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## Course Description

### Stability Design – Advanced Applications

March 16, 2015

This module investigates the application of modern structural analysis methods to several challenging stability design problems, including the sizing of general stability bracing for beams, beam-columns and frames, design of steel arches, and checking of stability during erection. The Direct Analysis Method as well as rigorous inelastic buckling analysis capabilities will be demonstrated.



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## Learning Objectives

- Apply the Direct Analysis Method (the DM) to the design of steel arches
- Assess stability of long-span trusses during erection using the DM
- Demonstrate the direct relationship between the DM & the AISC Appendix 6 relative (shear panel) bracing rules
- Apply the DM to the design of column lean-on bracing
- Assess column lean-on bracing by a rigorous in-plane inelastic buckling analysis
- Compare and contrast the DM and Buckling Analysis based approaches
- Demonstrate the application of rigorous inelastic buckling analysis procedures for assessment of general beam & beam-column stability bracing requirements as well as corresponding member strengths
- Select appropriate geometric imperfections for DM assessment of general stability design problems



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## Stability Design of Steel Structures – Applying Modern Methods of Structural Analysis

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### Session 7 Stability Design – Advanced Applications

Donald W. White, Ph.D.



## Course Overview

- Eight Sessions
  - Course Intro. and Modern Analysis (1 & 2)
  - Resources for Learning Stability by Analysis (3)
  - Second-Order Elastic Analysis (4)
  - Direct Analysis Method (5)
  - Low- and Medium-Rise Steel Buildings (6)
  - **Advanced Applications (7)**
  - Design by Inelastic Analysis (8)
- Presented by members of the Structural Stability Research Council (SSRC)
  - Don White and Ron Ziemian
  - Great to join AISC in this effort!

## Stability Design – Advanced Applications

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Topic 7.1 Stability Design of Arches by the DM

Topic 7.2 Stability of Long-Span Trusses during Erection

Topic 7.3 Bracing Design

7.3.1 Basic Column Bracing Design – Tie to the DM

7.3.2 Column Lean-On Bracing DM Design Example

7.3.3 Scrutinizing the Lean-On Bracing Calculations

7.3.4 Use of Inelastic Buckling Analysis, Comparison of Approaches

Topic 7.4 General Modeling of Geometric Imperfections

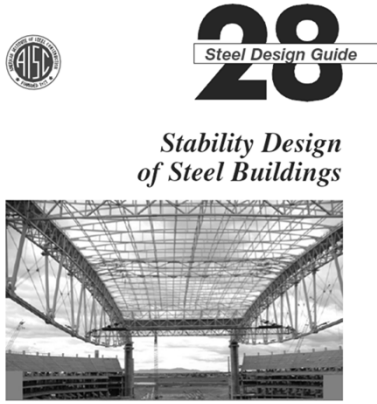
*i.e., how can we apply modern methods of analysis per the AISC Specification provisions to solve more general stability design problems*

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## Source Materials

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Stability Design – Advanced Applications

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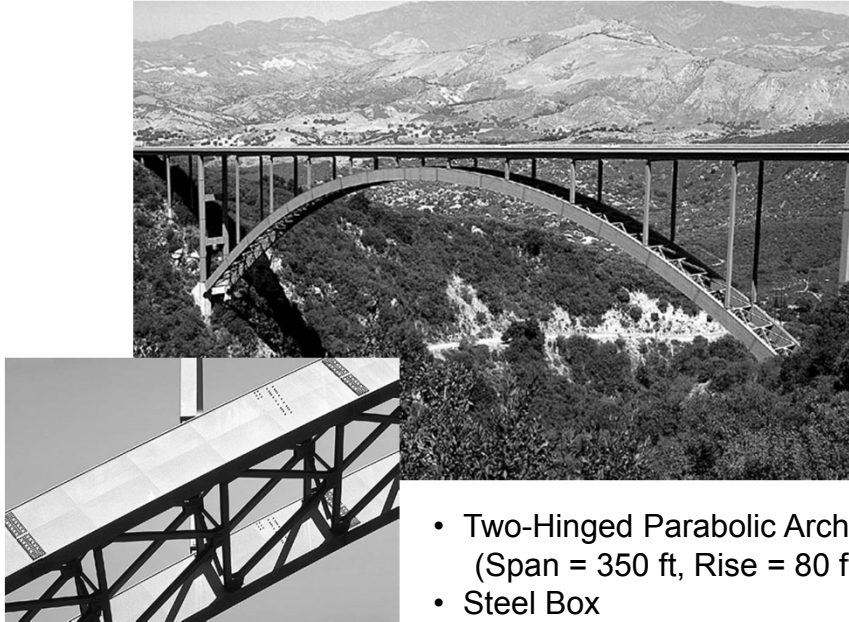
## TOPIC 7.1

STABILITY DESIGN OF STEEL ARCHES BY THE DIRECT  
ANALYSIS METHOD

(CREDITS: GREG DEIERLEIN, STANFORD UNIVERSITY)

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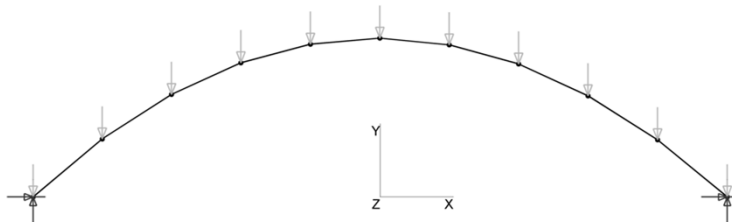




- Two-Hinged Parabolic Arch (Span = 350 ft, Rise = 80 ft)
- Steel Box (50x1x20x2 inch,  $F_y = 50$  ksi) <sup>13</sup>

## Problem Description

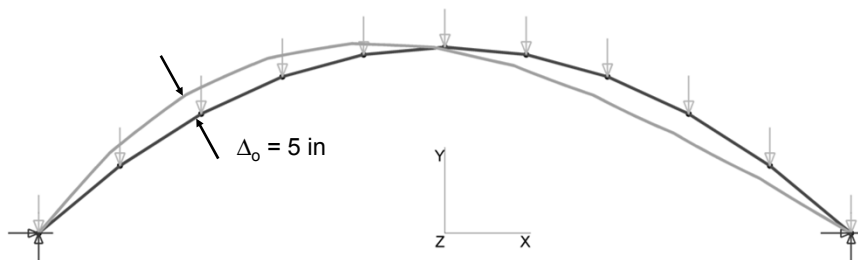
- Two-Hinged Parabolic Arch (Span = 350 ft, Rise = 80 ft)
- Steel Box (50 x 1 x 20 x 2 inch;  $F_y = 50$  ksi)
- Factored Load =  $w_u = 8.6$  kip/ft (horizontal projection)
- Reduced stiffness:  $0.8EI$  &  $0.8EA$  ( $\tau_b$  assumed equal to 1.0)
- Out-of-Straightness: Distorted Geometry  $\Delta_{o,max} = 5$  inches (approximately equal to the arc-span length / 1000)
- $\phi_c P_n = \phi_c P_y$  (due to consideration of out-of-straightness)
- Braced Out-of-Plane
- Analysis using MASTAN2 ([www.mastan2.com](http://www.mastan2.com))



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## Buckled Shape

- $w_{cr} = 1.64 w_u$  using nominal elastic stiffness

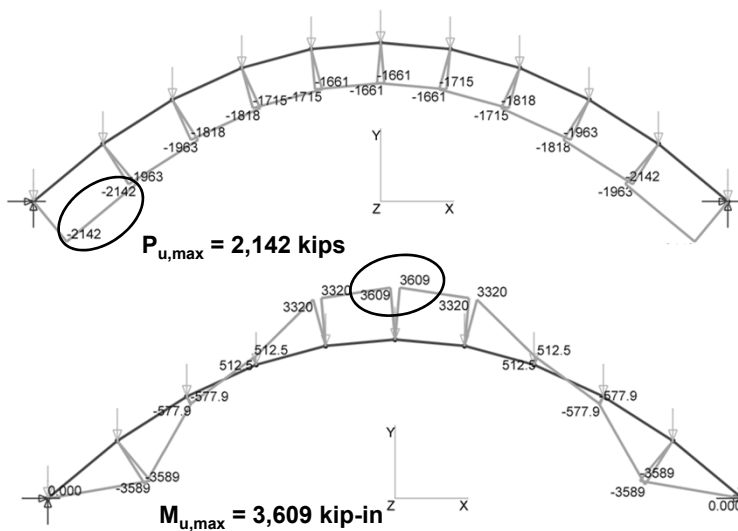


The buckled shape is an appropriate initial imperfection shape for this problem. The buckling deformations are scaled by the maximum assumed offset,  $\Delta_0$ .

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## First-Order Elastic Analysis Results

Using ideal geometry & nominal elastic stiffness

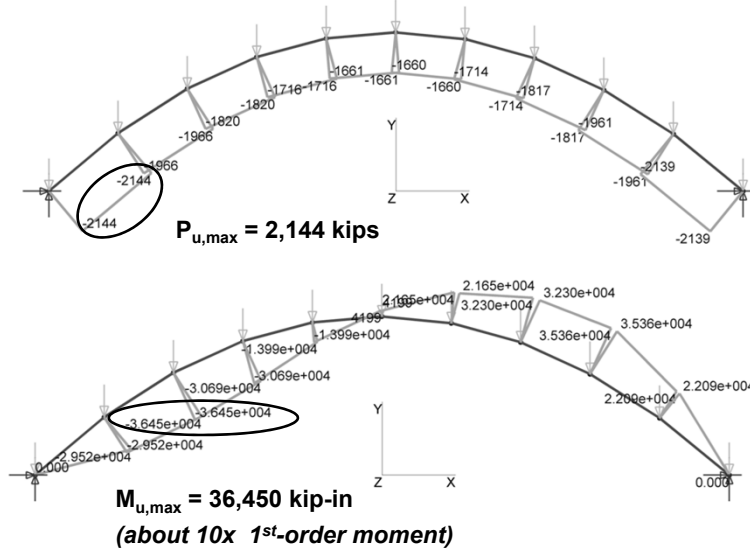


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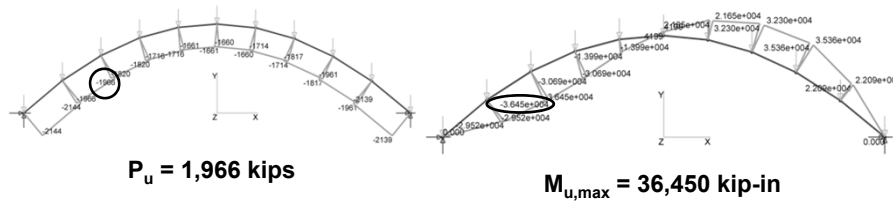
## 2<sup>nd</sup>-Order Analysis Results per DM

