs. (A-8-1) (A-8-3)

$C_m = 0.6 - 0.4\,(M_1/M_2)$ **Eq. (A-8-4)**

Conservative for combined
- large M_1/M_2 (double-curvature) &
- large P/P_{e1}

Unconservative by as much as 15 % at $P/P_{e1} = 0.7$ for
- small M_1/M_2 (single-curvature)

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General End-Restrained Members

End-loaded with no internal transverse loads, moment-connected to other framing

Given a member's maximum 1st-order end moment (M_2), the maximum internal 2^{nd}-order moment may be estimated using…

$$M_{2nd.max} \cong \overbrace{\frac{C_m}{1-P/P_{e1}}}^{B_1} M_2 \geq M_2 \qquad P_{e1} = \frac{\pi^2 EI^*}{(K_1 L)^2}$$

Eqs. (A-8-1) (A-8-3) (A-8-5)

$K_1 \leq 1$, $C_m = 0.6 - 0.4\, M_1/M_2$

Eq. (A-8-4)

However…

There is much more to the 2^{nd}-order interactions between adjacent members than this equation implies

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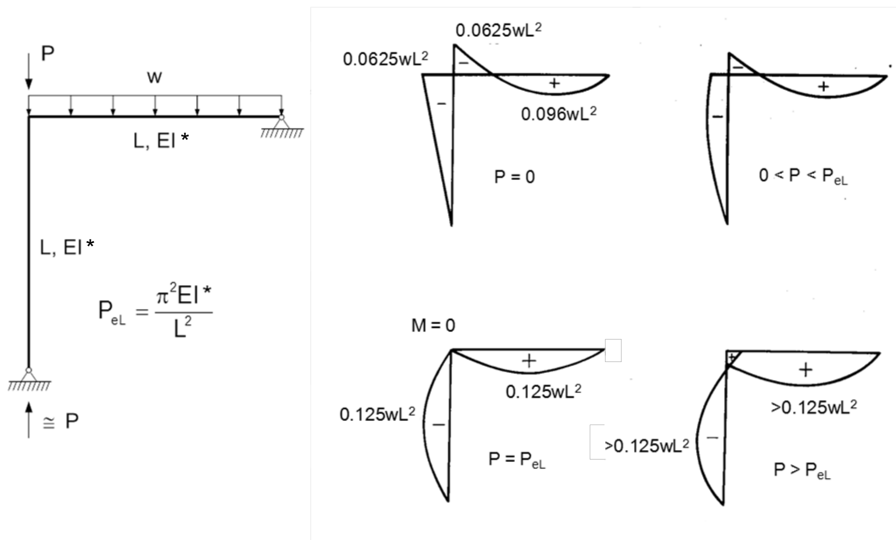
TOPIC 4.2

P-δ AMPLIFICATION
SYSTEM EFFECTS

Illustrative Braced Frame

adapted from Chen & Lui (1987)



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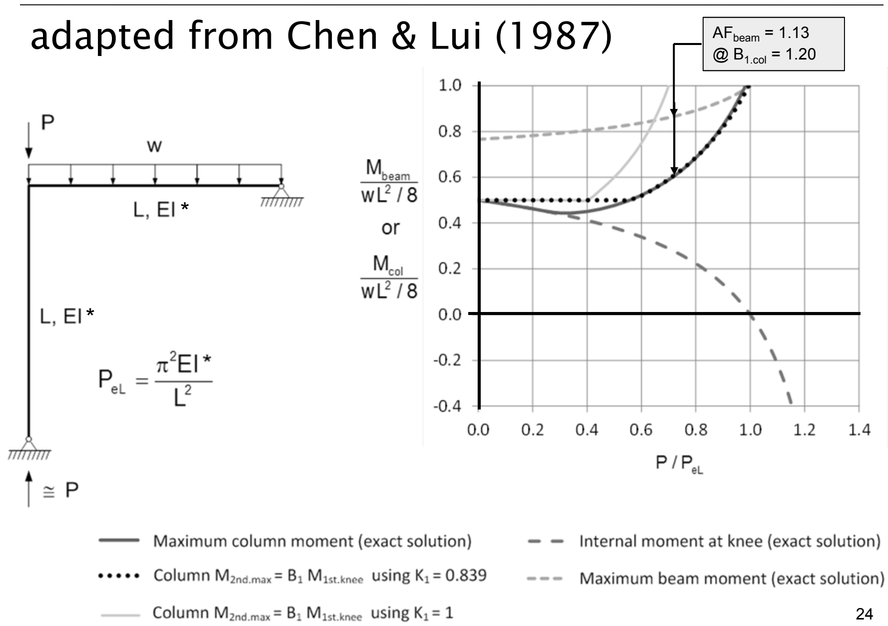
Illustrative Braced Frame

adapted from Chen & Lui (1987)



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Recommended Limits

… on the Use of B_1

For moment frames where **B_1 > 1.2** in members that have a significant effect on the response of the overall structure, the AISC Commentary recommends a **"rigorous" (i.e., general purpose) 2nd-order analysis**

… This is because

- B_1 can be relatively inaccurate in general at larger B_1 values
- For sidesway restrained frames, the reduction in the overall member flexural stiffness due to the P-δ moments can actually lead to:
  1. A *decrease* in the 2nd-order moments attracted to the columns (within redundant systems, less stiff members attract less force)
  2. An increase in the overall 2nd-order moments attracted to the beams (since the stiffness of the axially-loaded beam-columns decreases relative to the beam members)
- It is impossible to account for these effects by using a column B_1 amplifier

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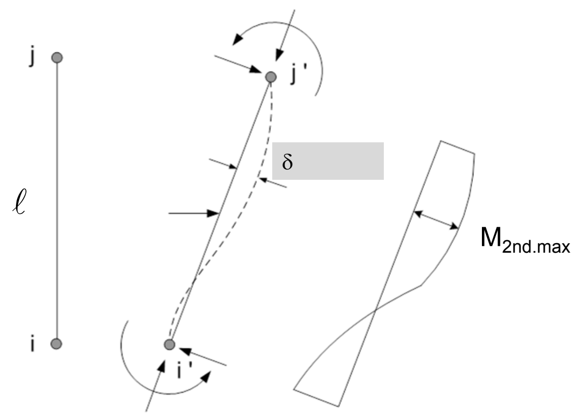
TOPIC 4.3

P-δ AMPLIFICATION
CALCULATION OF ELEMENT INTERNAL MOMENTS IN A GENERAL PURPOSE 2ND-ORDER ANALYSIS

Recommended Calculation of $M_{2nd.max}$

within frame elements, given accurate 2nd-order element end nodal moments and nodal rotations (discussed subsequently)



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Recommended Hand Estimate

$$M_{2nd.max} \cong \overbrace{\frac{C_m}{1 - P/P_{e\ell}}}^{B_1} M_{o.max} \geq M_{o.max} \qquad P_{e\ell} = \frac{\pi^2 EI^*}{\ell^2}$$

where: ℓ = element length

$M_{o.max}$ = maximum 1st-order moment between the nodes, based on the ***2nd-order nodal*** (element end) moments + any internal element loads, neglecting the internal P-δ moments between the nodes

For the calculation of C_m:

- Use **Eq. (A-8-4)** for elements with no internal transverse loads
- Otherwise, use C_m = 1 for elements having internal transverse loads

This approach is accurate to conservative for all cases
(It can be extremely conservative for large $P/P_{e\ell}$)

Use AISC **Table C-A-8.1** only with 1st-order analysis of members having internal transverse loads, and when the boundary conditions are close to ideally pinned or fixed

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Recommended Calculation

In *structural analysis software…*

1. Use enough elements to ensure accuracy of the 2nd-order nodal (element end) rotations (discussed subsequently)
2. Calculate δ_{1st} at several positions along the element length due to the *2nd-order nodal rotations* relative to the chord + the applied internal element loads
   - Calculate at close enough spacing "s" such that $P/P_{es} \le 0.02$
3. Calculate $\delta_{2nd} \cong \dfrac{\delta_{1st}}{1 - P/4P_{e\ell}}$ at each sampling point
4. Calculate $M_{2nd} = M_o + P\delta_{2nd}$ at each sampling point, where M_o is the 1st-order moment due to the nodal rotations + the applied element internal loads

This approach ensures less than 3% unconservative error and less than 6 % conservative error in the internal moments … far better accuracy than other similar methods (Guney and White, 2009)

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Definitions

ℓ = element length

s = spacing of sampling points along the element length

M_o = 1st-order internal moment at a given sampling point, generated by the element internal loads plus the element end rotations relative to its rotated (deflected) chord (calculated from the overall frame 2nd-order analysis)

M_{2nd} = 2nd-order internal moment at a given sampling point

$$P_{e\ell} = \frac{\pi^2 EI^*}{\ell^2}$$

$$P_{es} = \frac{\pi^2 EI^*}{s^2}$$

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Definitions (continued)

δ_{1st} = 1st-order transverse displacement relative to the chord between the element end nodes, calculated considering any element internal loads as well as the element end rotations relative to the chord, obtained from the overall 2nd-order analysis

δ_{2nd} = 2nd -order transverse displacement relative to the chord between the element end nodes

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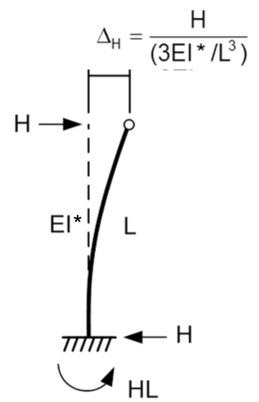
TOPIC 4.4

P-Δ AMPLIFICATION
FUNDAMENTALS

First-Order Sidesway Stiffness (P_{Lstory})



$$P_{Lstory} = \frac{H}{(\Delta_H / L)} = \frac{3EI^*}{L^2}$$

$$\frac{\Delta_H}{L} = \frac{H}{P_{Lstory}}$$ = 1st-order story drift ratio (due to H)

$$H = P_{Lstory}\frac{\Delta_H}{L}$$ = total story shear force

EI* = flexural rigidity used in the analysis ($0.8\tau_b EI$ for the Direct Analysis Method; EI for the Effective Length Method)

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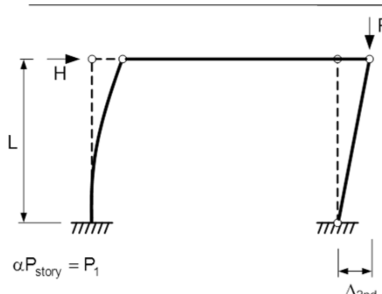
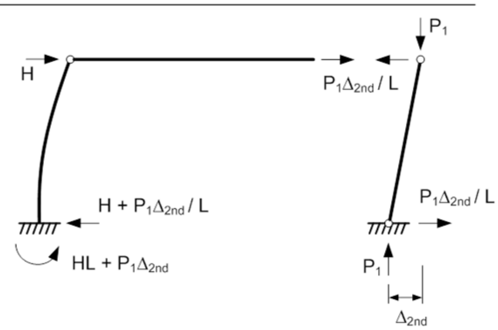
P-Δ Effect on Drift Ratio





$$\alpha P_{story} = P_1$$

$$\alpha = 1\,(LRFD),\ 1.6\,(ASD)$$

$$\Delta_{2nd} = \frac{\left(H + \alpha P_{story}\dfrac{\Delta_{2nd}}{L}\right)}{3EI^*/L^3} = \frac{\left(H + \alpha P_{story}\dfrac{\Delta_{2nd}}{L}\right)}{P_{Lstory}/L}$$

$$\frac{\Delta_{2nd}}{L} = \frac{\left(H + \alpha P_{story}\dfrac{\Delta_{2nd}}{L}\right)}{P_{Lstory}}$$

$$(P_{Lstory} - \alpha P_{story})\frac{\Delta_{2nd}}{L} = H$$

$$(P_{Lstory} - \alpha P_{story})\frac{\Delta_{2nd}}{L} = P_{Lstory}\frac{\Delta_H}{L}$$

$$\frac{\Delta_{2nd}}{L} = P_{Lstory}\frac{\Delta_H}{L}\frac{1}{(P_{Lstory} - \alpha P_{story})}$$

$$\boxed{\frac{\Delta_{2nd}}{L} = \frac{\Delta_H}{L}\frac{1}{\left(1 - \dfrac{\alpha P_{story}}{P_{Lstory}}\right)}}$$

Buckling occurs at $\gamma_{story}(\alpha P_{story}) = P_{Lstory}$

$$\boxed{\gamma_{story} = \frac{P_{Lstory}}{\alpha P_{story}}} \qquad \boxed{\frac{\Delta_{2nd}}{L} = \frac{\Delta_H}{L}\frac{1}{\left(1 - \dfrac{1}{\gamma_{story}}\right)}}$$

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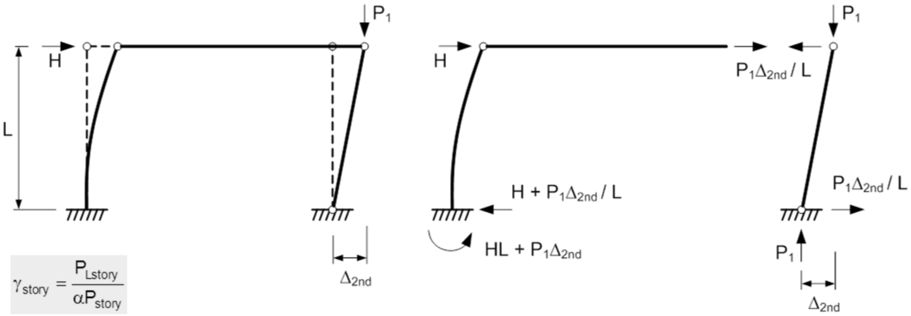
P−Δ Effect on Sway Moments



$$\gamma_{story} = \frac{P_{Lstory}}{\alpha P_{story}}$$

$$\frac{\Delta_{2nd}}{L} = \frac{\Delta_H}{L}\frac{1}{\left(1-\frac{1}{\gamma_{story}}\right)}$$

$$M_{2nd} = HL + \alpha P_{story}\Delta_{2nd} = HL + \alpha P_{story}\Delta_H \frac{1}{\left(1-\frac{\alpha P_{story}}{P_{Lstory}}\right)}$$

$$= HL\left[1+\frac{\alpha P_{story}}{H}\frac{\Delta_H}{L}\frac{1}{\left(1-\frac{\alpha P_{story}}{P_{Lstory}}\right)}\right]$$

$$M_{2nd} = HL\left[1+\frac{\alpha P_{story}}{P_{Lstory}\frac{\Delta_H}{L}}\frac{\Delta_H}{L}\frac{1}{\left(1-\frac{\alpha P_{story}}{P_{Lstory}}\right)}\right]$$

$$= HL\frac{1}{\left(1-\frac{\alpha P_{story}}{P_{Lstory}}\right)} \quad = HL\frac{1}{\left(1-\frac{1}{\gamma_{story}}\right)}$$