Using imperfect geometry, 0.8EA & 0.8EI



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## P & M Interaction Check



$$\frac{P_u}{\phi P_y} + \frac{8 M_u}{9 \phi M_p} \leq 1 \quad (P_u > 0.2 \phi P_y)$$

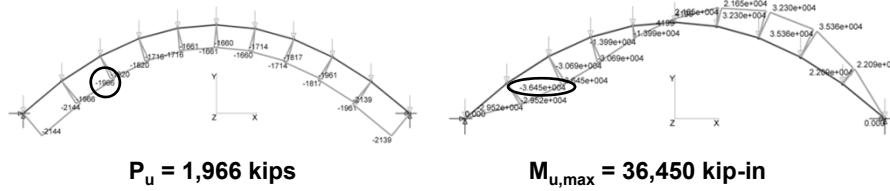
$$\frac{1,966}{(0.9)8,600} + \frac{8}{9} \frac{36,450}{(0.9)148,900} = 0.25 + 0.24 = 0.49 \leq 1 \therefore \text{OK}$$

Note:  $P_n = P_y$  &  $M_n = M_p$  assuming no web local buckling effects, & given full out-of-plane bracing & accounting for stiffness reductions & imperfections in the analysis

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## Second-Order Amplification



**Buckling Analysis:**

ideal elastic:  $w_u/w_{cr} = 0.61$   
 $w/\Delta_o$  &  $0.8E$ :  $w_u/w_{cr} = 0.77$



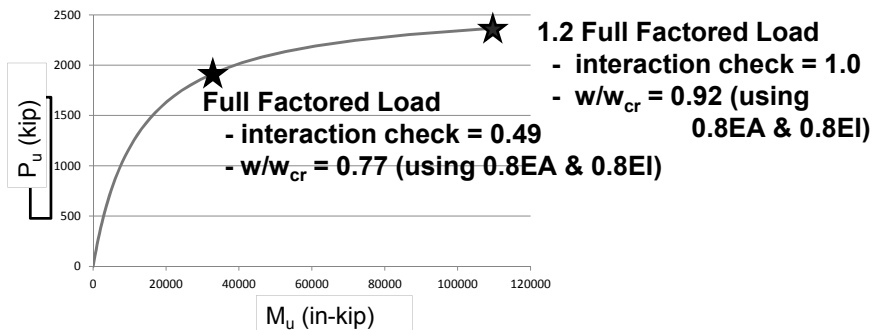
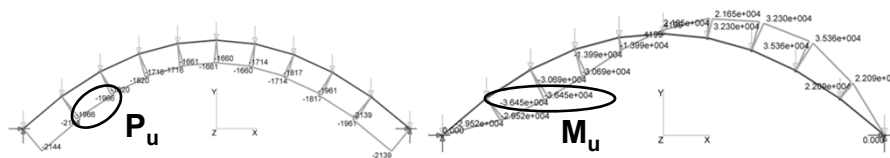
**Estimated Amplification Factor:**

$AF = 1/(1 - w_u/w_{cr})$   
 ideal elastic:  $AF = 2.6$   
 $w/\Delta_o$  &  $0.8E$ :  $AF = 4.3$

| Analysis                                     | P (kip) | AF  | M (k-in) | AF  | $\Delta_v$ (in)* | AF   |
|--|---------|-----|----------|-----|------------------|------|
| 1 <sup>st</sup> -order $w/\Delta_o$          | ~1,966  | --  | 9,076    | --  | 2.2              | --   |
| 2 <sup>nd</sup> -order $w/\Delta_o$          | ~1,966  | 1.0 | 22,850   | 2.5 | 2.3              | 1.05 |
| 2 <sup>nd</sup> -order $w/\Delta_o$ & $0.8E$ | ~1,966  | 1.0 | 36,450   | 4.0 | 3.3              | 1.5  |

\*  $\Delta_v$  = maximum vertical displacement 19

## Evolution of Internal Forces



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## TOPIC 7.2

STABILITY OF LONG-SPAN TRUSSES DURING ERECTION

(CREDITS: WILL JACOBS & CLINT REX, SDL ASSOCIATES,  
ATLANTA, GA)

see [www.aisc.org/webinars](http://www.aisc.org/webinars):

*Erection Engineering – The Science Behind the Art*

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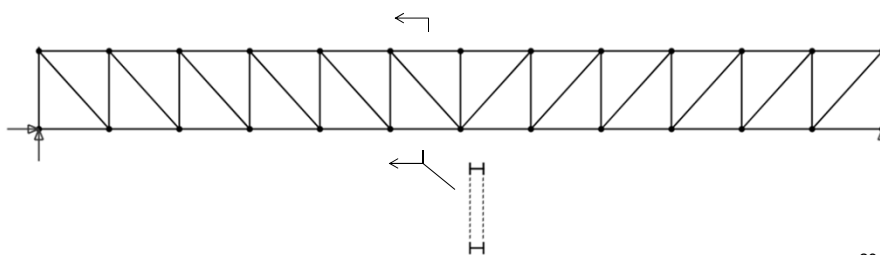


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## Example Truss

- Simple-span roof truss
- 120 ft long, 11 ft c-c of chords
- W12x65 top chord, W12x53 bottom chord, W12x45 webs, flanges in plane of truss
- Top & Bottom chord splices near mid-span
- Self-weight increased 20 % for misc. steel



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## What are the Issues?

- Stability during lifting typically is not the driving concern
- Stability after letting off the hook is often more of a concern:
  - Can I connect the ends and let it off the hook?
  - Can wind cause the truss to fail if it survives being let off the hook?
  - What magnitude of wind is a problem?
  - What size of bracing is needed when required?
  - What are the temporary splice forces if final truss splices are welded and made in the air?

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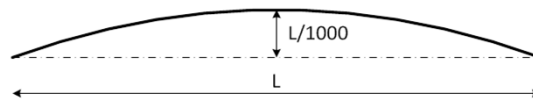
## Direct Analysis

- Model truss including weight associated with connections
- Use reduced stiffness  $0.8E$  (assuming  $\tau_b = 1$ )
- Sweep top and bottom chords
  - Note that these are continuous members
- Model web members as pinned or continuous as appropriate
- Determine wind loads
- Perform analysis & review unity checks
- Assess splice forces directly from analysis
- Model bracing, if required, in the analysis

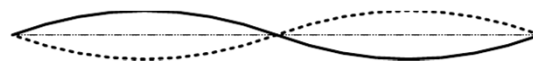
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## Direct Analysis

- For simple-span truss
  - Sweep top chord at  $L/1000$  out-of-plane
  - Sweep bottom chord in opposite direction (more extreme than common practice for I-section beams)



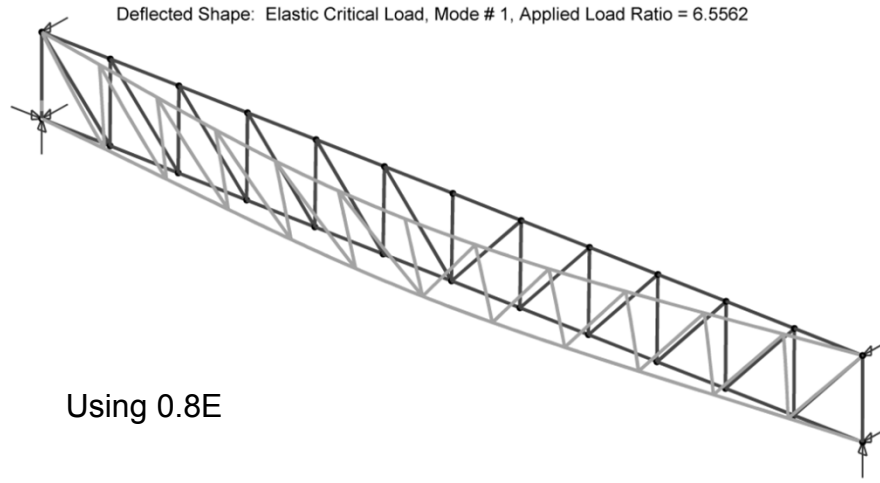
- If mid-span brace is needed, assess required brace using the above sweep
- After sizing brace, check using revised sweep that mimics the new buckled shape



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## Eigenvalue Buckling Analysis

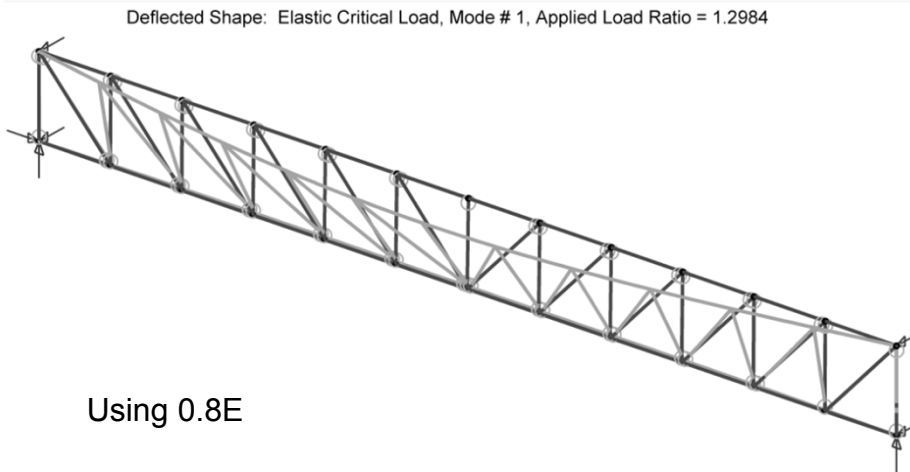
- $w_{cr} = 6.56 w_{SD}$ , web continuous



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## Eigenvalue Buckling Analysis