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Definitions

H	= total story shear force
L	= story height
M_{2nd}	= 2nd–order sidesway moment
P_1	= vertical load supported by the leaning column
P_{Lstory}	= 1st–order story sidesway stiffness, also equal to story elastic buckling load when the P–δ effects on drift are negligible
P_{story}	= total vertical load supported by the story
α	= AISC factor to convert required loads to strength load levels = 1.6 in ASD, 1.0 in LRFD
Δ_H	= 1st–order story drift due to the story shear H
Δ_{2nd}	= 2nd–order story drift
γ_{story}	= ratio of the theoretical story elastic sidesway buckling load to αP_{story}

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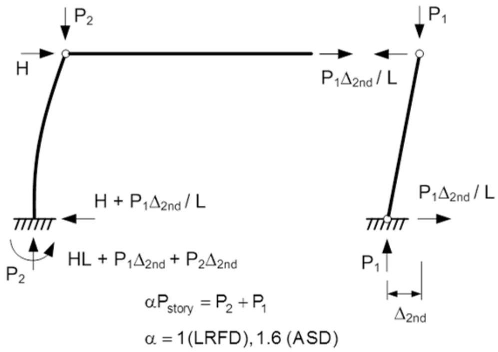
P–Δ + P–δ Effects



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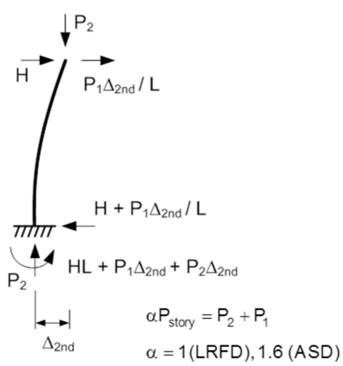
P–Δ + P–δ Effects



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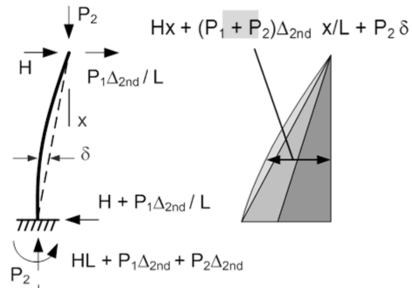
P-Δ + P-δ Effects on Drift Ratio



$\alpha P_{story} = P_2 + P_1$

$\alpha = 1\,(LRFD),\ 1.6\ (ASD)$

Assume $\delta = \Delta_{2nd}\left[\sin\left(\frac{\pi x}{2L}\right) - \frac{x}{L}\right]$

$$\Delta_{2nd} = \frac{1}{EI^*}\int_0^L \left[Hx + (P_1+P_2)\Delta_{2nd}\frac{x}{L} + P_2\delta\right] x\,dx$$

$$\frac{\Delta_{2nd}}{L} = \frac{H + (P_1+P_2)\frac{\Delta_{2nd}}{L}}{3EI^*/L^2} + P_2\frac{\Delta_{2nd}}{L}\frac{\left[\frac{3}{(\pi/2)^2} - 1\right]}{3EI^*/L^2}$$

$$\frac{\Delta_{2nd}}{L} = \frac{H + \alpha P_{story}\frac{\Delta_{2nd}}{L}}{P_{Lstory}} + P_2\frac{\Delta_{2nd}}{L}\frac{C_{L2}}{P_{Lstory}} \qquad C_{L2} = 0.216$$

$$\frac{\Delta_{2nd}}{L} = \frac{H + \alpha P_{story}\left(1+C_{Lavg}\right)\frac{\Delta_{2nd}}{L}}{P_{Lstory}}$$

$$\left[P_{Lstory} - \alpha P_{story}\left(1+C_{Lavg}\right)\right]\frac{\Delta_{2nd}}{L} = H$$

$$\left[P_{Lstory} - \alpha P_{story}\left(1+C_{Lavg}\right)\right]\frac{\Delta_{2nd}}{L} = P_{Lstory}\frac{\Delta_H}{L}$$

$$\frac{\Delta_{2nd}}{L} = P_{Lstory}\frac{\Delta_H}{L}\frac{1}{\left[P_{Lstory} - \alpha P_{story}\left(1+C_{Lavg}\right)\right]} = \frac{\Delta_H}{L}\frac{1}{\left(1-\frac{\alpha P_{story}\left(1+C_{Lavg}\right)}{P_{Lstory}}\right)}$$

$$\frac{\Delta_{2nd}}{L} = \frac{\Delta_H}{L}\frac{1}{\left(1-\frac{\alpha P_{story}}{R_M P_{Lstory}}\right)} \qquad \frac{1}{1+C_{Lavg}} = R_M \cong 1 - 0.15\frac{P_{mf}}{P_{story}}$$

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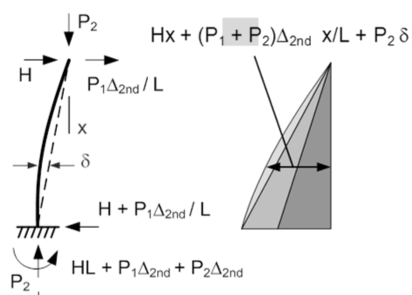
P-Δ + P-δ Effects on Sway Moments



$$\frac{\Delta_{2nd}}{L} = \frac{\Delta_H}{L}\underbrace{\frac{1}{\left(1-\frac{\alpha P_{story}}{R_M P_{Lstory}}\right)}}_{B_2} \qquad R_M \cong 1 - 0.15\frac{P_{mf}}{P_{story}}$$

Eqs. (A-8-1) (A-8-6) (A-8-7) (A-8-8)

Buckling occurs at $\gamma_{story}(\alpha P_{story}) = R_M P_{Lstory}$

$$\gamma_{story} = \frac{R_M P_{Lstory}}{\alpha P_{story}} \qquad \frac{\Delta_{2nd}}{L} = \frac{\Delta_H}{L}\underbrace{\frac{1}{\left(1-\frac{1}{\gamma_{story}}\right)}}_{B_2}$$

$$M_{2nd} = HL + \alpha P_{story}\Delta_{2nd} = HL + \alpha P_{story}\Delta_H\frac{1}{\left(1-\frac{\alpha P_{story}}{R_M P_{Lstory}}\right)} = HL\left[1+\frac{\alpha P_{story}}{H}\frac{\Delta_H}{L}\frac{1}{\left(1-\frac{\alpha P_{story}}{R_M P_{Lstory}}\right)}\right]$$

$$= HL\left[1+\frac{\alpha P_{story}}{P_{Lstory}\frac{\Delta_H}{L}}\frac{\Delta_H}{L}\frac{1}{\left(1-\frac{\alpha P_{story}}{R_M P_{Lstory}}\right)}\right] = HL\left[1+R_M\frac{\alpha P_{story}}{R_M P_{Lstory}}\frac{1}{\left(1-\frac{\alpha P_{story}}{R_M P_{Lstory}}\right)}\right] = HL\frac{1+(R_M-1)\frac{\alpha P_{story}}{R_M P_{Lstory}}}{\left(1-\frac{\alpha P_{story}}{R_M P_{Lstory}}\right)} = HL\frac{1+(R_M-1)\frac{1}{\gamma_{story}}}{\left(1-\frac{1}{\gamma_{story}}\right)}$$