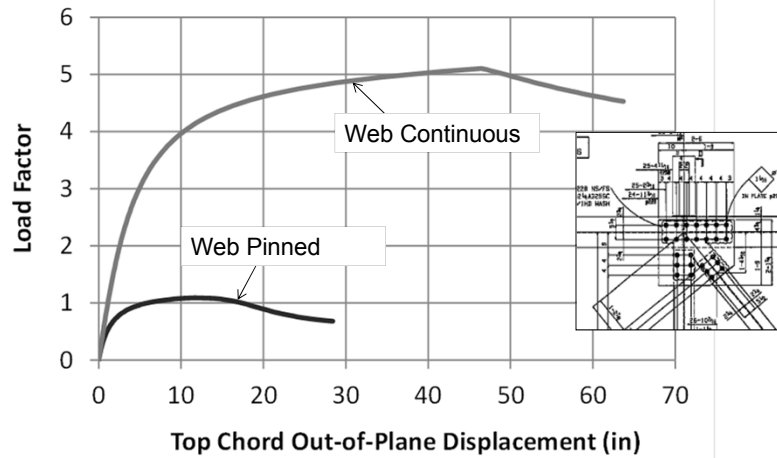
- $w_{cr} = 1.30 w_{SD}$ , web pinned



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## Direct Analysis - Load-Deflection

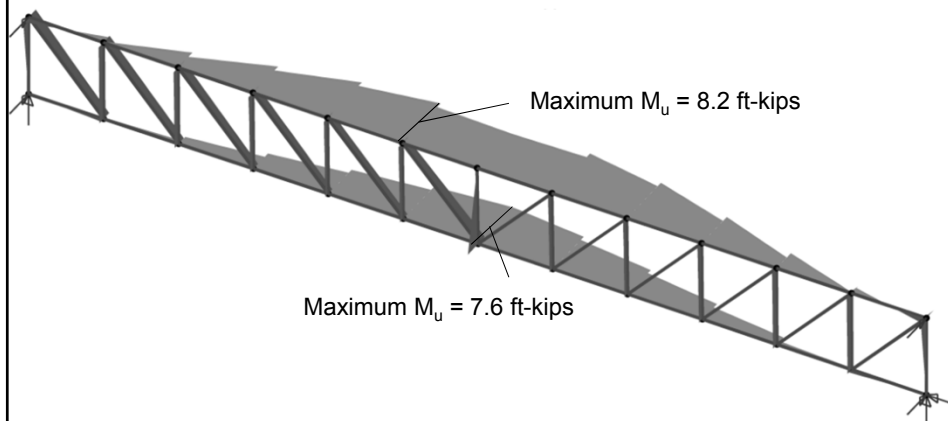
Self Weight Loading, using 0.8E & specified geometric imperfections



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## Direct Analysis - Moments @ LF = 1.4

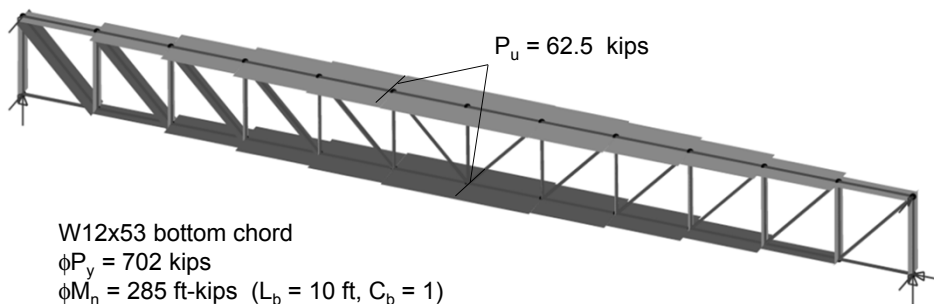
- Continuous webs



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## Direct Analysis – Axial Forces @ LF = 1.4

- Continuous webs



W12x53 bottom chord  
 $\phi P_y = 702$  kips  
 $\phi M_n = 285$  ft-kips ( $L_b = 10$  ft,  $C_b = 1$ )

$$\frac{P_u}{2\phi P_y} + \frac{M_u}{\phi M_n} \leq 1 \quad (P_u < 0.2\phi P_y)$$

$$\frac{62.5}{2(702)} + \frac{7.6}{285} = 0.04 + 0.03 = 0.07 \leq 1 \quad \therefore \text{OK under Dead Load}$$

... check Wind Combination

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## TOPIC 7.3.1

BASIC COLUMN BRACING DESIGN – TIE TO THE DIRECT ANALYSIS METHOD

(REFERENCE: AISC DESIGN GUIDE 28)

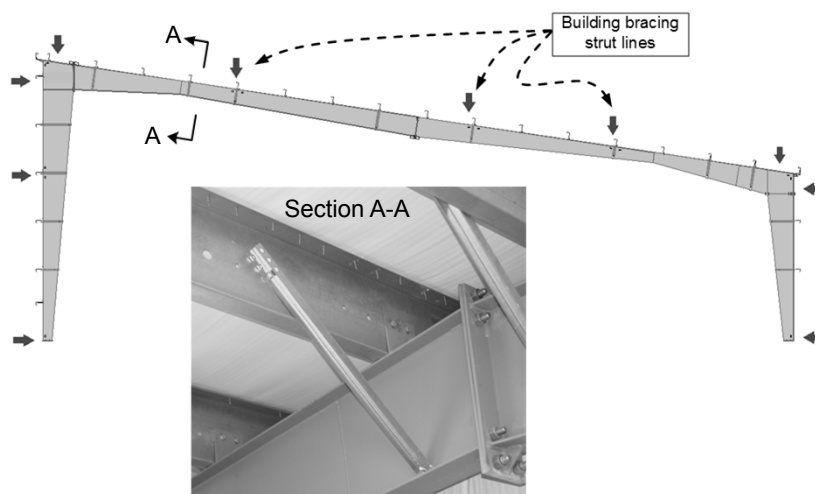
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## Definition of Stability Bracing

- Bracing systems in which the primary forces are all equal to zero
- Example: Out-of-plane bracing by purlins and flange diagonals

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## Frame Member (Flange) Bracing



Courtesy Igor Marinovic (Blue Scope Buildings)

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## Stability Bracing

- Appendix 6 of AISC addresses

– FORCE



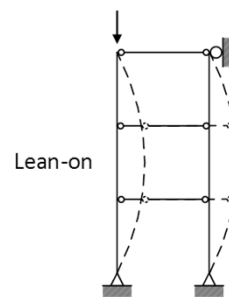
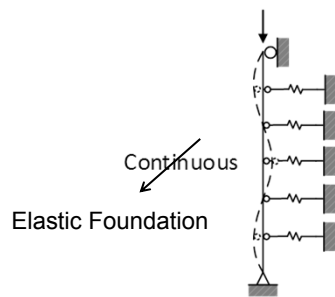
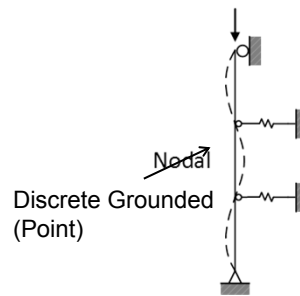
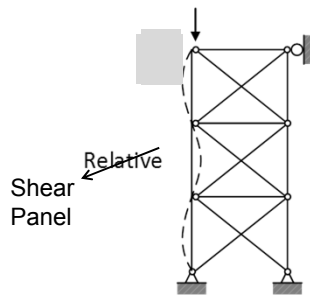
– & STIFFNESS



requirements for stability bracing

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## Types of Bracing



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## Overview

- Section A6.1 of AISC (2016) (under review) states:
  - “The required brace strength and stiffness can be obtained using a second-order analysis that satisfies the provisions of Appendix 1, Sections 1.1 or 1.2, and includes brace points displaced from their nominal locations in a pattern that provides for the greatest demand on the bracing”
- Analysis by the AISC Direct Analysis Method (DM) produces requirements that are consistent with the Appendix 6 column bracing provisions
- The DM is a tool the engineer can use to better understand, and where needed, to extend the Appendix 6 provisions to more general cases

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## Relative Bracing Model (ASD)

1<sup>st</sup>-order lateral deflection = 0 (no lat load)

$$\bar{\Delta}_{2tot} = \Delta_o + \bar{\Delta}_1$$

$$\Delta_1 = \bar{B}_2(\Delta_o + \bar{\Delta}_1) - \Delta_o$$

$$\bar{V}_{br} = \bar{H}_{P\Delta} = \frac{\alpha P_r}{L} \bar{\Delta}_{2tot} \frac{1}{\alpha} = \frac{P_r}{L} \bar{B}_2 (\Delta_o + \bar{\Delta}_1) \quad (1)$$

$$\bar{B}_2 = \frac{1}{1 - \frac{\alpha P_r}{0.8HL / \Delta_H}} = \frac{1}{1 - \frac{1.6P_r / L}{0.8H / \Delta_H}} \quad (2)$$

$$\beta_{act} = \frac{H}{\Delta_H} \quad (3)$$

$$\beta_i = \frac{\alpha P_r}{L} = \frac{1.6P_r}{L} \quad (4)$$

$$\bar{B}_2 = \frac{1}{1 - \frac{\beta_i}{0.8\beta_{act}}} \quad (5)$$

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## Definition of Displacement Terms

The Over-bar "—" indicates quantities that are impacted by the stiffness reduction of the DM

$\overline{\Delta}_0$  = initial out-of-plumbness

$\overline{\Delta}_1$  = relative transverse displacement between brace points due to applied loadings, determined from a 1<sup>st</sup>-order analysis of the geometrically perfect structure, using the reduced elastic stiffness of the DM model  
 (= 0 for the problem being considered here)

$\overline{\Delta}$  = additional relative transverse displacement due to stability effects, determined using the reduced elastic stiffness of the DM model

$\overline{\Delta}_{2tot}$  = total second-order relative transverse displacement between the brace points, determined using the reduced elastic stiffness of the DM model

$\overline{\Delta}_{o.total} = \overline{\Delta}_0 + \overline{\Delta}_1$

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## Other Definitions

$\overline{B}_2$  = sidesway amplification factor

$H$  = arbitrary applied lateral shear force within the panel, used to evaluate the shear panel stiffness

$H_{P\Delta}$  = P- $\Delta$  shear force associated with the effect of  $P_r$  acting through the panel displacement  $\overline{\Delta}_{2tot}$

$L$  = length between braced points

$P_r$  = required column axial strength (ASD)

$\overline{V}_{br}$  = required bracing panel shear strength

$\alpha$  = ASD/LRFD force level adjustment factor = 1.6 (ASD)

$\beta_{act}$  = Actual (provided) panel bracing stiffness

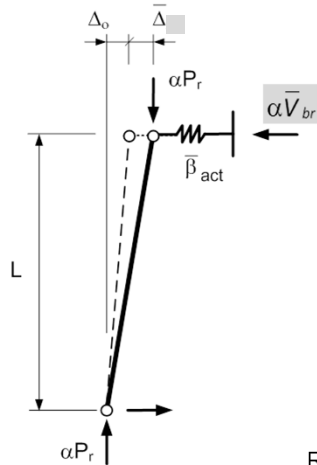
$\beta_i$  = Ideal bracing stiffness, i.e., the stiffness for which the member and its bracing system are at incipient buckling under the axial loading  $\alpha P_r$

$\Delta_H$  = relative displacement between the ends of the unbraced length due to the applied lateral load  $H$

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## Relative Bracing Model (ASD)



Substituting into Eq. (1)...

$$\begin{aligned} \bar{V}_{br} &= P_r \frac{1}{1 - \frac{\beta_i}{0.8\beta_{act}}} \frac{(\Delta_o + \bar{\Delta}_1)}{L} \\ &= P_r \frac{1}{1 - \frac{\beta_i}{0.8\beta_{act}}} \frac{\bar{\Delta}_{o,total}}{L} \end{aligned} \quad (6)$$

Definition, required bracing stiffness from AISC Appendix 6, Eq. (A-6-2):

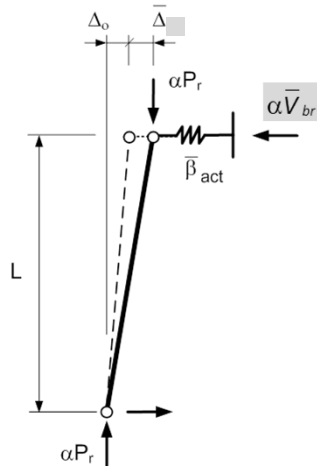
$$\beta_{br} = 2\Omega \frac{P_r}{L} = 2\Omega \frac{\beta_i}{\alpha} \quad (7)$$

Rearranging... 
$$\beta_i = \frac{\alpha\beta_{br}}{2\Omega} = \frac{1.6\beta_{br}}{2(2)}$$

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## Relative Bracing Model (ASD)

Substituting the result from Eq. (7) into Eq. (6)...



$$\bar{V}_{br} = P_r \frac{1}{1 - \frac{\beta_{br}}{2\beta_{act}}} \frac{\bar{\Delta}_{o,total}}{L} \quad (8, \text{AISC C-A-6-1})$$

However, Eq. (1) is the fundamental equation for the relative stability bracing strength requirements, where...

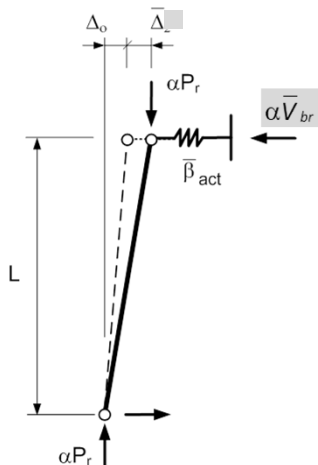
$$\begin{aligned} &1^{\text{st}}\text{-order lateral defl} \\ &= 0 \text{ (no lateral load)} \\ &\bar{\Delta}_{o,total} = \Delta_o + \bar{\Delta}_1 \end{aligned} \quad (9)$$

For general cases involving horizontal applied loads, H must be added to  $\bar{V}_{br}$  to obtain the total panel bracing force

If we let  $\Delta_o = 0.002L$  &  $\beta_{act} = \beta_{br} \dots$

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## Relative Bracing Model (ASD)



Then, substituting into Eq. (8) (or Eq. (1))...

$$\bar{V}_{br} = P_r(2)(0.002) = 0.004P_r$$

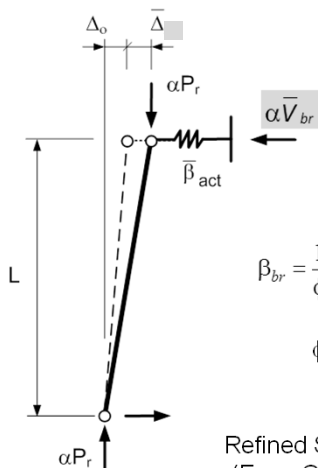
This is Appendix 6 Eq. (A-6-1)

Similarly, if derived in terms of LRFD...

we get the same result if we use a stiffness reduction factor of 0.75 rather than 0.8

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## AISC Relative Bracing Equations



In summary, the base equations are...

Strength:

$$\bar{V}_{br} = 0.004 P_r \quad \text{AISC (A-6-1)}$$

Stiffness:

$$\beta_{br} = \frac{1}{\phi} \left( \frac{2P_r}{L_b} \right) \text{ LRFD} \quad \beta_{br} = \Omega \left( \frac{2P_r}{L_b} \right) \text{ ASD} \quad \text{AISC (A-6-2)}$$

$$\phi = 0.75$$

$$\Omega = 2.00$$

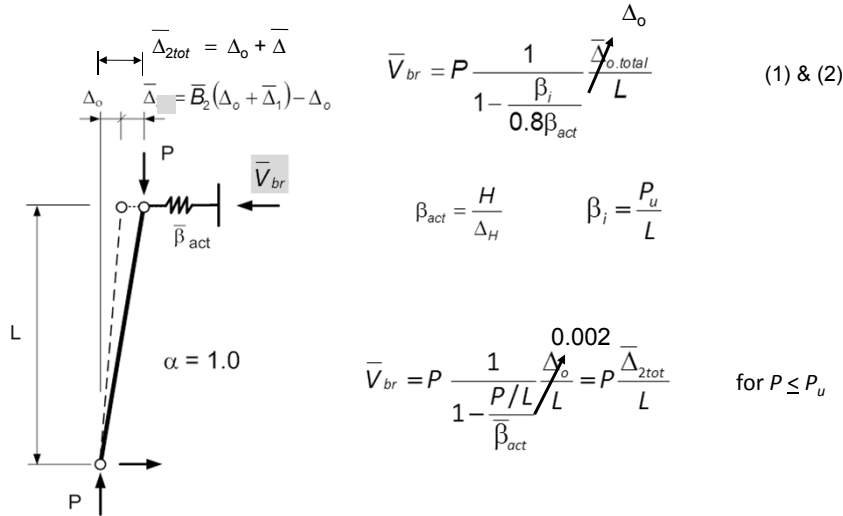
Refined Strength Eq...  
 (From Commentary)

$$\bar{V}_{br} = P_r \frac{1}{1 - \frac{\beta_{br}}{2\beta_{act}}} \frac{\bar{\Delta}_{o,total}}{L} \quad (8)$$

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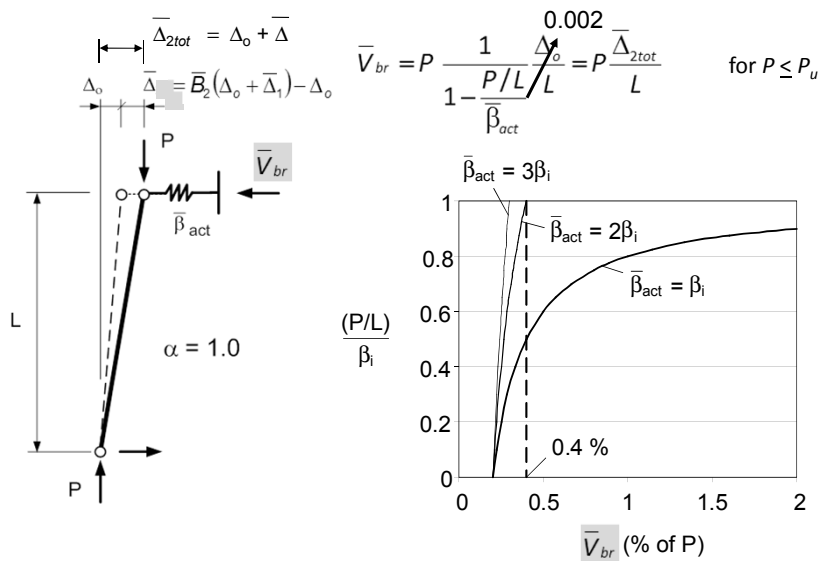


### Relative Bracing Behavior (LRFD)



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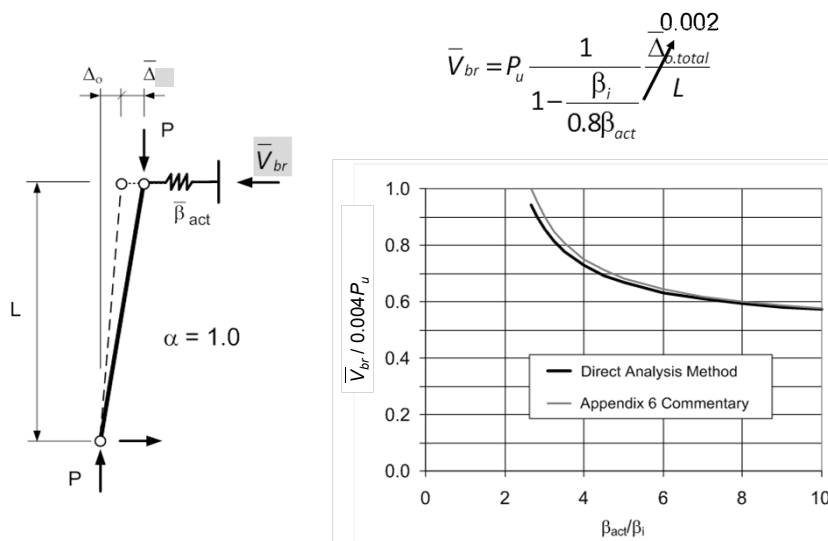
### Relative Bracing Behavior (LRFD)



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## Relative Bracing Behavior (LRFD)



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## Summary

- 1) When applied to the idealized “pinned-out” model of a column and its panel bracing system, the Direct Analysis Method produces the Appendix 6 equations for the panel bracing strength requirements
  - 2) The AISC relative bracing requirements address the *shear force* perpendicular to the column axis that must be resolved in *a given panel* of a bracing system
  - 3) The AISC relative bracing requirements *neglect* the help from the *EI* of the column(s) via the “pinned-out” idealization used in their development
  - 4) This idealization is accurate in many cases; the DM can be used to explicitly include column continuity effects if desired
  - 5) The DM can be used to gain additional design economy and more general design solutions compared to the base AISC Appendix 6 equations for various general bracing situations such as partial nodal bracing, unequal brace spacing, unequal brace stiffness, flexible end bracing conditions, and bracing of tapered members
- (See DG28 Appendix E for examples)