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Additional Definitions

x = coordinate from the top of the lateral load resisting column (the cantilever column)

B_2 = story sidesway amplification factor

C_{L2} = coefficient associated with the contribution of the lateral load resisting column $P\delta$ moments to the 2nd-order story drift

C_{Lavg} = weighted average C_L for the entire story

P_2 = vertical load supported by the lateral load resisting system (the cantilever column)

P_{mf} = total vertical load supported by the lateral load resisting columns in the story

R_M = coefficient quantifying the influence of column $P\delta$ moments on the 2nd-order story drift

δ = deflection of column relative to the chord between its top and bottom, due to bending

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Column Sidesway Eff. Len. Factor, K_2

Fundamental relationship → Axial load at incipient buckling = $\gamma_{story}\alpha P_r = \dfrac{\pi^2 EI}{(K_2 L)^2}$

Solve for K_2 → $K_2 = \sqrt{\dfrac{\pi^2 EI/L^2}{\gamma_{story}\alpha P_r}}$

Substitute equation for γ_{story} → $K_2 = \sqrt{\dfrac{\alpha P_{story}}{R_M P_{Lstory}} \dfrac{\pi^2 EI/L^2}{\alpha P_r}} = \sqrt{\dfrac{P_{story}}{P_r} \dfrac{\pi^2 EI/L^2}{R_M P_{Lstory}}}$

$$K_2 = \sqrt{\frac{P_{story}}{P_r}\frac{\pi^2 EI}{L^2}\frac{1}{R_M}\frac{\Delta_H/L}{H}} \geq \sqrt{\frac{\pi^2 EI}{L^2}\frac{\Delta_H/L}{1.7H_r}} \quad \text{or} \quad \frac{P_r}{P_{story}} \leq \frac{1.7}{R_M}\frac{H_r}{H}$$

EQ. (C-A-7-5)

(The above inequality is a limit developed by LeMessurier to ensure sufficient accuracy in determining the buckling load for stories with one or more relatively light columns, subjected to high axial load, compared to the other lateral-load resisting components… **It is acceptable to use $K_2 \geq 1$ as a simplification**.)

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Additional Definitions

EI = nominal flexural rigidity of the column under consideration

H_r = portion of the story shear force H resisted by the column under consideration

P_r = axial force in the column under consideration

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Sidesway Effective Length Factor, K_2

Note:
K = 1 for leaning columns

Also … K = 1 in the
Direct Analysis Method (the DM)

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Second-Order Elastic Analysis – Getting it Right **Don White**

TOPIC 4.5

INTELLIGENT APPLICATION OF B1 & B2
AMPLIFICATION FACTORS

Summary: Amplified 1st-Order Methods

- AISC Appendix 8 NT-LT
  (No Translation - Lateral Translation) Analysis
  ($M_r = B_1 M_{nt} + B_2 M_{lt}$; $P_r = P_{nt} + B_2 P_{lt}$)
- AISC Manual Section 2 Simplification (modified)
  ($M_r = \max(B_1, B_2) M_{1st}$; $P_r = P_{1st}$)
- Another often used estimate
  ($M_r = B_1 M_{gravity} + B_2 M_{lateral}$; $P_r = P_{1st}$)
  (neglecting sidesway under gravity load)
- Amplified Story Drift Method
  (White et al. 2007a & b)
  ($\Delta_{2nd} = B_2 (\Delta_o + \Delta_{1st})$; $H_{P\Delta} = \alpha P_{story} \Delta_{2nd}/L$)
  - Perform 1st-order analysis for $H_{P\Delta}$ forces
  - Add $H_{P\Delta}$ force effects to M_{1st} & P_{1st}
  - Multiply resulting member forces by B_1

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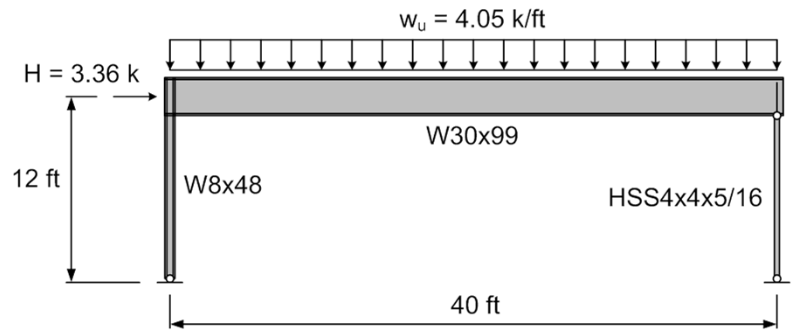
Comparison of Amplified 1st-Order Methods

Consider the analysis of LeMessurier's (1977) Example 3 for 1.2D + 1.6L_r + 0.5W (LRFD)



Using the DM, define a reduced elastic stiffness of 0.8E = 23,200 ksi

Assume that the sidesway amplification is less than 1.7 (1.5 based on the unreduced stiffness); thus, out-of-plumbness effects need not be considered

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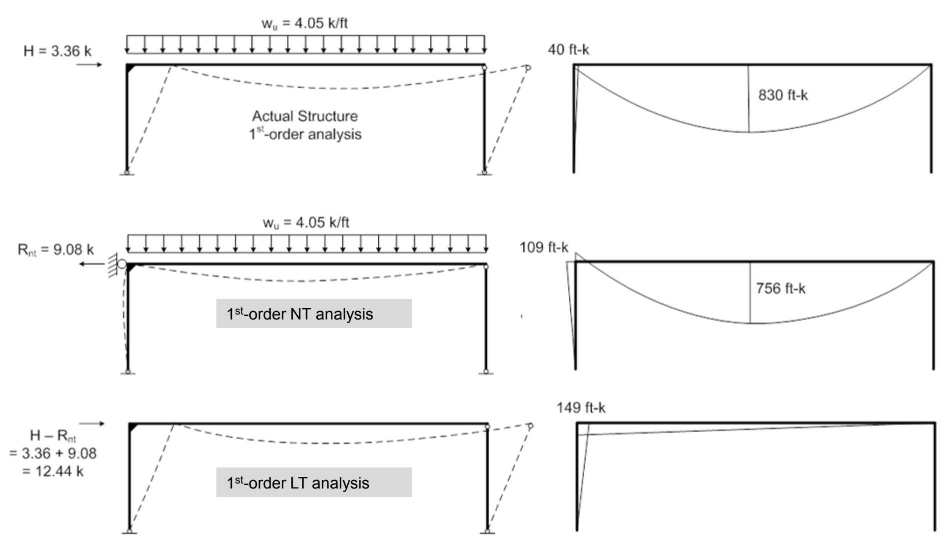
AISC Appendix 8 NT-LT Analysis