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Stability Design – Advanced Applications

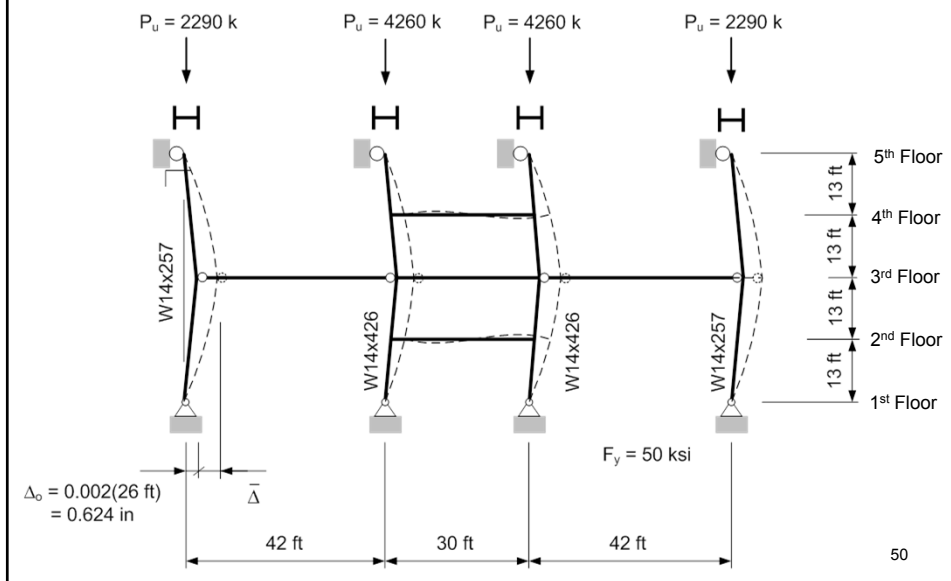
Don White

# TOPIC 7.3.2

COLUMN LEAN-ON BRACING DM DESIGN EXAMPLE

(REFERENCE: AISC DESIGN GUIDE 28)

## Problem: Bracing of Exterior Columns by Building Core



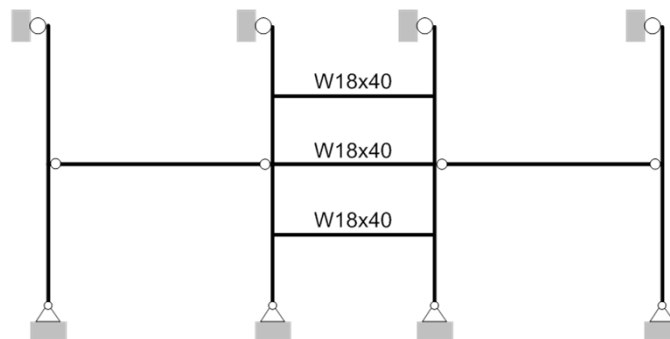
## Problem Statement

- Determine if the core framing has sufficient strength to brace the exterior columns at their mid-height
- If not, determine an appropriate increase in the floor beam sizes in the interior core
- Check the system by the DM
- Discuss broad implications of the results
- *Detailed presentation in AISC DG 28, Appendix E, Section E6*

51

## Floor Beams within the Core

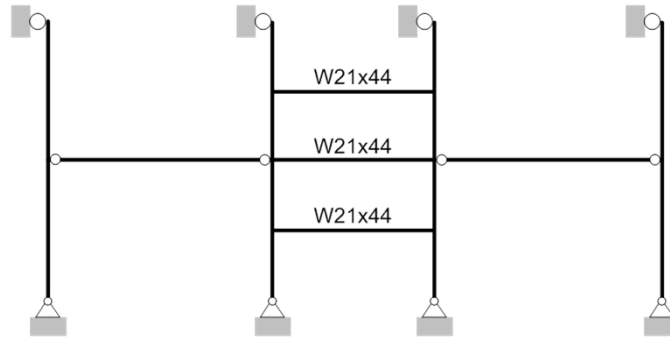
- Simply-supported W18x40 beams are sufficient to support LRFD gravity loads of 500 lbs/ft over the 30 ft core



52

## Floor Beams within the Core

- The engineer previously decided to increase the interior floor beam sizes to W21x44 and to install moment connections to develop sufficient lateral stiffness for the interior columns to support their loads



53

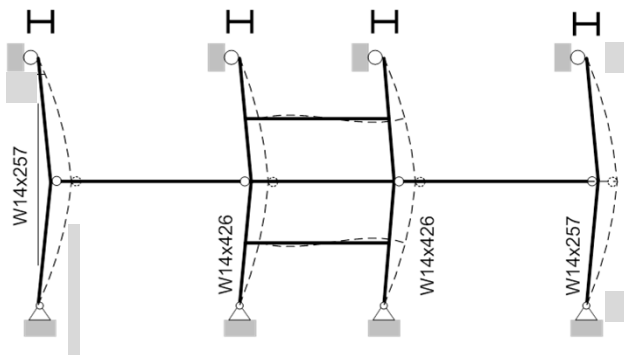
## Column Properties & Responses

- W14x257 Exterior Columns
  - $KL_x = 26$  ft
  - $KL_y = 13$  ft
  - In-plane flexural buckling governs,  
 $\phi_c P_{nx} = 2900$  kips
  - $P_u / P_y = 0.606 \rightarrow \tau_b = 0.955$
- W14x426 Interior Columns
  - $KL_x = 13$  ft
  - $KL_y = 13$  ft
  - Out-of-plane flexural buckling governs,  
 $\phi_c P_{ny} = 5120$  kips
  - $P_u / P_y = 0.682 \rightarrow \tau_b = 0.867$

54

## Key Attributes of the DM Analysis

- Stiffness Reduction
  - $0.8\tau_b$  for the column flexural rigidities  $EI$
  - 0.8 for all other stiffness contributions
- Column out-of-alignment affine to the expected primary buckling mode shown by the dashed lines



## Preliminary Design

- Idealize the bottom two stories of the interior frame as an equivalent single story
- Recognize symmetry about the middle floor level & consider just the bottom half of the structure
  - Asymmetrical buckling is checked subsequently
- Determine  $\bar{B}_2$

56

## $\bar{B}_2$ Calculation

$$\bar{B}_2 = \frac{1}{1 - \frac{P_{\text{story}}/L}{R_M H / \bar{\Delta}_H}} = \frac{1}{1 - \frac{P_{\text{story}}}{0.85 \sum \beta L}} = \frac{1}{1 - \frac{P_{\text{story}}}{\bar{P}_{\text{estory}}}}$$

$P_{\text{story}}$  = 13,100 kips, the total vertical load supported by all the columns

$L$  = 26 ft = 312 in, the height of the effective story being considered in the analysis

$H$  = story shear due to an arbitrary lateral load applied at the middle floor level

$\bar{\Delta}_H$  = story 1<sup>st</sup>-order elastic sidesway deflection due to  $H$ , calculated **using the DM reduced stiffnesses**

$R_M$  = 0.85 = reduction factor for the influence of P- $\delta$  effects on  $\Delta$

$\sum \bar{\beta}$  = total (reduced) 1<sup>st</sup>-order sidesway stiffness of the effective story

$\bar{P}_{\text{estory}}$  = total sidesway buckling capacity for the reduced stiffness DM model

57

## $\bar{B}_2$ Calculation

Contribution to the sidesway stiffness from each of the exterior columns:

$$\bar{\beta}_{\text{ext}} = \frac{3(0.8\tau_b EI)}{L^3} = \frac{3 \times 0.8 \times 0.955 \times 29000 \text{ ksi} \times 3400 \text{ in}^4}{(312 \text{ in})^3} = 7.44 \frac{\text{kips}}{\text{in}}$$

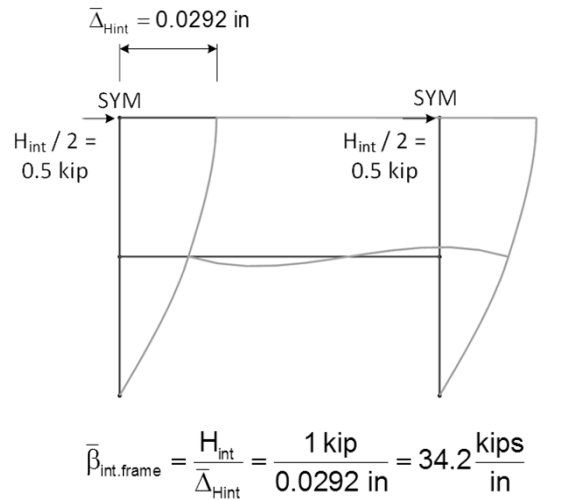
Contribution to the sidesway stiffness from the interior columns?...

58



## $\bar{B}_2$ Calculation

Subject a **reduced stiffness** model of the bottom two stories of the interior core frame to an arbitrary horizontal shear force of  $H_{int} = 1$  kip (associated with a lateral load at the middle floor of the 4-story frame)...



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## $\bar{B}_2$ Calculation

Sum the contributions from the exterior columns and the interior core framing:

$$\begin{aligned} \bar{P}_{estory} &= 0.85 (2\bar{\beta}_{ext} + \bar{\beta}_{int.frame}) L \\ &= 0.85 \left( 2 \times 7.44 \frac{\text{kips}}{\text{in}} + 34.2 \frac{\text{kips}}{\text{in}} \right) \times 312 \text{ in} \\ &= 13,000 \text{ kips} \end{aligned}$$

60



## Houston... we have a problem here

$$[ P_{\text{story}} = 13,100 \text{ kips} ] > [ \bar{P}_{\text{estory}} = 13,000 \text{ kips} ]$$

Larger internal beams will be needed so that the overall system will have sufficient sidesway stiffness to support the total gravity loads

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## Selection of Interior Floor Beam Size

- The interior floor beam at the middle floor doesn't provide any contribution to the overall lateral load resistance
  - Use a simply-supported W18x40 at the middle floor location
- Assume that the beam-column strength interaction equations for the interior columns (limiting the 2<sup>nd</sup>-order moment amplification in the beam-columns) will govern the selection of the 1<sup>st</sup> & 3<sup>rd</sup> story interior floor beams
  - Interior col.  $P_u / \phi_c P_n = 4260 \text{ k} / 5120 \text{ k} = 0.832$

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## Selection of Interior Floor Beam Size

For the W14x426 interior columns...

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9} \frac{M_u}{\phi_b M_n} \leq 1$$

$$P_u / \phi_c P_n = 0.832$$

For  $L_b = 13$  ft,  $\phi_b M_n = \phi_b M_p = 3260$  ft-kips

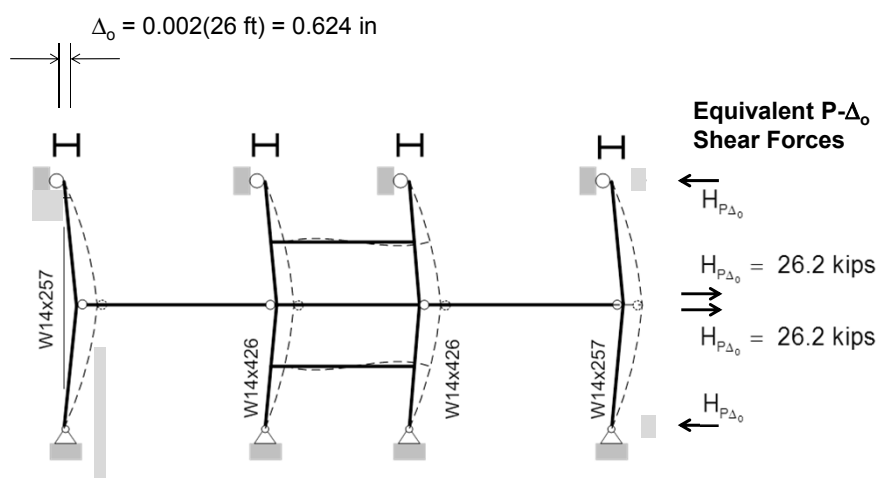
➔ Maximum tolerable  $M_u = 616$  ft-kips

Given the selected critical geometric imperfection pattern with  $\Delta_o = 0.624$  in = 0.002 (26 ft) at the middle floor level, the corresponding equivalent lateral P- $\Delta$  force in the bottom two stories is ...

$$H_{P\Delta_o} = 0.002 P_{\text{story}} = 0.002 \times 13,100 \text{ kips} = 26.2 \text{ kips}$$

63

## Geom. Imperfection for DM Analysis

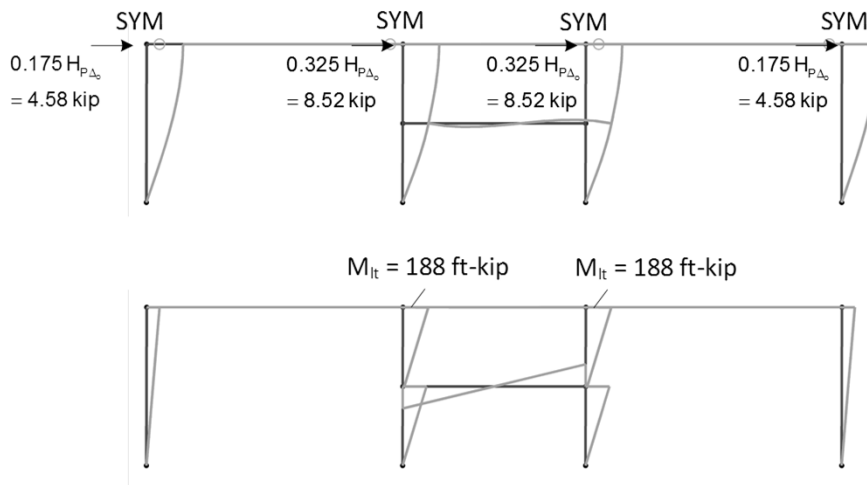


64



## Selection of Interior Floor Beam Size

$H_{P\Delta_0} = 26.2$  kips produces a maximum 1<sup>st</sup>-order moment of  
 $M_{lt} = 188$  ft-kips in the interior columns



## Selection of Interior Floor Beam Size

Therefore, assuming that the shape of the moment diagram isn't influenced significantly by the beam size, we can estimate that  $\bar{B}_2$  must be limited to

$$M_u / M_{lt} = 616 / 188 = 3.25$$

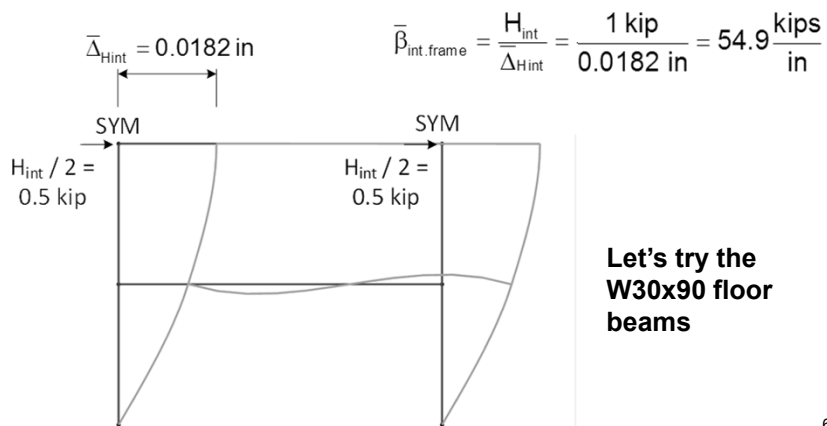
Given this limit for  $\bar{B}_2$ , we can solve for the required lateral stiffness of the internal frame using the expression:

$$[\bar{B}_2 = 3.25] \leq \left[ \frac{1}{1 - \frac{P_{\text{story}}}{0.85 (2\bar{\beta}_{\text{ext}} + \bar{\beta}_{\text{int.frame}}) L}} = \frac{1}{1 - \frac{13,100 \text{ kips}}{0.85 (2 \times 7.44 \text{ kips/in} + \bar{\beta}_{\text{int.frame}}) \times 312 \text{ in}} \right]$$

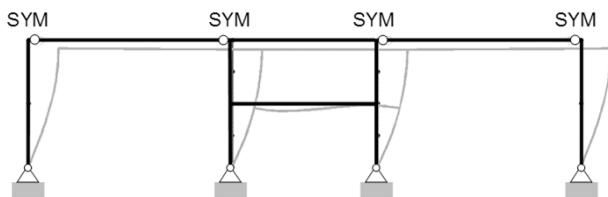
## Selection of Interior Floor Beam Size

Solving for  $\bar{\beta}_{int.frame}$ , we get  $\bar{\beta}_{int.frame} \geq 56.5$  kips/in

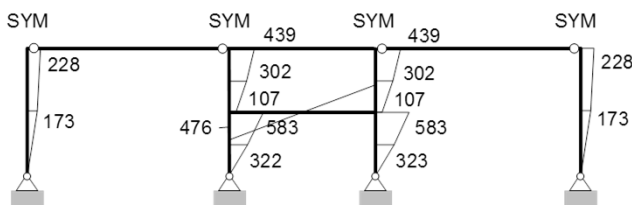
From Table 3-3 of the AISC Manual, a W30x90 nearly satisfies this requirement, i.e.,



## Rigorous 2<sup>nd</sup>-Order Analysis Results



(a) Deflected Shape



(b) 2<sup>nd</sup>-order internal moments (ft-kip)

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## DM Design Checks

W14x426 Interior Columns:

$$\frac{P_u}{\phi_c P_{ny}} + \frac{8 M_u}{9 \phi_b M_n} = \frac{4260 \text{ kips}}{5120 \text{ kips}} + \frac{8 \cdot 583 \text{ ft-kips}}{9 \cdot 3260 \text{ ft-kips}} = 0.832 + \frac{8}{9} \cdot 0.179 = 0.991$$

where  $\phi_c P_{ny} = 5120$  kips is based on  $KL_y = 1.0 \times 13$  ft, &  
 $\phi_b M_n = \phi_b M_p = 3260$  ft-kips is easily achieved by the interior columns  
 based on an unbraced length for LTB of  $L_b = 13$  ft

W14x257 Exterior Columns:

$$\frac{P_u}{\phi_c P_{nx}} + \frac{8 M_u}{9 \phi_b M_n} = \frac{2290 \text{ kips}}{2900 \text{ kips}} + \frac{8 \cdot 228 \text{ ft-kips}}{9 \cdot 1830 \text{ ft-kips}} = 0.790 + \frac{8}{9} \cdot 0.125 = 0.900$$

where  $\phi_c P_{nx} = 2900$  kips is based on  $KL_x = 1.0 \times 26$  ft, &  
 $\phi_b M_n = \phi_b M_p = 1830$  ft-kips is easily achieved by the exterior columns  
 based on an unbraced length for LTB of  $L_b = 13$  ft

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## DM Design Checks

W30x90 Floor Beams:

$$M_u = 476 \text{ ft-kips} < \phi_b M_n = 788 \text{ ft-kips, based on } L_b = 30 \text{ ft } C_b = 2.3$$

(additional end moments due to  $w_u = 500$  lb/ft  $\cong w_u L_b^2/12 = 38$  ft-kips  
 were not included in the analysis ... the floor beams work by inspection  
 for the combination of these gravity load moments with the above  
 lateral moments)

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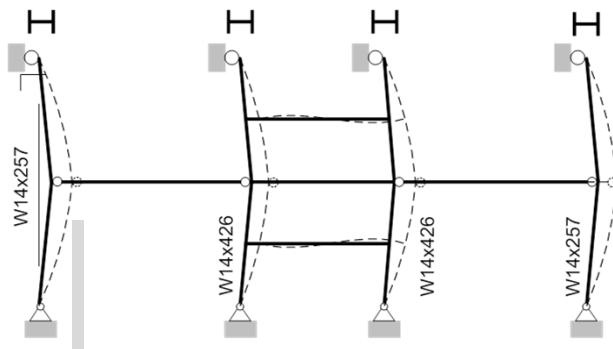
## TOPIC 7.3.3

### SCRUTINIZING THE LEAN-ON BRACING DESIGN CALCULATIONS

71

## $L_x$ for Interior Columns?

- All the beams are simply-connected at middle floor
- Exterior column is in fact leaning on the interior columns at the middle floor
- Should the interior columns be checked for  $KL_x = 26$  ft rather than 13 ft?