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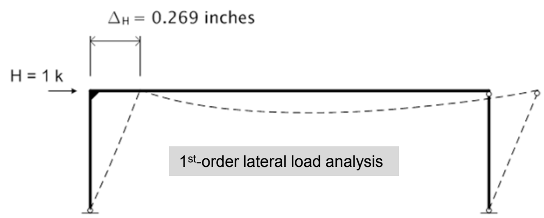
AISC Appendix 8 NT-LT Analysis



$$P_{L\,story} = \frac{HL}{\Delta_H} = \frac{1\,(144)}{0.269} = 535\text{ k / in} \qquad \alpha P_{story} = 4.05\,(40) = 162\text{ k}$$

$$R_M = 1 - 0.15\frac{P_{mf}}{P_{story}} = 1 - 0.15\frac{1}{2} = 0.925$$

Eq. (A-8-8)

$$B_2 = \frac{1}{1 - \dfrac{\alpha P_{story}}{R_M\,P_{L\,story}}} = \frac{1}{1 - \dfrac{162}{0.925(535)}} = 1.49$$

Eqs. (A-8-6) (A-8-7)

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AISC Appendix 8 NT-LT Analysis

W8x48 → I_x = 184 in^4, K_1 = 1 (conservative)

$$P_{e1} = \frac{\pi^2 (0.8E)\,I_x}{(K_1 L_x)^2} = \frac{\pi^2 (23{,}200)(184)}{144^2} = 2030\text{ k} \qquad B_1 = \frac{C_m}{1 - \dfrac{P_u}{P_{e1}}} = \frac{0.6}{1 - \dfrac{81}{2030}} = \cancel{0.62}\ \mathbf{1.0}$$

Eqs. (A-8-3) (A-8-4) (A-8-5)

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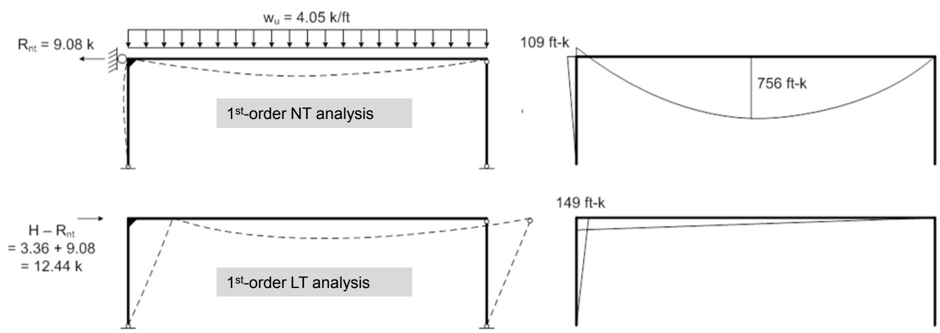
AISC Appendix 8 NT-LT Analysis



Moment at top of column & end of beam:

$M_u = B_1 M_{nt} + B_2 M_{lt}$ = 1.0 (-109) + 1.49 (149) = 113 ft-k **OK** 104 ft-k (exact)

Eq. (A-8-1)

Moment at beam mid-span:

M_u = 4.05 $(40)^2$ / 8 + 113/2 = 866 ft-kips **OK** 862 ft-k (exact)

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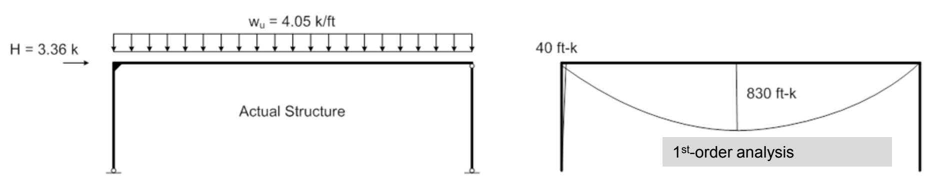
AISCM Section 2 Procedure (modified)



Moment at top of column & end of beam:

M_u = max (B_1, B_2) M_{1st} = 1.49 (40) = 60 ft-k **X** 104 ft-k (exact)

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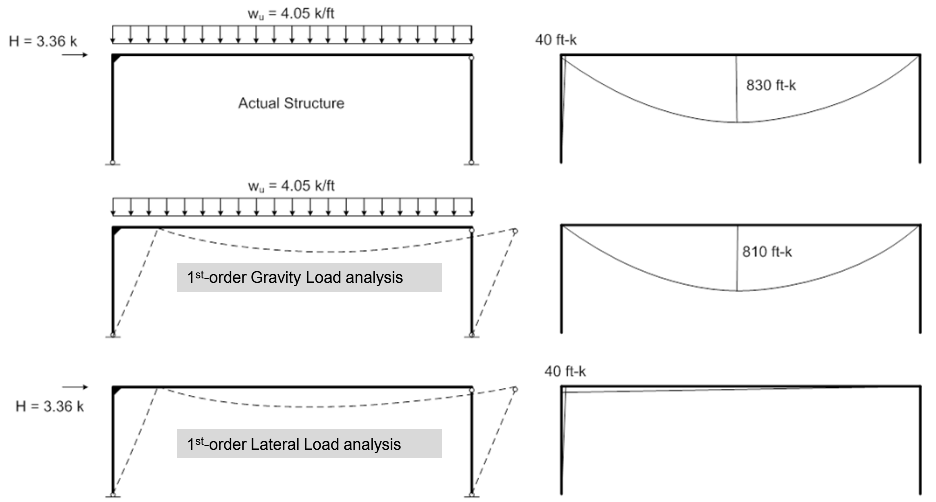
Another Often Used Estimate



Moment at top of column & end of beam:

$M_u = B_1 M_{gravity} + B_2 M_{lateral} = 1.0\ (0) + 1.49\ (40) = 60$ ft-k **X** 104 ft-k (exact)

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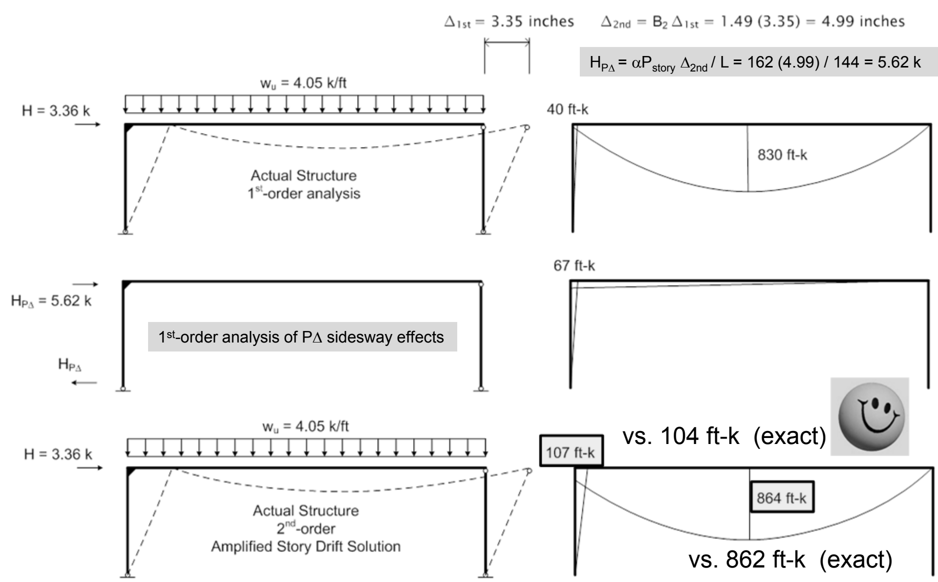
Amplified Story Drift Method