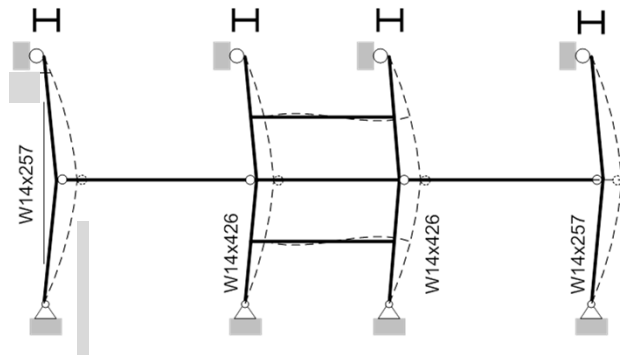


72



## $L_x$ for Interior Columns?

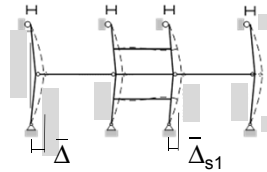
- $L_x = 13$  ft is the appropriate “L” in this problem
  - The exterior columns provide some (1<sup>st</sup>-order) resistance to lateral displacement of the middle floor
  - The deflections shown below are identical to those of a 2-story frame with 13 ft high stories & a rigid girder at its top story
  - Furthermore, the DM can be used with  $KL_x = 0$  if appropriate out-of-straightness imperfections are included in the analysis



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## Sideway of Middle Floor

- The DM 2<sup>nd</sup>-order analysis predicts an additional sideway deflection of the middle floor due to the applied gravity loads at the strength load level of  $\bar{\Delta} = 1.49$  in ( $\Delta_o = 0.63$  in)
  - $\bar{\Delta}_{tot} = \Delta_o + \bar{\Delta} = 2.12$  in
  - $\bar{\Delta} / \Delta_o = 2.39$
  - “ $B_2$ ” =  $\bar{\Delta}_{tot} / \Delta_o = 3.39$
  - $\bar{\Delta}_{s1} / L = 1.10$  in / 156 in = 1/142 in the 1<sup>st</sup> story
- Check sideway under 1.0D + 1.0L (using the nominal elastic stiffness)
  - $P_{ext} = 1530$  kips,  $P_{int} = 2840$  kips
  - $\Delta_{s1} / L = 0.210$  in / 156 in = 1/743 in the 1<sup>st</sup> story



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## Analysis Modeling Considerations

- With cubic Hermitian type displacement-based elements
  - 2 elements are required between each of the member intersections on the exterior columns
  - Only 1 element is required between each of the floor levels on the interior columns
- With a program having only P-large delta capabilities
  - $P_u/\bar{P}_{eL} = 0.30 \rightarrow$  3 elements are required between each of the member intersections on the exterior columns
  - $P_u/\bar{P}_{eL} = 0.08 \rightarrow$  Only 1 element is required between each of the floor levels on the interior columns

75



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## TOPIC 7.3.4

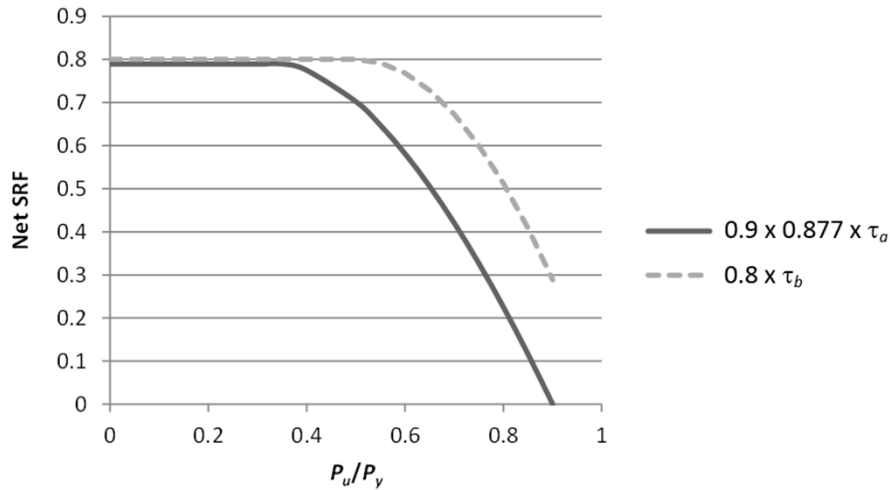
USE OF INELASTIC BUCKLING ANALYSIS TO ASSESS THE  
LEAN-ON BRACING DESIGN, COMPARISON OF APPROACHES

(CREDITS: AISC DG 28; TOGAY, JEONG & WHITE)

76



### Column Stiffness Reduction Factors (SRF)



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### Column $\tau_a$ Model

$$\phi_c P_n = 0.9 (0.877) P_{e\tau} = 0.9 (0.877) \tau_a P_e \quad \Rightarrow \quad P_{e\tau} = \tau_a P_e \quad \Rightarrow$$

$$\frac{\phi_c P_n}{\phi_c P_y} = 0.658 \frac{\tau_a P_y}{P_{e\tau}} \quad \text{for} \quad \frac{P_{e\tau}}{P_y} > \frac{4}{9} \quad \left( \frac{\phi_c P_n}{\phi_c P_y} > 0.390 \right) \quad \Rightarrow \quad P_{e\tau} = \frac{P_n}{0.877} \quad \Rightarrow$$

$$\frac{\phi_c P_n}{\phi_c P_y} = 0.658 \frac{0.877 \tau_a P_y}{P_n} \quad \Rightarrow$$

$$\ln \left( \frac{\phi_c P_n}{\phi_c P_y} \right) = \ln \left( 0.658 \frac{0.877 \tau_a P_y}{P_n} \right) \quad \Rightarrow$$

$$\ln \left( \frac{\phi_c P_n}{\phi_c P_y} \right) = 0.877 \tau_a \frac{P_y}{P_n} \ln(0.658) \quad \Rightarrow \quad \tau_a = -2.724 \frac{\phi_c P_n}{\phi_c P_y} \ln \left( \frac{\phi_c P_n}{\phi_c P_y} \right)$$

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## Significance of $\tau_a$

- If an inelastic buckling analysis is conducted using a reduced member rigidity of  $0.9 \times 0.877 \times \tau_a$ , i.e.:
  - The column internal forces at incipient buckling,  $\gamma P_u$ , are estimated
  - The column  $\tau_a$  factors are calculated given the  $\gamma P_u$  values
  - The column flexural rigidities are reduced by  $0.9 \times 0.877 \times \tau_a$
  - The buckling analysis is conducted to determine the multiple of the design load at buckling,  $\gamma$
  - Iterations are conducted until the  $\gamma$  assumed for the calculation of  $\tau_a$  is the same as the  $\gamma$  determined from the buckling analysis

the resulting  $\gamma P_u$  is rigorous calculation of  $\phi_c P_n$  accounting for all member continuity, bracing and/or end restraint effects

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## Inelastic Buckling Design Check

Calculate  $\tau_a$  factors

- $P/\phi_c P_y = 0.758$  &  $0.673$  for int. & ext. cols.

$$\text{➤ } \tau_a = -2.724 \frac{P}{\phi_c P_y} \ln \left( \frac{P}{\phi_c P_y} \right) = 0.572 \text{ \& } 0.726$$

Total Stiffness Reduction Factor (SRF):

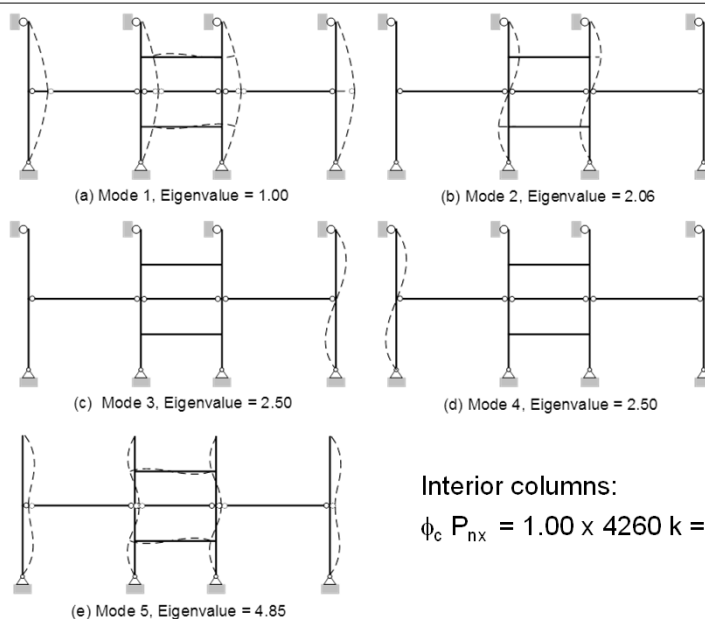
$$\phi_c \times 0.877 \times \tau_a = 0.9 \times 0.877 \times \tau_a = 0.789 \tau_a = 0.451 \text{ \& } 0.573$$

**Reduce the EI of the columns by these SRF values & run a buckling analysis of the structure**

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## Inelastic Buckling Design Check



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## Advantages & Disadvantages of the DM for Design of Stability Bracing

- Advantages
  - Rational assessment of both the stability bracing system stiffness and strength requirements
  - Greater generality, accuracy & economy of the bracing design
- Disadvantages
  - Detailed definition of geometric imperfections that cause the maximum strength demands on the bracing system

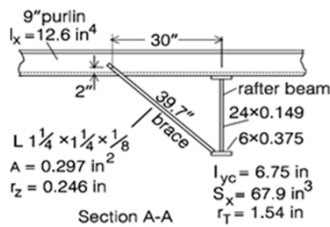
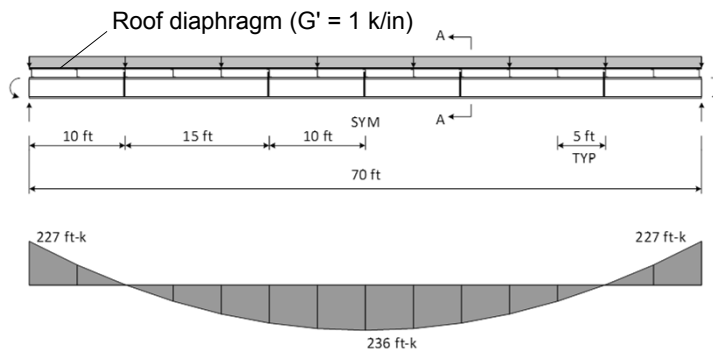
82