$B_1 = 1.0$

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Amplified Story Drift Method

Advantages:

- B_2 is applied just to the story drifts
- All the 2nd–order beam, connection, brace, panel zone & beam–column forces are obtained by direct addition of the $H_{P\Delta}$ force effects to the 1st–order analysis forces ***(superposition is applicable)***
- No separate mucking around with applying amplifiers to different 1st–order force effects
- No separate NT & LT analyses
- Improved accuracy of 2nd–order force estimates
- The method is simply a traditional P–Δ analysis … but using amplified story drifts directly (no iteration)

Disadvantages:

- A separate 1st–order analysis is necessary to determine the 2nd–order sidesway effects

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or … for greatest simplicity of application

- Use a general purpose 2nd–order analysis

- Use B_1 and B_2 as approximate indices to gage where the structure is relative to the intensity of the 2nd–order effects

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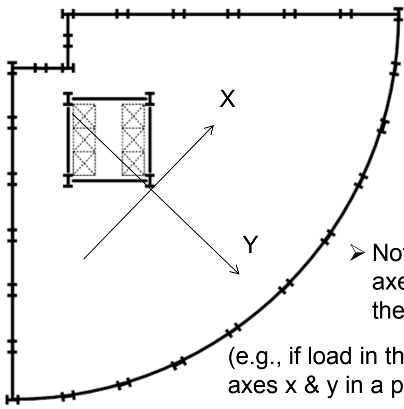
Proper 3D Application of B_1 & B_2



- Select orthogonal **X** & **Y** directions
- Apply separate B_2 values (e.g., $B_{2\mathbf{X}}$ & $B_{2\mathbf{Y}}$) for each story & each direction of lateral translation
- $B_{2\mathbf{X}}$ is applicable to all forces produced by story translation in the **X** direction
- Similar for $B_{2\mathbf{Y}}$
- Note: $B_{2\mathbf{X}}$ & $B_{2\mathbf{Y}}$ are associated with the global axes **X** & **Y**; they are completely unrelated to the direction of bending of individual members

(e.g., if load in the **X** direction causes moments M_x & M_y about axes x & y in a particular member, $B_{2\mathbf{X}}$ applies to both M_x & M_y)

- Apply separate B_1 values (B_{1x} & B_{1y}) for the x & y axis bending of every member subjected to compression & flexure
- B_{1x} is applicable to M_x regardless of the load that causes that moment
- Similar for B_{1y}

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Second-Order Elastic Analysis – Getting it Right

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TOPIC 4.6

ACCURACY CONSIDERATIONS FOR THE DESIGN ENGINEER WHEN USING GENERAL PURPOSE 2ND-ORDER ANALYSIS METHODS

Let's Consider Three Different Commonly-Used Methods

- Stability function based elements
- Cubic Hermitian elements
- P-Δ elements

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Stability Function Based Element

- Based on analytical solution of governing differential equations for a beam-column member
- Assumptions:
  – Elastic behavior
  – Prismatic geometry
  – Euler-Bernoulli kinematics (no shear deformation)
  – Finite displacements, but $\sin\theta \cong \theta$, where θ is the rotation relative to the element chord

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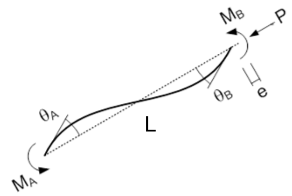
Stability Function Element, Stiffness Eqs.

$$\begin{Bmatrix} P \\ M_A \\ M_B \end{Bmatrix} = \frac{EI^*}{L} \begin{bmatrix} A/I & 0 & 0 \\ 0 & s_{ii} & s_{ij} \\ 0 & s_{ij} & s_{ii} \end{bmatrix} \begin{Bmatrix} e \\ \theta_A \\ \theta_B \end{Bmatrix}$$



$s_{ii} = s_{jj} = (kL \sin kL - (kL)^2 \cos kL) / (2 - 2 \cos kL - kL \sin kL)$

$s_{ij} = s_{ji} = [\ (kL)^2 - kL \sin kL\ ] / (2 - 2 \cos kL - kL \sin kL)$

$$kL = \sqrt{\frac{P}{EI^*}}\,L = \pi\sqrt{\frac{P}{P_e}} \qquad P_e = \frac{\pi^2 EI^*}{L^2}$$

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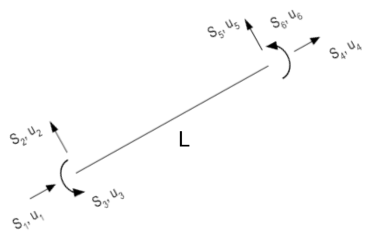
Stability Function Element, Stiffness Eqs.



$$\begin{Bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{Bmatrix} = \frac{EI^*}{L} \begin{bmatrix} A/I & 0 & 0 & -A/I & 0 & 0 \\ & \frac{2(s_{ii}+s_{ij})-(kL)^2}{L^2} & \frac{(s_{ii}+s_{ij})}{L} & 0 & \frac{-2(s_{ii}+s_{ij})+(kL)^2}{L^2} & \frac{(s_{ii}+s_{ij})}{L} \\ & & s_{ii} & 0 & -\frac{(s_{ii}+s_{ij})}{L} & s_{ij} \\ & & & A/I & 0 & 0 \\ & \text{Sym} & & & \frac{2(s_{ii}+s_{ij})-(kL)^2}{L^2} & \frac{-(s_{ii}+s_{ij})}{L} \\ & & & & & s_{ii} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix}$$

$$kL = \pi\sqrt{\frac{P}{P_e}} \qquad P_e = \frac{\pi^2 EI^*}{L^2}$$

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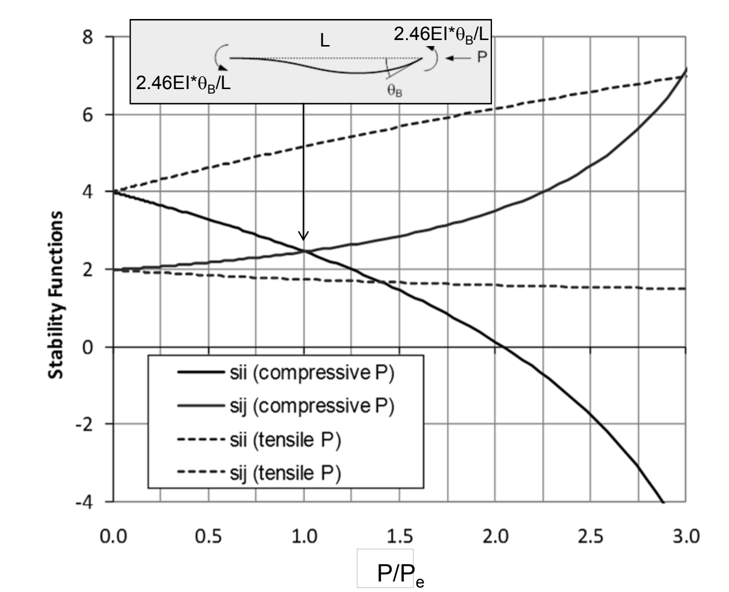
Stability Functions vs P/P_e ($P_e = \pi^2 EI^*/L^2$)



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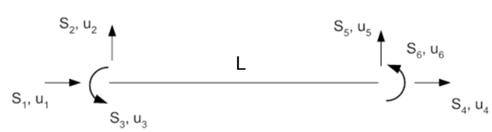
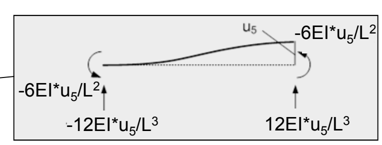
Cubic Hermitian Element, Stiffness Eqs.

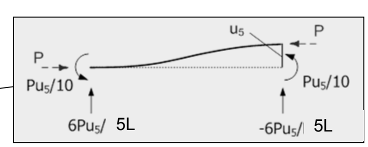
$(\mathbf{k}_E + \mathbf{k}_G)\,\mathbf{u} = \mathbf{S}$



$$\mathbf{k}_E = EI^*/L^3 \begin{bmatrix} AL^2/I & 0 & 0 & -AL^2/I & 0 & 0 \\ & 12 & 6L & 0 & -12 & 6L \\ & & 4L^2 & 0 & -6L & 2L^2 \\ & & & AL^2/I & 0 & 0 \\ & & & & 12 & -6L \\ \text{SYM.} & & & & & 4L^2 \end{bmatrix}$$



$$\mathbf{k}_G = -P/L \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 6/5 & L/10 & 0 & -6/5 & L/10 \\ & & 2L^2/15 & 0 & -L/10 & -L^2/30 \\ & & & 0 & 0 & 0 \\ & & & & 6/5 & -L/10 \\ & & & & & 2L^2/15 \end{bmatrix}$$



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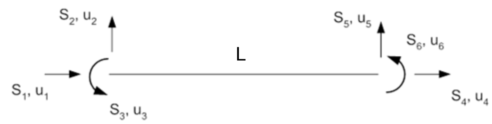
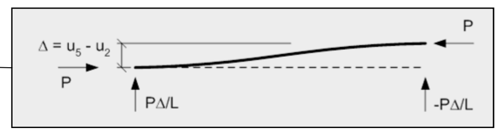
P-Δ Element, Stiffness Equations

$(\mathbf{k}_E + \mathbf{k}_G)\ \mathbf{u} = \mathbf{S}$



$\mathbf{k}_E$ = same as in Cubic Hermitian Element

$$\mathbf{k}_G = -P/L \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & 1 & 0 & 0 & -1 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 1 & 0 \\ & & & & & 0 \end{bmatrix}$$



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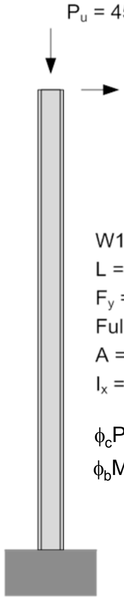
Example P-Δ Analysis Sidesway Error

P_u = 452 kips

$H = H_u + N_i$ = 1.254 kips + 0.002 (452 kips) = 2.158 kips



W10x60
L = 15 ft
F_y = 50 ksi
Fully braced out-of-plane
A = 17.6 in^2
I_x = 341 in^4

$\phi_c P_n$ = 704 kip
$\phi_b M_n$ = 3360 in-kip

$P_u/P_y = 0.514$, $\tau = 0.999 \cong 1.0$
$P_{eL} = \pi^2 EI_x/L^2$ = 3012 kips
$P_u/P_{eL} = 0.15$, $P_u/P_{cr} = 0.60$
$0.8\tau E$ = 23,200 ksi
$u = P_u L^2/(0.8\tau EI_x) = 1.361$
Exact Moment Amplification = tan u / u = 3.45
Exact Displacement Amplification = 3(tan u – u) / u^3 = 3.97
1st order drift Δ_{1st} = 0.531 in (using reduced stiffness)

Exact 2nd-order drift Δ_{2nd} = 2.11 in
Exact M_u = 1340 kip-in
Exact $P_u/\phi_c P_n + 8/9\ M_u/\phi_b M_n = 1.0$

HL/Δ_{1st} = 731.9 kips
Single element P-Δ Moment & Displacement Amplification = $1/[1 - P_u/(HL/\Delta_{1st})] = 2.61$

P-Δ Estimate of 2nd-order drift = 1.39 in **(-34 % error)**
P-Δ Estimate of M_u = 1013 kip-in **(-24 % error)**
P-Δ Estimate of $P_u/\phi_c P_n + 8/9\ M_u/\phi_b M_n = 0.912$ **(-9 % error)**

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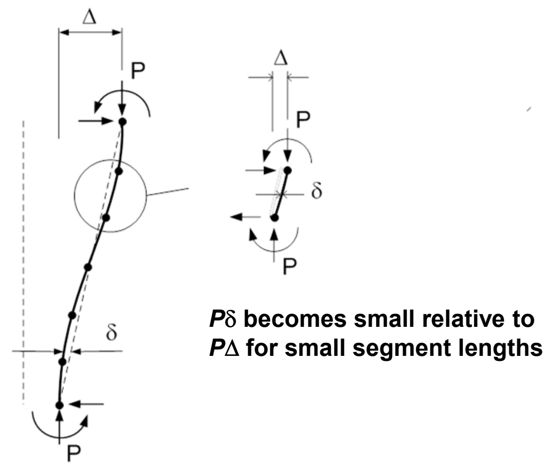
Capture of P-δ by a P-Δ Analysis

- General purpose **P-Δ analysis methods** can capture P-δ effects sufficiently in members with significant P-δ effects **if enough elements are used**



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Required # Elements/Member

(from AISC Design Guide 25)

- Criteria:
  - 5 % accuracy in the member end displacements and internal member max nodal displacements
  - 5 % accuracy in the member end rotations
  - 3 % accuracy in maximum internal forces

for

- $\alpha P_r/P_{cr}$ up to 0.7
  (P_{cr} = maximum member axial force at theoretical elastic buckling)
- Comprehensive range of member end conditions

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Required # Elements/Member

- Caveats:
  - The element internal moments are determined accurately, e.g., using the ($M_o + P\delta_{2nd}$) method discussed previously
  - The accuracy estimates are based on:
    - The best achievable theoretical solutions
    - The worst-case member boundary conditions (causing the largest error for a given category)
  - The actual accuracy depends on
    - Solution algorithms
    - Implementation details
    - Similarity of the actual boundary conditions to the worst-case boundary conditions

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Required # Elements/Member

- Definition of terms:
  - ℓ = element length
  - L = member length
  - s = spacing of "sampling points," where element internal moments are evaluated

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Required # Elements/Member

Stability Function Based Elements

- Generally, one element/member is sufficient
- The element internal moments must be determined analytically, or using the $M_o + P\delta_{2nd}$ procedure, ... sampling at an interval "s" such that $P/P_{es} \leq 0.02$