## Advantages & Disadvantages of Inelastic Buckling Analysis for Design of Stability Bracing

- Advantages
  - The procedure is simply a refined application of the ELM, but without any explicit calculation of effective length factors
  - Similar to the DM, an Inelastic Buckling Analysis that uses the  $\tau_a$  factor allows for detailed consideration of continuity effects and any member bracing or end restraint effects ... the buckling analysis gives the corresponding  $\phi_c P_n$  directly
  - The need to define geometric imperfections is avoided
  - A similar approach can be used for beams and beam-columns
- Disadvantages
  - Inelastic buckling analysis does not provide bracing system force requirements; however, when a bracing system stiffness of  $2/\phi$  times the ideal bracing stiffness (obtained from a  $\tau_a$ -based buckling analysis) is used, the bracing system forces can be calculated reasonably well using simple estimates similar to the AISC Appendix 6 strength requirements

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## Roof Girder Example (AISC 2002)



Panel bracing at top flange:  
 $L_{br} = 5 \text{ ft}$ ,  $s = 25 \text{ ft}$ ,  $\beta = 5 \text{ k/in}$


Torsional bracing from  
 purlins & flange diagonals:  
 $\beta_T = 6400 \text{ in-k/rad}$

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### SABRE2 Critical Inelastic Buckling Mode

Credits: Woo Yong Jeong

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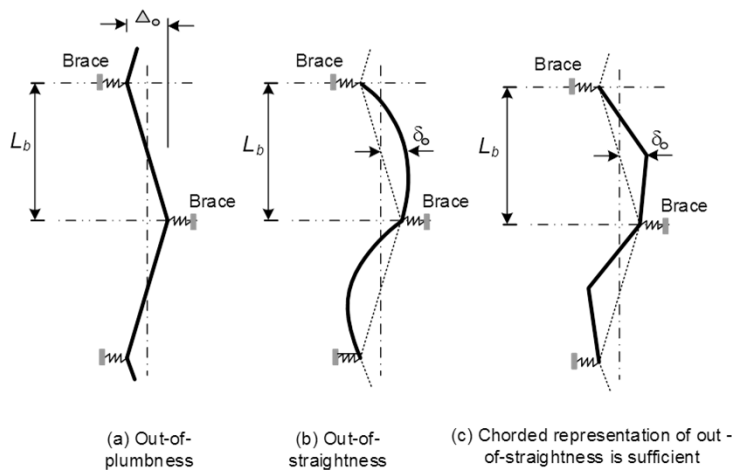
## TOPIC 7.4

### GENERAL MODELING OF GEOMETRIC IMPERFECTIONS

86



## Magnitude of $\Delta_o$ & $\delta_o$ ??



$$\Delta_o = L_b / 500 \quad \& \quad \delta_o = L_b / 1000 \quad (\text{AISC COSP}) \quad \Rightarrow \quad P_n = QP_y$$

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## Appropriate $\Delta_o$ & $\delta_o$ pattern ??...

- In cases where the displacement “mode” of the structure under the applied loads has substantial affinity with the displacement of the structure at its strength limit state
  - The response is a load–deflection problem
  - The ultimate strength is associated with a progressive growth of the deflections
  - The appropriate pattern of  $\Delta_o$  &  $\delta_o$  = the pattern of the structure’s displacements under the applied loading

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## Appropriate $\Delta_o$ & $\delta_o$ pattern ??...

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- In cases where the strength limit state, neglecting initial geometric imperfections, involves a bifurcation from a primary equilibrium path
  - Distributions of  $\Delta_o$  &  $\delta_o$  that have significant affinity with the corresponding buckling modes must be considered

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## Appropriate $\Delta_o$ & $\delta_o$ pattern ??...

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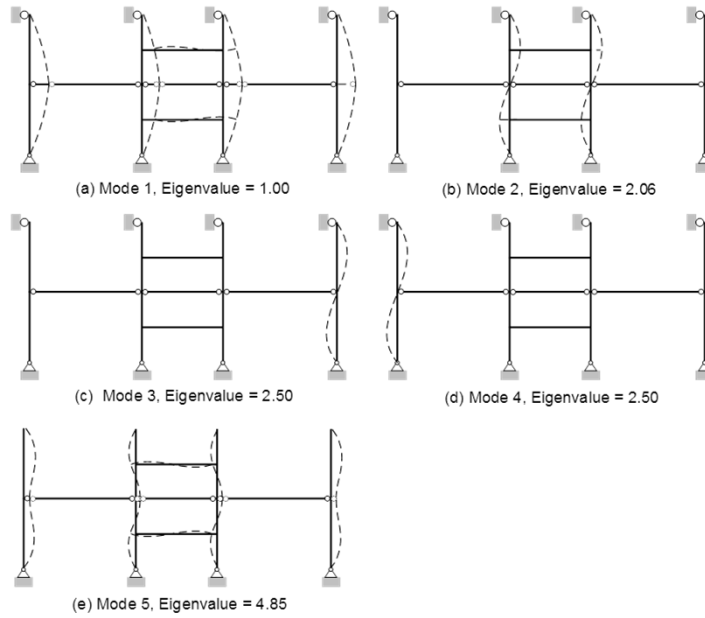
- When assessing the strength demands on bracing components, one has to consider an out-of-alignment,  $\Delta_o$ , at the bracing under consideration

(The structure's critical buckling mode may not exhibit any displacement at the critical brace location)

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### Inelastic Buckling Eigenvalues & Modes



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### DM Imperfections (for use of $P_{nx} = P_y$ )

#### Imperfection 1

$$\delta_o = 0.002(13 \text{ ft}) = 0.312 \text{ in}$$

$$\Delta_o = 0.002(26 \text{ ft}) = 0.624 \text{ in}$$

$$P_u/P_{eL} < 0.10$$

#### Imperfection 2

$$\delta_o = 0.002(13 \text{ ft}) = 0.312 \text{ in}$$

$$\Delta_o = 0.002(26 \text{ ft}) = 0.624 \text{ in}$$

$$P_u/P_{eL} < 0.10$$

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## Internal Moments & UCs (for $P_{nx} = P_y$ )

### Using Imperfection 1:

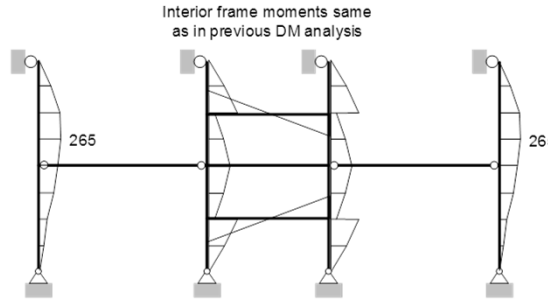
W14x257 Exterior Columns:

$$\frac{P_u}{\phi_c P_{ny}} + \frac{8 M_u}{9 \phi_c M_n} =$$

$$\frac{2290 \text{ kips}}{3060 \text{ kips}} + \frac{8 \cdot 265 \text{ ft-kips}}{9 \cdot 1830 \text{ ft-kips}} =$$

$$0.748 + \frac{8}{9} \cdot 0.145 = 0.877$$

vs. **0.900** using  $P_n = P_{nx}(L_x=26\text{ft})$



### Using Imperfection 2:

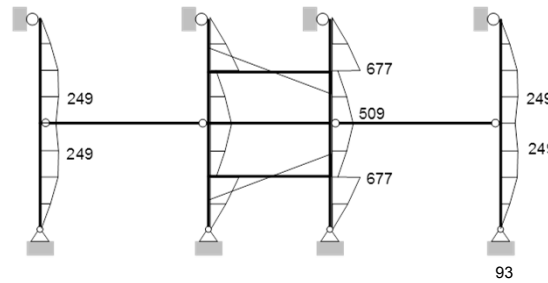
W14x426 Interior Columns:

$$\frac{P_u}{\phi_c P_{ny}} + \frac{8 M_u}{9 \phi_c M_n} =$$

$$\frac{4260 \text{ kips}}{5120 \text{ kips}} + \frac{8 \cdot 677 \text{ ft-kips}}{9 \cdot 3260 \text{ ft-kips}} =$$

$$0.832 + \frac{8}{9} \cdot 0.208 = 1.017$$

vs. **0.991** using imperfection from Slide 50



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**THAT'S IT!**

**Thanks For Attending!!**



## References

- AISC (2002). "Example Problems Illustrating the Use of the New Bracing Provisions – Section C3, Spec. and Commentary," Ad hoc Committee on Stability Bracing, November.
- Griffis, L. and White, D.W. (2013). **Stability Design of Steel Buildings**, AISC Design Guide 28, American Institute of Steel Construction, 175 pp.
- Jacobs, W.P and Rex, C.L. (2012). "Erection Engineering: The Science Behind the Art," Session N3, North American Steel Construction Conference, AISC, Grapevine, TX, April 18–19.
- Togay, O., Jeong, W.Y. and White, D.W. (2015). "Toward a More Comprehensive Approach for Design using Buckling Analysis," *Proceedings of the Annual Stability Conference*, SSRC, Nashville, TN, March 25–27.

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## Up Next...

- Session 8: March 30 –  
**More Opportunities – Design by Inelastic Analysis**  
by R.D. Ziemian, PE, PhD
- In the 14th edition of the AISC Specification, the provisions for employing inelastic analysis in design have been fully revised. Certain levels of inelastic analysis may now be used in place of the Specification's design equations when confirming the adequacy of the structural system and its components. The associated requirements for this approach will be reviewed and examples presented.

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## 8-Session Registrants

Access to the quiz: Information for accessing the quiz will be emailed to you by Thursday. It will contain a link to access the quiz. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG

Quiz and Attendance records: Posted Tuesday mornings.  
[www.aisc.org/nightschool](http://www.aisc.org/nightschool) - click on Current Course Details.

Reasons for quiz:

- EEU – must take all quizzes and final to receive EEU
- CEUs/PDHS – If you watch a recorded session you must take quiz for CEUs/PDHS.
- REINFORCEMENT – Reinforce what you learned tonight. Get more out of the course.

NOTE: If you attend the live presentation, you do not have to take the quizzes to receive CEUs/PDHS.



There's always a solution in Steel

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## 8-Session Registrants

**Access to the recording:** Information for accessing the recording will be emailed to you by this Wednesday. The recording will be available for two weeks. For 8-session registrants only. EMAIL COMES FROM NIGHTSCHOOL@AISC.ORG.

**CEUs/PDHS** – If you watch a recorded session you must take AND PASS the quiz for CEUs/PDHS.



There's always a solution in Steel

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# Thank You

Please give us your feedback!  
*Survey at conclusion of webinar.*