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Required # Elements/Member

For Cubic Displacement-Based Elements
(e.g., Mastan, Fastrak, ETABS, GT-Strudl, LARSA, SAP, RAM Elements)

- End-loaded members with sidesway unrestrained
  → Generally, one element/member is sufficient
- Non-sway members
  → Generally, two elements/member are sufficient
  → One element is sufficient if $P/P_{eL} \leq 0.17$
- To avoid the need for element internal ($M_o + P\delta_{2nd}$) calculations, one must satisfy $P/P_{e\ell} \leq 0.02$
- Special case: when torsional warping is included (14 dof elem.), use 8 elem. in ea. unbraced length

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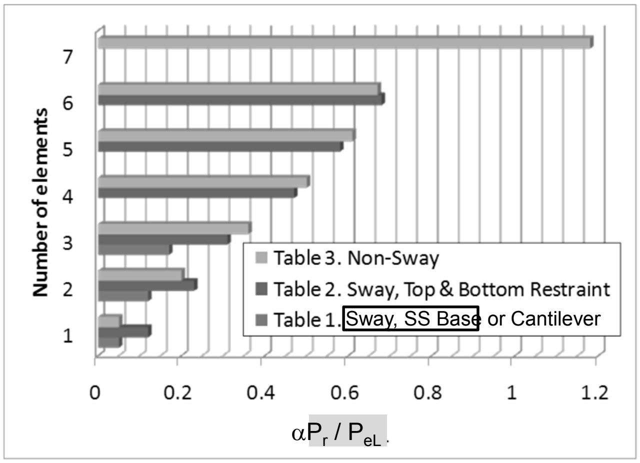
Required # Elements/Member

For P-Δ Elements (see prev. slide for cubic elem.)

- Generally, many more elements are required compared to cubic displacement-based elements



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Conditional Upper-Bound on P-Δ Analysis Sidesway Error

If $\frac{P_{mf}}{P_{story}} \le \frac{1}{3}$ and $B_2 \le 1.7$,

the error in neglecting P-δ effects on Δ is generally less than 4 %

$$R_M = 1 - 0.15\frac{P_{mf}}{P_{story}} = 1 - 0.15\left(\frac{1}{3}\right) = 0.95$$

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{L\,story}}} = 1.7 \quad \Rightarrow \quad \frac{\alpha P_{story}}{P_{L\,story}} = 0.412$$

$$\Rightarrow \quad B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{R_M P_{L\,story}}} = 1.766 \qquad 100\frac{1.766 - 1.7}{1.766} = 3.7\%$$

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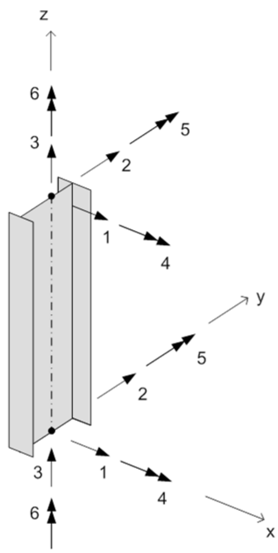
3D Frame Element Torsional Stiffness Gotchas



Typical 3D Frame Element …

… what about the torsional stiffness due to restraint of warping?

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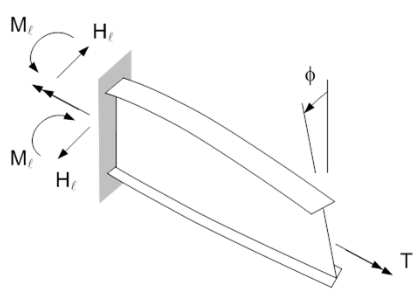
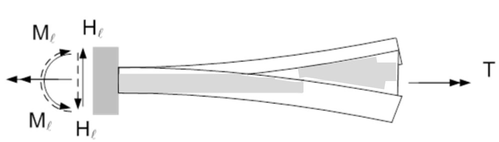
Torsion of an I-Section Member



W14x22, L = 10 ft, T = 2 in-k, J = 0.208 in^4, C_w = 314 in^6

$$\phi = \frac{T}{GJ}\left[L - \frac{\sinh(pL)}{p\cosh(pL)}\right] = 0.052 \text{ rad}$$

$$p = \sqrt{\frac{GJ}{EC_w}}$$



Top View

Using an analysis that does not include warping stiffness…

$$\phi = \frac{TL}{GJ} = 0.103 \text{ rad}$$

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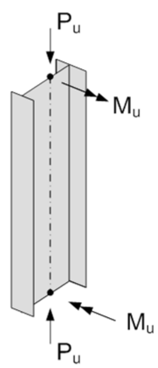
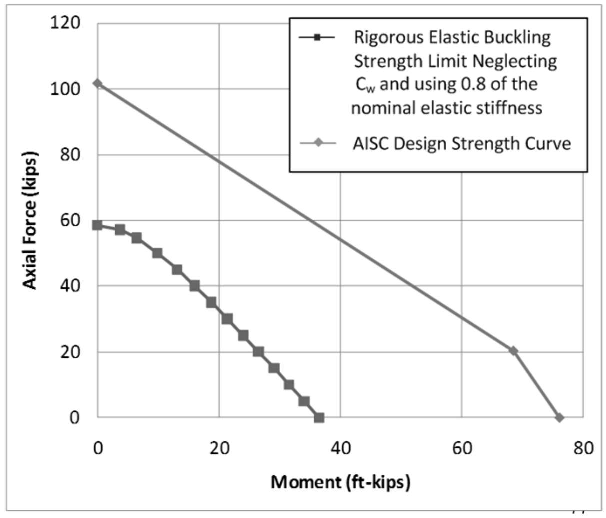
Example Problem Case

W14x22, simply-supported flexurally & torsionally,
$L_b = L_r = 10.4$ ft, $K_x = K_y = K_z = 1.0$





Recommendation

For open-section members where the model indicates buckling at small load levels (because the warping rigidity (EC_w) is neglected in the analysis):

- Consider using a larger "effective" member GJ
  - Option available in RISA 3D
  - Suggested for grid analysis of curved & skewed bridges in the AASHTO LRFD Specifications

  or

- Ideally… use an analysis that correctly models both the St. Venant & member warping and torsional rigidities (both EC_w & GJ)

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Second-Order Elastic Analysis – Getting it Right Don White

THAT'S IT!

Thanks For Attending!!

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References

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Up Next…

- Session 5: March 2 –
  The AISC Direct Analysis Method from Soup to Nuts
  by D.W. White, PhD
- This session presents a comprehensive overview of the Direct Analysis Method of design, which was first introduced in Appendix 7 of the 2005 AISC 13th Edition Specification and is referred to as the preferred method of design in Chapter C of the 2010 AISC 14th Edition Specification. The lecture emphasizes the key fundamental concepts and the practical application of the method to building frames.

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www.aisc.org/nightschool - click on Current Course Details.

Reasons for quiz:

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- CEUs/PDHS – If you watch a recorded session you must take quiz for CEUs/PDHs.
- REINFORCEMENT – Reinforce what you learned tonight. Get more out of the course.

NOTE: If you attend the live presentation, you do not have to take the quizzes to receive CEUs/PDHs.

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Thank You

Please give us your feedback!
Survey at conclusion of webinar.

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